Vacuum instabilities around the corner

destabilizing the vacuum at the SUSY scale

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www.haaretz.com

"I think, we have it!"
Rolf Heuer

www.thedailystar.net

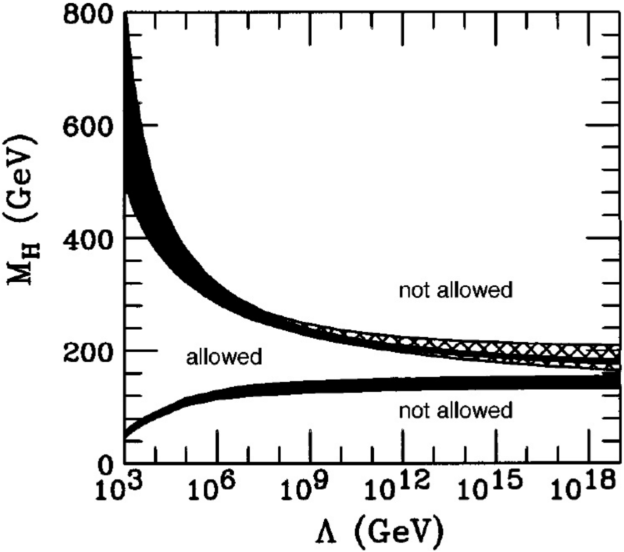
“Eureka!”

Archimedes

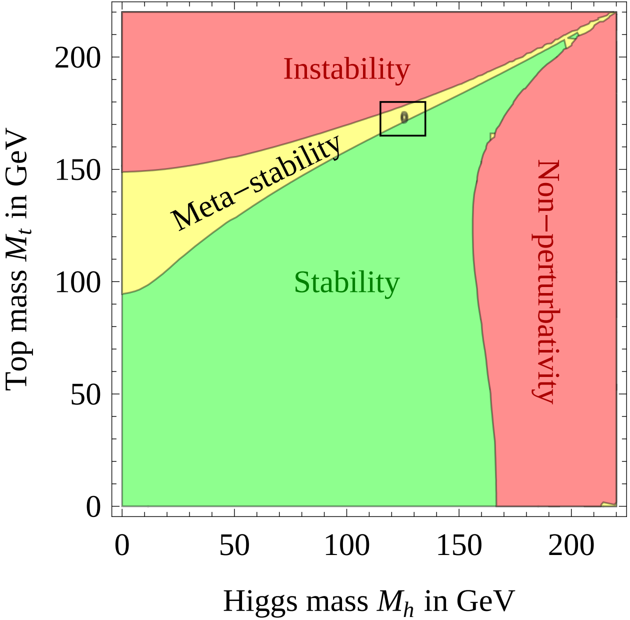
What do we have?

[particlezoo]

Having pinned down the missing piece...



[Hambye, Riesselmann: Phys.Rev. D55 (1997) 7255]



[Degrassi et al. JHEP 1208 (2012) 098]

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The SM Higgs sector

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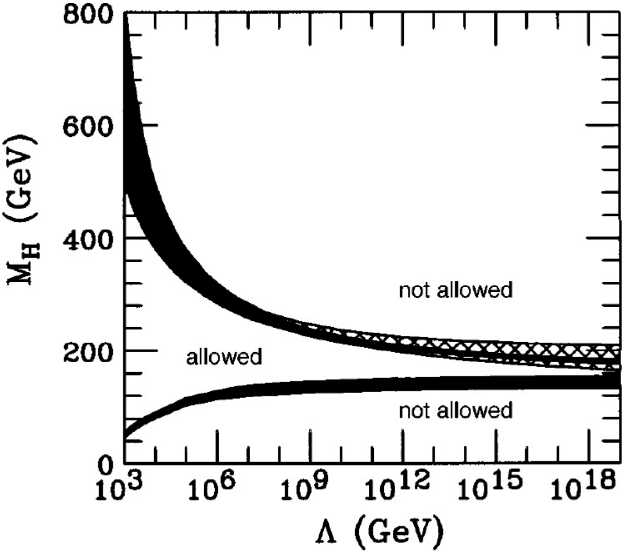

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Theoretical considerations

- $\lambda < 4\pi \quad \hookrightarrow$ perturbativity
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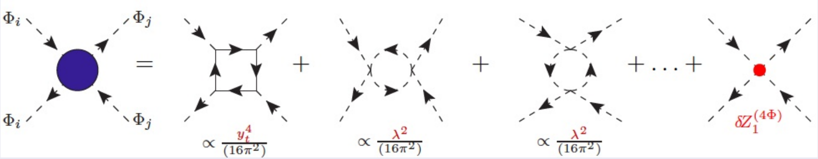
trivial at the classical (i.e. tree) level

$$V(\Phi) = m^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2$$

Quantum level



[courtesy of Max Zoller]

- dominant contribution: top quark ($y_t \sim 1$)
- dependence on the energy scale: β function!

Scale-independent loop-corrected effective potential

$$Q\frac{\mathrm{d}}{\mathrm{d}\,Q}V_{\mathsf{loop}}(\lambda_i,\phi,Q)=0$$

Approximation for large field values

$$V_{\mathsf{loop}}(\phi)=\lambda(\phi)\phi^4,$$

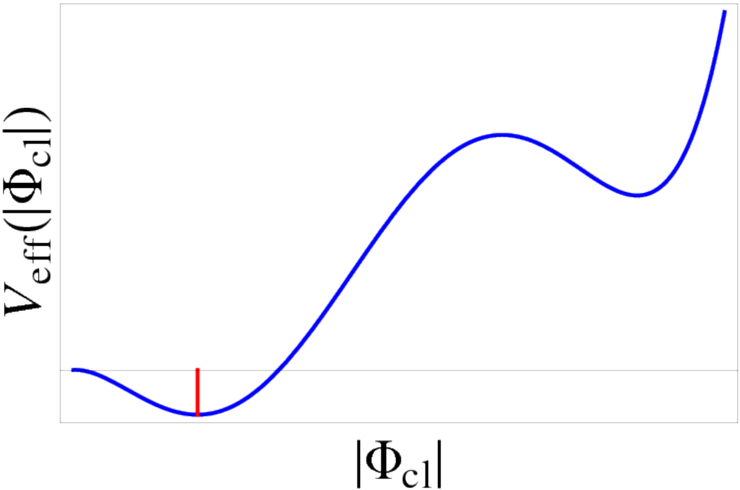
evaluated at $Q=\phi$

β function for coupling λ_i

$$\beta_i(\lambda_i)=Q\frac{\mathrm{d}\,\lambda_i(Q)}{\mathrm{d}\,Q}$$

- running of λ determines stability of the loop potential
- upper bound: Landau pole; lower bound: $\lambda>0$

Stability, instability or metastability?



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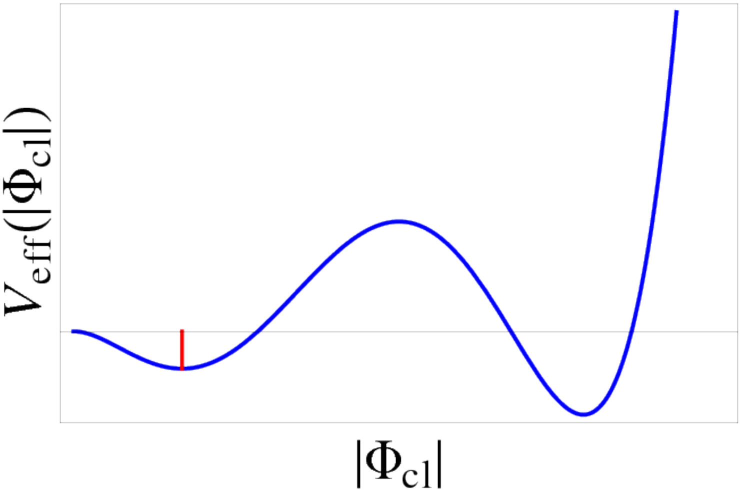

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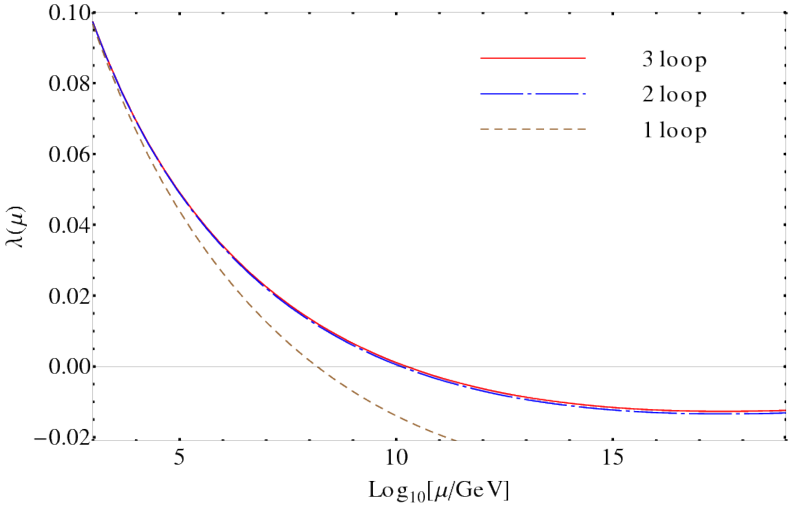
Precise analysis: up to three loops!



[Zoller 2014]

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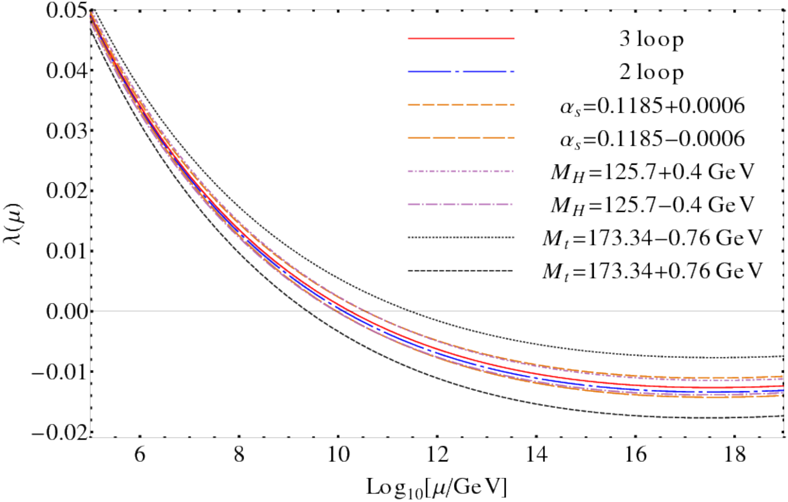

[Zoller 2014]

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Why Supersymmetry?

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- metastability $\rightarrow$ absolute stability

Task: Do not introduce further instabilities!

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(generically difficult)

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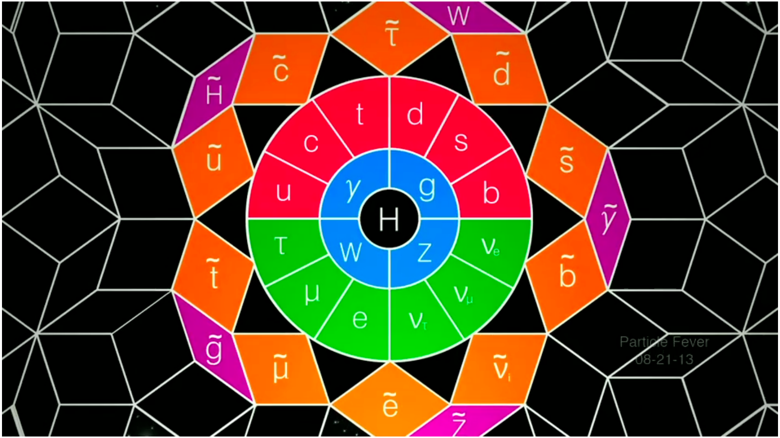

[Particle Fever]

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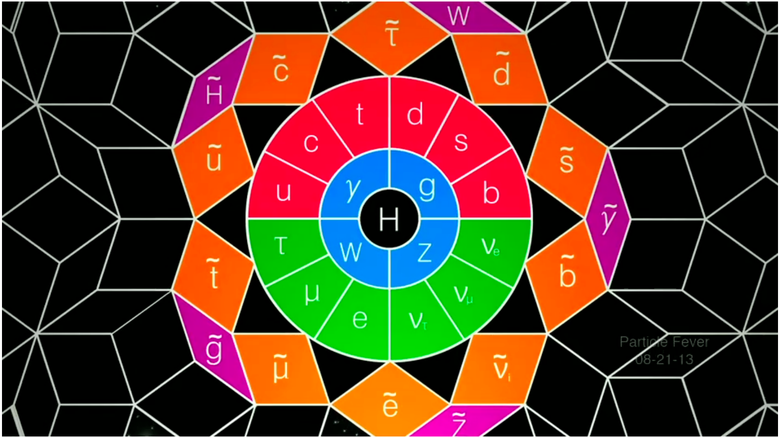

[Particle Fever]

$$\mathcal{W}_{\text{MSSM}} = -Y^e_{ij} H_d \cdot L_{L,i} \bar{E}_{R,j} + Y^u_{ij} H_u \cdot Q_{L,i} \bar{U}_{R,j} - Y^d_{ij} H_d \cdot Q_{L,i} \bar{D}_{R,j}$$

$$
\begin{aligned}
-\mathcal{L}^{\nu\mathsf{MSSM}}_{\cancel{\mathsf{SUSY}}} = {} & \tilde{q}^*_{\mathsf{L},i}\left(\tilde{\boldsymbol{m}}^2_Q\right)_{ij}\tilde{q}_{\mathsf{L},j} + \tilde{u}^*_{\mathsf{R},i}\left(\tilde{\boldsymbol{m}}^2_u\right)_{ij}\tilde{u}_{\mathsf{R},j} + \tilde{d}^*_{\mathsf{R},i}\left(\tilde{\boldsymbol{m}}^2_d\right)_{ij}\tilde{d}_{\mathsf{R},j} \\
& + \tilde{\ell}^*_{\mathsf{L},i}\left(\tilde{\boldsymbol{m}}^2_\ell\right)_{ij}\tilde{\ell}_{\mathsf{L},j} + \tilde{e}^*_{\mathsf{R},i}\left(\tilde{\boldsymbol{m}}^2_e\right)_{ij}\tilde{e}_{\mathsf{R},j} \\
& + \Big[h_\mathsf{d}\cdot\tilde{q}_{\mathsf{L},i}A^\mathsf{d}_{ij}\tilde{d}^*_{\mathsf{R},j} + \tilde{q}_{\mathsf{L},i}\cdot h_\mathsf{u}A^\mathsf{u}_{ij}\tilde{u}^*_{\mathsf{R},j} + \\
& \quad h_\mathsf{d}\cdot\tilde{\ell}_{\mathsf{L},i}A^\mathsf{e}_{ij}\tilde{e}^*_{\mathsf{R},j} + \text{h.c.} \Big] \\
& + m^2_{h_\mathsf{d}}|h_\mathsf{d}|^2 + m^2_{h_\mathsf{u}}|h_\mathsf{u}|^2 + \left(B_\mu\, h_\mathsf{d}\cdot h_\mathsf{u} + \text{h.c.}\right)
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$$

Higgs potential of 2HDM type II

$$V = m_{11}^2\; H_d^\dagger H_d + m_{22}^2\; H_u^\dagger H_u + \left(m_{12}^2\; H_u \cdot H_d + \text{h.c.}\right)$$
$$+ \frac{\lambda_1}{2}\left(H_d^\dagger H_d\right)^2 + \frac{\lambda_2}{2}\left(H_u^\dagger H_u\right)^2$$
$$+ \lambda_3\left(H_u^\dagger H_u\right)\left(H_d^\dagger H_d\right) + \lambda_4\left(H_u^\dagger H_d\right)\left(H_d^\dagger H_u\right) + \{\lambda_5, \lambda_6, \lambda_7\}$$

In the MSSM: tree potential calculated from D-terms and $\mathcal{L}_{\text{soft}}$

$$m_{11}^{2\,\text{tree}} = |\mu|^2 + m_{H_d}^2, \qquad \lambda_1^{\text{tree}} = \lambda_2^{\text{tree}} = -\lambda_3^{\text{tree}} = \frac{g^2 + g'^2}{4},$$
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Unbounded from below requirements

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}$$

and others. . . [Gunion, Haber 2003]

- always fulfilled in the MSSM @ tree

Extending the tree

- loop corrections? [Gorbahn, Jäger, Nierste, Trine 2011]

- integrating out heavy SUSY particles
- requirement of large SUSY scale $M_{\mathrm{SUSY}} \gg M_A \sim v_{\mathrm{ew}}$
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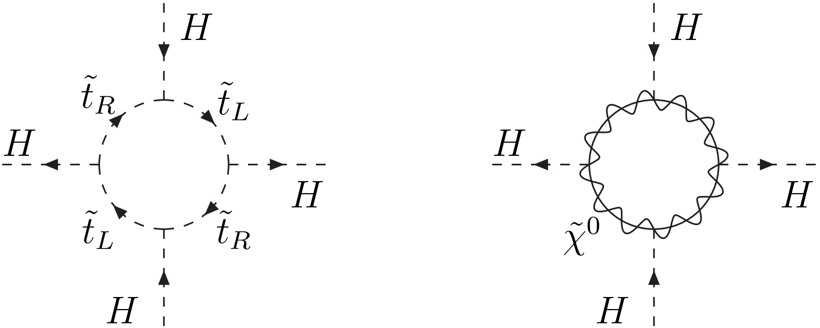


collecting all SUSY contributions:

$$\lambda_i = \lambda_i(\tan\beta, \mu, M_1, M_2, \tilde{m}_Q^2, \tilde{m}_u^2, \tilde{m}_d^2, \tilde{m}_L^2, \tilde{m}_e^2, A_{\mathsf{u}}, A_{\mathsf{d}}, A_{\mathsf{e}}).$$

simple check:

$$\lambda_1 > 0\,, \qquad \lambda_2 > 0\,, \qquad \lambda_3 > -\sqrt{\lambda_1 \lambda_2},$$

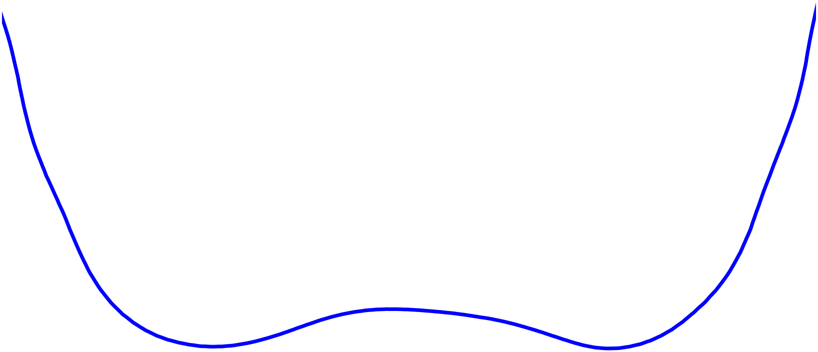
where now

$$\lambda_i = \lambda_i^{\text{tree}} + \frac{\lambda_i^{\text{ino}} + \lambda_i^{\text{sferm}}}{16\pi^2}\,.$$

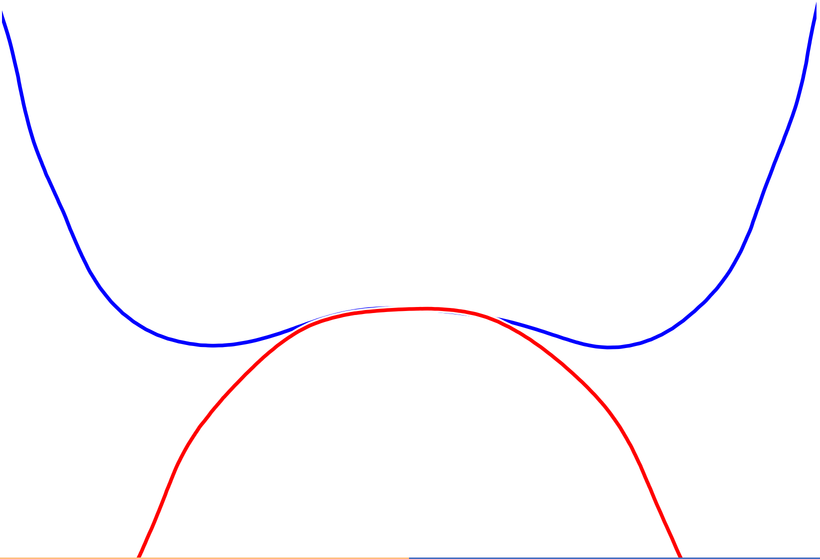
Severe UFB limits

Bounds on $\lambda_{1,2,3}$ transfer into bounds on m_0, A_{t}, μ, ...

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4$$

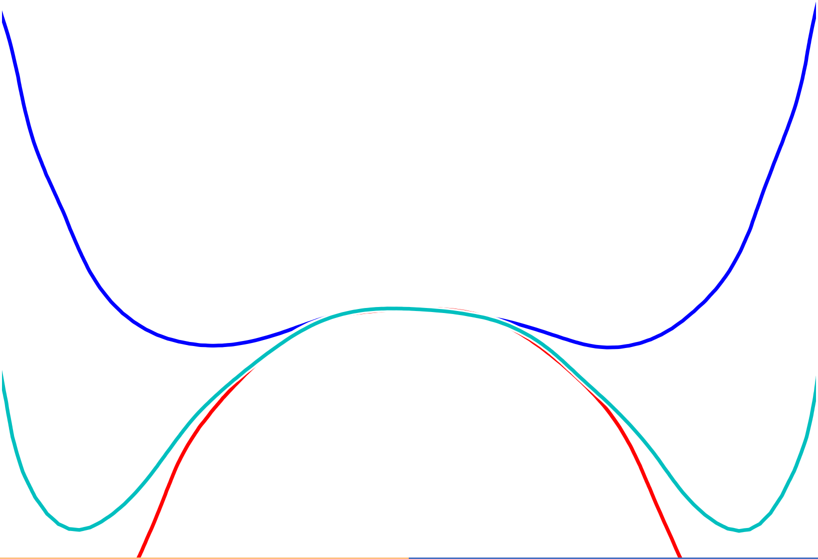


$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4$$

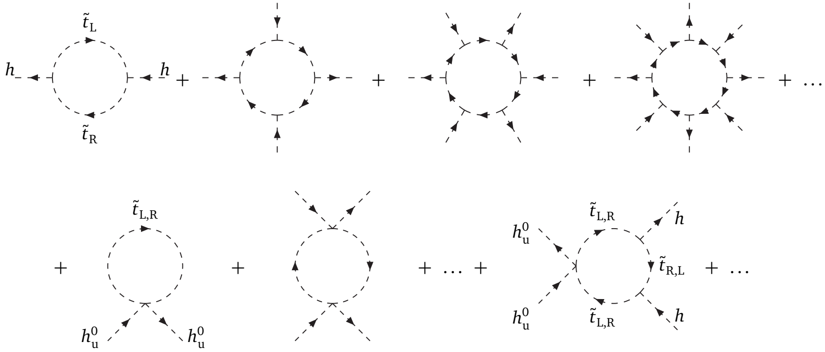


Recovery from unbounded from below???

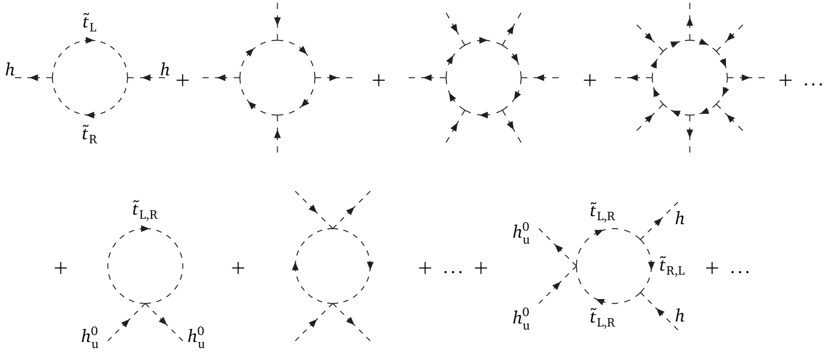
$$V(\phi) = -\mu^2\phi^2 - \lambda\phi^4 + \lambda^{(6)}\phi^6$$



Calculating the 1-loop effeective potential



- dominant contribution from third generation squarks
- quadrilinear couplings ($\sim |Y_{\mathsf{t}}|^2$)
- trilinear coupling to a linear combination $(\mu^* Y_{\mathsf{t}} h_{\mathsf{d}}^{\dagger} - A_{\mathsf{t}} h_{\mathsf{u}}^{0})$
- series summable to an infinite number of external legs



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- **Do not stop after renormalizable / dim 4 terms!**

- 1-loop effective potential [Coleman, Weinberg 1973]

$$V_1(h_u, h_d) = \frac{1}{64\pi^2} \,\mathrm{STr}\, \mathcal{M}^4(h_u, h_d) \left[\ln\left(\frac{\mathcal{M}^2(h_u, h_d)}{Q^2} \right) - \frac{3}{2} \right]$$

  - *field dependent mass* $\mathcal{M}(h_u, h_d)$
  - STr accounts for spin degrees of freedom
- same result can be obtained by the tadpole method

$$T \sim \frac{\partial}{\partial h} V_1(h) \quad \hookrightarrow \quad V_1(h) \sim \int \mathrm{d}h \; T(h)$$

[Lee, Sciaccaluga 1975]

- functional methods: effective potential for arbitrary number of scalars: $V_1(\phi_1, \phi_2, \ldots \phi_n)$ [Jackiw 1973]

The effective potential

$V_{\text{eff}}(\phi)$: average energy density

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The ground state of the theory

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Technically:

generating function for 1PI n-point Green's functions:

$$V_{\text{eff}}(\phi) = -\sum_{n=2}^{\infty} \tilde{G}^{(n)}(p_i = 0)\phi^n$$

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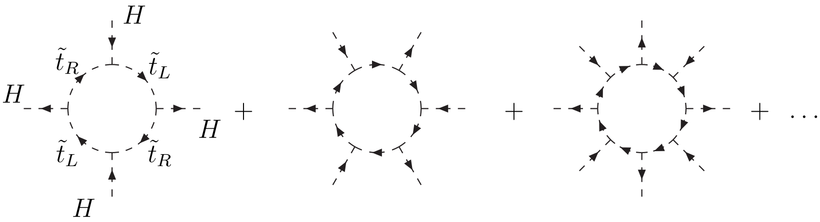
(with subleties)

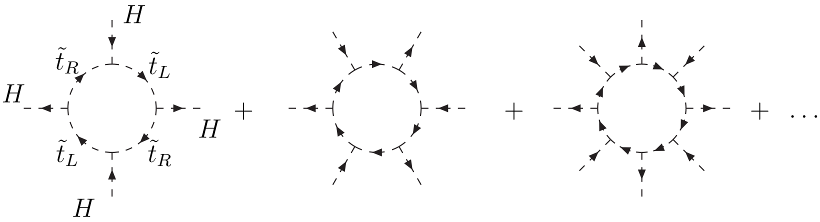


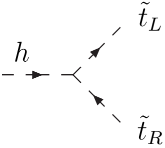
- most dominant contribution from top Yukawa y_t and A_t
- can be easily summed for $m_{\tilde{t}_R} = m_{\tilde{t}_L} \equiv M$
- 1-PI potential as generating function for 1-PI Green's functions

$$-V_{1-\mathrm{PI}}(\phi) = \Gamma_{1-\mathrm{PI}}(\phi) = \sum_n \frac{1}{n!} G_n(p_{\mathrm{ext}} = 0)\phi^n$$

- "classical" field value $\phi \to \langle 0|\phi|0\rangle$
- $\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0$ determines ground state of the theory

Summing up external legs





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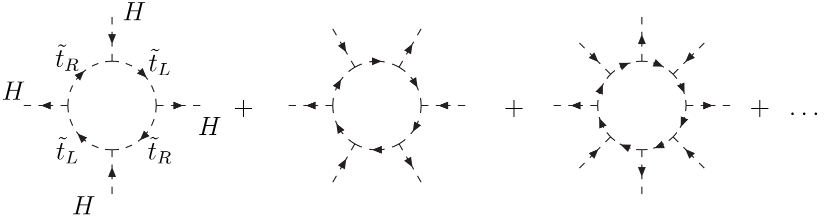

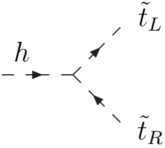

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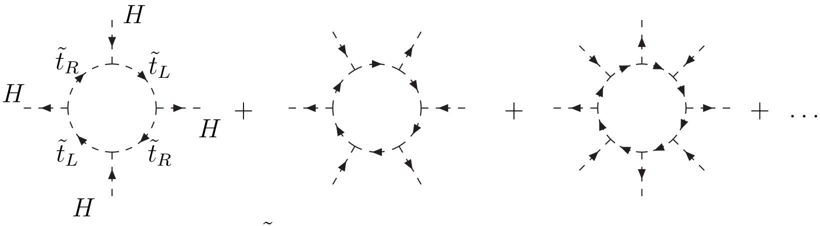

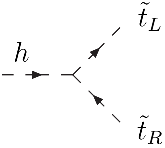

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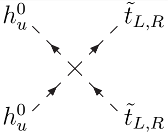
$$\boxed{V_1 = \frac{N_c M^4}{32\pi^2} \left[(1+x)^2 \log(1+x) + (1-x)^2 \log(1-x) - 3x^2\right]}$$

$Q^2 = M^2$ $\qquad$ $x^2 = |\mu Y_t|^2 h^\dagger h / M^4$, $m^2_{\tilde{t}_L} = m^2_{\tilde{t}_R} = M^2$

Field dependent stop mass

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde{t}_R} + |Y_t h_u^0|^2 \end{pmatrix}$$

- trilinear $\sim h(h_d^0, h_u^0)$, quadrilinear $\sim |h_u^0|^2$
- diagrams with mixed contributions



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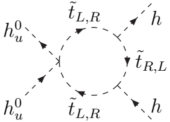
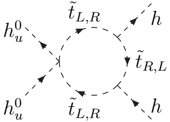
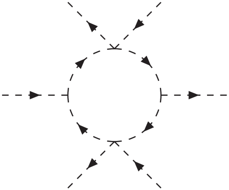


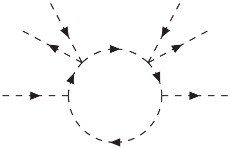
Field dependent stop mass

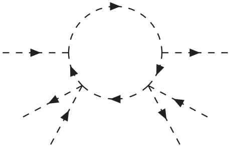
$$\mathcal{M}^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^{0*}_d \\ A^*_t h^{0*}_u - \mu Y^*_t h^0_d & m^2_{\tilde{t}_R} + |Y_t h^0_u|^2 \end{pmatrix}$$

- trilinear $\sim h(h^0_d, h^0_u)$, quadrilinear $\sim |h^0_u|^2$
- diagrams with mixed contributions









Summing up (ctd.)

Field dependent stop mass

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} m^2_{\tilde{t}_L} + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & m^2_{\tilde{t}_R} + |Y_t h_u^0|^2 \end{pmatrix}$$

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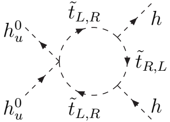
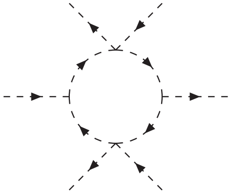
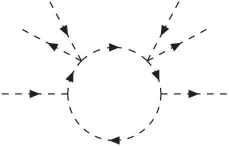
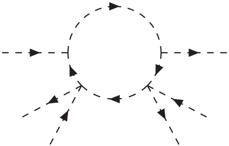


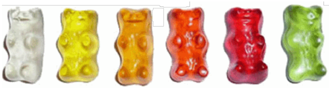






- gummi bear factor

$$\frac{(2n+k-1)!}{k!(2n-1)!}$$



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Field dependent stop mass

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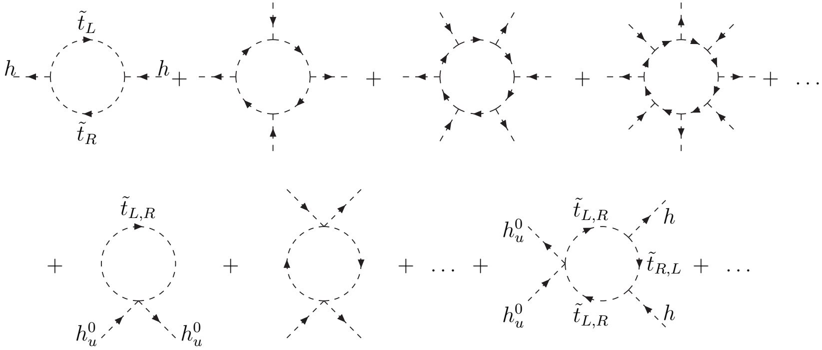

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$$\begin{aligned} V_1 &\sim \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_{kn}\, x^{2n} y^k\,, \qquad x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4}, y = \frac{|Y_t h_u^0|^2}{M^2} \\ &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{1}{n(2n+k-1)(2n+k-2)} \frac{(2n+k-1)!}{k!(2n-1)!}\, x^{2n} y^k \\ &= \Big[\, (1+y+x)^2 \log(1+y+x) \\ &\quad + (1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y)\Big] \end{aligned}$$

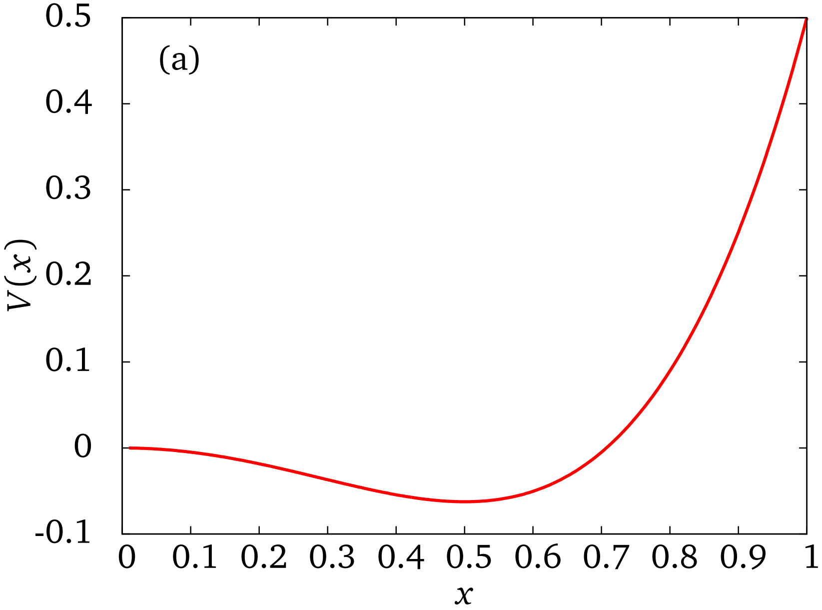
Features of the resummed series

$$V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2}\Big[(1+y+x)^2\log(1+y+x) + (1+y-x)^2\log(1+y-x) - 3(x^2+y^2+2y)\Big]$$

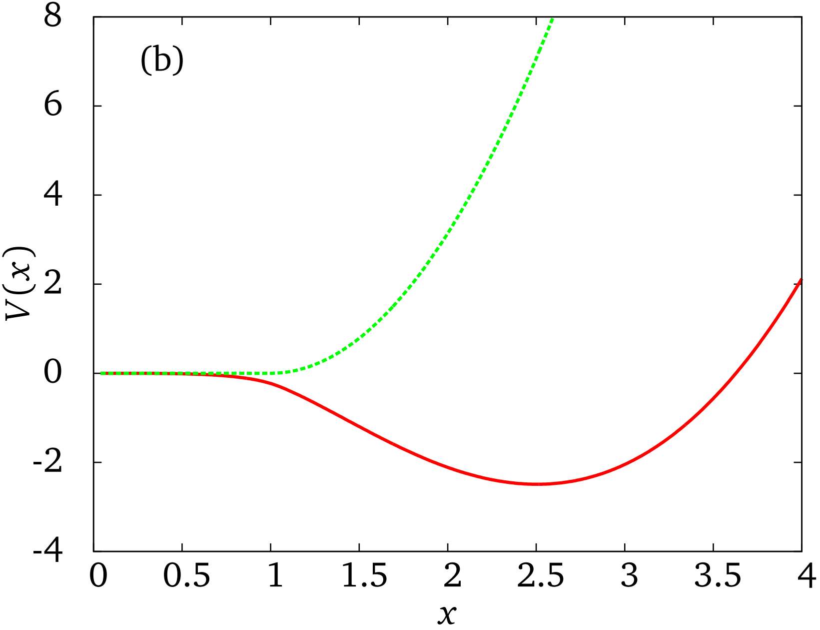
$$x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4},\ h = h_d^{0*} - \frac{A_t}{\mu^* Y_t} h_u^0, \qquad y = \frac{|Y_t h_u^0|^2}{M^2}$$

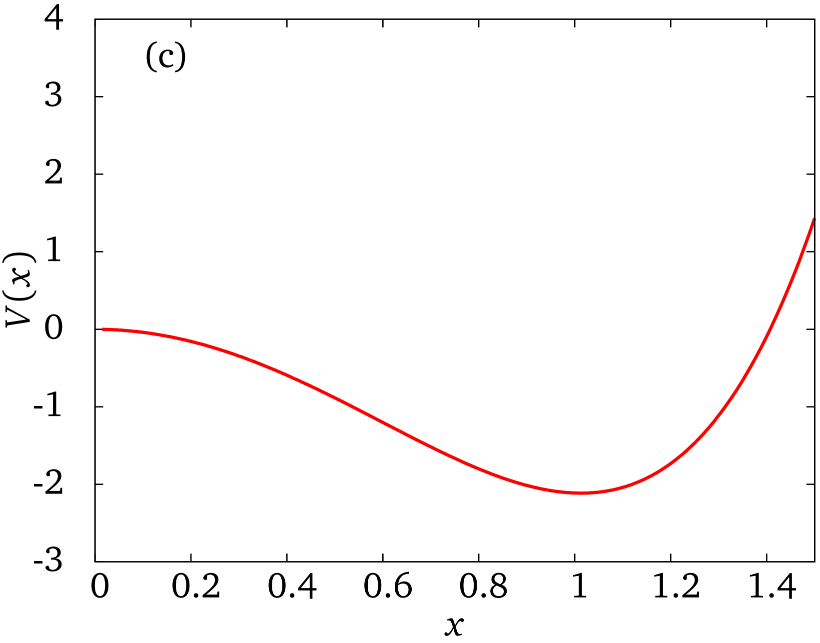
- branch cut at $x - y = 1$: take real part (analytic continuation)
- ignore imaginary part: $\log(1+y-x) = \frac{1}{2}\log\left((1+y-x)^2\right)$
- always bounded from below
- minimum independent of Higgs parameters from tree potential
- minimum determined by SUSY scale parameters

Tree, loop and tree + loop

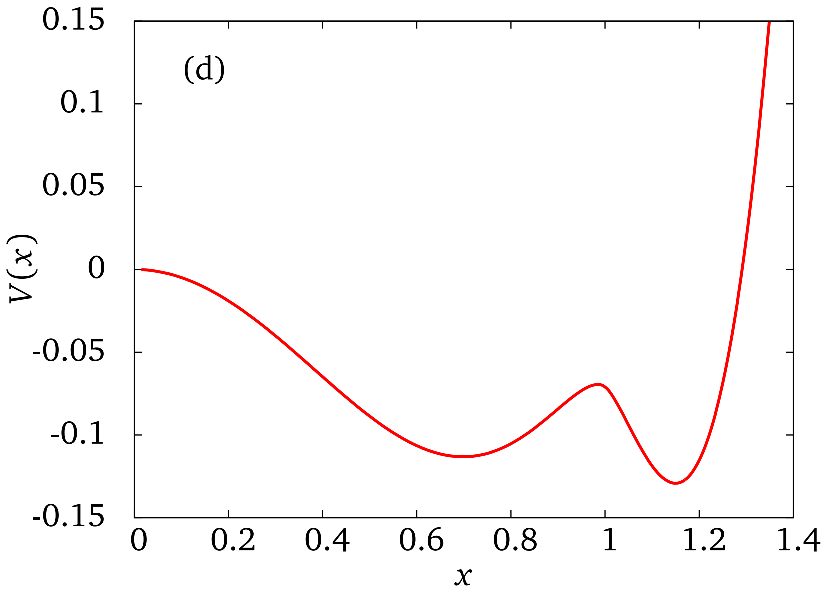


Tree, loop and tree + loop




(c)
4
3
2
1
0
-1
-2
-3
$V(x)$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x

Tree, loop and tree + loop



Minimum at the electroweak scale $v = 246\,\mathrm{GeV}$

$$m_{11}^{2\,\mathrm{tree}} = m_{12}^{2\,\mathrm{tree}} \tan\beta - \frac{v^2}{2}\cos(2\beta)\lambda_1^{\mathrm{tree}} - \frac{1}{v\cos\beta}\,\frac{\delta}{\delta\phi_d}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}},$$

$$m_{22}^{2\,\mathrm{tree}} = m_{12}^{2\,\mathrm{tree}} \cot\beta + \frac{v^2}{2}\cos(2\beta)\lambda_1^{\mathrm{tree}} - \frac{1}{v\sin\beta}\,\frac{\delta}{\delta\phi_u}V_1\bigg|_{\substack{\phi_{u,d}\to 0\\ \chi_{u,d}\to 0}}.$$

$m_h = 126\,\mathrm{GeV}$

- using `FeynHiggs 2.10.0` to determine light Higgs mass by adjusting A_t (several solutions: $\mathrm{sign}\,A_t = -\,\mathrm{sign}\,\mu$)
- connection to potential: m_A
- pseudoscalar mass m_A less dependent on higher loops
- decoupling limit: $m_A, m_{H^\pm}, m_H \gg m_h$

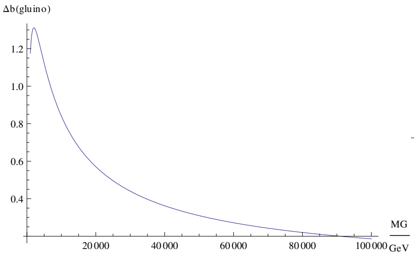
- include sbottom (drives minimum), take $A_b = 0$

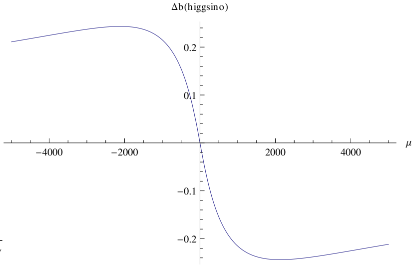
Yukawa coupling not given directly by the mass

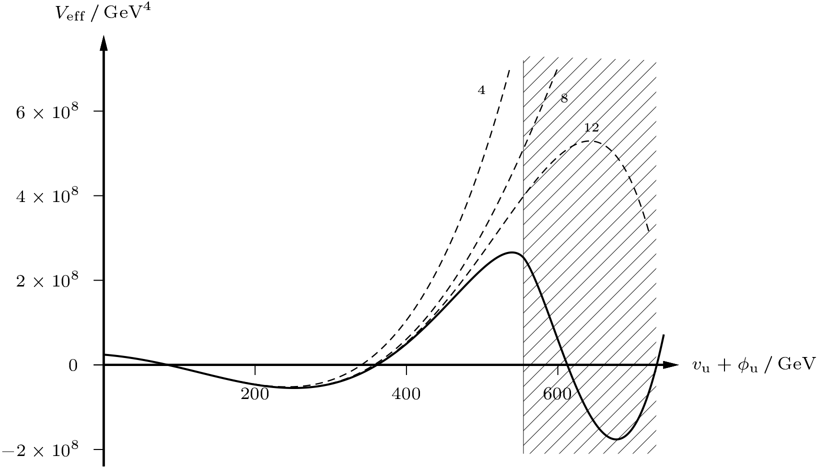
$$y_b = \frac{m_b}{v_d(1+\Delta_b)}$$

$$\Delta_b^{\text{gluino}} = \frac{2\alpha_s}{3\pi}\mu M_{\tilde{G}} \tan\beta C_0(\tilde{m}_{\tilde{b}_1}, \tilde{m}_{\tilde{b}_2}, M_{\tilde{G}}),$$

$$\Delta_b^{\text{higgsino}} = \frac{Y_t^2}{16\pi^2}\mu A_t \tan\beta C_0(\tilde{m}_{\tilde{t}_1}, \tilde{m}_{\tilde{t}_2}, \mu).$$

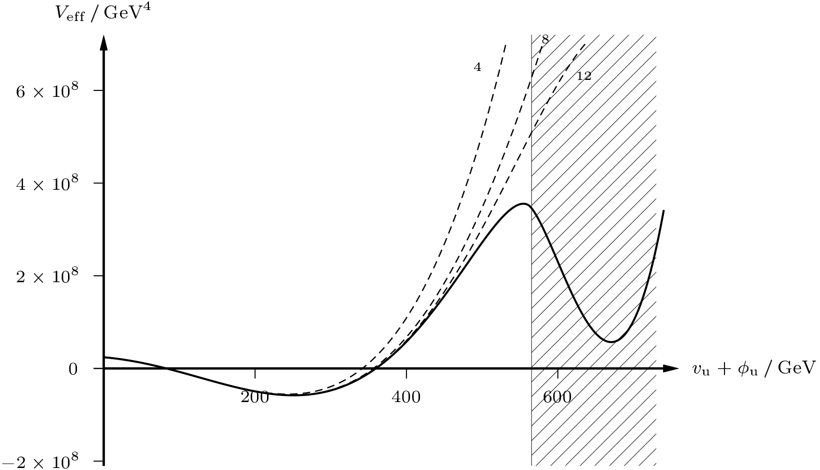






- $\tan\beta = 40$
- $m_A = 800\,\text{GeV}$
- $M = 1\,\text{TeV}$
- $\mu = 3.83\,\text{TeV}$
- $A_t \simeq -1.5\,\text{TeV}$

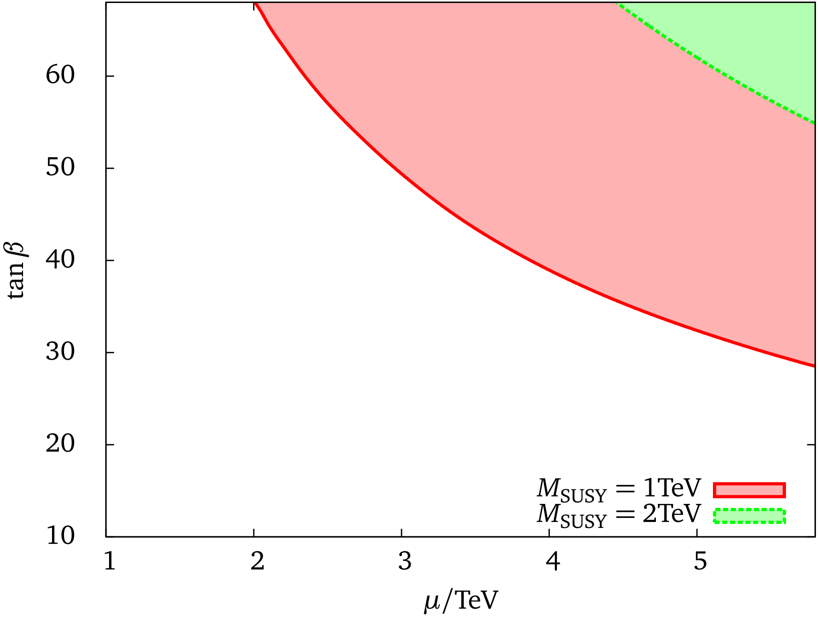
Discussion of stability



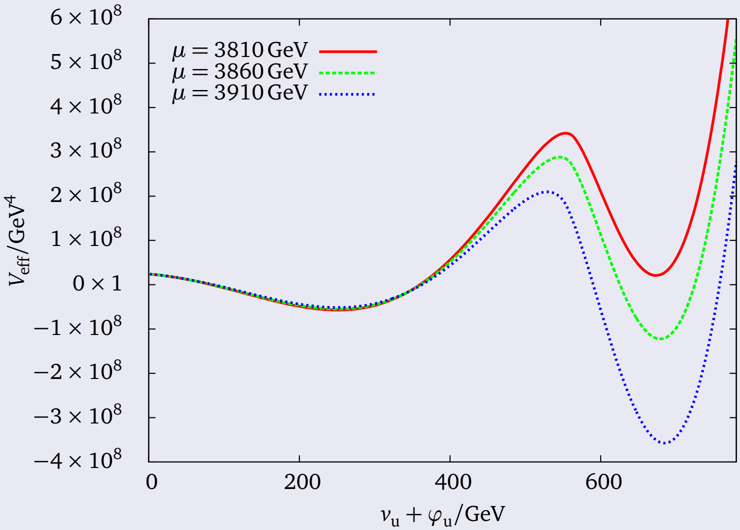
- $\tan\beta = 40$
- $m_A = 800\,\text{GeV}$
- $M = 1\,\text{TeV}$
- $\mu = 3.75\,\text{TeV}$
- $A_t \simeq -1.5\,\text{TeV}$

[www.bbcamerica.com]

Naive exclusions: Constraint in μ-$\tan\beta$



Access to Charge and Color breaking minima



Access to Charge and Color breaking minima

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_b h_d^0|^2 & A_b h_d^0 - \mu^* Y_b h_u^{0*} \\ A_b^* h_d^{0*} - \mu Y_b^* h_u^0 & \tilde{m}_b^2 + |Y_b h_d^0|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev

Access to Charge and Color breaking minima

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{pmatrix}$$

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- non-trivial behaviour of sfermions masses with Higgs vev:

$$m^2_{\tilde{b}_{1,2}}(h_u^0, h_d^0) = \frac{\tilde{m}_Q^2 + \tilde{m}_b^2}{2} + |Y_b h_d^0|^2 \pm \frac{1}{2}\sqrt{(\tilde{m}_Q^2 - \tilde{m}_b^2)^2 + 4|A_b h_d^0 - \mu^* Y_b h_u^{0*}|^2}$$

- expand theory around new minimum: $m^2_{\tilde{b}_2} < 0$

Access to Charge and Color breaking minima

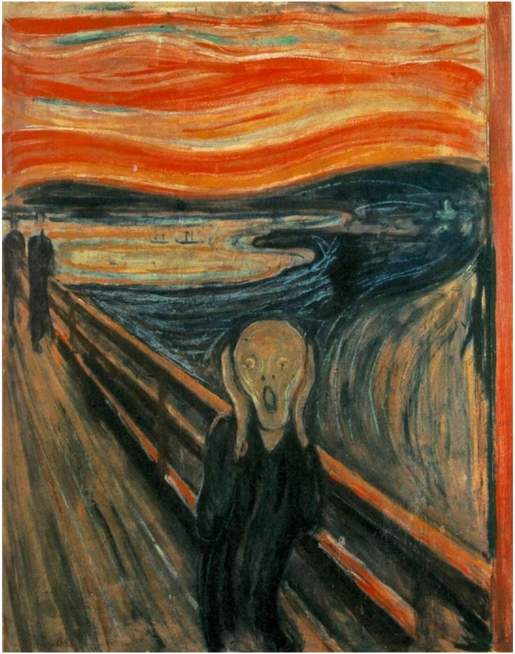
$$\mathcal{M}^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}^2_Q + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^{0*}_d \\ A^*_t h^{0*}_u - \mu Y^*_t h^0_d & \tilde{m}^2_t + |Y_t h^0_u|^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}}(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}^2_Q + |Y_b h^0_d|^2 & A_b h^0_d - \mu^* Y_b h^{0*}_u \\ A^*_b h^{0*}_d - \mu Y^*_b h^0_u & \tilde{m}^2_b + |Y_b h^0_d|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev:

$$m^2_{\tilde{b}_{1,2}}(h^0_u, h^0_d) = \frac{\tilde{m}^2_Q + \tilde{m}^2_b}{2} + |Y_b h^0_d|^2 \pm \frac{1}{2}\sqrt{(\tilde{m}^2_Q - \tilde{m}^2_b)^2 + 4|A_b h^0_d - \mu^* Y_b h^{0*}_u|^2}$$

- expand theory around new minimum: $m^2_{\tilde{b}_2} < 0$
- **tachyonic squark mass!**



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What does a tachyonic mass mean?

- mass $\Leftrightarrow$ second derivative: $m_\phi^2 = \partial^2 V / \partial \phi^2$
- $m_\phi^2 < 0 \quad \Leftrightarrow \quad$ negative curvature
- non-convex potential: imaginary part
- $\log(1 + y - x) \sim \log(m_\phi^2)$

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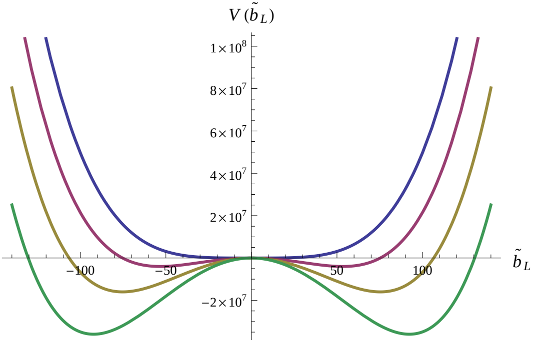

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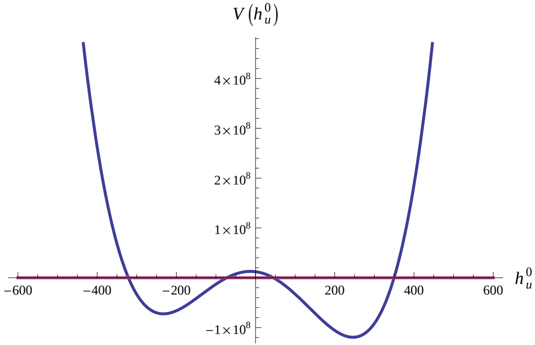


Including colored directions

$$V_{\tilde{b}}^{\text{tree}} = \tilde{b}_{\mathrm{L}}^{*}(M_{\tilde{Q}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{L}} + \tilde{b}_{\mathrm{R}}^{*}(M_{\tilde{\mathrm{b}}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{R}}$$

$$- \left[\tilde{b}_{\mathrm{L}}^{*}(\mu^{*} Y_{\mathrm{b}} h_{\mathrm{u}}^{0\dagger} - A_{\mathrm{b}} v_{\mathrm{d}})\tilde{b}_{\mathrm{R}} + \text{h. c.}\right] + |Y_{\mathrm{b}}|^{2}|\tilde{b}_{\mathrm{L}}|^{2}|\tilde{b}_{\mathrm{R}}|^{2}$$

$$+ \ D\text{-terms.}$$

Including colored directions

$$V_{\tilde{b}}^{\text{tree}} = \tilde{b}_{\mathrm{L}}^{*}(M_{\tilde{Q}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{L}} + \tilde{b}_{\mathrm{R}}^{*}(M_{\tilde{\mathrm{b}}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{R}} - \left[\tilde{b}_{\mathrm{L}}^{*}(\mu^{*} Y_{\mathrm{b}} h_{\mathrm{u}}^{0\dagger} - A_{\mathrm{b}} v_{\mathrm{d}})\tilde{b}_{\mathrm{R}} + \text{h. c.}\right] + |Y_{\mathrm{b}}|^{2}|\tilde{b}_{\mathrm{L}}|^{2}|\tilde{b}_{\mathrm{R}}|^{2} + D\text{-terms.}$$

D-flat direction

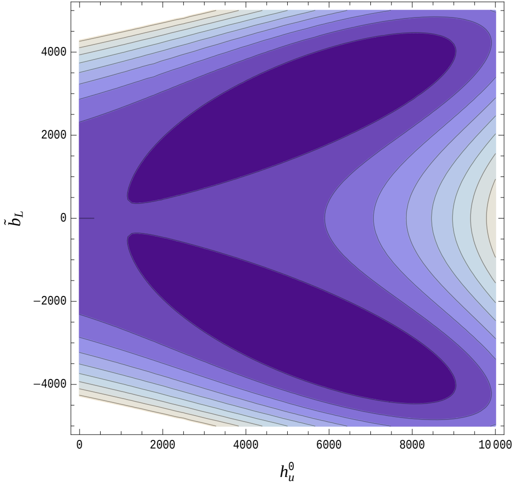
- D-terms: $g^2\phi^4$

$$V_D = \frac{g_1^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2\right)^2 + \frac{g_2^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 - |\tilde{b}_L|^2\right)^2 + \frac{g_3^2}{6}\left(|\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2.$$

- will always take over
- take e.g. $\tilde{b}_L = \tilde{b}_R = h_{\mathrm{u}}^0$ and $h_{\mathrm{d}}^0 \approx 0$

Preliminary results

- previously "safe" false but CCB conserving minima turn into to deep global minima with $\langle \tilde{b}_L \rangle = \langle \tilde{b}_R \rangle \neq 0$ and $\langle h^0_\mathrm{u} \rangle \neq v_\mathrm{u}$



- the Higgs potential in the SM is (un/meta)stable
- MSSM: multi-scalar theory, has several unwanted minima
- formation of new CCB conserving minima at the 1-loop level
- stability of the electroweak vacuum: bounds on μ-$\tan\beta$
- instability of electroweak vacuum by second minimum in "standard model direction" $\sim v_u$: global CCB minimum
- global analysis shows more severe bounds (preliminary)
- squark contribution to the effective Higgs potential:
  - only third generation squarks light
  - A_t fixed by m_h, $A_b \equiv 0$ (for simplicity)
  - $\mathrm{sign}(A_t) = -\,\mathrm{sign}\,\mu$ for CCB conserving minimum
  - free parameters: $M_{\mathrm{SUSY}} = m_{\tilde{t}} = m_{\tilde{b}}$, $\tan\beta$, μ
  - no new insights if $m_{\tilde{t}_L,\tilde{b}_L} \neq m_{\tilde{t}_R,\tilde{b}_R}$
  - gluino and electroweak gauginos heavy
- FeynHiggs: determines right light Higgs mass
- new CCB bounds exist for all μ-A_t sign combinations

Greetings from Señor Higgs
(*courtesy of Jens Hoff*)

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Slides