

# Finding hints of New Physics in Tritium molecular spectra?

Wolfgang Gregor Hollik

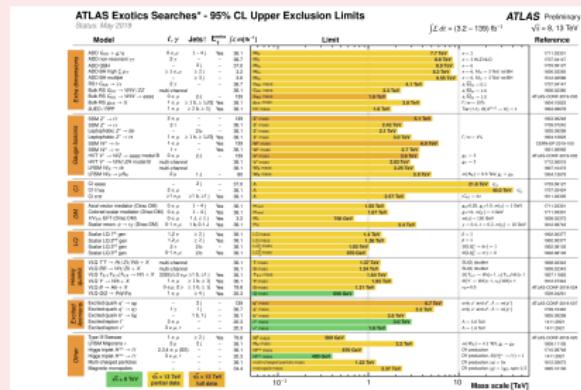


IKP & TTP @ KIT  
KIT-Centrum für Elementarteilchen- und Astroteilchenphysik (KCETA)

26 September 2019 | DESY Theory Workshop 2019 | Hamburg

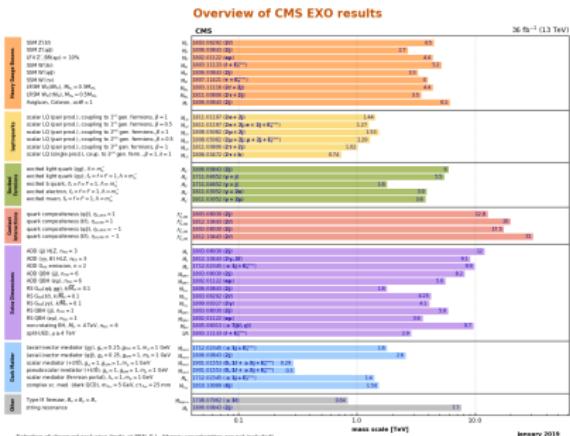
## New ways of New Physics

## No New Physics at high energies?



\*Only a selection of the available status levels on some states or pharmaceuticals is shown.

(Small-radius/large-radius) jets are denoted by the letter J (S).



Selection of observed exclusion limits at 95% C.L. theory uncertainties are not included

## High Energy vs. Precision Frontier

- New machines, higher collider energies, more luminosity
  - Lower energies, more precision, cheaper experiments

## Room for New Physics in T<sub>2</sub>?

Fundamental vibrational splittings:  $Q(J)$  in T<sub>2</sub> [cm<sup>-1</sup>]

Line	experiment	theory (nonrel)	diff.
Q(0)	2 464.5052 (4)	2 464.5021	0.0031 (4)
Q(1)	2 463.3494 (3)	2 463.3463	0.0031 (3)
Q(2)	2 461.0388 (4)	2 461.0372	0.0016 (4)
Q(3)	2 457.5803 (4)	2 457.5795	0.0008 (4)
Q(4)	2 452.9817 (4)	2 452.9803	0.0014 (4)
Q(5)	2 447.2510 (4)	2 447.2492	0.0017 (4)

exp.: [Trivikram, Schlösser, Ubachs, Salumbides: PRL120, 163002 (2018)]

theo.: [Pachucki, Komasa: JCP143, 034111 (2015)]

### “Interpretation” by the experimentalists

[PRL120,163002]

- missing QED and relativistic corrections for T<sub>2</sub>
- “measured” contribs. larger than known ones for H<sub>2</sub> and D<sub>2</sub>

- absence of New Physics in High Energy Physics experiments
- quest for open problems and issues in the Standard Model
  - A possible solution to the strong CP problem: Axions?
  - A hidden sector: dark photons and kinetic mixing?
  - Small neutrino masses and sterile neutrinos?
- ultralight and ultraweakly coupled particles still weakly constrained
- e. g. modification of coulomb interaction by a light scalar  $\phi$  (*axion*)

$$V_{AB}(\vec{r}) = -g_A g_B \frac{e^{-m_\phi r}}{4\pi r}$$

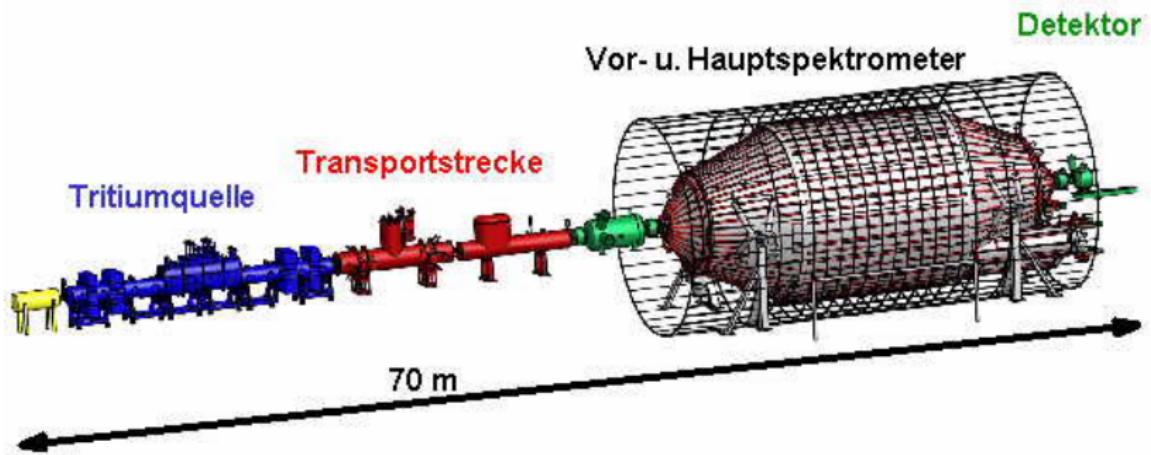
- length scale–mass relation:  $1\text{\AA} \approx 1\text{keV}$ , where molecular bond lengths are  $\mathcal{O}(\text{\AA})$
- sensitivity up to several keV massive particles possible with large couplings
- Neutrino pair exchange: Dirac vs. Majorana?
- Modification of Coulomb force by Dark Photons?

# Why Tritium!?

# Tritium spectroscopy



[stern.de]

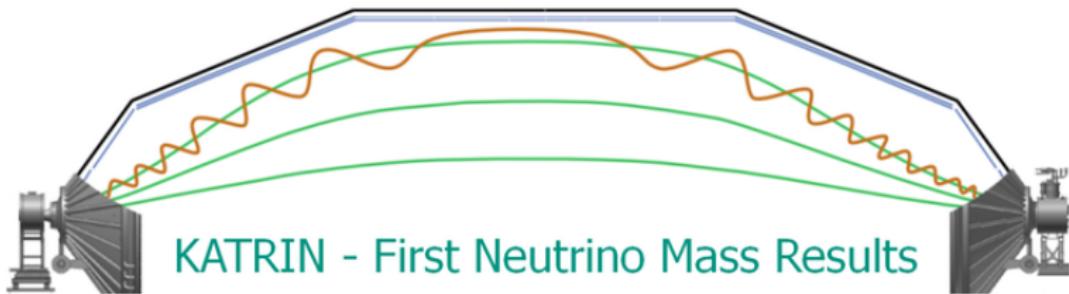


[[www.itep.kit.edu](http://www.itep.kit.edu)]

# Tritium spectroscopy



[KIT KATRIN]



## KATRIN - First Neutrino Mass Results

We are pleased to inform you about the results of our first neutrino mass measurement. The KATRIN experiment and its collaborators provide high quality data of beta decay electrons from molecular tritium. The first neutrino mass measurement campaign, which took place this year in spring, made a significant scientific contribution by setting a new upper limit for the absolute mass scale of neutrinos.

We derive a upper limit of **1.1 eV (90% confidence level)** on the absolute neutrino mass scale from our first 28 data days. In the coming years we will collect more data on the decay of molecular tritium and further reduce this limit.

13th / 16th September 2019

Toyama / Eggenstein-Leopoldshafen (*near Karlsruhe*)

[arXiv:1909.06048]

## Tritium Laboratory Karlsruhe

- world's largest amount of civil tritium
- 25 g of gaseous tritium ( $T_2$ )
- research on nuclear fusion (discontinued) and  $\beta$ -decay

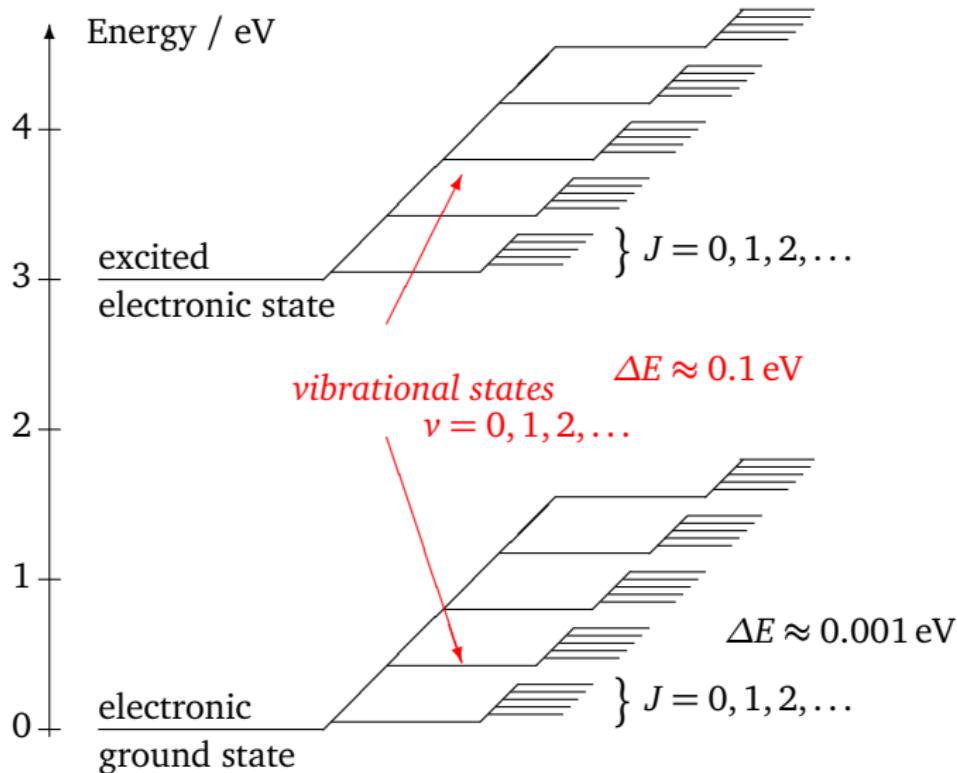
## Hydrogen spectroscopy

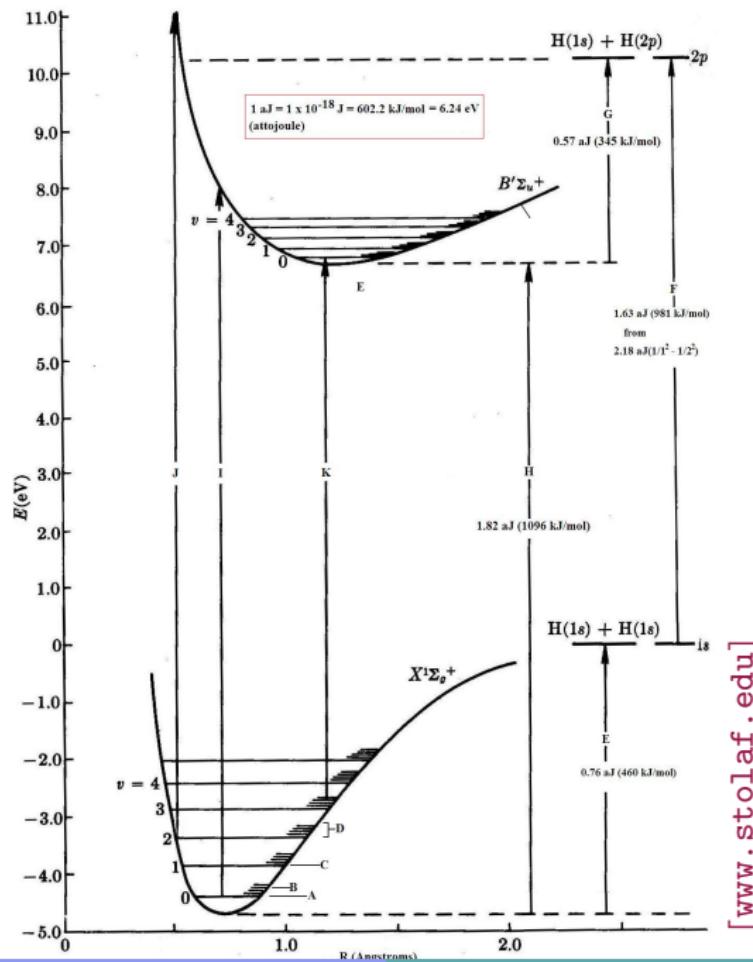
- very precise measurements
- Laser–Raman spectroscopy
- $H_2$  molecule and variants: clean theoretical prediction (?)

## New Physics in molecules?

- test of different length and mass (energy) scales than atoms
- isotopic effects:  $T_2$ ,  $D_2$ ,  $H_2$ , DT, HT, HD

## Schematic molecular energy level diagram (not to scale)





[www.stolaf.edu]

**Atomic Units (*atomic and molecular physics*)**Hartree:  $e = m_e = \hbar = k_e = k_B = 1$  $c = \frac{1}{\alpha}$  (*speed of light*)Coulomb's constant:  $k_e = \frac{1}{4\pi\epsilon_0}$ 

$$\text{length } \ell_A = \frac{\hbar^2 4\pi\epsilon_0}{m_e e^2} \approx 5.292 \cdot 10^{-11} \text{ m} \approx 0.5 \text{ \AA} \quad (\text{Bohr radius})$$

**Natural Units (*particle physics*)**

$$\hbar = c = 1 \text{ e.g. } m_e = 511 \text{ keV, } m_p \sim 1 \text{ GeV}$$

typical bond length  $\text{H}_2 \sim 0.75 \text{ \AA}$ 

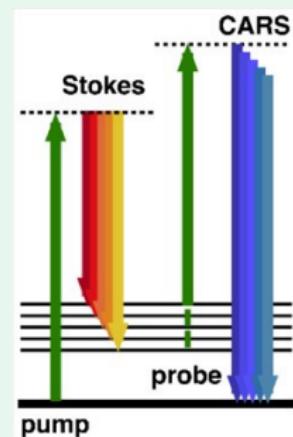
$$1 \text{ \AA} = 10^{-10} \text{ m}$$

**Mass-energy-length relation: keV!**

$$1 \underbrace{\text{ eV}^{-1}}_{10^3 \text{ keV}} = 1.97 \cdot \underbrace{10^{-7} \text{ m}}_{10^3 \text{ \AA}} \equiv \frac{\hbar c}{\text{eV}}$$

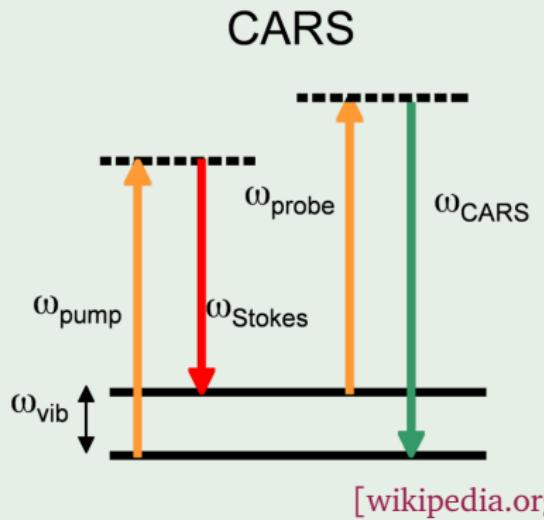
## Coherent Anti-Stokes Raman Scattering

“CARS is a nonlinear variant of normal Raman vibrational spectroscopy that allows label-free chemical imaging at (sub)micron resolution. Owing to the inherently small CARS volume, depth profiling of a sample is possible without the need for a confocal set-up. The high sensitivity of the technique due to coherence effects overcomes the difficulty of low Raman scattering cross sections, making CARS three to eight orders of magnitude more sensitive than normal Raman scattering. Thus, vibrational fingerprints of the sample are acquired extremely fast with typical integration times between 20 and 100 ms per voxel.”



[Max-Planck-Institut für Polymerforschung, Mainz]

## CARS principle and advantage



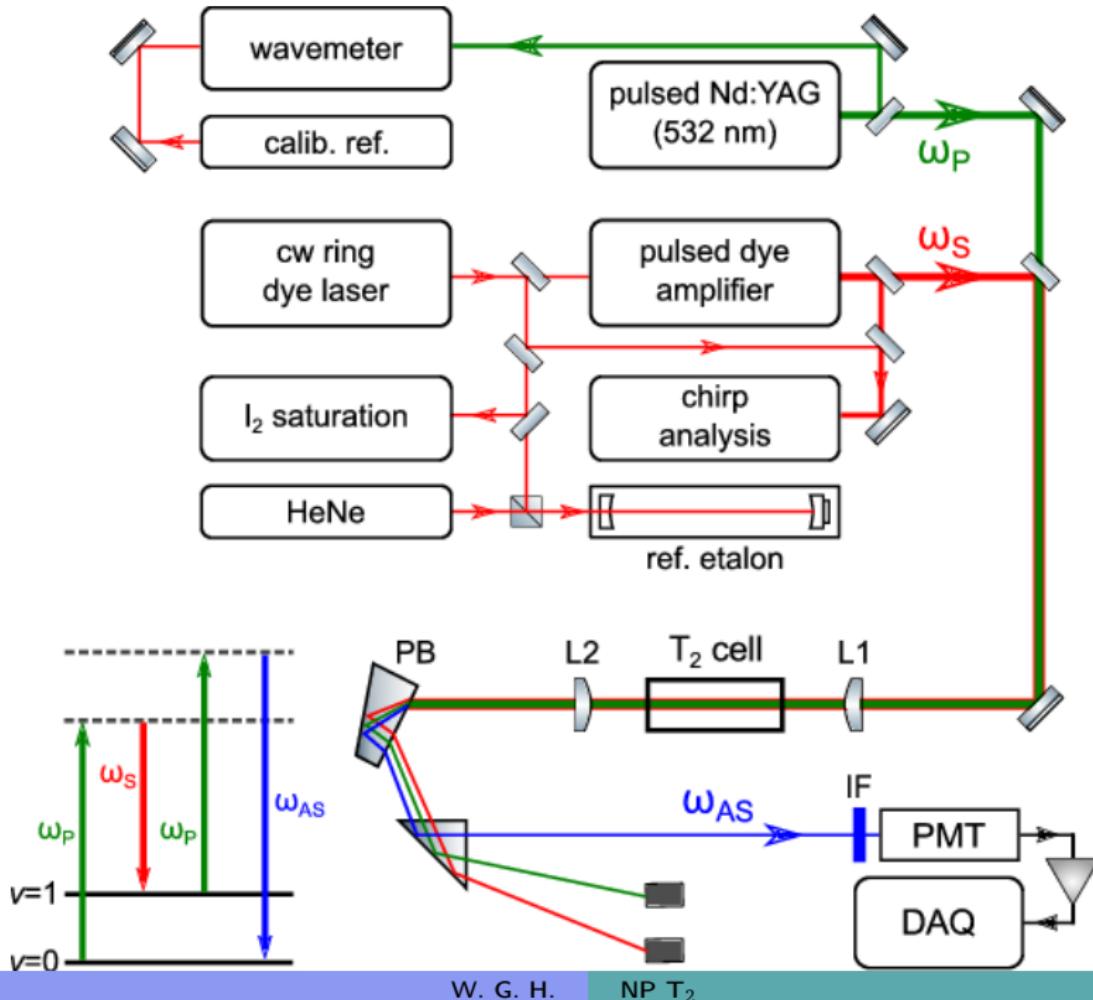
- quantum mechanical (an)harmonic oscillator:  
 $\omega_{\text{vib}}, \nu = 0 \rightarrow 1$
- two laser beams:
  - pump  $\omega_{\text{pump}}$
  - Stokes  $\omega_{\text{Stokes}}$
- difference frequency  
 $\omega_{\text{pump}} - \omega_{\text{Stokes}} \rightarrow \omega_{\text{vib}}$  resonance
- coherent superposition of  $|0\rangle$  and  $|1\rangle$

Measurements with Tritium ( $T_2$ , DT, HT) done by the molecular physics group and LaserLaB, Vrije Universiteit Amsterdam

[W. Ubachs et al.]

Tritium source by KIT-TLK

[M. Schlösser et al.]



## Is it feasible to search for New Physics effects in molecules?

### Theory:

constrain New Physics models

- Matthias Linster, KIT-TTP
  - Mustafa Tabet, KIT-TTP
  - Sonia Rani, IIT Bombay
  - Aman Sardwal, IIT Bombay
  - Wolfgang G. Hollik, KIT-IKP/TTP
  - Ulrich Nierste, TTP-KIT
- 
- light new particles: ALPs, dark photons, sterile neutrinos?

### Experiment:

measure precise spectra

- Magnus Schlösser, KIT-TLK
- Edcel Salumbides, VU Amsterdam
- Wim Ubachs, VU Amsterdam

## Rotational Energies

- q. m. rotator: two masses  $m_{1,2}$ , distance  $r = r_1 + r_2$
- center of mass:  $m_1 r_1 = m_2 r_2$
- moment of inertia:  $I = m_1 r_1^2 + m_2 r_2^2 = \frac{m_1 + m_2}{m_1 m_2} (r_1^2 + r_2^2) = \mu r^2$

$$\begin{array}{lll} \text{angular momentum} & \vec{L} = I\vec{\omega}/\hbar & \\ \text{energy} & E = I\omega^2/2 & \end{array} \quad \left. \right\} \quad E = \frac{\vec{L}^2 \hbar^2}{2I}$$

$|\vec{L}| = \hbar \sqrt{J(J+1)}$ , spinless masses:  $J = 0, 1, 2, \dots$

- quantized energy:  $E_J = J(J+1) \hbar^2 / (2I) \equiv B J(J+1)$
- energy separation between rotational levels  $\sim 10^{-3}$  eV (mw)

## Vibrational Modes: level separation $\sim 0.1$ eV (IR)

anharmonic oscillator:  $E_v = (\nu + \frac{1}{2}) \hbar\omega - (\nu + \frac{1}{2})^2 x_e \hbar\omega$

Q-band spectroscopy:  $\Delta J = 0, \nu = 0 \rightarrow 1$

CARS technique

Hydrogen-like molecules: 2 nuclei  $A$  and  $B$ , 2 electrons

$$H = \underbrace{\frac{\vec{P}_1^2}{2m} + \frac{\vec{P}_2^2}{2m}}_{\text{electrons}} + \underbrace{\frac{\vec{P}_A^2}{2M_A} + \frac{\vec{P}_B^2}{2M_B}}_{\text{nuclei } A+B} + \alpha \left\{ -\frac{1}{r_{1A}} - \frac{1}{r_{2B}} + \left( \frac{1}{r_{12}} + \frac{1}{R_{AB}} - \frac{1}{r_{1B}} - \frac{1}{r_{2A}} \right) \right\}$$

coulombic terms:

- attractive: electron<sub>1/2,2/1</sub> at nucleus<sub>A/B,B/A</sub>  $\sim \frac{1}{r_{1A/2B,1B,2A}}$
- repulsive: electron-electron  $\sim \frac{1}{r_{12}}$ , nucleus-nucleus  $\sim \frac{1}{R_{AB}}$

### Heitler–London approach

[Heitler/London 1927; Born/Oppenheimer 1927]

- neglection motion of nuclei (heavy wrt electrons)
- calculate electronic wave function  
→ effective potential between nuclei

## Electronic wave functions, nuclei at fixed positions

- $\psi_X(\vec{r}_j)$ : electron  $j$  at nucleus  $X$ ; two solutions: gerade, ungerade

$$\Phi_{\pm}(\vec{r}_1, \vec{r}_2) = \frac{\psi_A(\vec{r}_1)\psi_B(\vec{r}_2) \pm \psi_B(\vec{r}_1)\psi_A(\vec{r}_2)}{\sqrt{2(1 \pm |S_{AB}|^2)}}$$

- overlap  $S_{AB} = \int d^3 r_1 \psi_A^*(\vec{r}_1) \psi_B(\vec{r}_1)$

Perturbation: “wrong” Hamiltonian  $\hookleftarrow$  energy shift

$$E_{11} = \int dr_1 dr_2 \left\{ \left( \frac{\alpha}{r_{12}} + \frac{\alpha}{R} \right) \frac{u_A(r_1)^2 u_B(r_2)^2 + u_A(r_2)^2 u_B(r_1)^2}{2} - \left( \frac{\alpha}{r_{1A}} + \frac{\alpha}{r_{2B}} \right) \frac{u_A(r_2)^2 u_B(r_1)^2}{2} - \left( \frac{\alpha}{r_{2A}} + \frac{\alpha}{r_{1B}} \right) \frac{u_A(r_1)^2 u_B(r_2)^2}{2} \right\}$$

$$E_{12} = \int dr_1 dr_2 \left( \frac{2\alpha}{r_{12}} + \frac{2\alpha}{R} - \sum_{x=1A,2A,1B,2B} \frac{\alpha}{r_x} \right) \frac{u_A(r_1)u_A(r_2)u_B(r_1)u_A(r_2)}{2}$$

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## Perturbation: “wrong” Hamiltonian $\hookrightarrow$ energy shift

$$E_{\pm} = \frac{E_{11} \pm E_{12}}{1 \pm S_{AB}}$$

- Coulomb integral:  $E_{11}$
- exchange integral:  $E_{12}$
- potential energy curve: Morse potential  $V(r) = D_0 (1 - e^{-\alpha(r-r_0)})^2$

## Electronic wave functions, nuclei at fixed positions

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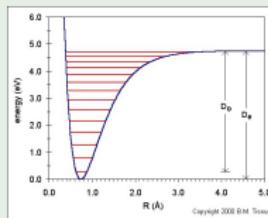
$$\Phi_{\pm}(\vec{r}_1, \vec{r}_2) = \frac{\psi_A(\vec{r}_1)\psi_B(\vec{r}_2) \pm \psi_B(\vec{r}_1)\psi_A(\vec{r}_2)}{\sqrt{2(1 \pm |S_{AB}|^2)}}$$

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## Full Hamiltonian

$$H = \sum_{k=1}^2 \frac{P_k^2}{2M_k} + \frac{1}{2} \sum_{i=1}^2 \frac{p_i^2}{2m} + V_C(r, R) = \sum_{k=1}^2 \frac{P_k^2}{2M_k} + H^0$$

- electronic problem solved
- eigenvalues  $E_n^0(R)$  and eigenstates  $\varphi_n(r, R)$  known:

$$(H^0 - E_n^0(R)) \varphi_n(r, R) = 0$$

## Nuclear wave function

semi-separation ansatz: nuclear wavefunction  $\psi_n(R)$

$$\Psi(r, R) = \sum_n \psi_n(R) \varphi_n(r, R)$$

solve

$$(H - E) \Psi(r, R) = 0$$

## Integrating electronic coordinates $\int dr \varphi_{n'}^*(r, R)(H - E)\Psi(r, R)$

$$\int dr \varphi_{n'}^*(r, R) \left( \sum_{k=1}^2 \frac{P_k^2}{2M_k} + H^0 - E \right) \sum_n \psi_n(R) \varphi_n(r, R) = 0,$$

$$\int dr \varphi_{n'}^*(r, R) \left( \sum_{k=1}^2 \frac{P_k^2}{2M_k} + E_n^0(R) - E \right) \sum_n \psi_n(R) \varphi_n(r, R) = 0,$$

$$\left( \sum_{k=1}^2 \frac{P_k^2}{2M_k} + E_{n'}^0(R) - E \right) \psi_{n'}(R) - \sum_n C_{n'n} \psi_n(R) = 0,$$

## Nuclear distance $R$ not constant anymore

$$A_{n'n}^{(k)} = \int dr \varphi_{n'}^*(r, R) P_k \varphi_n(r, R) \quad B_{n'n}^{(k)} = \frac{1}{2} \int dr \varphi_{n'}^*(r, R) P_k^2 \varphi_n(r, R)$$

Schrödinger equation for the nuclei  $\psi(R)$   $C_{n'n} = \sum_k \frac{1}{M_k} (A_{n'n}^{(k)} P_k + B_{n'n}^{(k)})$

$$\left( \sum_{k=1}^2 \frac{P_k^2}{2M_k} + U_n(X) - E \right) \psi_n + \sum_{n' \neq n} C_{nn'} \psi_{n'}(R) = 0$$

## Separating electronic and nuclear motion

$$H_0 = -\frac{1}{2}(\Delta_{r_1} + \Delta_{r_2}) + V,$$

$$H_1 = -\frac{m}{8\mu}(\vec{\nabla}_{r_1} + \vec{\nabla}_{r_2})^2,$$

$$H_2 = -\frac{m}{2\mu}\langle 0 | \Delta_R | 0 \rangle,$$

$$H_3 = -\frac{m}{2\tilde{\mu}}\langle 0 | \vec{\nabla}_R (\vec{\nabla}_{r_1} + \vec{\nabla}_{r_2}) | 0 \rangle,$$

- reduced mass  $\mu = M_a M_b / (M_a + M_b)$  and  $\tilde{\mu} = M_a M_b / (M_a - M_b)$

- electronic ground state  $|0\rangle$  i. e. two 1s H atoms

elliptic coordinates

[Hylleraas 1931]

$$\Psi = \frac{1}{2\pi} \sum_{i,n} c_{i,n} \psi_i(\xi_1, \eta_1, \xi_2, \eta_2, \phi) h_n(R)$$

with

$$\xi_i = \frac{r_{ai} + r_{bi}}{R} \quad \text{and} \quad \eta_i = \frac{r_{ai} - r_{bi}}{R}$$

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$$H_1 = -\frac{m}{8\mu}(\vec{\nabla}_{r_1} + \vec{\nabla}_{r_2})^2,$$

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non-adiabatic theory: dissociation energy [Kołos, Wolniewicz 1963 ff]

variational method:  $\delta \int d\tau_1 d\tau_2 d^3 R \Psi^* H \Psi = 0$

“trial” wave functions

$$\psi_i = \exp[-\alpha(\xi_1 + \xi_2)] \left( \xi_1^{p_i} \eta_1^{q_i} \xi_2^{r_i} \eta_2^{s_i} + \xi_1^{r_i} \eta_1^{s_i} \xi_2^{p_i} \eta_2^{q_i} \right) \left( \frac{2r_{12}}{R} \right)^{m_i}$$

## A short wrap-up

[source: Paweł Czachorowski PhD 2019 Warsaw]

- *ab initio* calculations
- large sets of trial wave functions: higher precision, more CPU
- non-relativistic QED [Pachucki 2004, 2005]

$$E(\alpha) = \alpha^2 E^{(2)} + \alpha^4 E^{(4)} + \alpha^5 E^{(5)} + \alpha^6 E^{(6)} + \alpha^7 E^{(7)} + \dots$$

- leading relativistic correction: Breit–Pauli [Puchalski et al. 2018]
- non-adiabatic perturbation theory
- nuclear nuclear size correction: accuracy of charge radii
- relativistic non-adiabatic [Czarochowski et al. 2018]
- explicitly correlated exponentials vs. correlated Gaussian
- usable computer code: H2spectre [www.fuw.edu.pl/~kpl]

**Total theoretical error budget**       $D_0^{\text{H}_2} = 36118.06945(53) \text{ cm}^{-1}$

e. g. dissociation energy H<sub>2</sub>: relative error  $\simeq 1.46 \times 10^{-8}$

## Dissociation energies

	$\frac{D_0}{\text{cm}^{-1}}$	H <sub>2</sub>	HD	D <sub>2</sub>
Th.	36 118.069 636(26)	36 405.782 39(19)	36 748.362 28(15)	
E. 1	36 118.069 62(37)	36 405.783 66(36)	36 748.362 86(68)	
Δ 1	+0.000 02(37)	-0.001 27(41)		-0.000 58(70)
E. 2	36 118.069 45(31)			
Δ 2	+0.000 19(31)			

[Czachorowski PhD 2019 and references therein!]

- theoretical error estimates?
- missing non-adiabatic QED  $E^{(5,1)}$  corrections?
- full nonadiabatic  $E^{(2)}$  energies for rovibrational levels
- long-term perspective:  $E^{(7)}$  and  $E^{(6,1)}$  contributions
- tackling the proton charge radius puzzle: independent determination of  $r_p$  from Hydrogen-isotoper spectroscopy

[Czachorowski PhD 2019]

## Simplest scenario: Yukawa potential

$$V_{\text{Yuk}} = \frac{g_{\text{NP}}}{2\pi} \frac{e^{-mr}}{r}$$

- massive particle with mass  $m$ , New Physics coupling  $g_{\text{NP}}$
- exponential drop-off
- e. g. dark photon [Jaeckel, Roy 2010]
- e. g. “fifth force” ( $m \rightarrow 1/\lambda$  length) [Salumbides et al. 2013]
- axion-like particles (small couplings) [Jaeckel, Ringwald 2010]
- constraints on electron-electron, nucleon-nucleon,  
electron-nucleon generic couplings [WGH et al.]

## Simple task(?)

Yukawa potential as perturbation to Coulomb potentials

$$\langle \Delta E_{\text{NP}} \rangle = \frac{g_{\text{NP}}}{2\pi} \left[ \left\langle \Psi_{v_f, J_f} \left| \frac{e^{-mr_x}}{r_x} \right| \Psi_{v_f, J_f} \right\rangle - \left\langle \Psi_{v_i, J_i} \left| \frac{e^{-mr_x}}{r_x} \right| \Psi_{v_i, J_i} \right\rangle \right]$$

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wave functions from H2spectre

[Pachucki et al.]

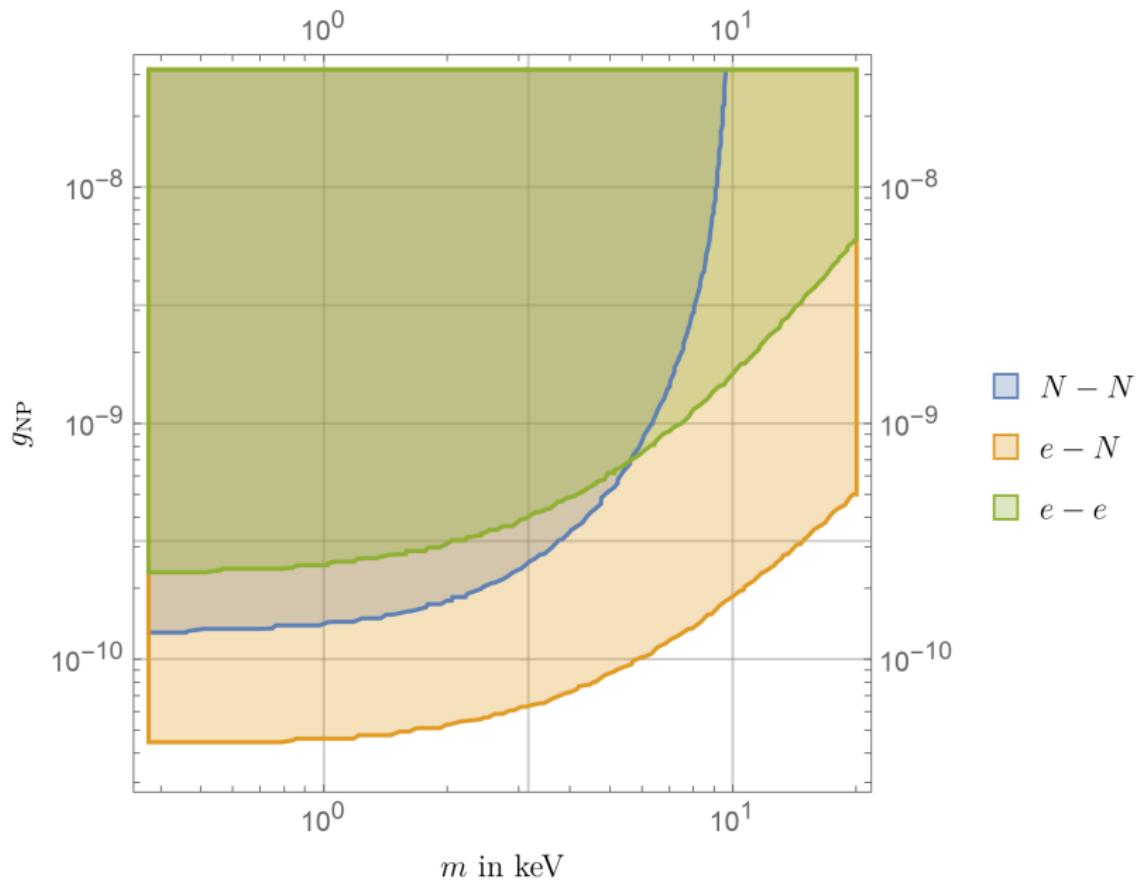
## Potential for New Physics

compare “old“ and new physics with experiment

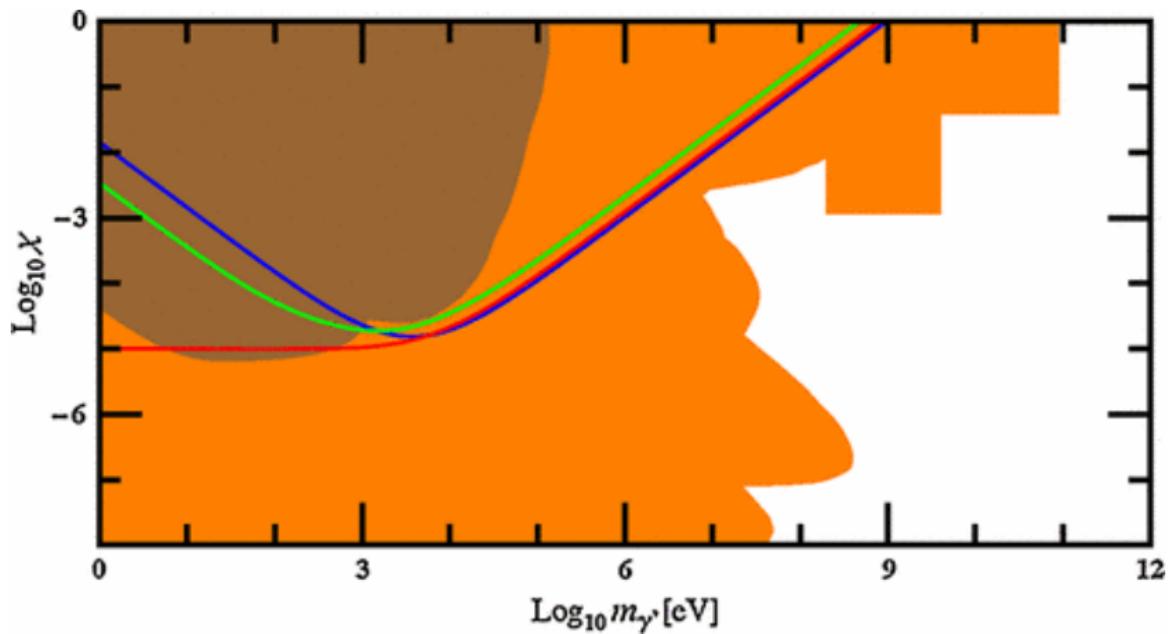
$$E_{\text{exp}} + \delta E_{\text{exp}} \quad \text{vs.} \quad E_{\text{theo}} + \delta E_{\text{theo}} + \Delta E_{\text{NP}}$$

constraints on mass vs. coupling

$$V_{\text{NP}} = \frac{g_{\text{NP}}}{2\pi} \frac{\exp(-mr)}{r}$$

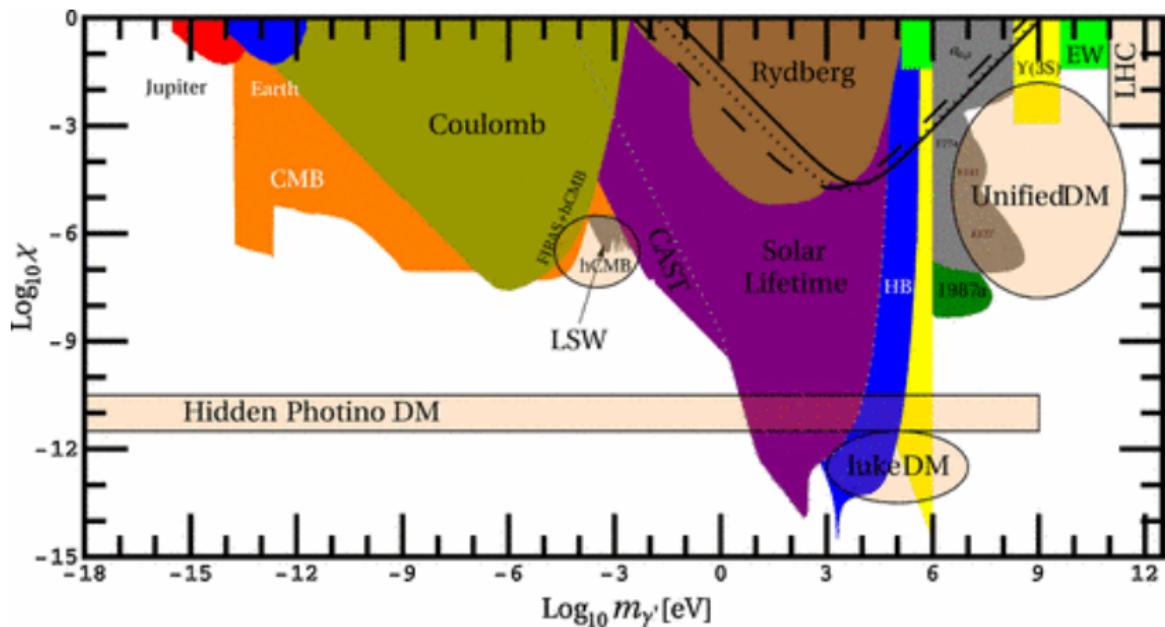


## Comparison with similar bounds



[Jaeckel, Roy: "Spectroscopy as a test of Coulomb's law: A probe of the hidden sector", PRD82,125020(2010)]

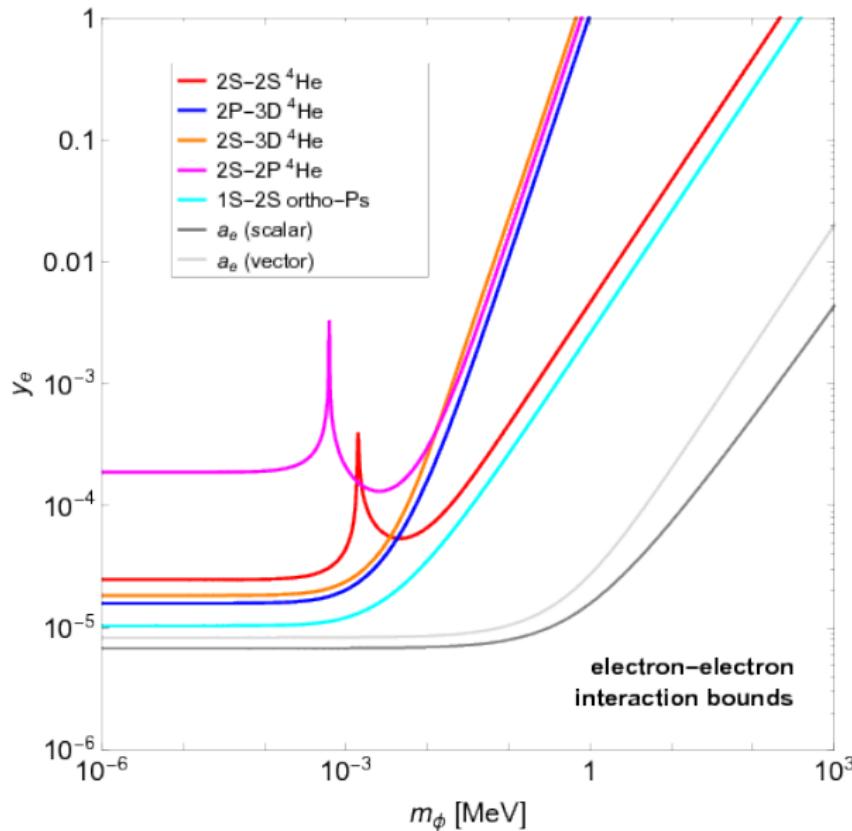
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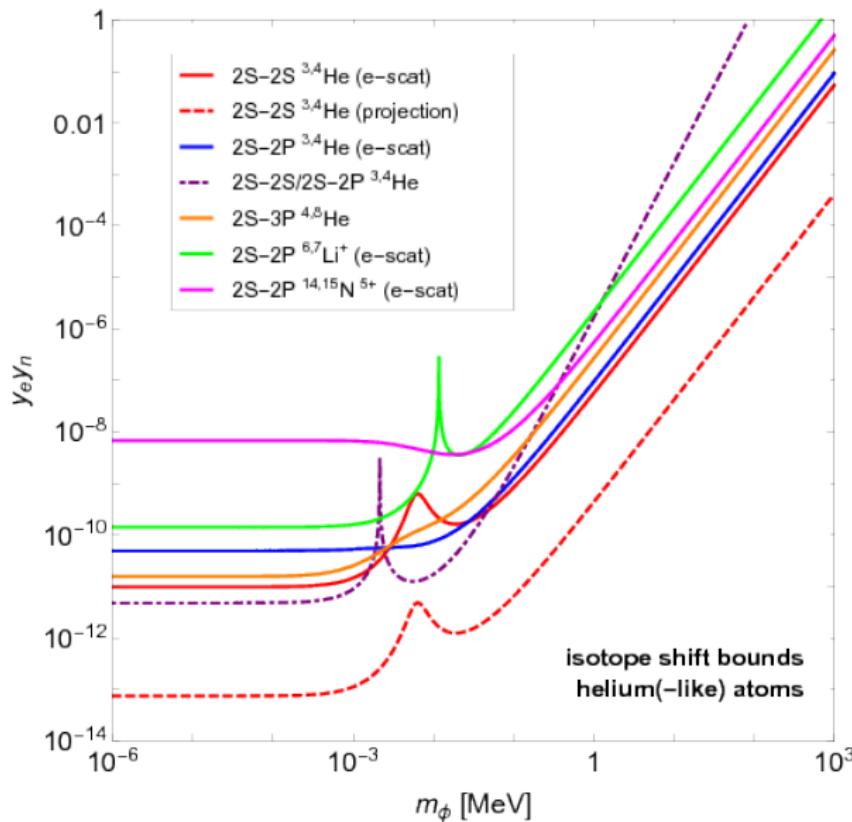
# Atom spectroscopy with few electrons

[Delaunay, Frugiuele, Fuchs, Soreq 2017]



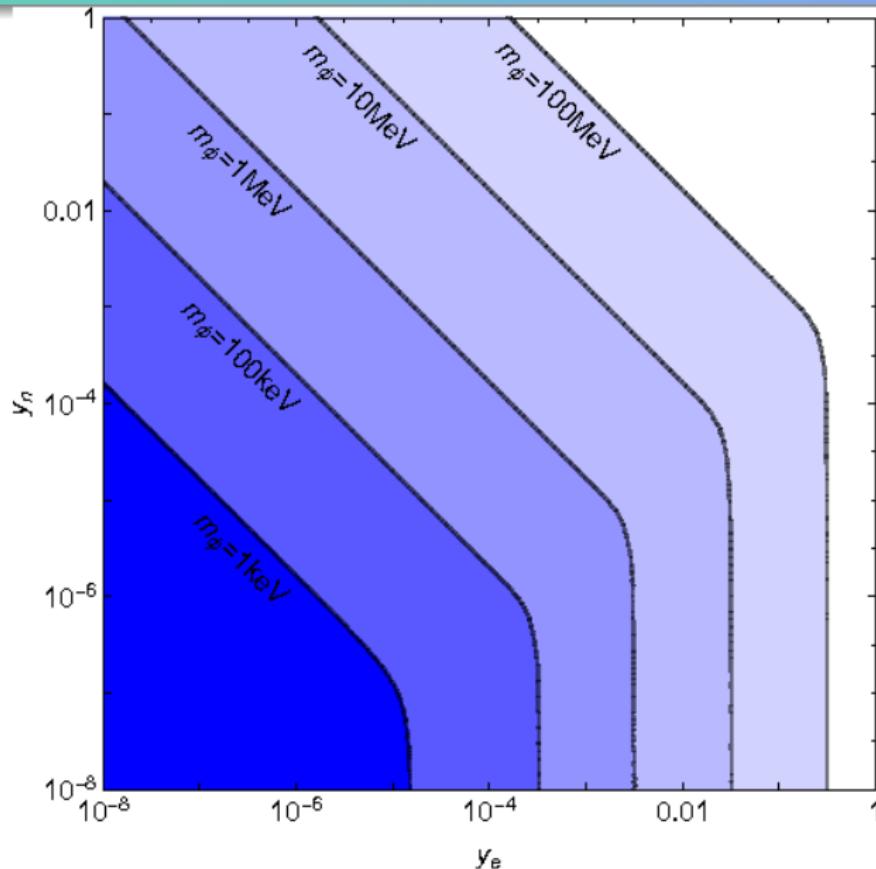
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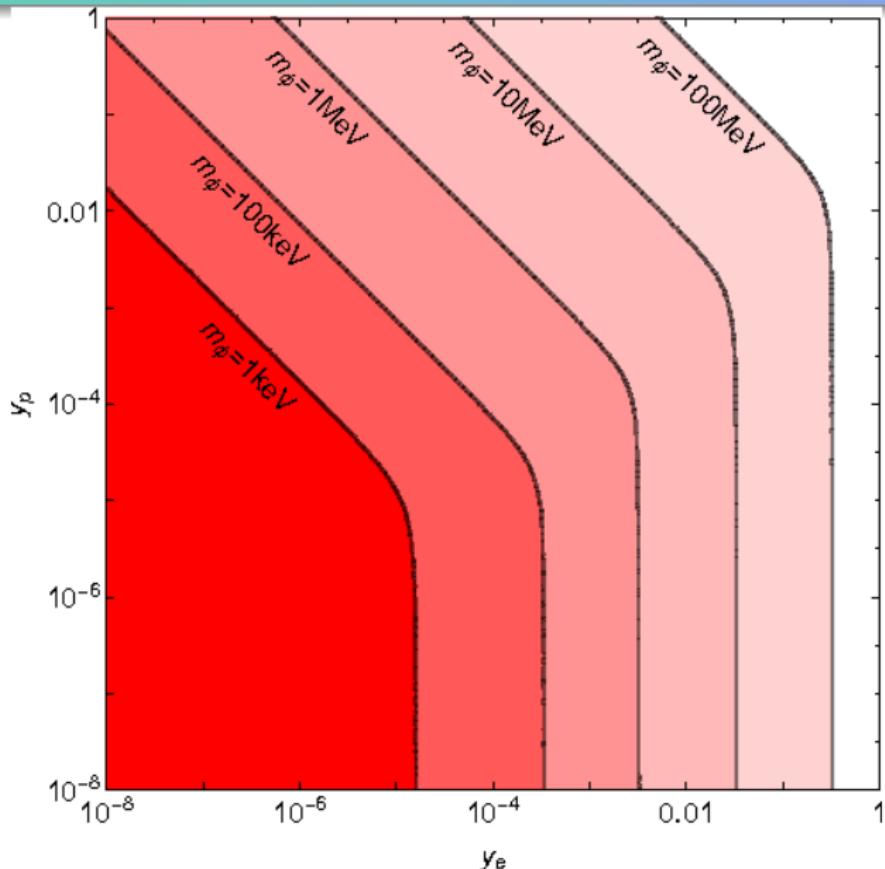
# Atom spectroscopy with few electrons

[Delaunay, Frugiuele, Fuchs, Soreq 2017]



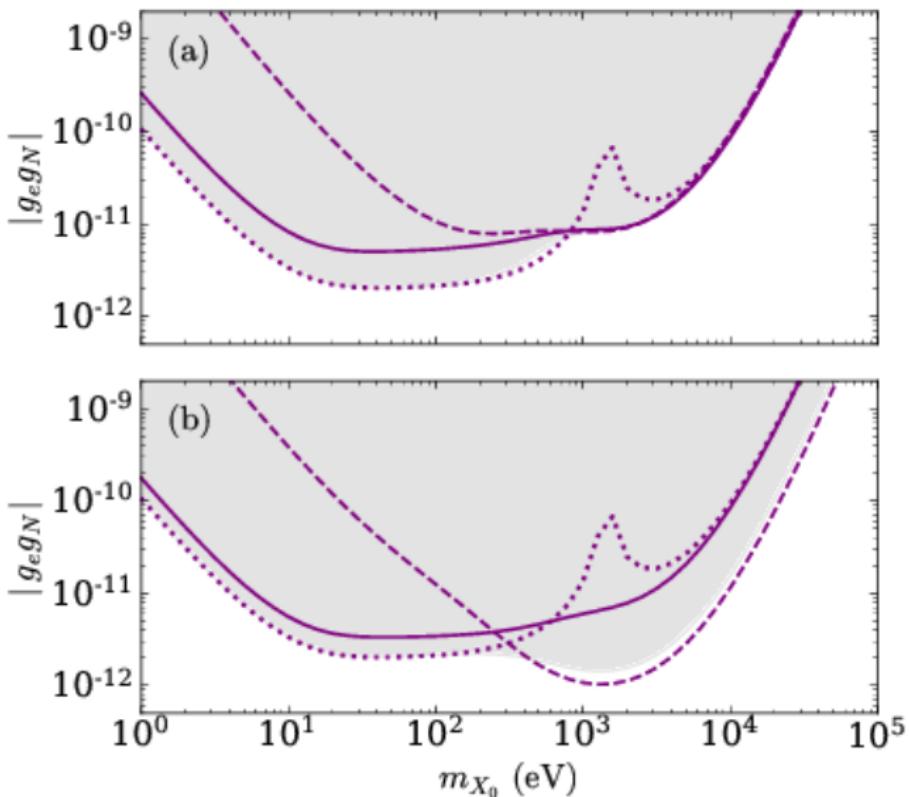
# Atom spectroscopy with few electrons

[Delaunay, Frugiuele, Fuchs, Soreq 2017]



# Rydberg states of atomic hydrogen

[Jones, Potvliege, Spannowsky arXiv:1909.09194]



## Summary and Conclusions

- rovibrational spectroscopy as a probe of New Physics
- Hydrogen isotopomers: H<sub>2</sub>, D<sub>2</sub>, T<sub>2</sub>, HD, HT, DT
- first step: constrain simple New Physics as Yukawa potential

$$V_{\text{NP}} = \frac{g_{\text{NP}}}{2\pi} \frac{e^{-mr}}{r}$$

## Outlook

- isotopes for correlations [WGH et al.]
- improvement in theoretical *ab initio* calculations expected
- experimental refinements
- measurement of DT and HT to be published
- test of different potential types (also with spin) [WGH et al.]