Avoiding Death By Vacuum Decay

Cutting on the MSSM parameter space

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DESY Hamburg Theory

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The end is nigh...

www.haaretz.com

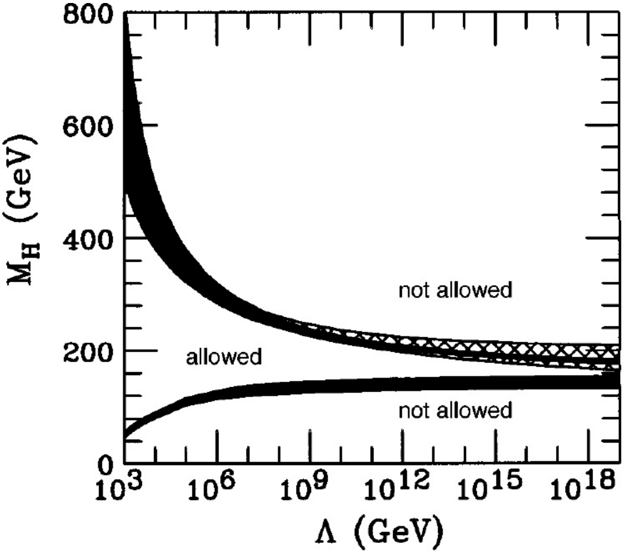
"I think, we have it!"
Rolf Heuer

“Eureka!”

Archimedes

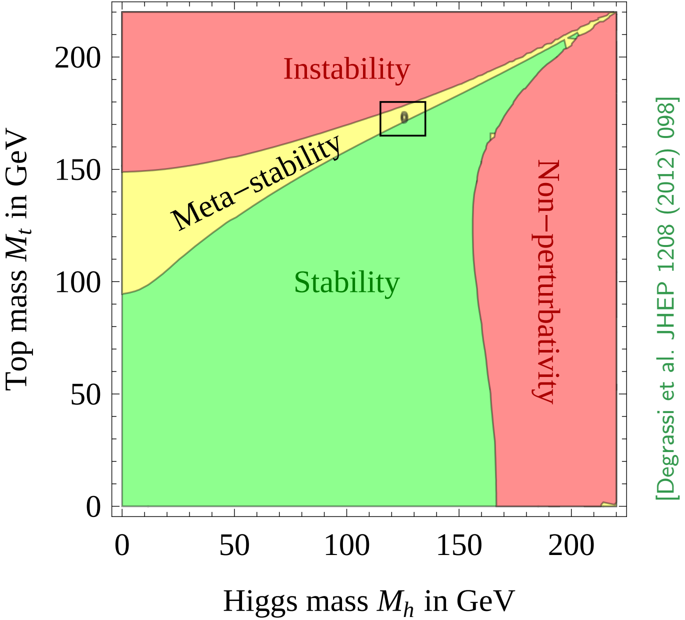
What do we have?

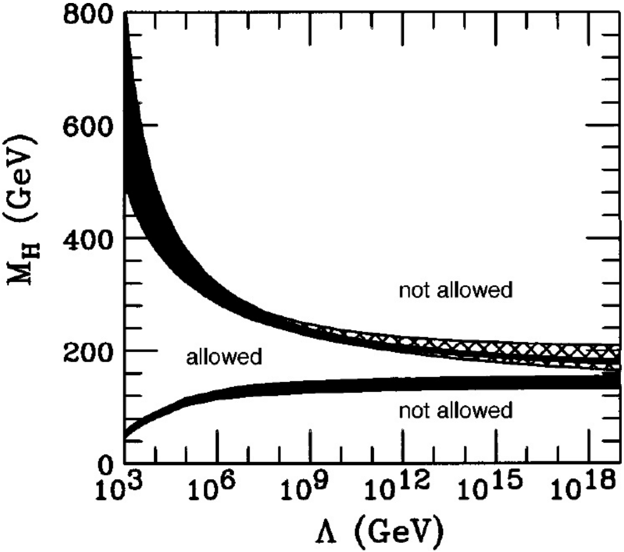
[particlezoo]



[Hambye, Riesselmann: Phys.Rev. D55 (1997) 7255]

The SM phase diagram





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Theoretical considerations

- $\lambda < 4\pi \quad \hookrightarrow$ perturbativity
- $\lambda > 0 \quad \hookrightarrow$ unbounded from below, aka vacuum stability

Constraining the SM Higgs sector

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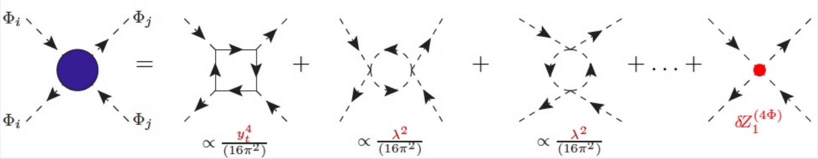
trivial at the classical (i.e. tree) level

$$V(\Phi) = m^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

Quantum level



[courtesy of Max Zoller]

- dominant contribution: top quark ($y_t \sim 1$)
- dependence on the energy scale: β function!

Energy scale dependence

Scale-independent loop-corrected effective potential

$$Q \frac{\mathrm{d}}{\mathrm{d}\, Q} V_{\text{loop}}(\lambda_i, \phi, Q) = 0$$

Approximation for large field values

$$V_{\text{loop}}(\phi) = \lambda(\phi)\phi^4,$$

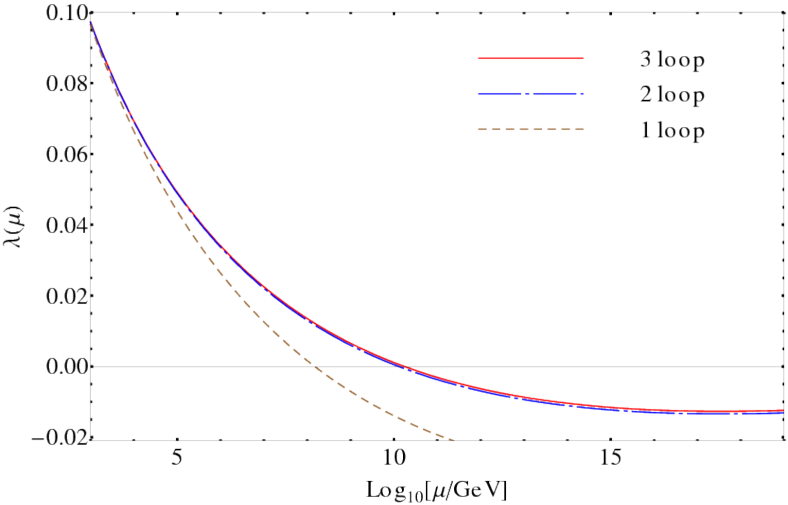
evaluated at $Q \sim \phi$

β function for coupling λ_i

$$\beta_i(\lambda_i) = Q \frac{\mathrm{d}\, \lambda_i(Q)}{\mathrm{d}\, Q}$$

- running of λ determines stability of the loop potential
- upper bound: Landau pole; lower bound: $\lambda > 0$

Precise analysis: up to three loops!



[Zoller 2014]

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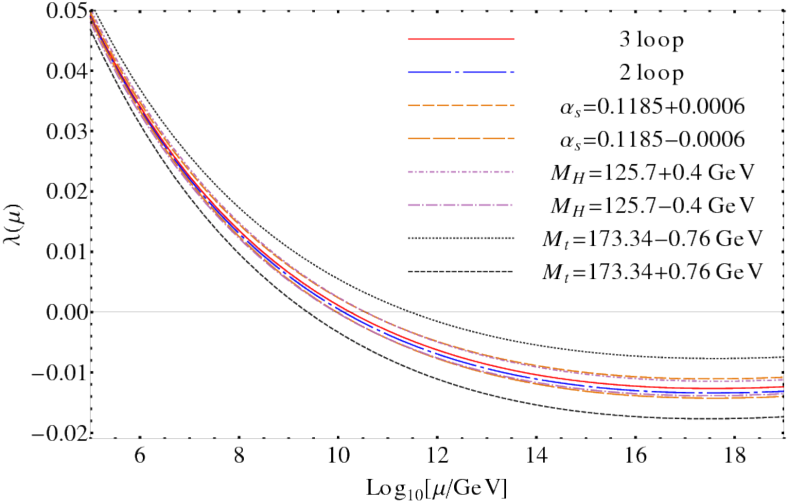

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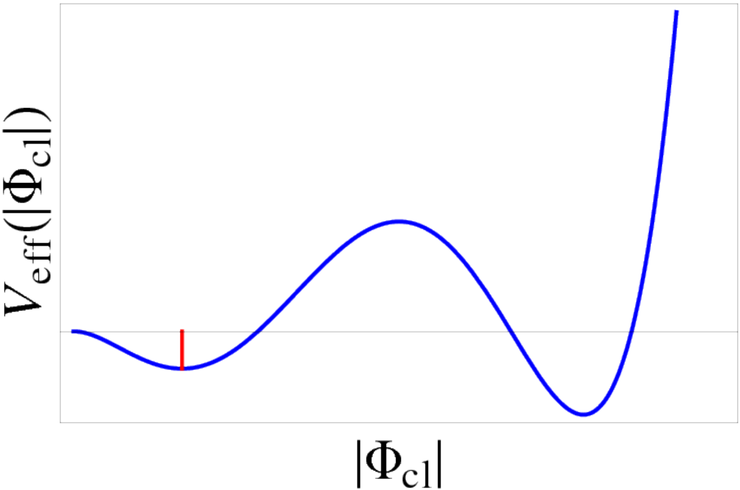
Stability, instability or metastability?



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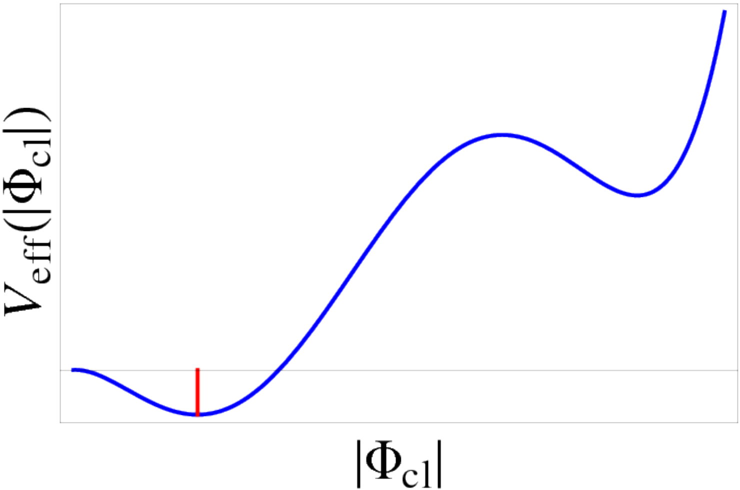

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- metastability: decay time of false vacuum large
- instability scale around $10^{10\ldots12}\,\mathrm{GeV}$
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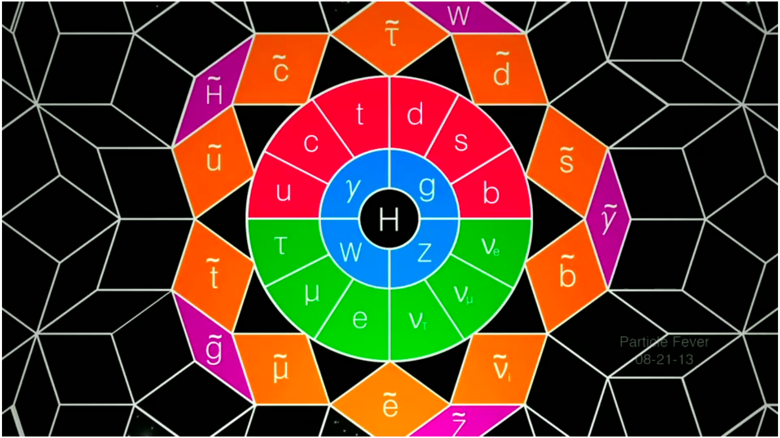

[Particle Fever]

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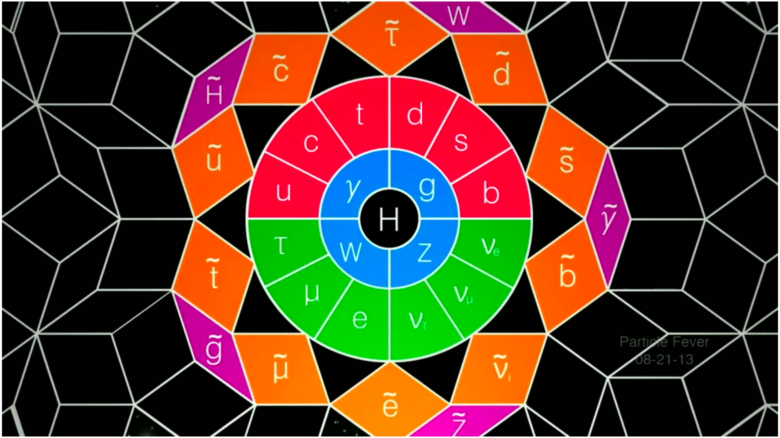

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$$\mathcal{W}_{\text{MSSM}} = \mu H_d \cdot H_u - Y^e_{ij} H_d \cdot L_{L,i} \bar{E}_{R,j} + Y^u_{ij} H_u \cdot Q_{L,i} \bar{U}_{R,j} - Y^d_{ij} H_d \cdot Q_{L,i} \bar{D}_{R,j}$$

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Higgs potential of 2HDM type II

$$\begin{aligned} V = {} & m_{11}^2\; H_d^\dagger H_d + m_{22}^2\; H_u^\dagger H_u + \left(m_{12}^2\; H_u \cdot H_d + \text{h.c.}\right) \\ & + \frac{\lambda_1}{2}\left(H_d^\dagger H_d\right)^2 + \frac{\lambda_2}{2}\left(H_u^\dagger H_u\right)^2 \\ & + \lambda_3 \left(H_u^\dagger H_u\right)\left(H_d^\dagger H_d\right) + \lambda_4 \left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right) + \{\lambda_5, \lambda_6, \lambda_7\} \end{aligned}$$

In the MSSM: tree potential calculated from D-terms and $\mathcal{L}_{\text{soft}}$

$$\begin{aligned} m_{11}^{2\;\text{tree}} &= |\mu|^2 + m_{H_d}^2, & \lambda_1^{\text{tree}} = \lambda_2^{\text{tree}} = -\lambda_3^{\text{tree}} &= \frac{g^2 + g'^2}{4}, \\ m_{22}^{2\;\text{tree}} &= |\mu|^2 + m_{H_u}^2, & \lambda_4^{\text{tree}} &= \frac{g^2}{2}, \\ m_{12}^{2\;\text{tree}} &= B_\mu, & \lambda_5^{\text{tree}} = \lambda_6^{\text{tree}} = \lambda_7^{\text{tree}} &= 0. \end{aligned}$$

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The Higgs sector of the MSSM and its stability

$$\mathcal{W}_{\mathsf{MSSM}} = \mu H_d \cdot H_u - Y^e_{ij} H_d \cdot L_{L,i} \bar{E}_{R,j} + Y^u_{ij} H_u \cdot Q_{L,i} \bar{U}_{R,j} - Y^d_{ij} H_d \cdot Q_{L,i} \bar{D}_{R,j}$$

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Unbounded from below requirements

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}$$

and others... [Gunion, Haber 2003]

- always fulfilled in the MSSM @ tree

Extending the tree

- loop corrections? [Gorbahn, Jäger, Nierste, Trine 2011]

- integrating out heavy SUSY particles
- requirement of large SUSY scale $M_{\mathrm{SUSY}} \gg M_A \sim v_{\mathrm{ew}}$
- effective theory: generic 2HDM, λ_i calculated from SUSY loops

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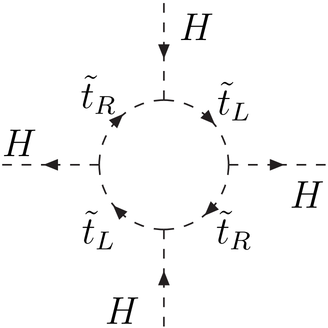
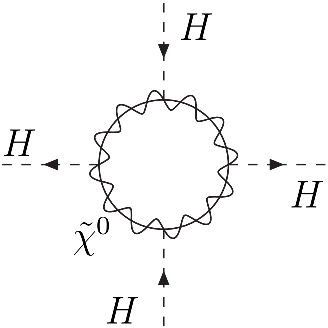




collecting all SUSY contributions:

$$\lambda_i = \lambda_i(\tan\beta, \mu, M_1, M_2, \tilde{m}_Q^2, \tilde{m}_u^2, \tilde{m}_d^2, \tilde{m}_L^2, \tilde{m}_e^2, A_{\mathsf{u}}, A_{\mathsf{d}}, A_{\mathsf{e}}).$$

Triviality bounds on the effective 2HDM

simple check:

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1\lambda_2},$$

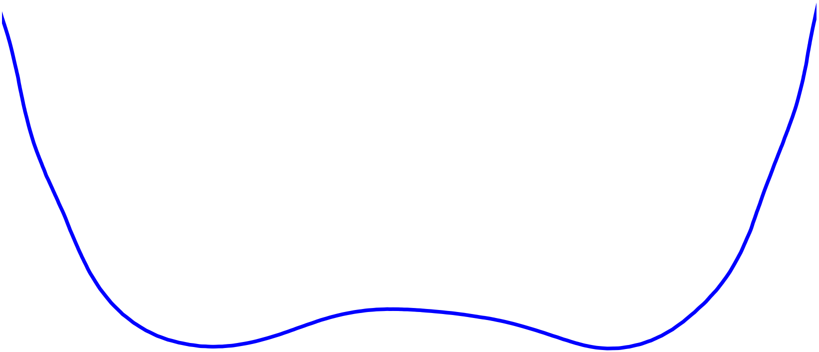
where now

$$\lambda_i = \lambda_i^{\mathrm{tree}} + \frac{\lambda_i^{\mathrm{ino}} + \lambda_i^{\mathrm{sferm}}}{16\pi^2}\,.$$

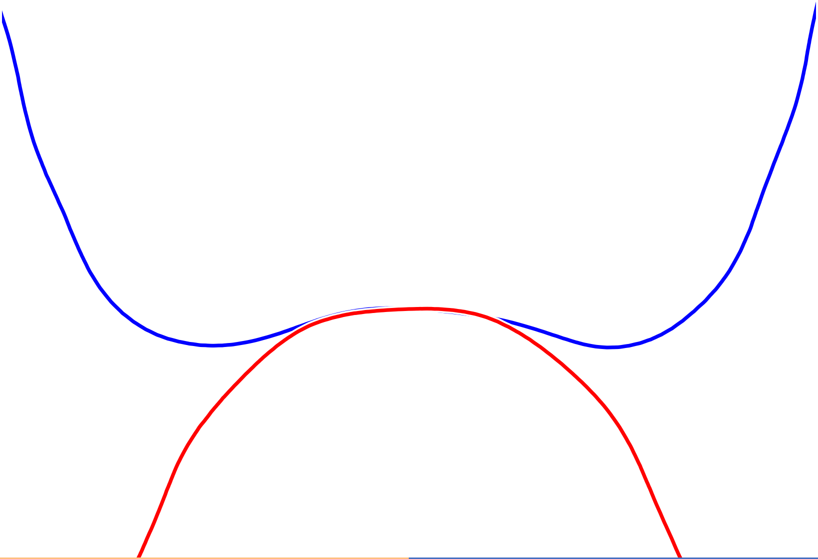
Severe UFB limits

Bounds on $\lambda_{1,2,3}$ transfer into bounds on m_0, A_{t}, μ, ...

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4$$

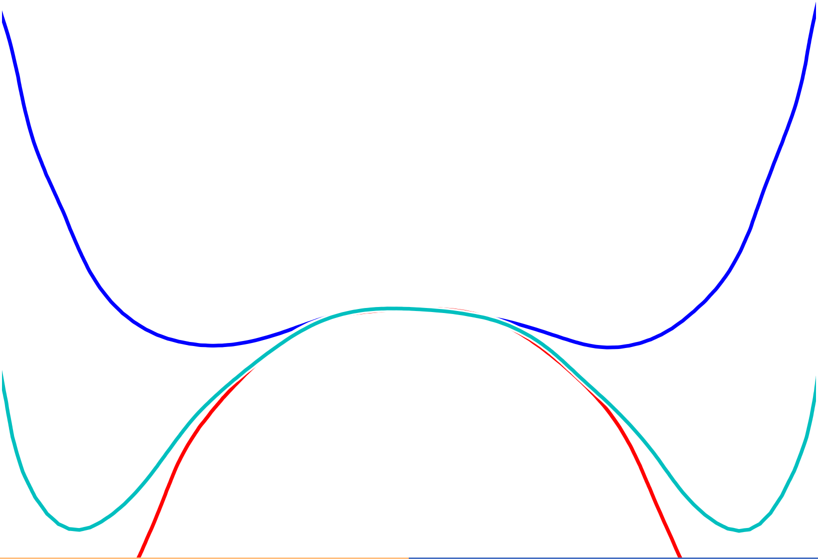


$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4$$

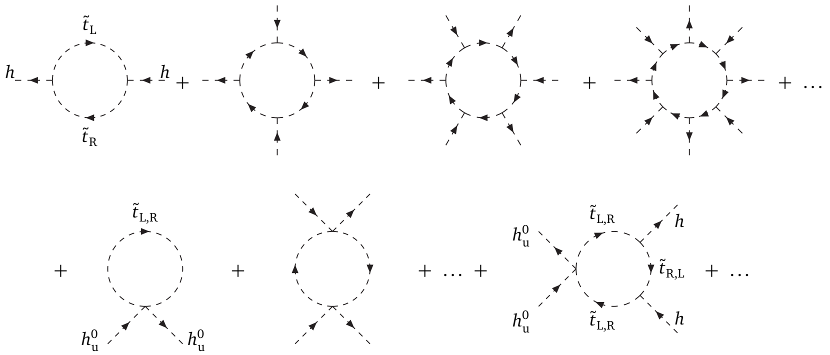


Recovery from unbounded from below???

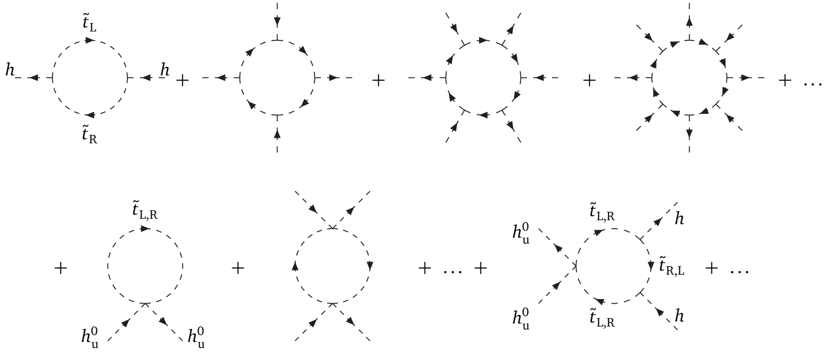
$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4 + \lambda^{(6)}\phi^6$$



Calculating the 1-loop effeective potential



- dominant contribution from third generation squarks
- quadrilinear couplings ($\sim |Y_\mathrm{t}|^2$)
- trilinear coupling to a linear combination $(\mu^* Y_\mathrm{t} h_\mathrm{d}^\dagger - A_\mathrm{t} h_\mathrm{u}^0)$
- series summable to an infinite number of external legs



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- **Do not stop after renormalizable / dim 4 terms!**

- 1-loop effective potential [Coleman, Weinberg 1973]

$$V_1(h_u, h_d) = \frac{1}{64\pi^2}\,\mathrm{STr}\,\mathcal{M}^4(h_u, h_d)\left[\ln\left(\frac{\mathcal{M}^2(h_u, h_d)}{Q^2}\right) - \frac{3}{2}\right]$$

  - *field dependent mass* $\mathcal{M}(h_u, h_d)$
  - STr accounts for spin degrees of freedom

- same result can be obtained by the tadpole method

$$T \sim \frac{\partial}{\partial h} V_1(h) \quad \hookrightarrow \quad V_1(h) \sim \int \mathrm{d}h\, T(h)$$

[Lee, Sciaccaluga 1975]

- functional methods: effective potential for arbitrary number of scalars: $V_1(\phi_1, \phi_2, \ldots \phi_n)$ [Jackiw 1973]

The effective potential

$V_{\text{eff}}(\phi)$: average energy density

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The ground state of the theory

V_{eff} minimized:

$$\left.\frac{\mathrm{d}\,V_{\text{eff}}}{\mathrm{d}\,\phi}\right|_{\phi=v} = 0$$

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Technically:

generating function for 1PI n-point Green's functions:

$$V_{\text{eff}}(\phi) = -\sum_{n=2}^{\infty} \tilde{G}^{(n)}(p_i = 0)\phi^n$$

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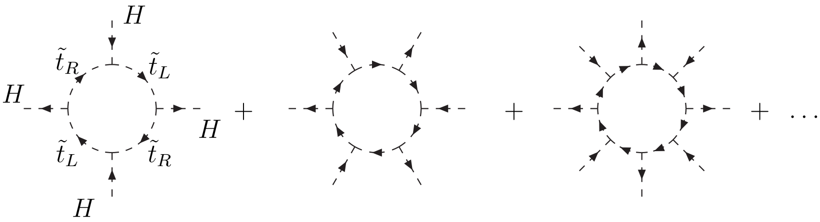
(with subleties)

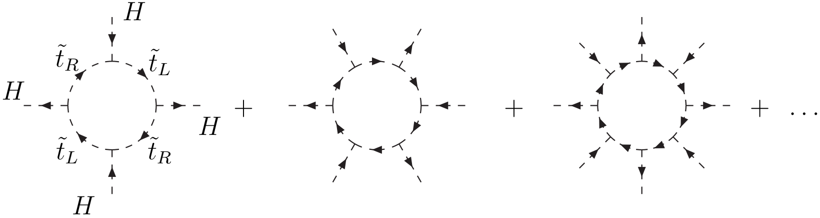


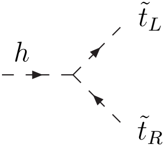
- most dominant contribution from top Yukawa y_t and A_t
- can be easily summed for $m_{\tilde{t}_R} = m_{\tilde{t}_L} \equiv M$
- 1-PI potential as generating function for 1-PI Green's functions

$$-V_{1-\mathrm{PI}}(\phi) = \Gamma_{1-\mathrm{PI}}(\phi) = \sum_n \frac{1}{n!} G_n(p_{\mathrm{ext}} = 0)\phi^n$$

- "classical" field value $\phi \to \langle 0|\phi|0\rangle$
- $\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0$ determines ground state of the theory

Summing up external legs





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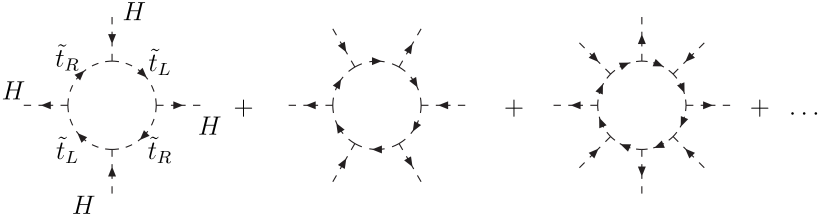

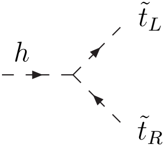

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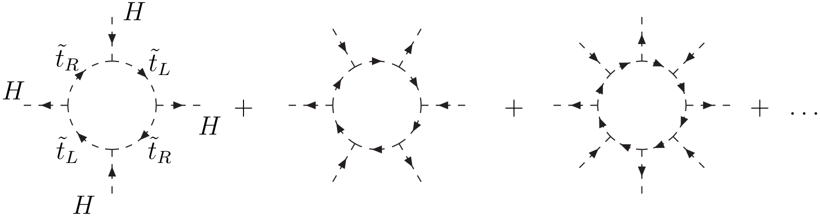

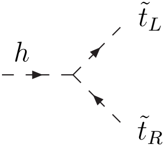

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$$V_1 = \frac{N_c M^4}{32\pi^2} \left[(1+x)^2 \log(1+x) + (1-x)^2 \log(1-x) - 3x^2\right]$$

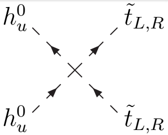
$Q^2 = M^2$ $\qquad$ $x^2 = |\mu Y_t|^2 h^\dagger h / M^4$, $m^2_{\tilde{t}_L} = m^2_{\tilde{t}_R} = M^2$

Summing up (ctd.)

Field dependent stop mass

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde{t}_R} + |Y_t h_u^0|^2 \end{pmatrix}$$

- trilinear $\sim h(h_d^0, h_u^0)$, quadrilinear $\sim |h_u^0|^2$
- diagrams with mixed contributions



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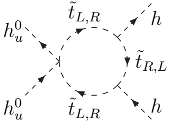

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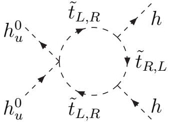
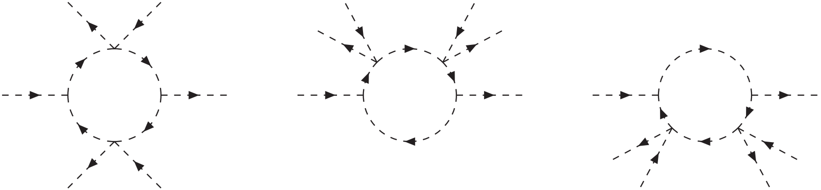


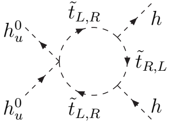


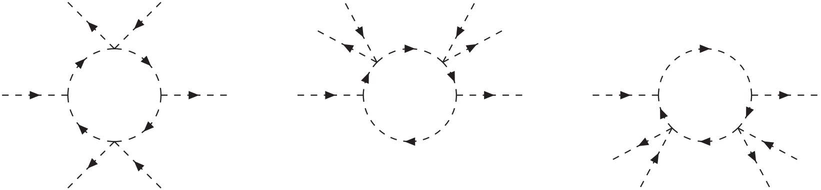
Summing up (ctd.)

Field dependent stop mass

$$\mathcal{M}^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^{0*}_d \\ A^*_t h^{0*}_u - \mu Y^*_t h^0_d & m^2_{\tilde{t}_R} + |Y_t h^0_u|^2 \end{pmatrix}$$

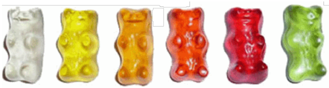
- trilinear $\sim h(h^0_d, h^0_u)$, quadrilinear $\sim |h^0_u|^2$
- diagrams with mixed contributions





- gummi bear factor

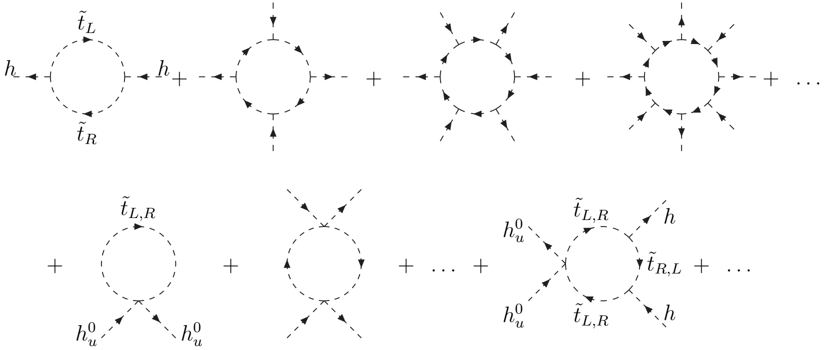
$$\frac{(2n+k-1)!}{k!(2n-1)!}$$



www.idn.uni-bremen.de/biologiedidaktik

Field dependent stop mass

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde{t}_R} + |Y_t h_u^0|^2 \end{pmatrix}$$



Field dependent stop mass

$$\mathcal{M}^2_{\tilde t}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde t_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde t_R} + |Y_t h_u^0|^2 \end{pmatrix}$$

$$V_1 \sim \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{kn}\, x^{2n} y^k \;, \qquad x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4}, y = \frac{|Y_t h_u^0|^2}{M^2}$$

$$= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{1}{n(2n+k-1)(2n+k-2)} \frac{(2n+k-1)!}{k!(2n-1)!}\, x^{2n} y^k$$

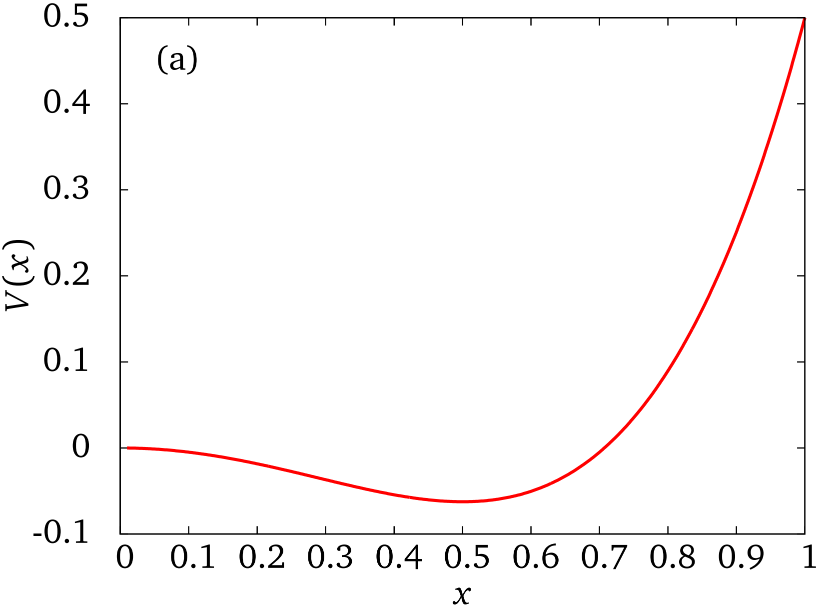
$$= \Big[\, (1+y+x)^2 \log(1+y+x) + (1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y) \Big]$$

Features of the resummed series

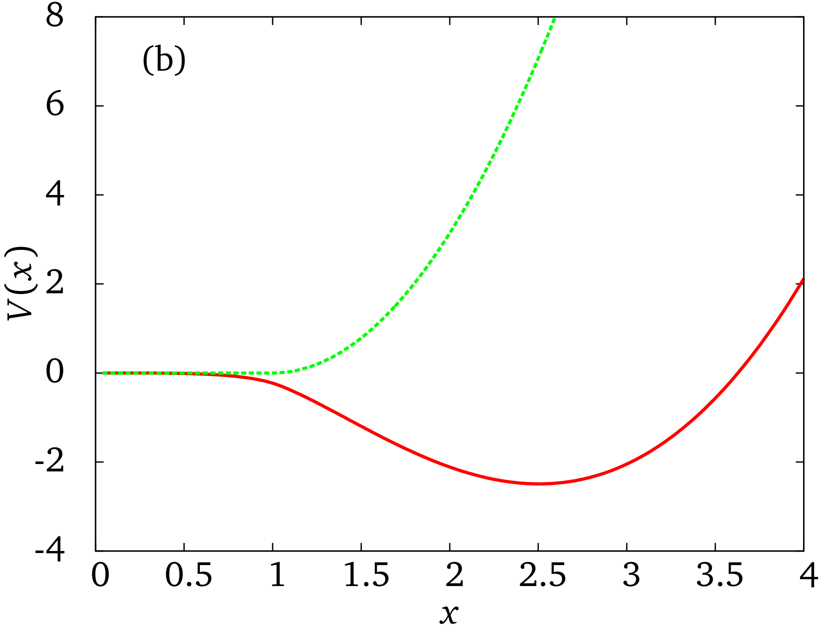
$$V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2}\Big[(1+y+x)^2\log(1+y+x) \\ + (1+y-x)^2\log(1+y-x) \\ - 3(x^2+y^2+2y)\Big]$$

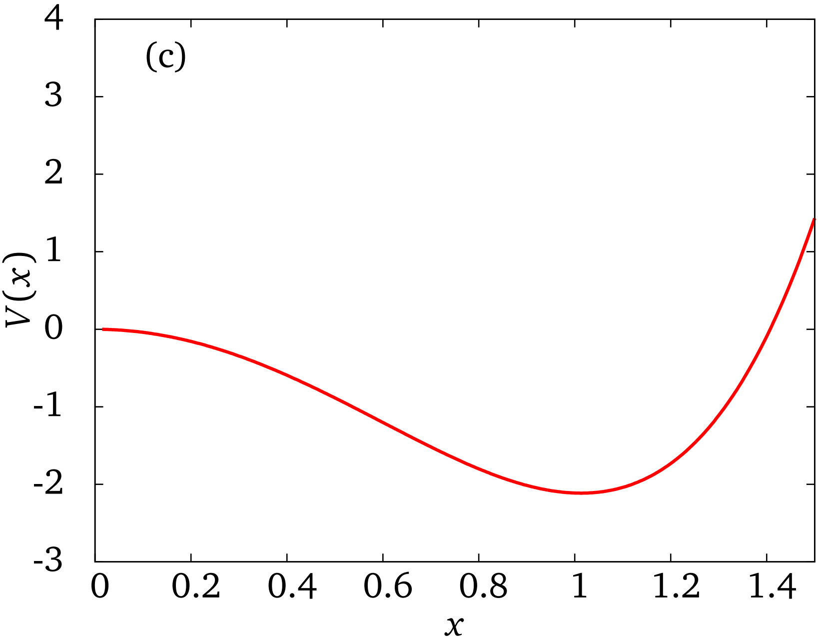
$$x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4}, \; h = h_d^{0*} - \frac{A_t}{\mu^* Y_t} h_u^0, \qquad y = \frac{|Y_t h_u^0|^2}{M^2}$$

- branch cut at $x - y = \pm 1$: take real part (analytic cont.)
- ignore imaginary part: $\log(1+y-x) = \frac{1}{2}\log\left((1+y-x)^2\right)$
- always bounded from below
- minimum independent of Higgs parameters from tree potential
- minimum determined by SUSY scale parameters

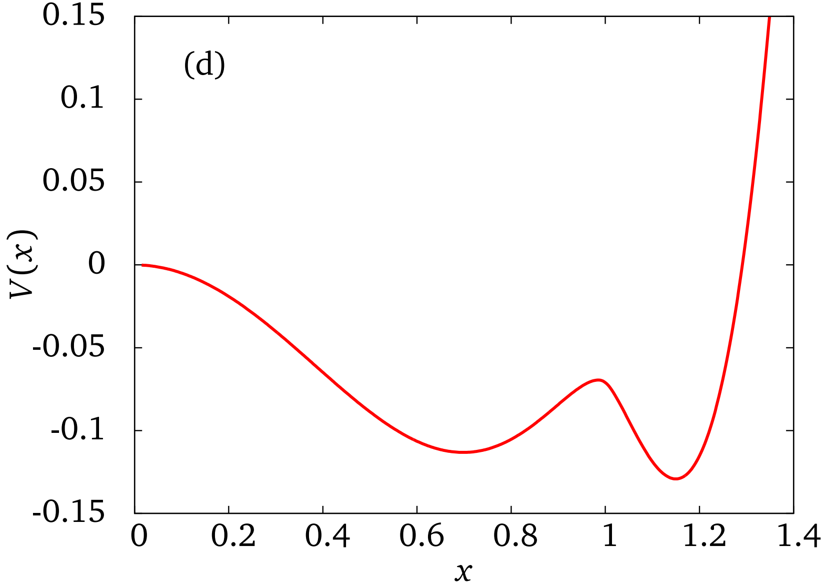


(a)
$V(x)$
x
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


(b)
8
6
4
2
0
-2
-4
$V(x)$
0
0.5
1
1.5
2
2.5
3
3.5
4
x


(c)
$V(x)$
x
4
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Tree, loop and tree + loop



Minimum at the electroweak scale $v = 246\,\mathrm{GeV}$

$$m_{11}^{2\,\text{tree}} = m_{12}^{2\,\text{tree}}\,\tan\beta - \frac{v^2}{2}\cos(2\beta)\lambda_1^{\text{tree}} - \frac{1}{v\cos\beta}\,\frac{\delta}{\delta\phi_d}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}},$$

$$m_{22}^{2\,\text{tree}} = m_{12}^{2\,\text{tree}}\,\cot\beta + \frac{v^2}{2}\cos(2\beta)\lambda_1^{\text{tree}} - \frac{1}{v\sin\beta}\,\frac{\delta}{\delta\phi_u}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}}.$$

$m_h = 125\,\mathrm{GeV}$

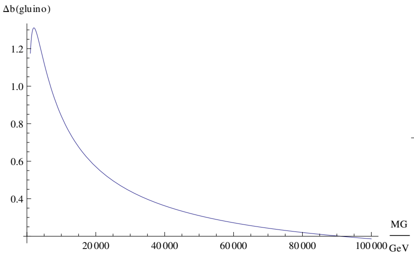
- using `FeynHiggs 2.10.0` to determine light Higgs mass by adjusting A_t (several solutions: $\operatorname{sign} A_t = -\operatorname{sign}\mu$)
- connection to potential: m_A
- pseudoscalar mass m_A less dependent on higher loops
- decoupling limit: $m_A, m_{H^\pm}, m_H \gg m_h$

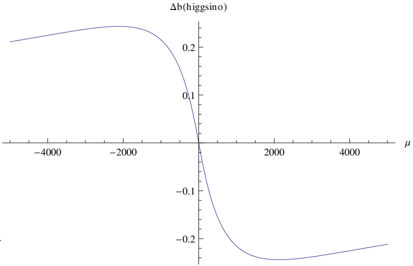
- include sbottom (drives minimum), take $A_b = 0$

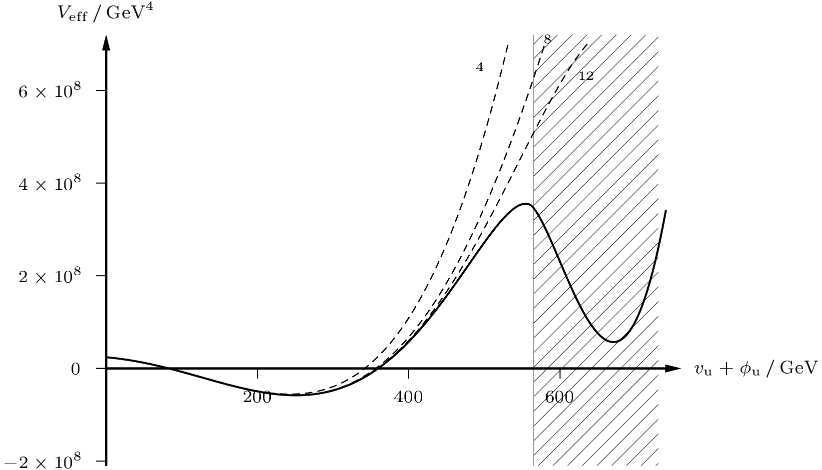
Yukawa coupling not given directly by the mass

$$y_b = \frac{m_b}{v_d(1+\Delta_b)}$$

$$\Delta_b^{\text{gluino}} = \frac{2\alpha_s}{3\pi}\mu M_{\tilde{G}} \tan\beta C_0(\tilde{m}_{\tilde{b}_1}, \tilde{m}_{\tilde{b}_2}, M_{\tilde{G}}),$$
$$\Delta_b^{\text{higgsino}} = \frac{Y_t^2}{16\pi^2}\mu A_t \tan\beta C_0(\tilde{m}_{\tilde{t}_1}, \tilde{m}_{\tilde{t}_2}, \mu).$$

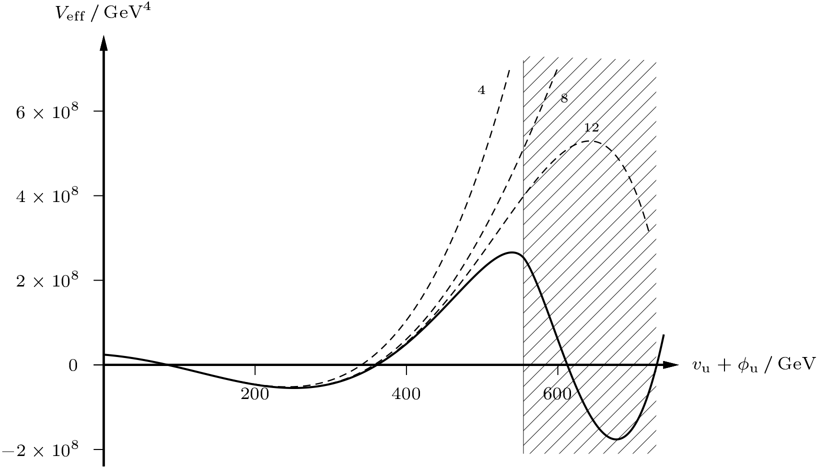






- $\tan\beta = 40$
- $m_A = 800\,\mathrm{GeV}$
- $M = 1\,\mathrm{TeV}$
- $\mu = 3.75\,\mathrm{TeV}$
- $A_t \simeq -1.5\,\mathrm{TeV}$

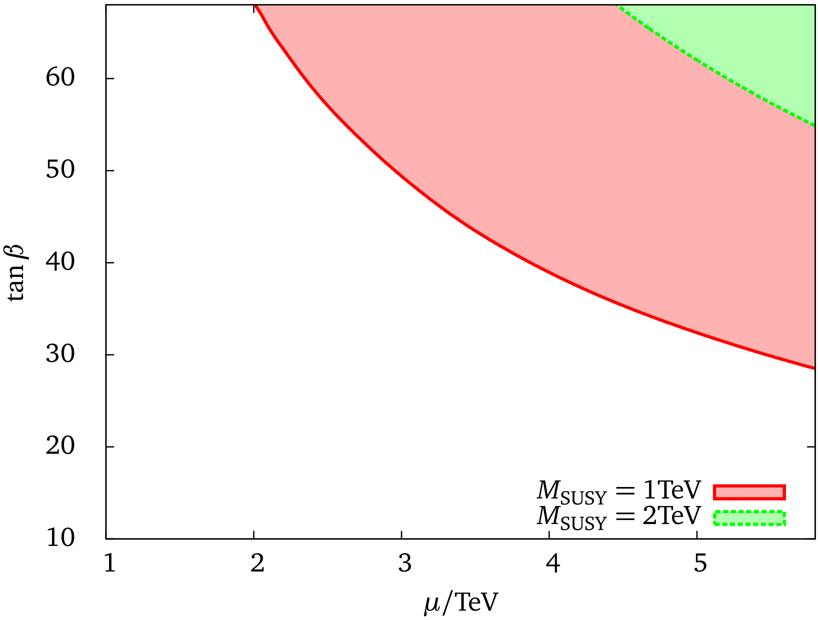
Discussion of stability



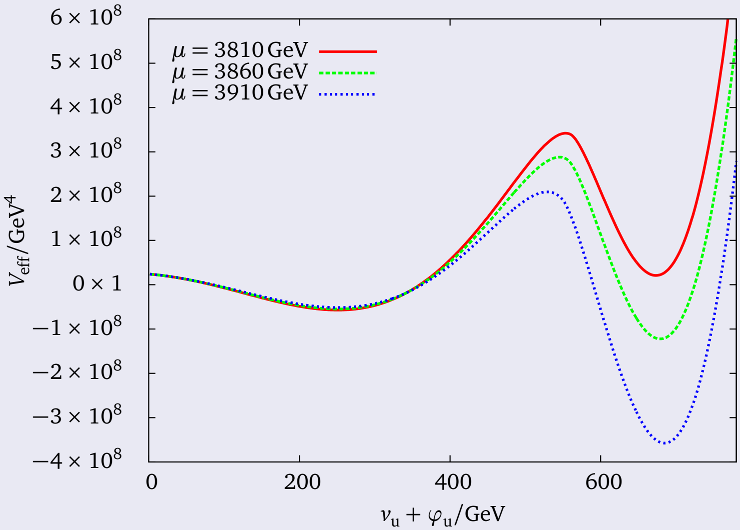
- $\tan\beta = 40$
- $m_A = 800\,\mathrm{GeV}$
- $M = 1\,\mathrm{TeV}$
- $\mu = 3.83\,\mathrm{TeV}$
- $A_t \simeq -1.5\,\mathrm{TeV}$

[www.bbcamerica.com]

Naive exclusions: Constraint in μ-$\tan\beta$



Access to Charge and Color breaking minima



Access to Charge and Color breaking minima

$$\mathcal{M}^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}^2_Q + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^{0*}_d \\ A^*_t h^{0*}_u - \mu Y^*_t h^0_d & \tilde{m}^2_t + |Y_t h^0_u|^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}}(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}^2_Q + |Y_b h^0_d|^2 & A_b h^0_d - \mu^* Y_b h^{0*}_u \\ A^*_b h^{0*}_d - \mu Y^*_b h^0_u & \tilde{m}^2_b + |Y_b h^0_d|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev

Access to Charge and Color breaking minima

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_b h_d^0|^2 & A_b h_d^0 - \mu^* Y_b h_u^{0*} \\ A_b^* h_d^{0*} - \mu Y_b^* h_u^0 & \tilde{m}_b^2 + |Y_b h_d^0|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev:

$$m^2_{\tilde{b}_{1,2}}(h_u^0, h_d^0) = \frac{\tilde{m}_Q^2 + \tilde{m}_b^2}{2} + |Y_b h_d^0|^2 \pm \frac{1}{2}\sqrt{(\tilde{m}_Q^2 - \tilde{m}_b^2)^2 + 4|A_b h_d^0 - \mu^* Y_b h_u^{0*}|^2}$$

- expand theory around new minimum: $m^2_{\tilde{b}_2} < 0$

Access to Charge and Color breaking minima

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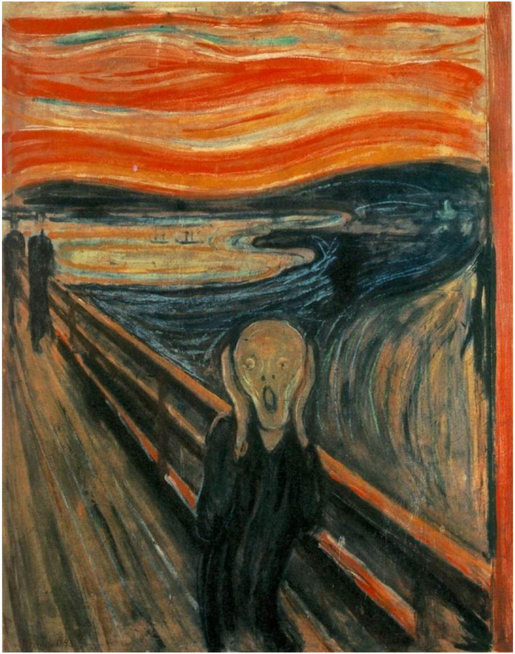
- expand theory around new minimum: $m^2_{\tilde{b}_2} < 0$
- **tachyonic squark mass!**



[commons.wikimedia.org]

What does a tachyonic mass mean?

- mass $\Leftrightarrow$ second derivative: $m_\phi^2 = \partial^2 V/\partial\phi^2$
- $m_\phi^2 < 0 \quad \Leftrightarrow \quad$ negative curvature
- non-convex potential: imaginary part
- $\log(1 + y - x) \sim \log(m_\phi^2)$

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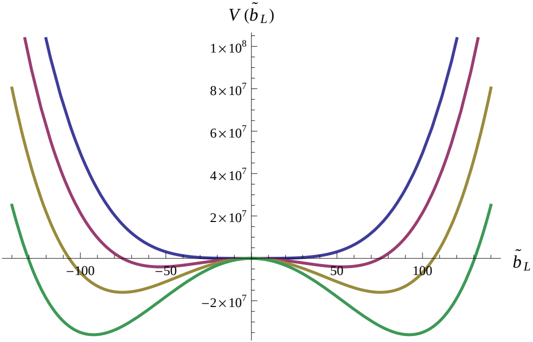

Including colored directions

$$V_{\tilde{b}}^{\text{tree}} = \tilde{b}_{\mathrm{L}}^{*}(M_{\tilde{Q}}^{2} + |Y_{\mathrm{b}}v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{L}} + \tilde{b}_{\mathrm{R}}^{*}(M_{\tilde{\mathrm{b}}}^{2} + |Y_{\mathrm{b}}v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{R}}$$
$$- \left[\tilde{b}_{\mathrm{L}}^{*}(\mu^{*}Y_{\mathrm{b}}h_{\mathrm{u}}^{0\dagger} - A_{\mathrm{b}}v_{\mathrm{d}})\tilde{b}_{\mathrm{R}} + \text{h. c.}\right] + |Y_{\mathrm{b}}|^{2}|\tilde{b}_{\mathrm{L}}|^{2}|\tilde{b}_{\mathrm{R}}|^{2}$$
$$+ \; D\text{-terms.}$$

A more complete picture

Including colored directions

$$V_{\tilde{b}}^{\text{tree}} = \tilde{b}_{\mathrm{L}}^{*}(M_{\tilde{Q}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{L}} + \tilde{b}_{\mathrm{R}}^{*}(M_{\tilde{\mathrm{b}}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{R}} - \left[\tilde{b}_{\mathrm{L}}^{*}(\mu^{*} Y_{\mathrm{b}} h_{\mathrm{u}}^{0\dagger} - A_{\mathrm{b}} v_{\mathrm{d}})\tilde{b}_{\mathrm{R}} + \text{h. c.}\right] + |Y_{\mathrm{b}}|^{2}|\tilde{b}_{\mathrm{L}}|^{2}|\tilde{b}_{\mathrm{R}}|^{2} + D\text{-terms.}$$

D-flat direction: minimizing D-term contribution

- D-terms: $g^2\phi^4$

$$V_D = \frac{g_1^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2\right)^2 + \frac{g_2^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 + |\tilde{b}_L|^2\right)^2 + \frac{g_3^2}{6}\left(|\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2.$$

- will always take over for large field values
- take e.g. $\tilde{b}_L = \tilde{b}_R = \tilde{b}$ and $|h_{\mathrm{d}}^0|^2 = |h_u^0|^2 + |\tilde{b}|^2|$

A more complete picture

Including colored directions

$$V_{\tilde{b}}^{\text{tree}} = \tilde{b}_{\mathrm{L}}^{*}(M_{\tilde{Q}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{L}} + \tilde{b}_{\mathrm{R}}^{*}(M_{\tilde{\mathrm{b}}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{R}}$$
$$- \left[\tilde{b}_{\mathrm{L}}^{*}(\mu^{*} Y_{\mathrm{b}} h_{\mathrm{u}}^{0\dagger} - A_{\mathrm{b}} v_{\mathrm{d}})\tilde{b}_{\mathrm{R}} + \text{h. c.}\right] + |Y_{\mathrm{b}}|^{2}|\tilde{b}_{\mathrm{L}}|^{2}|\tilde{b}_{\mathrm{R}}|^{2}$$
$$+ \ D\text{-terms.}$$

D-flat direction: minimizing D-term contribution

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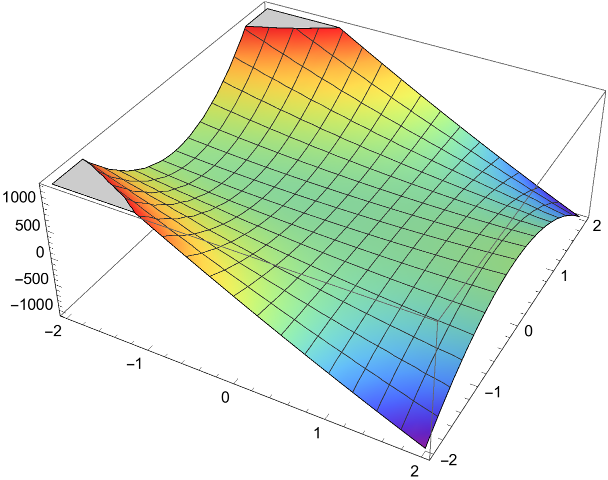
$$V_D = \frac{g_1^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2\right)^2$$
$$+ \frac{g_2^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 + |\tilde{b}_L|^2\right)^2 + \frac{g_3^2}{6}\left(|\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2.$$

- will always take over for large field values
- or $h_d^0 = 0$ and $\tilde{b} = h_u^0$ [large D-term]

From CCC to CCB

- previously “safe” false but CC conserving minima turn into to deep global minima with $\langle\tilde{b}_L\rangle = \langle\tilde{b}_R\rangle \neq 0$ and $\langle h_{\mathrm{u}}^0\rangle \neq v_{\mathrm{u}}$



From CCC to CCB

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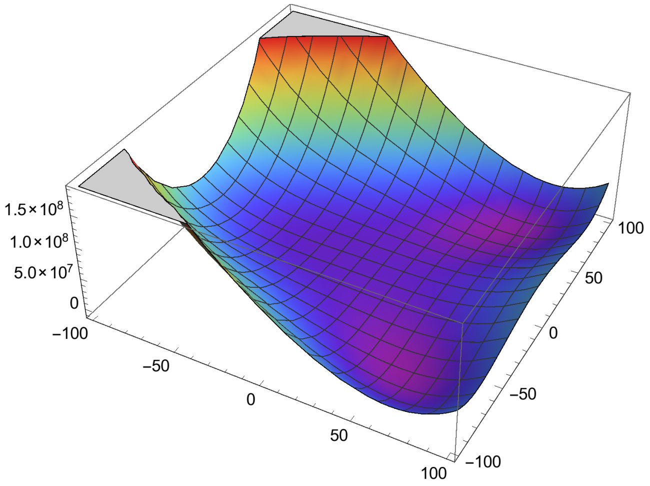

1. choose appropriate direction $\hookrightarrow$ one-field problem

$$V_\phi^{\mathsf{tree}} = \bar{m}^2\phi^2 - A\phi^3 + \lambda\phi^4$$

$h_\mathrm{u}^0 = \tilde{b}$, $h_\mathrm{d}^0 = 0$

2. identify parameters, e.g. $\bar{m}^2 = \tilde{m}_Q^2 + \tilde{m}_b^2 + m_{H_\mathrm{u}}^2 + \mu^2$, $\lambda = Y_\mathrm{b}^2 + \frac{g_1^2+g_2^2}{2}$, $A = 2\mu Y_\mathrm{b}$
3. necessary condition: $\bar{m}^2 > \frac{A^2}{4\lambda}$ $\hookrightarrow$ second minimum not below first (trivial) one)

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$h_\mathrm{u}^0 = \tilde{b},\ h_\mathrm{d}^0 = 0$

$$m_{H_\mathrm{u}}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu Y_\mathrm{b})^2}{Y_\mathrm{b}^2 + (g_1^2 + g_2^2)/2}$$

1. choose appropriate direction $\hookrightarrow$ one-field problem

$$V_\phi^{\text{tree}} = \bar{m}^2\phi^2 - A\phi^3 + \lambda\phi^4$$

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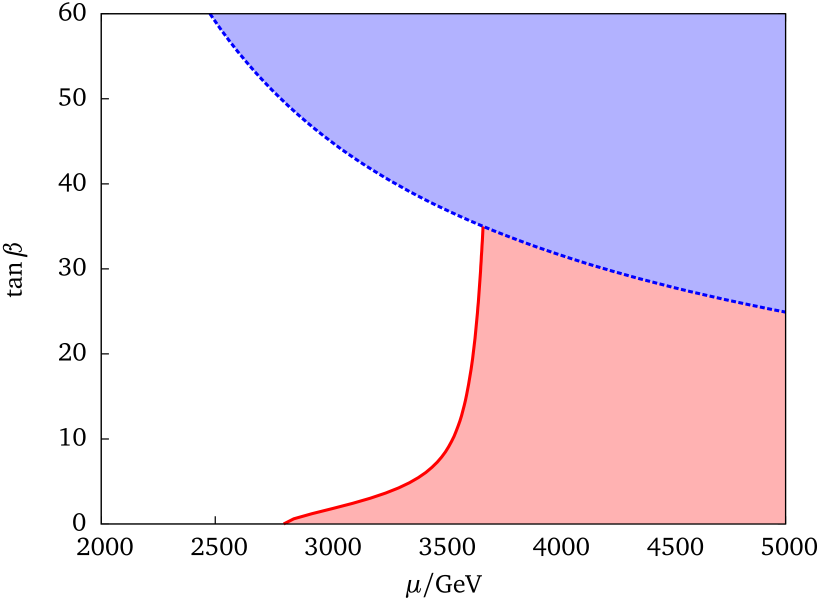
3. necessary condition: $\bar{m}^2 > \frac{A^2}{4\lambda} \hookrightarrow$ second minimum not below first (trivial) one)

$|h_{\mathrm{d}}^0|^2 = |h_{\mathrm{u}}^0|^2 + |\tilde{b}|^2,\ \tilde{b} = \alpha h_{\mathrm{u}}^0$

$$m_{11}^2(1+\alpha^2) + m_{22}^2 \pm 2m_{12}^2\sqrt{1+\alpha^2} + \alpha^2(\tilde{m}_Q^2 + \tilde{m}_b^2) > \frac{\mu^2\alpha^4}{2+3\alpha^2}$$


60
50
40
30
20
10
0
2000
2500
3000
3500
4000
4500
5000
$\tan\beta$
μ/GeV

- the Higgs potential in the SM is meta/un/stable
- MSSM: multi-scalar theory, has several unwanted minima
- formation of new CCB *conserving* minima at the 1-loop level
- stability of the electroweak vacuum: bounds on μ-$\tan\beta$
- instability of electroweak vacuum by second minimum in "standard model direction" $\sim v_u$: global CCB minimum
- more severe bounds [new CCB constraints]
- CCB constraints for non-vanishing D-Terms
- quantum tunneling: either very long- or very short-lived

Greetings from Señor Higgs
(*courtesy of Jens Hoff*)

Backup

Slides

References

- Wolfgang Gregor Hollik: *"Charge and color breaking constraints in the Minimal Supersymmetric Standard Model associated with the bottom Yukawa coupling"* Physics Letters B **752** (2016) 7 – 12
- Wolfgang Gregor Hollik: *"Neutrinos meet Supersymmetry: Quantum Aspects of Neutrinophysics in Supersymmetric Theories"*, PhD Thesis, Karlsruhe 2015
- Markus Bobrowski, Guillaume Chalons, Wolfgang G. Hollik, Ulrich Nierste: *"Vacuum stability of the effective Higgs potential in the Minimal Supersymmetric Standard Model"* Physical Review D **90**, 035025 (2014)