A Flavour Physics View on the Strong CP Problem

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DESY Hamburg Theory

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More flavour!

Unknown knowns: 19 parameters of the SM

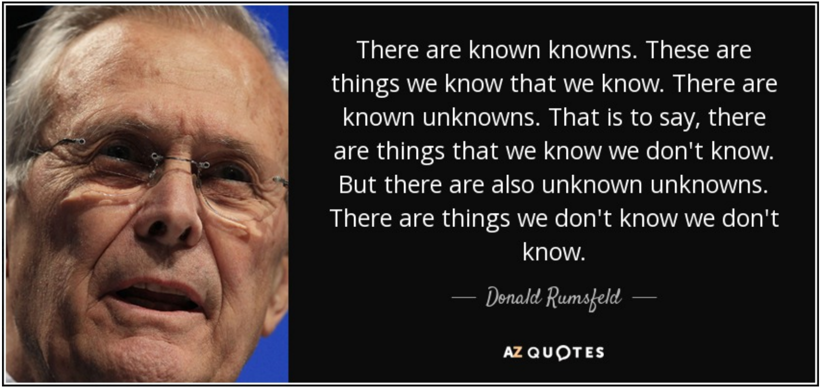
- 3 gauge couplings
- 1 scalar mass & 1 self-coupling
- 3 lepton masses
- 6 quark masses
- 3 mixing angles & 1 CP phase (CKM matrix)
- strong CP violation?

Known unknowns: flavour sector

Yukawa couplings

- Why 3 generations?
- Why hierarchical?
- Why preference of left-handed fermions?
- Quarks only: 18 (!) parameters $\rightarrow$ 10 relevant in the SM

Who knows about the unknown?



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- kinetic terms: flavour blind $\overline{\psi}_i \partial\!\!\!/ \psi_i$
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- Yukawa interactions: responsible for masses, couplings differ for each family $y_{ij} \overline{\psi}_{L,i} \Phi \psi_{R,j}$

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Yukawa sector of the Standard Model

Fermion content: $Q_{L,i}$, $u_{R,i}$, $d_{R,i}$, $L_{L,i}$, $\ell_{R,i}$

$$\mathcal{L}_{\mathrm{Y}} = y^d_{ij} \bar{Q}_{L,i} \Phi d_{R,j} + y^u_{ij} \bar{Q}_{L,i} \tilde{\Phi} u_{R,j} + y^e_{ij} \bar{L}_{L,i} \Phi \ell_{R,j} + \text{h. c.}$$

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Singular Value Decomposition: $\boldsymbol{Y} \to \boldsymbol{L}^\dagger \boldsymbol{\Sigma} \boldsymbol{R}$: $\quad \boldsymbol{\Sigma} = \boldsymbol{L}\boldsymbol{Y}\boldsymbol{R}^\dagger$

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Rotate fields in flavour space:

$$Q_L \to \boldsymbol{L}_Q\, Q_L,$$
$$u_R \to \boldsymbol{R}_u\, u_R,$$
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How the fermion mixing enters the charged current?

- Yukawa couplings break $\mathrm{SU}(2)_L$ invariance in the fermions:

$$\mathcal{L}_{\mathrm{Y}}^{q} = \bar{Q}_L \, \boldsymbol{Y}_d \, \Phi \, d_R + \bar{Q}_L \, \boldsymbol{Y}_u \, \tilde{\Phi} \, u_{R,k} + \text{ h. c.}$$

  in general $\boldsymbol{Y}_d \neq \boldsymbol{Y}_u$
- right-handed rotations unobservable
- $\boldsymbol{L}_d$ and $\boldsymbol{L}_u$ fixed via $\boldsymbol{Y}_{u,d} = \boldsymbol{L}^{\dagger}_{u,d} \, \boldsymbol{\Sigma}_{u,d} \, \boldsymbol{R}_{u,q}$ by

$$\mathrm{diag}(m^2_{u,d}, m^2_{c,s}, m^2_{t,b}) = \boldsymbol{L}_{u,d} \, \boldsymbol{M}_{u,d} \boldsymbol{M}^{\dagger}_{u,d} \, \boldsymbol{L}^{\dagger}_{u,d}$$

- Charged current interaction:

$$W^+_\mu \, \overline{u}_L \gamma^\mu d_L \rightarrow W^+_\mu \, \overline{u}_L \, \boldsymbol{L}^{\dagger}_{u} \boldsymbol{L}_u \, \gamma^\mu \, \boldsymbol{L}^{\dagger}_{d} \boldsymbol{L}_d d_L$$

$$\rightarrow W^+_\mu \, \overline{u}'_L \, \boldsymbol{L}_u \, \gamma^\mu \, \boldsymbol{L}^{\dagger}_{d} d'_L \equiv W^+_\mu \, \overline{u}'_L \, \boldsymbol{V}_{\mathrm{CKM}} \, \gamma^\mu \, d_L$$

What is allowed to model flavour

- massless Standard Model: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

$$\mathrm{U}(3)_Q \times \mathrm{U}(3)_u \times \mathrm{U}(3)_d$$

- (quarks only)
- 3 generations (why?)
- gauge couplings U(3)-invariant for complex triplets $\mathbf{3}, \bar{\mathbf{3}}$
- $\overline{\psi}_i \not{D} \psi_i$ for $i = 1, \ldots, 3$ generations
- broken by Yukawa couplings (mix gauge representations)

$$\overline{\psi}_i^L \, Y_{ij} \, \psi_j^R$$

- $\mathrm{U}(3)^2$ freedom of rotating Yukawas

$$\overline{\psi}_{i'}^L \, {U_{i'i}^L}^* \, Y_{ij} \, U_{jj'}^R \, \psi_{j'}^R \quad \xrightarrow{\text{diagonalization}} \quad \overline{\psi}_i^L \, m_{ii} \, \psi_i^R$$

The strong CP problem with quark masses

The non-trivial QCD vacuum

non-vanishing winding number

$$n = \frac{g_s^2}{32\pi^2} \int \mathrm{d}^4 x \, F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu}$$

can be interpreted as additional Lagrangian term

$$\mathcal{L}_\theta = \theta_{\mathrm{QCD}} \frac{g_s^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu}$$

Axial anomaly $\psi \to \exp(-\mathrm{i}\,\gamma_5 \theta_{\mathrm{QFD}})\psi$

$$\partial^\mu J^5_\mu = \frac{g_s^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{a\,\mu\nu}$$

The effective theta term

Contributions from QCD vacuum and axial anomaly

$$\theta_{\mathrm{QCD}} \to \bar{\theta} = \theta_{\mathrm{QCD}} + \theta_{\mathrm{QFD}}$$

Quark Flavour Dynamics

$$\theta_{\mathrm{QFD}} = \arg\det(\boldsymbol{M}_u) + \arg\det(\boldsymbol{M}_d) = \arg\det(\boldsymbol{M}_u \boldsymbol{M}_d)$$

The strong CP problem

- $\bar{\theta}$ violates CP and induces a neutron electric dipole moment
- measurement: $\bar{\theta} < 10^{-10}$
- Why do θ_{QCD} and θ_{QFD} cancel so precisely?

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Weak CPV

Jarlskog invariant:

$$J_q \sim \mathrm{Im}\left[\det\left(\left[\boldsymbol{M}_u\boldsymbol{M}_u^\dagger, \boldsymbol{M}_d\boldsymbol{M}_d^\dagger\right]\right)\right]$$
$$\sim \mathrm{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right)$$

[Ellis, Gaillard 1979]

renormalization of θ_{QCD} from CP violations in weak interactions

- Kobayashi–Maskawa model: 3 generations, 1 CP phase δ
- first finite renormalization in 4th order

$$\Delta\theta \approx \left(\frac{\alpha}{\pi}\right)^2 s_1^2 s_2 s_3 \sin\delta \, \mathcal{O}\left(\frac{m_q}{m_W}\right)^4$$

estimate: $\Delta\theta \approx \mathcal{O}(10^{-16} \ldots 10^{-12})$

- “infinite” (logarithmic) divergence in 14th order

$$\Delta\theta^{\log} \approx \left(\frac{\alpha}{\pi}\right)^7 \left(\frac{m_t^4 m_b^4 m_s^2 m_c^2}{m_W^{12}}\right) s_1^2 s_2 s_3 \sin\delta \, \ln\frac{\mu_0}{1\,\text{GeV}}$$

- renormalization at some “relaxation scale” $\mu_0 \lesssim M_P$

$$\Delta\theta^{\inf} = 10^{-32}$$

What is behind θ_{QFD}?

$$\theta_{\mathrm{QFD}} = \arg\det(\boldsymbol{M}_u) + \arg\det(\boldsymbol{M}_d) = \arg\det(\boldsymbol{M}_u \boldsymbol{M}_d)$$

Simple matrix algebra, exploiting $\boldsymbol{M} = \boldsymbol{L}^\dagger \boldsymbol{\Sigma} \boldsymbol{R}$, $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \sigma_3)$

$$\theta_{\mathrm{QFD}} = \arg\det\left(\boldsymbol{L}_u^\dagger \boldsymbol{\Sigma}_u \boldsymbol{R}_u \boldsymbol{L}_d^\dagger \boldsymbol{\Sigma}_d \boldsymbol{R}_d\right)$$

knowing that $\det(\boldsymbol{A}\boldsymbol{B}) = \det\boldsymbol{A}\det\boldsymbol{B}$ and $\arg(xy) = \arg x + \arg y$;
U(3) rotations: $\det\boldsymbol{U} = e^{\mathrm{i}\varphi}$

$$\theta_{\mathrm{QFD}} = -\varphi_u^L + \varphi_u^R - \varphi_d^L + \varphi_d^R$$

(singular values are real and positive: $\arg\det\boldsymbol{\Sigma} = 0$)

Only global phases from U(3) flavour groups

$$\arg\det\boldsymbol{X}_q = \varphi_q^X$$

Can we have weak CPV ($J_q \neq 0$) and $\theta_{\mathrm{QFD}} = 0$?

Special unitary groups are enough!

$$U(\theta,\delta,\eta) = \begin{pmatrix} e^{\mathrm{i}\,\eta}\cos\theta & e^{-\mathrm{i}\,\delta}\sin\theta \\ e^{\mathrm{i}\,\delta}\sin\theta & e^{-\mathrm{i}\,\eta}\cos\theta \end{pmatrix}$$

Gatto–Sartori–Tonin-like mixing angles

$$\tan\theta = \sqrt{m_1/m_2}$$

a certain ansatz [Cheng, Sher 1987]

$$\boldsymbol{M} = \begin{pmatrix} 0 & \sqrt{m_1 m_2}e^{-\mathrm{i}\,\delta} \\ -\sqrt{m_1 m_2}e^{\mathrm{i}\,\delta} & (m_2 - m_1) \end{pmatrix}$$

no global phase (even $\eta = 0$ in diagonalization)

generalization possible for 3×3 matrices

More towards a model

[“flavour blind principle”, Saldaña-Salazar 2015]

$$S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to S_{2A}$$

["flavour blind principle", Saldaña-Salazar 2015]

$$S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to S_{2A}$$

An estimated guess (democratic couplings)

$$\boldsymbol{Y} = \frac{y}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \to \quad y\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

["flavour blind principle", Saldaña-Salazar 2015]

$$S_{3L} \times S_{3R} \to S_{2L} \times S_{2R} \to S_{2A}$$

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First (mild) perturbation

$$\begin{pmatrix} \alpha & \alpha & \beta_1 \\ \alpha & \alpha & \beta_1 \\ \beta_2 & \beta_2 & x \end{pmatrix}$$

$S_{2L} \times S_{2R}$ symmetry left $\qquad \alpha, \beta_{1,2}, x \ll y$

break remaining symmetry by *anti*symmetric permutations

A heuristic trial [WGH, Saldaña-Salazar 2014]

- start with mixing angles as functions of mass ratios

$$\tan\theta_{23}^{(0)} = \sqrt{\frac{m_2}{m_3}}, \quad \tan\theta_{12}^{(0)} = \sqrt{\frac{m_1}{m_2}}, \quad \tan\theta_{13}^{(0)} = \sqrt{\frac{m_1}{m_3}},$$

- include correcting rotations, since

$$\frac{m_1}{m_3} \approx \left(\frac{m_2}{m_3}\right)^2 \approx \left(\frac{m_1}{m_2}\right)^2$$

- allow for CP phases $\delta \in \{0, \frac{\pi}{2}, \pi\}$ in [e. g. Masina and Savoy 2006]

$$U(\theta,\delta) = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-\mathrm{i}\delta} \\ -\sin\theta\, e^{\mathrm{i}\delta} & \cos\theta \end{pmatrix}$$

We deconstruct the CKM matrix as $\boldsymbol{V}_{\mathrm{CKM}} = \boldsymbol{L}^u \left(\boldsymbol{L}^d\right)^\dagger$ with

[WGH and Saldaña-Salazar 2014]

$$\begin{aligned}
\boldsymbol{L}^u &= \boldsymbol{L}^u_{12}\left(\frac{m_\mathrm{u}}{m_\mathrm{c}}\right)\boldsymbol{L}^u_{13}\left(\frac{m_\mathrm{u}m_\mathrm{c}}{m_\mathrm{t}^2}\right)\boldsymbol{L}^u_{13}\left(\frac{m_\mathrm{c}^2}{m_\mathrm{t}^2}\right)\boldsymbol{L}^u_{13}\left(\frac{m_\mathrm{u}}{m_\mathrm{t}}\right) \\
&\quad \times \boldsymbol{L}^u_{23}\left(\frac{m_\mathrm{u}m_\mathrm{c}}{m_\mathrm{t}^2}\right)\boldsymbol{L}^u_{23}\left(\frac{m_\mathrm{u}}{m_\mathrm{t}}\right)\boldsymbol{L}^u_{23}\left(\frac{m_\mathrm{c}}{m_\mathrm{t}}\right), \\
\boldsymbol{L}^{d\dagger} &= \boldsymbol{L}^{d\,\dagger}_{23}\left(\frac{m_\mathrm{s}}{m_\mathrm{b}}, 0\right)\boldsymbol{L}^{d\,\dagger}_{23}\left(\frac{m_\mathrm{d}}{m_\mathrm{b}}, \pi\right)\boldsymbol{L}^{d\,\dagger}_{23}\left(\frac{m_\mathrm{d}m_\mathrm{s}}{m_\mathrm{b}^2}, \pi\right) \\
&\quad \times \boldsymbol{L}^{d\,\dagger}_{13}\left(\frac{m_\mathrm{d}}{m_\mathrm{b}}, 0\right)\boldsymbol{L}^{d\,\dagger}_{13}\left(\frac{m_\mathrm{s}^2}{m_\mathrm{b}^2}, \pi\right)\boldsymbol{L}^{d\,\dagger}_{13}\left(\frac{m_\mathrm{d}m_\mathrm{s}}{m_\mathrm{b}^2}, \pi\right) \\
&\quad \times \boldsymbol{L}^{d\,\dagger}_{12}\left(\frac{m_\mathrm{d}}{m_\mathrm{s}}, \frac{\pi}{2}\right),
\end{aligned}$$

leading to (Standard Paramametrization à la PDG)

$$\delta_{\mathrm{CKM}} \approx \arctan\left[\sqrt{\frac{\frac{m_\mathrm{d}}{m_\mathrm{s}}\left(1+\frac{m_\mathrm{d}}{m_\mathrm{s}}\right)}{\frac{m_\mathrm{u}}{m_\mathrm{c}}\left(1+\frac{m_\mathrm{u}}{m_\mathrm{c}}\right)}}\right] \approx (1.38 \pm 0.10)\,\mathrm{rad}$$

[Díaz-Cruz, WGH, Saldaña-Salazar 2016]

We don't give a model! (But requirements a good model should fulfill.)

Left-Right symmetry trivially fulfills

[Mohapatra, Senjanović '83; Babu, Mohapatra '90; Kuchimanchi '10; Senjanović, Tello '15]

- Yukawa couplings Hermitian: $\boldsymbol{Y} = \boldsymbol{Y}^\dagger$ because of $L \longleftrightarrow R$ in
$$-\mathcal{L}_Y = \bar{\psi}_L \boldsymbol{Y} \Phi \psi_R + \text{h. c.}$$
- holds for radiative Yukawa couplings

[Gabrielli, Marzola, Raidal '16]

Explaining the flavour hierarchy from quantum corrections

Yukawa couplings forbidden by symmetry at tree-level
$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_Y$$

one Higgs doublet for each SU(2) [Gabrielli, Marzola, Raidal '16]
$$O_{Y_f} = \frac{1}{\Lambda^f_{\text{eff}}} \left(\bar{\psi}^f_L H_L\right)\left(H^\dagger_R \psi^f_R\right) + \text{h. c.}$$

Spontaneous non-abelian flavour symmetry breaking

$$\mathcal{G}_f = \mathrm{SU}(3)_Q \times \mathrm{SU}(3)_u \times SU(3)_d$$

treat Yukawa couplings as *dynamical* fields under $\mathcal{G}_f$

$$-\mathcal{L} = y_u \bar{Q}_L Y_u u_R \tilde{H} + y_d \bar{Q}_L Y_d d_R H + \mathrm{h.\,c.}$$

with

$$Q_L = (\mathbf{3}, 1, 1), \quad u_R = (1, \mathbf{3}, 1), \quad d_R = (1, 1, \mathbf{3}),$$
$$H = (1, 1, 1), \quad Y_u = (\mathbf{3}, \bar{\mathbf{3}}, 1), \quad Y_d = (\mathbf{3}, 1, \bar{\mathbf{3}})$$

write down potential for Y_u, Y_d and solve for minima
$\hookrightarrow$ observed Yukawa couplings are vevs
condition for phases

$$\arg\det[\langle Y_u\rangle\langle Y_d\rangle] = \langle\delta_u\rangle + \langle\delta_d\rangle = 0 (\mathrm{mod}\, 2\pi)$$

[Fong and Nardi 2013]

Axions from family symmetry breaking: *familons* [Wilczek 1982]

Serving the axion in a Froggatt–Nielsen way

Flavour hierarchies from exponential suppression

$$-\mathcal{L} = \lambda_{ij}^{d} \left(\frac{\Phi}{\Lambda} \right)^{n_{ij}^{d}} \bar{Q}_{L,i} \cdot H d_{R,j} + \lambda_{ij}^{u} \left(\frac{\Phi}{\Lambda} \right)^{n_{ij}^{u}} \bar{Q}_{L,i} \cdot \tilde{H} u_{R,j}$$

with $n_{ij}^{d} = q_{Q_{L,i}} - q_{d_{R,j}}$ and $n_{ij}^{u} = q_{Q_{L,i}} - q_{u_{R,j}}$ and the Froggatt–Nielsen charges $q_{Q_{L,i}}$, $q_{d_{R,i}}$, $q_{u_{R,i}}$ of the fields $Q_{L,i}$, $d_{R,i}$ and $u_{R,i}$ under $\mathrm{U}(1)_{\mathrm{FN}}$

the flavoured axion

the pseudoscalar component of the flavon field Φ serves as axion

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field ϕ heavy and removed from low energy pheno

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Flaxion [Ema, Hamaguchi, Moroi and Nakayama 2016]

Axiflavon [Calibbi, Goertz, Redigolo, Ziegler, Zupan 2016]

Conclusions

- Strong CP problem is more than just axions.
- If the QCD axion is not found, that doesn't mean that there is no solution to the strong CP problem.

- most reasonable flavour models have no strong CP problem

$$\bar{\theta} = \theta_{\mathrm{QCD}} + \theta_{\mathrm{QFD}}$$

- $\theta_{\mathrm{QFD}} = -\varphi_u^L + \varphi_u^R - \varphi_d^L + \varphi_d^R$

- There are also mdels with flavoured axions. [Flaxion, Axiflavon]
- The idea to make $\arg\det(\boldsymbol{M}_u \boldsymbol{M}_d)$ vanish without vanishing masses is rather old (spontaneous CPV) [Nelson '84, Barr '84]

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- condition trivially fulfilled by LR symmetry [Mohapatra et al. '83ff]

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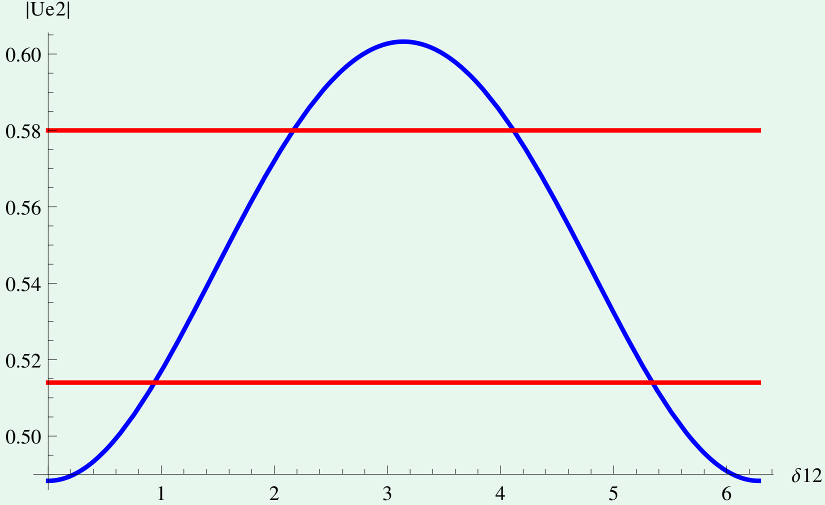
- Thanks to Lorenzo Díaz-Cruz and Ulises Saldaña-Salazar!

Backup

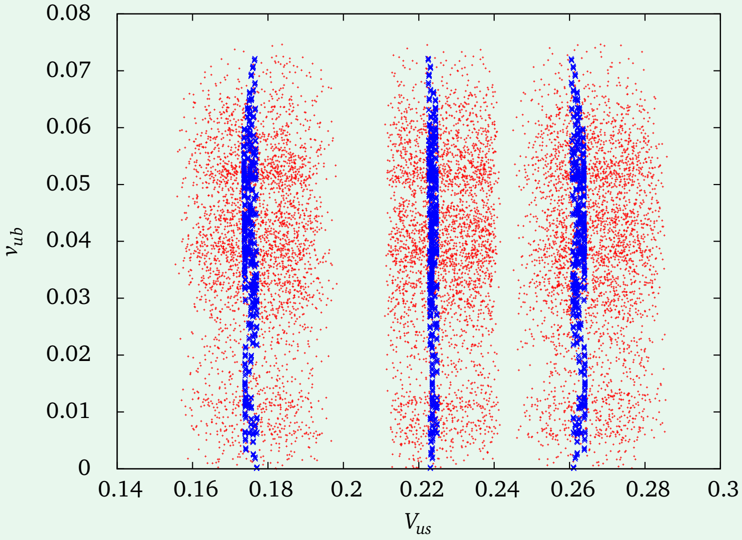
Slides

Final mixing matrix elements depent on the phases

e.g. PMNS mixing, $U_{e2} = 0.514\ldots0.580$ @ 3σ



Finally only *one* out of 2187 combinations allowed



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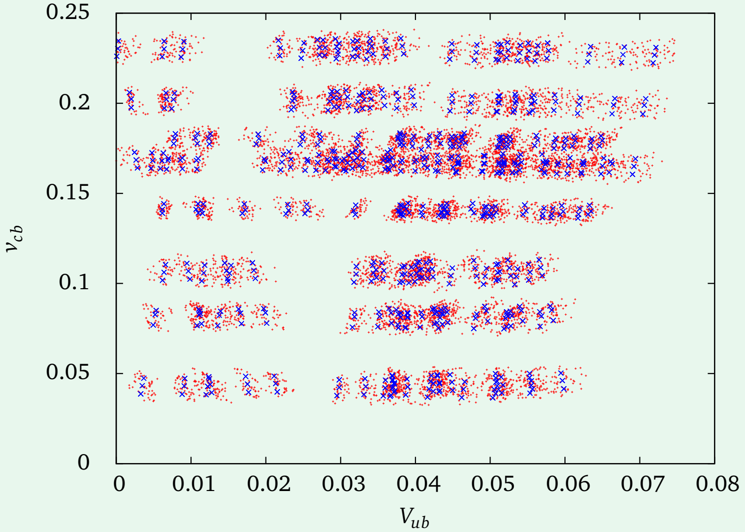

- on one hand, we can (nearly) predict everything
- on the other hand: fixing the phases by a look into data sets viable patterns/textures for the mass matrices
- "fitting" the phase combinations point towards the underlying flavor symmetry

Either minimal (i.e. no) or maximal CP violation

choose $\delta_{ij}^{a(x)} \in \{0, \frac{\pi}{2}, \pi\}$

	δ_{12}	$\delta_{13}^{(0)}$	$\delta_{13}^{(1)}$	$\delta_{13}^{(2)}$	$\delta_{23}^{(0)}$	$\delta_{23}^{(1)}$	$\delta_{23}^{(2)}$
CKM	$\frac{\pi}{2}$	0	π	π	0	π	π
PMNS	$\frac{\pi}{2}$	0	π	π	π	π	0

- only one non-vanishing CP-phase: $\delta_{12} = \frac{\pi}{2}$

[see also Masina, Savoy 2006]

CKM matrix (our values)

$$|V^{\text{th}}_{\text{CKM}}| = \begin{pmatrix} 0.974^{+0.004}_{-0.003} & 0.225^{+0.016}_{-0.011} & 0.0031^{+0.0018}_{-0.0015} \\ 0.225^{+0.016}_{-0.011} & 0.974^{+0.004}_{-0.003} & 0.039^{+0.005}_{-0.004} \\ 0.0087^{+0.0010}_{-0.0008} & 0.038^{+0.004}_{-0.004} & 0.9992^{+0.0002}_{-0.0001} \end{pmatrix}$$

Jarlskog invariant: $J_q = \mathrm{Im}(V_{us} V_{cb} V^*_{ub} V^*_{cs}) = (2.6^{+1.3}_{-1.0}) \times 10^{-5}$

PMNS matrix (our values)

$$|U^{\text{th}}_{\text{PMNS}}| = \begin{pmatrix} 0.83^{+0.04}_{-0.05} & 0.54^{+0.06}_{-0.09} & 0.14 \pm 0.03 \\ 0.38^{+0.04}_{-0.06} & 0.57^{+0.03}_{-0.04} & 0.73 \pm 0.02 \\ 0.41^{+0.04}_{-0.06} & 0.61^{+0.03}_{-0.04} & 0.67 \pm 0.02 \end{pmatrix}$$

$J_\ell = \mathrm{Im}(U_{e2} U_{\mu 3} U^*_{e3} U^*_{\mu 2}) = 0.031^{+0.006}_{-0.007}$

$\Rightarrow$ our prediction: $|\delta^{\text{PMNS}}_{\text{Dirac}}| = 90^\circ \pm 20^\circ$

CKM matrix (PDG)

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97427^{+0.00014}_{-0.00014} & 0.22536^{+0.00061}_{-0.00061} & 0.00355^{+0.00015}_{-0.00015} \\ 0.22522^{+0.00061}_{-0.00061} & 0.97343^{+0.00015}_{-0.00015} & 0.0414^{+0.0012}_{-0.0012} \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914^{+0.00005}_{-0.00005} \end{pmatrix}$$

Jarlskog invariant: $J_q = \mathrm{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*) = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$

PMNS matrix (nu-fit.org, $3\,\sigma$)

$$|U_{\mathrm{PMNS}}| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}$$

$J_\ell^{\max} = 0.033 \pm 0.010$

$\Rightarrow$ our prediction: $|\delta_{\mathrm{Dirac}}^{\mathrm{PMNS}}| = 90^\circ \pm 20^\circ$

Empirical relation [Gatto, Sartori, Tonin 1968]

Cabbibo angle:

$$\theta_C \approx \sqrt{\frac{m_d}{m_s}}$$

+ small correction from $\frac{m_u}{m_c}$
approximation: large hierarchy $m_d \ll m_s$

GST-like mixing angles follow from mass matrices with a structure

$$|\mathbf{M}| = \begin{pmatrix} 0 & \sqrt{m_1 m_2} \\ \sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix}$$

Hierarchy: $\sqrt{\frac{m_1}{m_2}} = \varepsilon \ll 1$, most general mass matrix

$$|\mathbf{M}| \sim \begin{pmatrix} \mathcal{O}(\varepsilon^2) & \mathcal{O}(\varepsilon) \\ \mathcal{O}(\varepsilon) & 1+\mathcal{O}(\varepsilon^2) \end{pmatrix}, |\mathbf{M}\mathbf{M}^\dagger| \sim \begin{pmatrix} \mathcal{O}(\varepsilon^2) & \mathcal{O}(\varepsilon) \\ \mathcal{O}(\varepsilon) & 1+\mathcal{O}(\varepsilon^2) \end{pmatrix}$$

Singular Value Decomposition

$$-\mathcal{L}_Y \supset Y_{ij}\bar{L}_i \cdot \Phi\, R_j + \text{h.c.}$$

diagonalize $\mathbf{Y}$ as $\mathbf{S}_L \mathbf{Y}\, \mathbf{S}_R^\dagger = \circ$

Large hierarchy in singular values: $\Sigma_{11} \ll \Sigma_{22} \ll \Sigma_{33}$

Schmidt–Eckart–Young–Mirsky theorem

lower-rank approximation, take $\mathbf{S}_{L/R} = [\vec{s}_{L/R,1}, \vec{s}_{L/R,2}, \vec{s}_{L/R,3}]$:

$$\mathbf{M} = m_3 \left[\left(\vec{s}_{L,1} \frac{m_1}{m_2} \vec{s}_{R,1}^{\,\dagger} + \vec{s}_{L,2} \vec{s}_{R,2}^{\,\dagger} \right) \frac{m_2}{m_3} + \vec{s}_{L,3} \vec{s}_{R,3}^{\,\dagger} \right]$$

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Rank one approximation

$$\hat{\mathbf{M}} = \vec{s}_{L,3} \vec{s}_{R,3}^{\,\dagger} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Singular Value Decomposition

$$-\mathcal{L}_Y \supset Y_{ij}\bar{L}_i \cdot \Phi R_j + \text{h.c.}$$

diagonalize $\mathbf{Y}$ as $\mathbf{S}_L \mathbf{Y}\, \mathbf{S}_R^\dagger = ^\circ$

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The origin of the CKM matrix

$$\mathcal{L}_{\mathrm{CC}} = -\frac{\mathrm{i}g_2}{\sqrt{2}} W^+_\mu \bar{u}_L \gamma^\mu d_L + \mathrm{h.c.} \to -\frac{\mathrm{i}g_2}{\sqrt{2}} W^+_\mu \bar{u}'_L \boldsymbol{S}^u_L \gamma^\mu \boldsymbol{S}^{d\dagger}_L d_L + \mathrm{h.c.}$$

V_{CKM}

$$\boldsymbol{V}_{\mathrm{CKM}} = \boldsymbol{S}^u_L\, \boldsymbol{S}^{d\dagger}_L$$

$$\boldsymbol{V}_{\mathrm{CKM}} = \boldsymbol{V}_{23}(\theta^{\mathrm{CKM}}_{23}) \boldsymbol{V}_{13}(\theta^{\mathrm{CKM}}_{13}, \delta_{\mathrm{CKM}}) \boldsymbol{V}_{12}(\theta^{\mathrm{CKM}}_{12})$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{-\mathrm{i}\,\delta_{\mathrm{CKM}}} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{\mathrm{i}\,\delta_{\mathrm{CKM}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\mathrm{i}\,\delta_{\mathrm{CKM}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathrm{CKM}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathrm{CKM}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathrm{CKM}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathrm{CKM}}} & c_{23}c_{13} \end{pmatrix}$$

We deconstruct the CKM matrix as $\boldsymbol{V}_{\text{CKM}} = \boldsymbol{S}_{\text{L}}^{u} \left(\boldsymbol{S}_{\text{L}}^{d}\right)^{\dagger}$ with

$$
\begin{aligned}
\boldsymbol{S}_{\text{L}}^{u} = {} & \boldsymbol{S}_{12}^{\text{L},\,u}\left(\frac{m_{\text{u}}}{m_{\text{c}}}\right) \boldsymbol{S}_{13}^{\text{L},\,u}\left(\frac{m_{\text{u}} m_{\text{c}}}{m_{\text{t}}^{2}}\right) \boldsymbol{S}_{13}^{\text{L},\,u}\left(\frac{m_{\text{c}}^{2}}{m_{\text{t}}^{2}}\right) \boldsymbol{S}_{13}^{\text{L},\,u}\left(\frac{m_{\text{u}}}{m_{\text{t}}}\right) \\
& \times \boldsymbol{S}_{23}^{\text{L},\,u}\left(\frac{m_{\text{u}} m_{\text{c}}}{m_{\text{t}}^{2}}\right) \boldsymbol{S}_{23}^{\text{L},\,u}\left(\frac{m_{\text{u}}}{m_{\text{t}}}\right) \boldsymbol{S}_{23}^{\text{L},\,u}\left(\frac{m_{\text{c}}}{m_{\text{t}}}\right), \\
\boldsymbol{S}_{\text{L}}^{d\dagger} = {} & \boldsymbol{S}_{23}^{\text{L},\,d\dagger}\left(\frac{m_{\text{s}}}{m_{\text{b}}}, \delta_{23}^{(0)}\right) \boldsymbol{S}_{23}^{\text{L},\,d\dagger}\left(\frac{m_{\text{d}}}{m_{\text{b}}}, \delta_{23}^{(1)}\right) \boldsymbol{S}_{23}^{\text{L},\,d\dagger}\left(\frac{m_{\text{d}} m_{\text{s}}}{m_{\text{b}}^{2}}, \delta_{23}^{(2)}\right) \\
& \times \boldsymbol{S}_{13}^{\text{L},\,d\dagger}\left(\frac{m_{\text{d}}}{m_{\text{b}}}, \delta_{13}^{(0)}\right) \boldsymbol{S}_{13}^{\text{L},\,d\dagger}\left(\frac{m_{\text{s}}^{2}}{m_{\text{b}}^{2}}, \delta_{13}^{(1)}\right) \boldsymbol{S}_{13}^{\text{L},\,d\dagger}\left(\frac{m_{\text{d}} m_{\text{s}}}{m_{\text{b}}^{2}}, \delta_{13}^{(2)}\right) \\
& \times \boldsymbol{S}_{12}^{\text{L},\,d\dagger}\left(\frac{m_{\text{d}}}{m_{\text{s}}}, \delta_{12}\right),
\end{aligned}
$$