Physics of Strong Interactions

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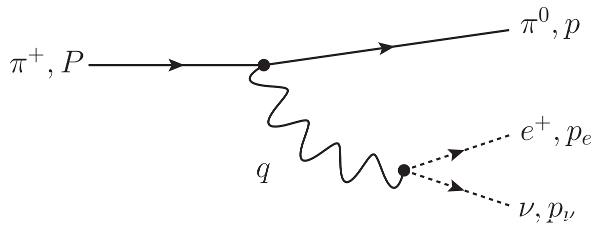
Exercise Sheet 3

Due 13.11.2015

Problem 1 - Semileptonic Pion Decay

The hadronic decay of a π^+ to π^0 and leptons can be computed "exactly" because the momentum transfer to the leptons is very small compared to Λ_{QCD}. In this exercise, we will compute the decay rate and determine $|V_{ud}|$ using the known values:

$$\tau_{\pi^+} = 2.6033 \times 10^{-8}\,\text{s}, \qquad m_{\pi^+} = 139.57\,\text{MeV}, \qquad m_{\pi^0} = 134.98\,\text{MeV},$$
$$R(\pi^+ \to \pi^0 + e^+ + \nu) = 1.036 \times 10^{-8},$$
$$G_F = 1.1664 \times 10^{-5}\,\text{GeV}^{-2}.$$



1) Write down the amplitude for this decay in terms of the pion form factor $F^{\pi^+\to\pi^0}(q^2)$ in the limit $q^2 \to 0$. Note that the scale provided by the W mass is large so we can disregard terms of order $\frac{q^2}{M_w^2}$ and the leptons can be considered massless.

2) Show that the squared matrix element can be written as

$$|\mathcal{M}|^2 = 32 G_F^2 |V_{ud}|^2 m_{\pi^+}^2 E_\nu E_e (1 + \cos\theta), \tag{1}$$

where θ is the angle between the electron and neutrino.

3) Integrate over the phase space to find the decay rate $\Gamma(\pi^+ \to \pi^0 + e^+ + \nu)$.

4) Use the branching ratio $R(\pi^+ \to \pi^0 + e^+ + \nu)$ to extract a value of $|V_{ud}|$ from your result.

5) How does your result compare to the world average of $|V_{ud}| = 0.97425 \pm 0.00022$? We made a number of approximations, $q^2 \to 0$, $M_w \to \infty$, $m_e = 0$, which do you expect will provide the most significant correction to $|V_{ud}|$? Why?

Problem 2 - Energy Momentum Tensor

In class we used conservation and transformation properties of the hadronic matrix element to show that it can be written in terms of the form factor $F_+(q^2)$,

$$\langle H_2|\overline{u}\gamma^\mu(1+\gamma^5)d|H_1\rangle = F_+(q^2)\left[p_2^\mu + p_1^\mu - \frac{m_2^2 - m_1^2}{q^2}q^\mu\right]. \tag{2}$$

The energy momentum tensor, $T^{\mu\nu}(x)$ is symmetric and conserved. Use arguments analogous to those from the notes for the computation of the hadronic current to show that the matrix elements of $T^{\mu\nu}$ between spin-less particle states can be written as

$$\langle H(p_2)|T^{\mu\nu}|H(p_1)\rangle = F_1\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) + F_2 k^\mu k^\nu, \tag{3}$$

where $q = p_1 - p_2$ and $k = p_1 + p_2$.

The energy momentum tensor is also related to the 4-momentum operators by

$$P^\mu = \int T^{\mu 0}(x) d^3x. \tag{4}$$

Use this identity with T^{00} to show that at $q^2 = 0$, $F_2 = \frac{1}{2}$.