Low energy phenomenology of an inflationary NMSSM

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work done in collaboration with

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Higgs mass @ 2loop
Higgs production & decays

Inflationary model based on

[1] M. B. Einhorn and D. R. T. Jones, *"Inflation with Non-minimal Gravitational Couplings in Supergravity"*, JHEP **1003**, 026 (2010) [arXiv:0912.2718]

[2] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, *"Jordan Frame Supergravity and Inflation in NMSSM"*, Phys. Rev. D **82**, 045003 (2010) [arXiv:1004.0712]

[3] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, *"Superconformal Symmetry, NMSSM, and Inflation"*, Phys. Rev. D **83**, 025008 (2011) [arXiv:1008.2942] [FKLMvP]

Higgs inflation

- inflation is a cosmological necessity
- instead of introducing a new field:

$$\text{(SM) Higgs} = \text{inflaton}$$

- non-minimal couplings of the scalar field to gravity
- SM becomes "unnatural" [cf. Einhorn, Jones]
- a viable candidate might be the scale-free (Next-to) Minimal Supersymmetric Standard Model [FKLMvP]

Canonical Superconformal Supergravity (CSS)

- scale invariance of global supersymmetry → local SUSY
- modified SUGRA Lagrangian [Einhorn, Jones]

$$\mathcal{L} = -6 \int \mathrm{d}^2\,\theta\,\mathcal{E}\left[R - \frac{1}{4}\left(\bar{\mathcal{D}}^2 - 8R\right)\Phi^\dagger\Phi + P(\Phi)\right] + \text{h. c.} \; + \dots$$

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$$\mathcal{L} = -6 \int \mathrm{d}^2\,\theta\,\mathcal{E}\left[R + X(\Phi)R - \frac{1}{4}\left(\bar{\mathcal{D}}^2 - 8R\right)\Phi^\dagger\Phi + P(\Phi)\right] + \text{h. c.} + \ldots$$

Superconformal symmetry breaking

- $X(\Phi)$
- dimensionless coupling (!)
- only function of chiral superfields (Φ, not $\Phi^\dagger$)

Jordan frame → Einstein frame, $M_P = 1$

- frame function $\Omega = \phi_i^* \phi_i - 3$
- Kähler potential $K = -3\log(-\Omega/3)$
- non-minimal coupling

$$\Omega_\chi = \Omega - \frac{3}{2}\left(X(\phi) + \text{h.c.}\right)$$

NMSSM superconformal symmetry breaking

$$\Omega = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \frac{3}{2}\chi\left(H_u \cdot H_d + \text{h.c.}\right)$$

Enlarged Higgs sector

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \quad S$$

Superpotential, $\mathbb{Z}_3$-invariant:

$$\mathcal{W}_{\text{Higgs}} = \lambda\, S H_u \cdot H_d + \frac{\kappa}{3}\, S^3,$$

where $H_u \cdot H_d = H_u^+ H_d^- - H_u^0 H_d^0$

The NMSSM solves the "μ-problem"

$$\mathcal{W}_{\text{MSSM}} = \mu\, H_u \cdot H_d + \text{Yukawa}$$

only dimensionful parameter μ has to be $\sim$ electroweak scale

$$\mathcal{W}_{\text{NMSSM}} \supset \lambda\, S H_u \cdot H_d + \frac{\kappa}{3} S^3$$

dynamical μ-term: $\lambda \langle S \rangle = \mu_{\text{eff}}$

$\mathbb{Z}_3$ symmetry forbids dimensionful couplings (bilinear, tadpole terms)

local U(1) $\mathcal{R}$ symmetry

- part of the superconformal SU(2, 2|1)
- χ term breaks continous $\mathcal{R}$ *and* discrete $\mathbb{Z}_3$ symmetry
- breaking at dimension 6: $\sim \chi \frac{\lambda^2 h^6}{M_P^2}$

Corrected Superpotential

$$\mathcal{W}_{\text{eff}} \to \mathcal{W} e^{X(\Phi)/M_P^2} = \mathcal{W} + \frac{\langle \mathcal{W}_{\text{hid}} \rangle}{M_P^2} X(\Phi)$$
$$\simeq \mathcal{W} + m_{3/2} X(\Phi)$$

The iNMSSM

$$\mathcal{W}_{\text{eff}} = \lambda\, S H_u \cdot H_d + \frac{\kappa}{3} S^3 + \frac{3}{2} \chi m_{3/2} H_u \cdot H_d$$

Cosmo pheno requires $|\chi/\lambda| \simeq 10^5$

like the NMSSM with an extended effective μ term

$$\mu'_{\text{eff}} = \lambda \langle S \rangle + \frac{3}{2} \chi m_{3/2} = \mu_{\text{eff}} + \mu$$

Additional soft SUSY breaking term

$$V_{\text{soft}} = \lambda A_\lambda \, S H_u \cdot H_d + \frac{1}{3} \kappa A_\kappa \, S^3 + \frac{3}{2} B_\mu \chi m_{3/2} \left(H_u \cdot H_d + \text{h.c.} \right)$$

Higgs potential of the iNMSSM

$$V = \left[m_{H_d}^2 + (\mu + \lambda S)^2 \right] |H_d|^2 + \left[m_{H_u}^2 + (\mu + \lambda S)^2 \right] |H_u|^2 + m_S^2 S^2 + \tfrac{2}{3} \kappa A_\kappa S^3 + \left[\kappa S^2 + \lambda H_u \cdot H_d \right]^2 + 2 \left(B_\mu \mu + \lambda A_\lambda S \right) H_u \cdot H_d + \tfrac{g_1^2 + g_2^2}{8} \left(|H_d|^2 - |H_u|^2 \right)^2 + \tfrac{g_2^2}{2} |H_d^\dagger H_u|^2$$

Stabilization of the inflationary trajectory

- only neutral components ("truncation")

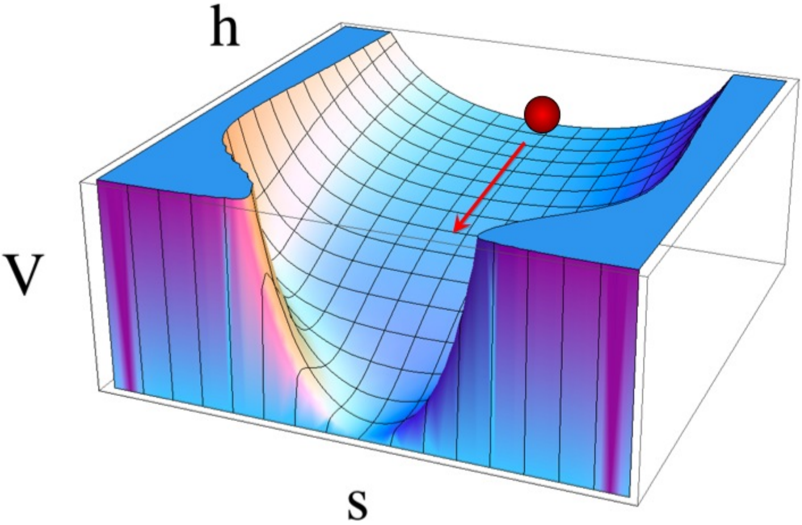
$$S = se^{i\alpha}/\sqrt{2}, \quad H_u^0 = h_2 e^{i\alpha_1}/\sqrt{2}, \quad H_d^0 = h_1 e^{i\alpha_2}/\sqrt{2},$$

with $h_1 = h\cos\beta$ and $h_2 = h\sin\beta$; $\tan\beta = h_2/h_1$

- D-flat direction:

$$\beta = \pi/4 \qquad h_1^2 = h_2^2 = h^2$$

- "simplest" direction: $s = 0$, $\alpha_{1,2} = 0$ [FLKMvP]
- ↯ tachyonic singlet directions [Einhorn, Jones]
- add $-\zeta(S\bar{S})^2$ to the frame function



stabilization for $\zeta > \frac{2|\lambda\kappa|}{\lambda^2 h^2} + 0.0327$ [FLKMvP]

Flat potential $V(\phi, \ldots)$

slow roll parameters $\epsilon, \eta \gg 1$:

$$\epsilon = \frac{1}{2}\left(\frac{1}{V}\frac{\partial V}{\partial \phi}\right)^2$$

$$\eta = \frac{1}{V}\frac{\partial^2 V}{\partial \phi^2}$$

inflationary NMSSM [FLKMvP]

$$\epsilon \simeq -\frac{64}{3\chi^2 h^4}, \qquad \eta \simeq -\frac{16}{3\chi h^2}$$

slow roll ends when $\epsilon, \eta \simeq 1$, thus

$$h_{\text{end}} \simeq 2.2/\sqrt{\chi} \approx 0.007$$

in Planck units!

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A SUSY electroweak model

$$V = \left[m_{H_d}^2 + (\mu + \lambda S)^2\right]|H_d|^2 + \left[m_{H_u}^2 + (\mu + \lambda S)^2\right]|H_u|^2 + m_S^2 S^2$$
$$+ \tfrac{2}{3}\kappa A_\kappa S^3 + \left[\kappa S^2 + \lambda H_u \cdot H_d\right]^2 + 2\left(B_\mu \mu + \lambda A_\lambda S\right) H_u \cdot H_d$$
$$+ \tfrac{g_1^2+g_2^2}{8}\left(|H_d|^2 - |H_u|^2\right)^2 + \tfrac{g_2^2}{2}|H_d^\dagger H_u|^2$$

$$m_{H_d}^2 = -(\mu + \mu_{\rm eff})^2 - v^2 \lambda^2 s_\beta^2 - \tfrac{1}{2} M_Z^2 c_{2\beta} + a_1 t_\beta\,,$$
$$m_{H_u}^2 = -(\mu + \mu_{\rm eff})^2 - v^2 \lambda^2 c_\beta^2 + \tfrac{1}{2} M_Z^2 c_{2\beta} + a_1 / t_\beta\,,$$
$$m_S^2 = a_4 - a_5 - a_7 - v^2 \lambda^2 - \left(\nu + 2\mu_{\rm eff}\frac{\kappa}{\lambda}\right),$$

with $\langle h_u^0\rangle_{\rm ew} = v_u/\sqrt{2}$, $\langle h_d^0\rangle_{\rm eq} = v_d/\sqrt{2}$, $\langle s^0\rangle_{\rm ew} = \mu_{\rm eff}/\lambda$.

Minimisation conditions are in general misleading!

$$\frac{\partial V}{\partial h_u}\Big|_{\rm vev} = 2 m_{H_u}^2 v_u + \dots$$
$$\frac{\partial V}{\partial h_d}\Big|_{\rm vev} = 2 m_{H_d}^2 v_d + \dots$$
$$\frac{\partial V}{\partial h_u}\Big|_{\rm vev} = 2 m_S^2 v_s + \dots$$

linear equations for soft SUSY breaking masses $m_{H_u}^2$, $m_{H_d}^2$, m_S^2, can be solved uniquely; determine numerical values for those

- different phenomenology than pure $\mathbb{Z}_3$-invariant NMSSM
- tachyonic directions in both
- additonal μ-term allows for *more* allowed (i. e physical) parameter space
- selection for sign μ_{eff} ($\mu > 0$ by construction)
- scenarios with *alternative vevs* possible
  - $\langle h_u \rangle \neq v_u/\sqrt{2}$, $\langle h_d \rangle \neq v_d/\sqrt{2}$, $\langle s \rangle \neq \mu_{\text{eff}}/\lambda$
  - in general: $h_u \simeq h_d \gg v$ or 0 and/or $s \gg \mu_{\text{eff}}/\lambda$
  - vacuum tunneling: mostly long lifetimes
- SM-like Higgs mass @ 125 GeV!
- no (too) light singlets (can be shifted with A_κ)
- not much viable space left

Constraining the parameter space

- different phenomenology than pure $\mathbb{Z}_3$-invariant NMSSM
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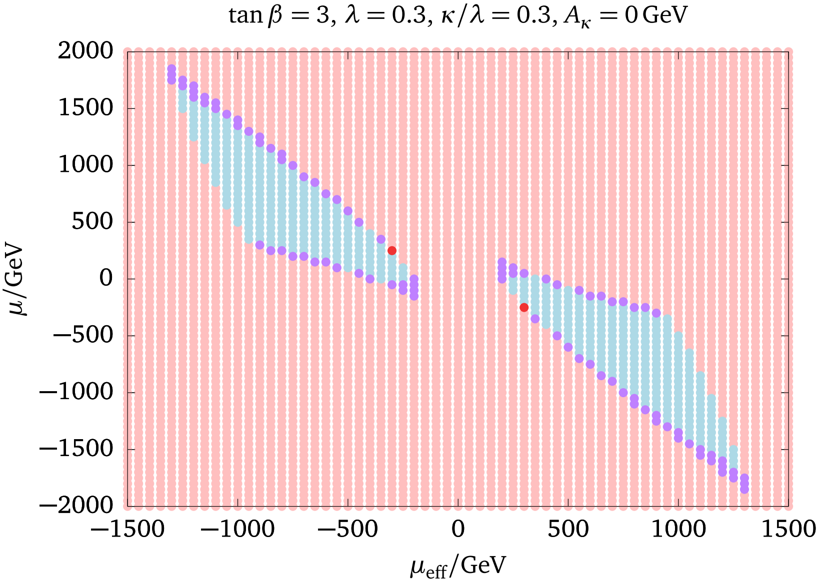
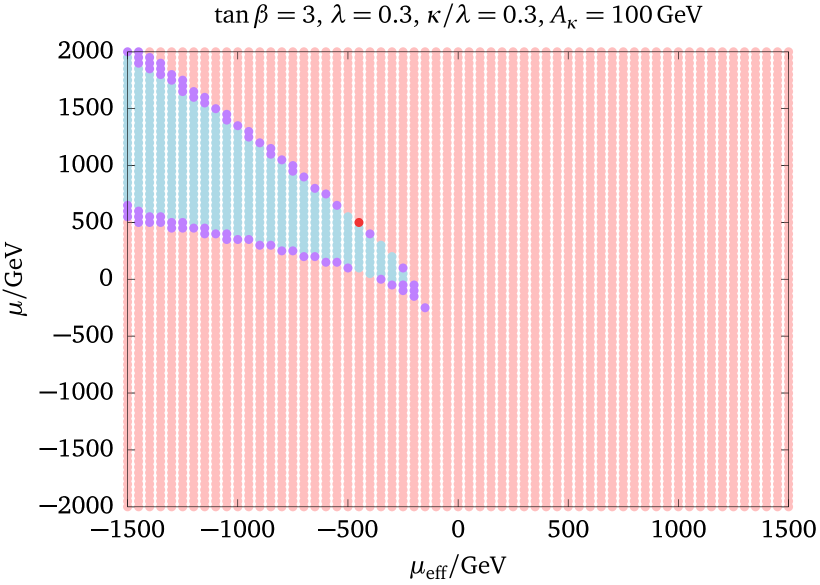


tan β = 3, λ = 0.3, κ/λ = 0.3, A_κ = 0 GeV
2000
1500
1000
500
0
−500
−1000
−1500
−2000
μ/GeV
−1500
−1000
−500
0
500
1000
1500
μ_eff/GeV


tan β = 3, λ = 0.3, κ/λ = 0.3, A_κ = 100 GeV
2000
1500
1000
500
0
−500
−1000
−1500
−2000
μ/GeV
−1500
−1000
−500
0
500
1000
1500
$\mu_{\rm eff}$/GeV

Main feactures can be seen from tree-level

$$\mathcal{M}_S^2 = \begin{pmatrix} M_Z^2 c_\beta^2 + a_1 t_\beta & \left(2 v^2 \lambda^2 - M_Z^2\right) c_\beta s_\beta - a_1 & a_2 c_\beta - a_3 s_\beta \\ \cdot & M_Z^2 s_\beta^2 + a_1/t_\beta & a_2 s_\beta - a_3 c_\beta \\ \cdot & \cdot & a_4 + a_5 \end{pmatrix}$$

$$\mathcal{M}_P^2 = \begin{pmatrix} a_1 t_\beta & a_1 & -a_6 s_\beta \\ \cdot & a_1/t_\beta & -a_6 c_\beta \\ \cdot & \cdot & a_4 - 3 a_5 - 2 a_7 \end{pmatrix}$$

with

$$a_1 = B_\mu \mu + \mu_{\rm eff} \left(\frac{\kappa}{\lambda} \mu_{\rm eff} + A_\lambda\right) \qquad a_2 = 2 v \lambda (\mu + \mu_{\rm eff})$$

$$a_3 = v \lambda \left(2 \frac{\kappa}{\lambda} \mu_{\rm eff} + A_\lambda\right)$$

$$a_4 = \frac{1}{\mu_{\rm eff}} \left[v^2 \lambda^2 c_\beta s_\beta \left(\frac{\kappa}{\lambda} \mu_{\rm eff} + A_\lambda\right) - v^2 \lambda^2 \mu\right]$$

$$a_5 = 4 \left(\frac{\kappa}{\lambda}\right)^2 \mu_{\rm eff}^2 + \frac{\kappa}{\lambda} \left[\mu_{\rm eff} A_\kappa - v^2 \lambda^2 c_\beta s_\beta\right]$$

$$a_6 = v \lambda \left(2 \frac{\kappa}{\lambda} \mu_{\rm eff} - A_\lambda\right) \qquad a_7 = -6 \left(\frac{\kappa}{\lambda}\right)^2 \mu_{\rm eff}^2$$

Choosing reasonable input parameters

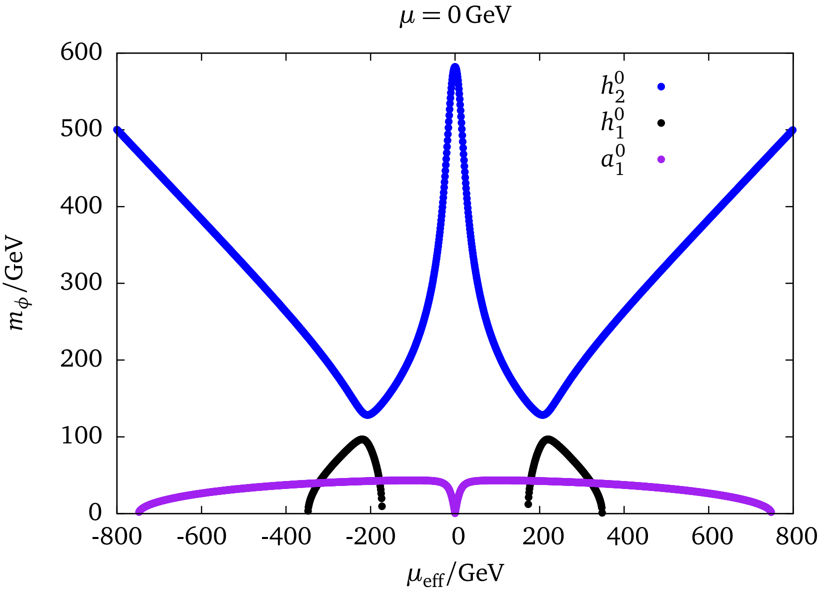
Avoid tachyonic charged Higgs by definition

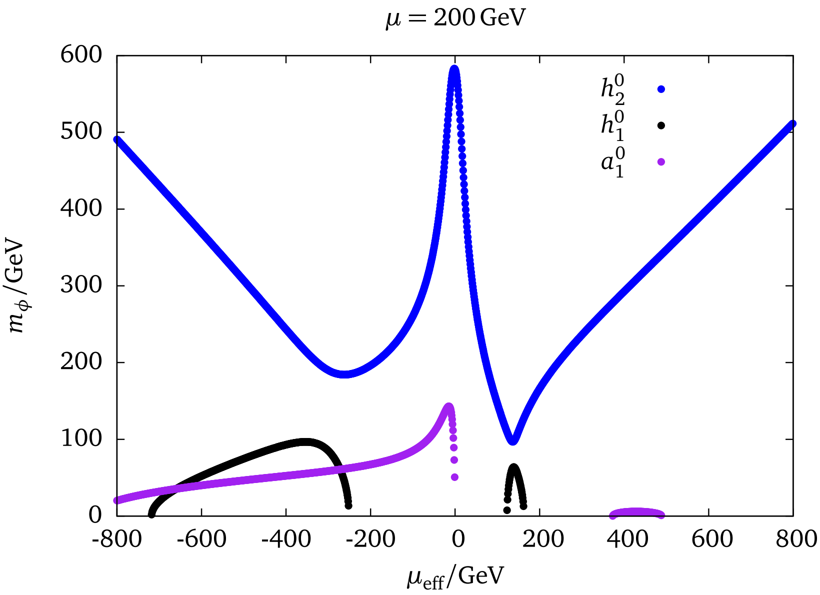
$$m_{H^\pm}^2 = M_W^2 - v^2\,\lambda^2 + \frac{a_1}{c_\beta\, s_\beta}$$

$$a_1 = B_\mu\,\mu + \mu_{\text{eff}}\left(\tfrac{\kappa}{\lambda}\,\mu_{\text{eff}} + A_\lambda\right)$$

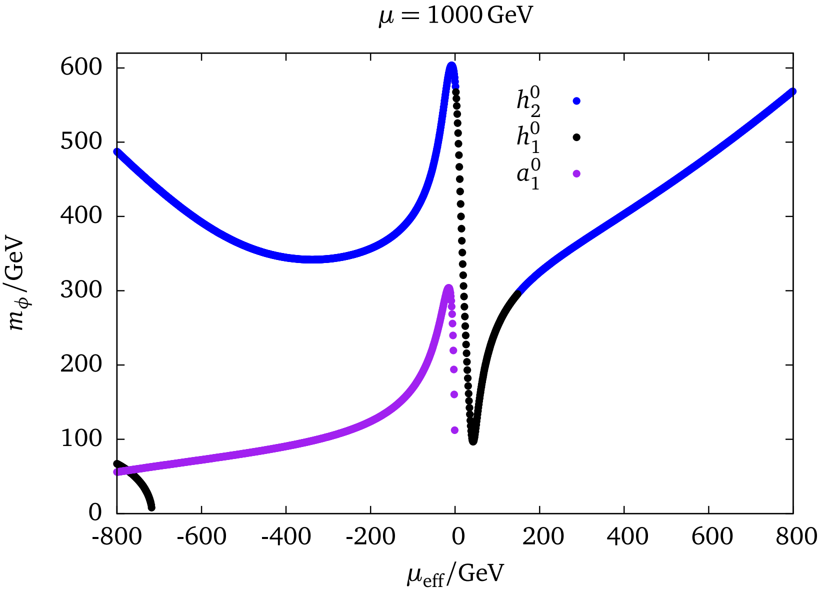
$$A_\lambda = \frac{c_\beta\, s_\beta}{\mu_{\text{eff}}}\left(m_{H^\pm}^2 - M_W^2 + v^2\,\lambda^2\right) - \frac{B_\mu\,\mu}{\mu_{\text{eff}}} - \mu_{\text{eff}}\,\frac{\kappa}{\lambda}$$

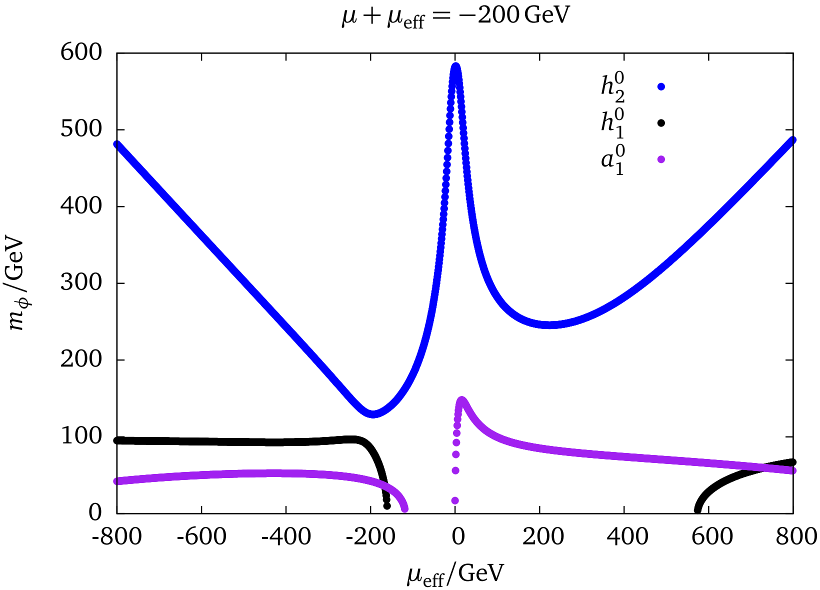
- small $\tan\beta$: large NMSSM-effect on light Higgs mass ($\Delta m_{h^0}^2 \sim \lambda^2 v^2 \sin^2 2\beta$)
- large $m_{H^\pm} = 800\,\text{GeV}$ (although not needed for small $\tan\beta$)
- $\text{sign}\,A_\kappa$ selects $\text{sign}\,\mu_{\text{eff}}$
- $\mu + \mu_{\text{eff}}$ as effective μ-term
- single μ_{eff} contributions


$\mu = 0\,\mathrm{GeV}$
h_2^0
h_1^0
a_1^0
600
500
400
300
200
100
0
-800
-600
-400
-200
0
200
400
600
800
m_ϕ/GeV
$\mu_{\mathrm{eff}}/\mathrm{GeV}$

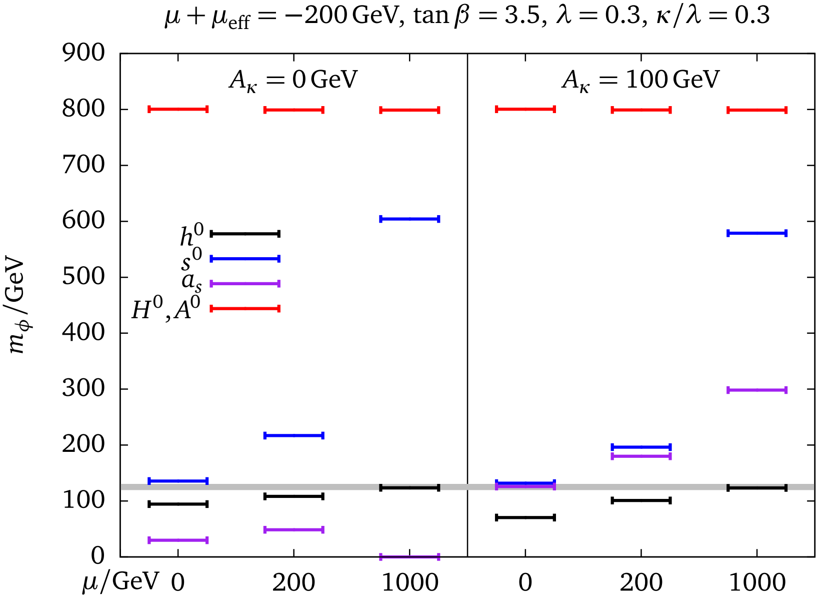


$\mu = 200\,\mathrm{GeV}$
h_2^0
h_1^0
a_1^0
m_ϕ/GeV
$\mu_{\mathrm{eff}}/\mathrm{GeV}$
0
100
200
300
400
500
600
-800
-600
-400
-200
0
200
400
600
800


$\mu = 1000\,\mathrm{GeV}$
h_2^0
h_1^0
a_1^0
m_ϕ/GeV
$\mu_{\mathrm{eff}}/\mathrm{GeV}$
0
100
200
300
400
500
600
-800
-600
-400
-200
0
200
400
600
800



$\mu + \mu_{\text{eff}} = -200\,\text{GeV}$
m_ϕ/GeV
$\mu_{\text{eff}}/\text{GeV}$
h_2^0
h_1^0
a_1^0
0
100
200
300
400
500
600
-800
-600
-400
-200
0
200
400
600
800


$\mu + \mu_{\text{eff}} = -200\,\text{GeV}$, $\tan\beta = 3.5$, $\lambda = 0.3$, $\kappa/\lambda = 0.3$
$A_\kappa = 0\,\text{GeV}$
$A_\kappa = 100\,\text{GeV}$
m_ϕ/GeV
900
800
700
600
500
400
300
200
100
0
h^0
s^0
a_s
H^0, A^0
μ/GeV
0
200
1000
0
200
1000

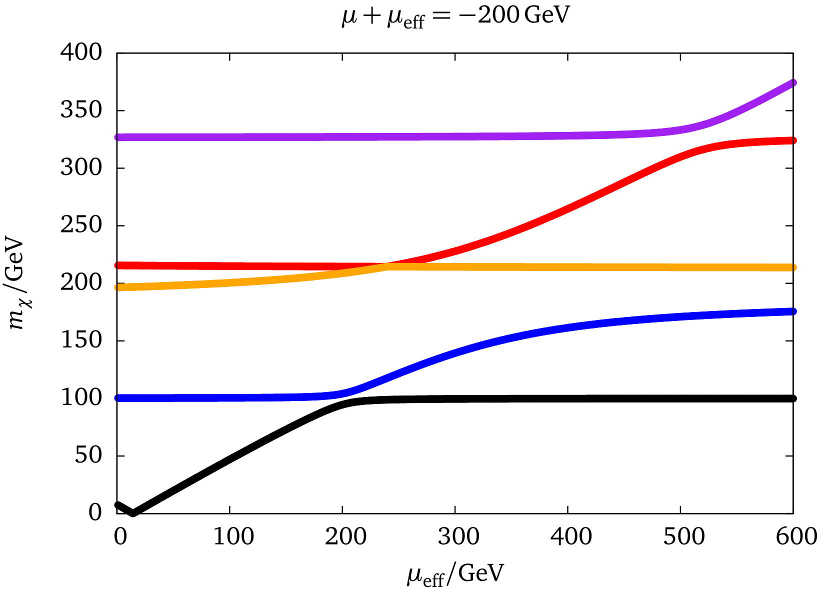
A different sector: Neutralinos!

- as in NMSSM: 5 Neutralino states
- different scaling behaviour with μ, μ_{eff}
- lightest state probably dark matter candidate
- generically heavy Singlino!

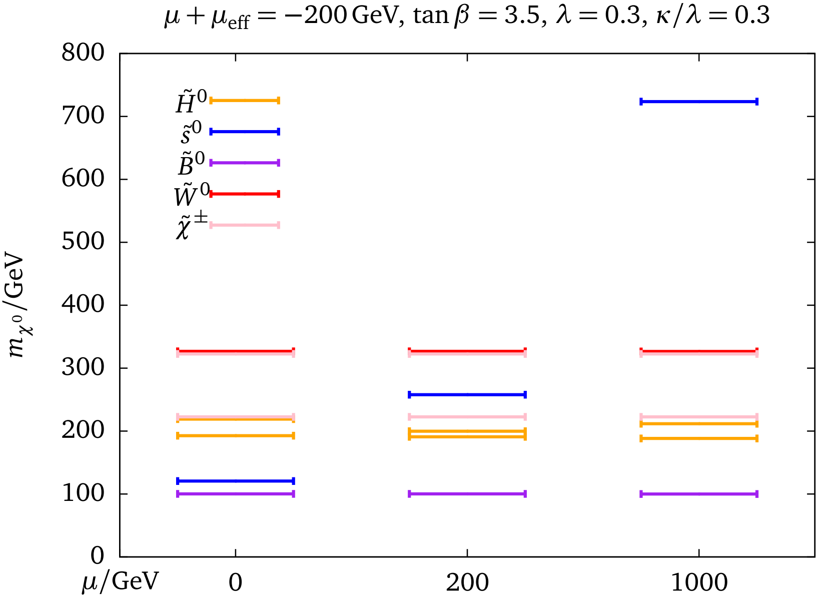
$$\mathcal{M}_\chi = \begin{pmatrix} M_1 & 0 & -M_Z s_\text{w} c_\beta & M_Z s_\text{w} s_\beta & 0 \\ \cdot & M_2 & M_Z c_\text{w} c_\beta & -M_Z c_\text{w} s_\beta & 0 \\ \cdot & \cdot & 0 & -(\mu_\text{eff} + \mu) & -\lambda v s_\beta \\ \cdot & \cdot & \cdot & 0 & -\lambda v c_\beta \\ \cdot & \cdot & \cdot & \cdot & 2 \frac{\kappa}{\lambda} \mu_\text{eff} \end{pmatrix}$$

Possible distinct scenarios

- physical Higgsino mass $\sim (\mu_\text{eff} + \mu)$
- small $\mu_\text{eff} + \mu$
- large cancellation possible: Singlino mass $\nearrow$!


$\mu + \mu_{\text{eff}} = -200\,\text{GeV}$
m_χ/GeV
400
350
300
250
200
150
100
50
0
0
100
200
300
400
500
600
$\mu_{\text{eff}}/\text{GeV}$

A Neutralino spectrum

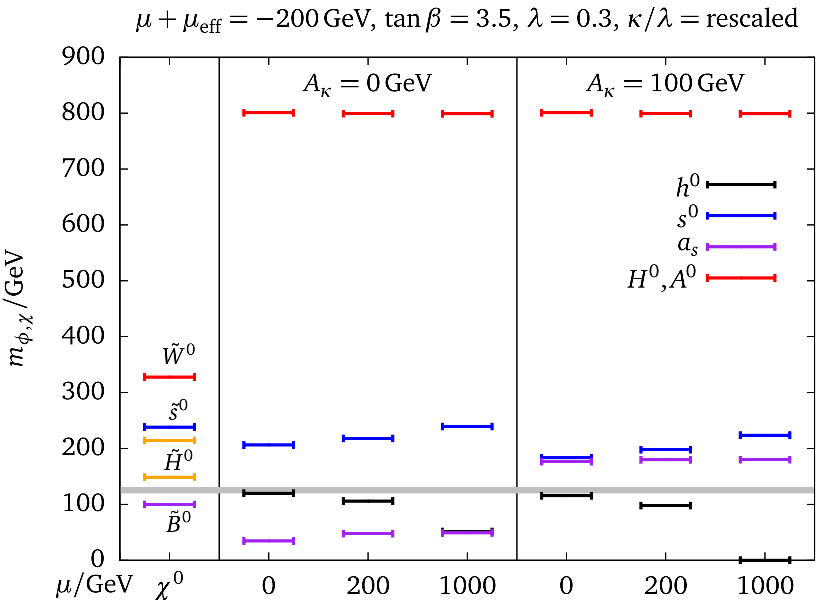


$$\mathcal{M}_\chi = \begin{pmatrix} M_1 & 0 & -M_Z\, s_\mathrm{w}\, c_\beta & M_Z\, s_\mathrm{w}\, s_\beta & 0 \\ \cdot & M_2 & M_Z\, c_\mathrm{w}\, c_\beta & -M_Z\, c_\mathrm{w}\, s_\beta & 0 \\ \cdot & \cdot & 0 & -(\mu_\mathrm{eff} + \mu) & -\lambda\, v\, s_\beta \\ \cdot & \cdot & \cdot & 0 & -\lambda\, v\, c_\beta \\ \cdot & \cdot & \cdot & \cdot & 2\,\frac{\kappa}{\lambda}\,\mu_\mathrm{eff} \end{pmatrix}$$

"Liebler" rescaling [G. Weiglein]

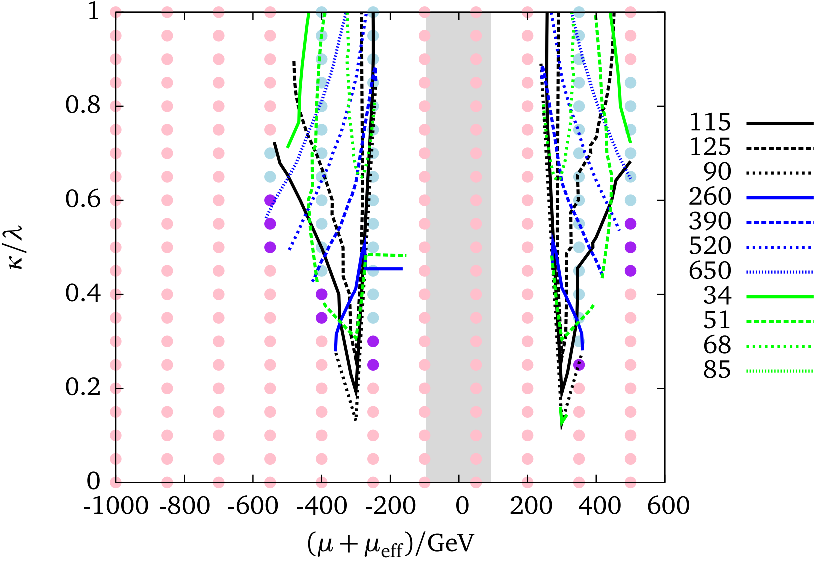
- only 5-5 elements depends on κ
- keep $\mu_\mathrm{eff} + \mu$ fixed
- rescale $\frac{\kappa}{\lambda}$ such that $\left(\mathcal{M}_\chi\right)_{55}$ stays the same

Liebler rescaling

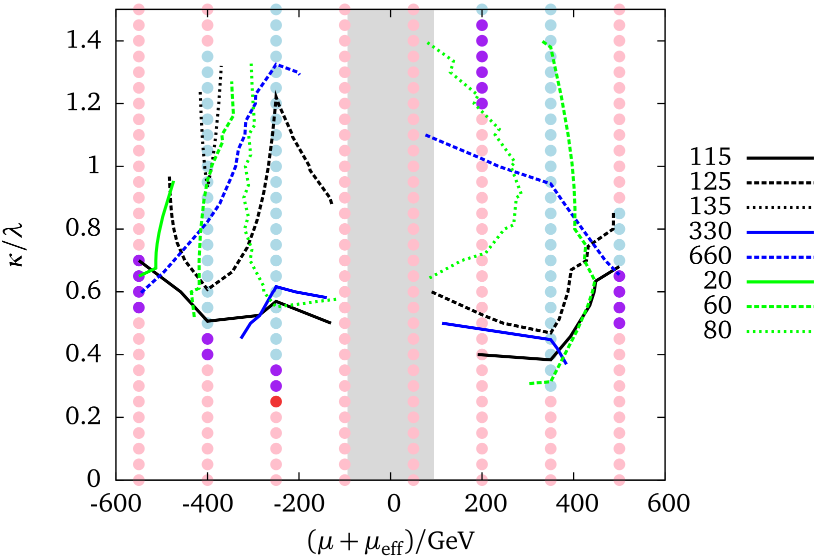


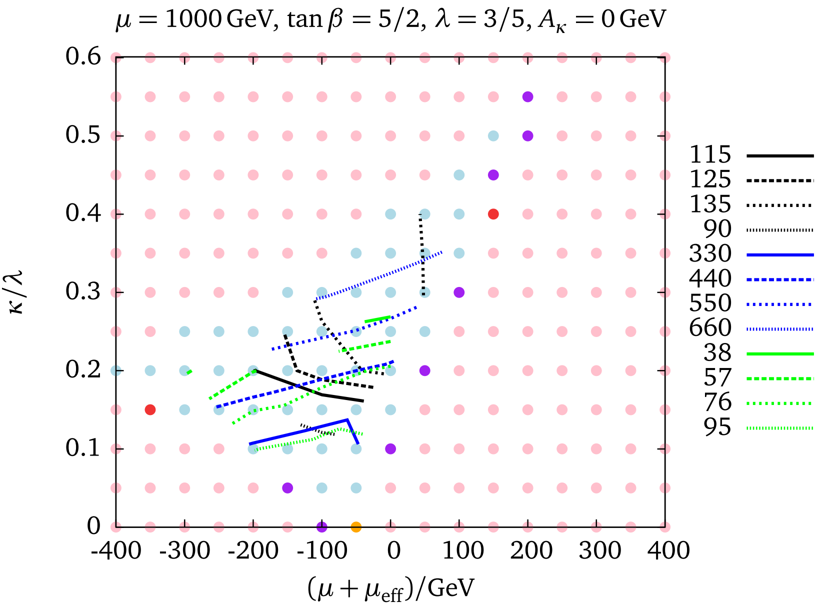
Boiling down the parameter space…

$\mu = 0\,\text{GeV},\ \tan\beta = 5/2,\ \lambda = 1/2,\ A_\kappa = 0\,\text{GeV}$

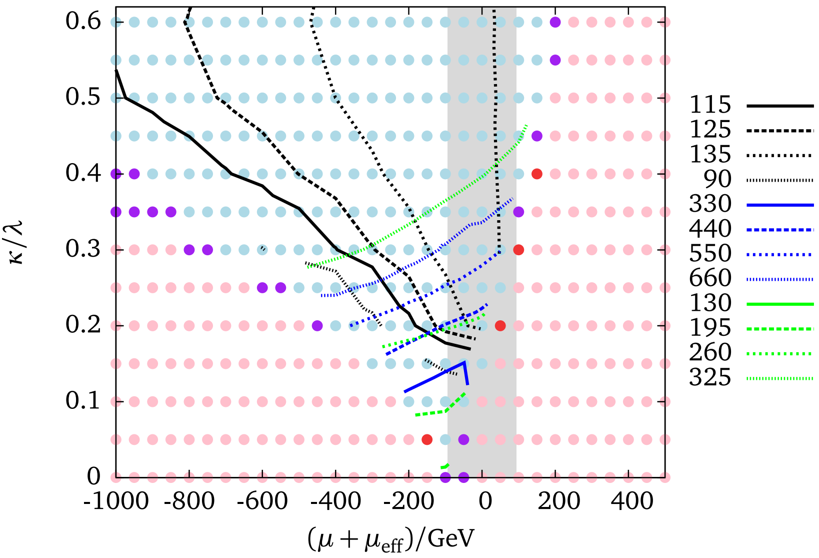


$\tan\beta = 5/2,\ \lambda = 3/5,\ A_\kappa = 0\,\mathrm{GeV}$

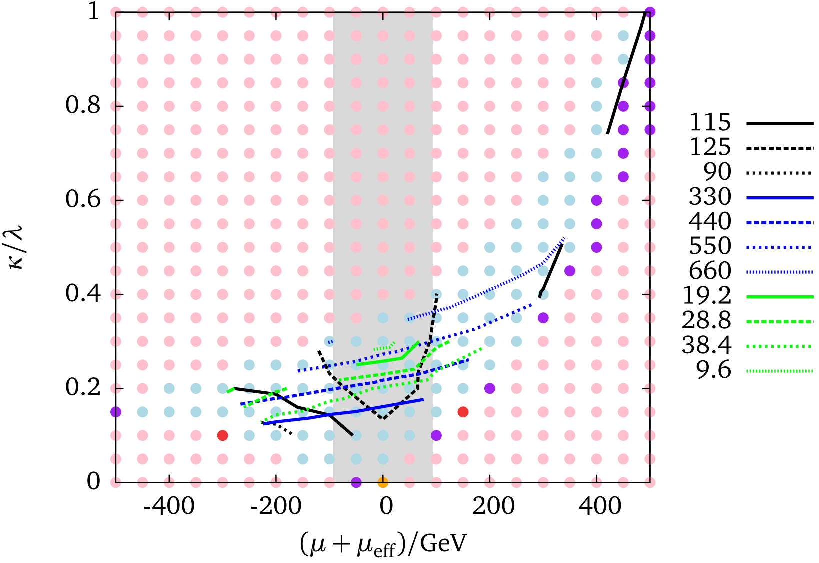


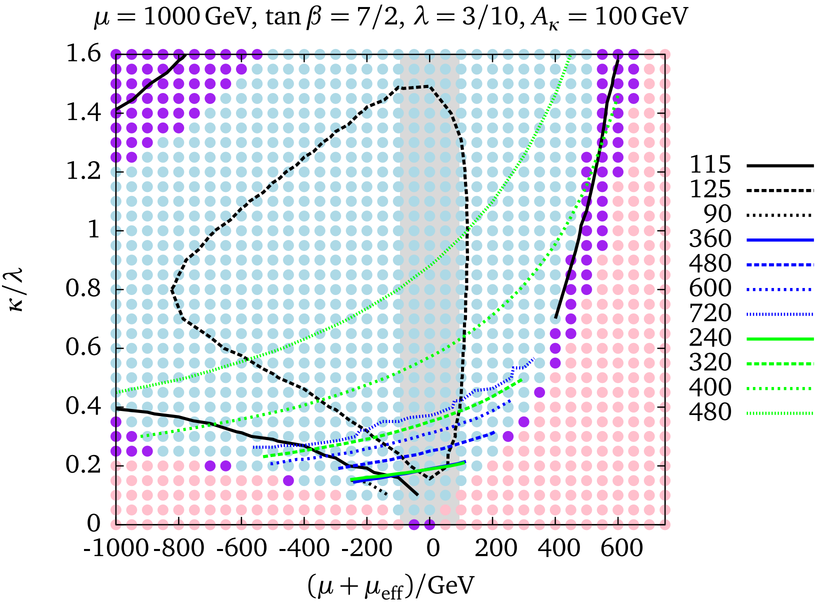

$\mu = 1000\,\text{GeV}, \tan\beta = 5/2, \lambda = 3/5, A_\kappa = 0\,\text{GeV}$
0.6
0.5
0.4
0.3
0.2
0.1
0
-400
-300
-200
-100
0
100
200
300
400
κ/λ
$(\mu + \mu_{\text{eff}})/\text{GeV}$
115
125
135
90
330
440
550
660
38
57
76
95

$\mu = 1000\,\mathrm{GeV}$, $\tan\beta = 5/2$, $\lambda = 3/5$, $A_\kappa = 100\,\mathrm{GeV}$



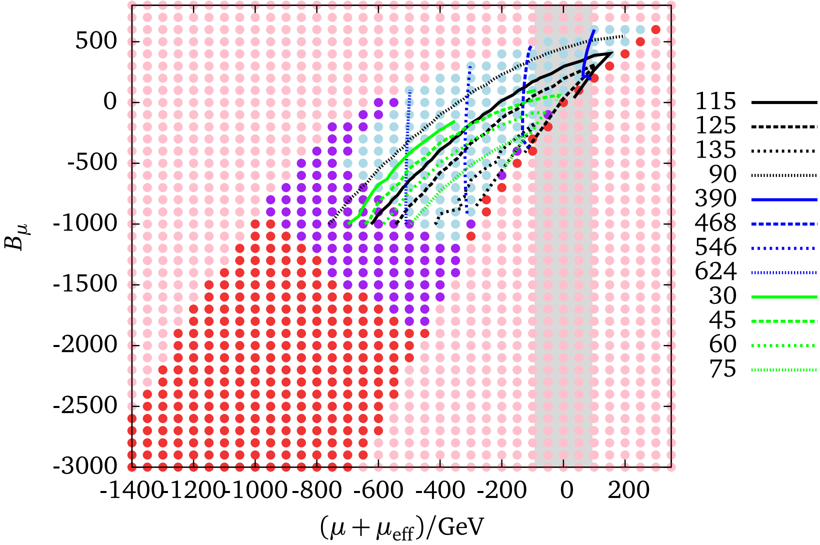
$\mu = 1000\,\text{GeV}$, $\tan\beta = 7/2$, $\lambda = 3/10$, $A_\kappa = 0\,\text{GeV}$

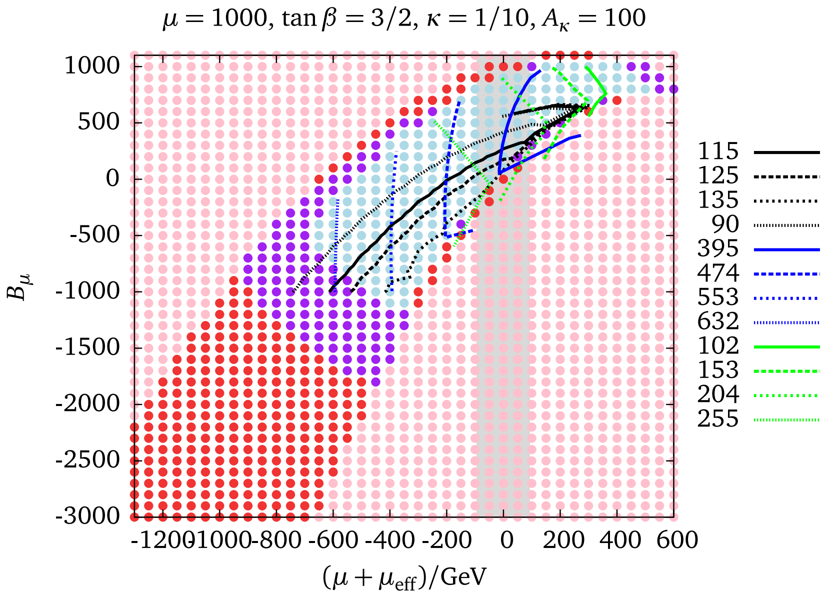



$\mu = 1000\,\mathrm{GeV}$, $\tan\beta = 7/2$, $\lambda = 3/10$, $A_\kappa = 100\,\mathrm{GeV}$
κ/λ
$(\mu + \mu_{\mathrm{eff}})/\mathrm{GeV}$
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1000
-800
-600
-400
-200
0
200
400
600
115
125
90
360
480
600
720
240
320
400
480

Additional soft $\mathbb{Z}_3$ breaking leads to severe instabilities.

$\mu = 1000, \tan\beta = 3/2, \kappa = 1/10, A_\kappa = 0$




$\mu = 1000, \tan\beta = 3/2, \kappa = 1/10, A_\kappa = 100$
B_μ
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
-1200
-1000
-800
-600
-400
-200
0
200
400
600
$(\mu + \mu_{\rm eff})/\mathrm{GeV}$
115
125
135
90
395
474
553
632
102
153
204
255

Higgs Inflation in the NMSSM

- the MSSM is not enough
- Singlet direction to stabilize inflationary trajectory
- inflaton formed out of doublet Higgses

A μ term from gravity

$$\mathcal{W}_{\text{iNMSSM}} = \mathcal{W}_{\text{NMSSM}} + \mu H_u \cdot H_d$$

Caveats and features

- tachyonic Higgs directions
- alternative vevs
- Neutralino sector different from NMSSM
- Higgs-to-Higgs decays! (not talked about)
- parameter region of light Higgsinos favoured!

Backup

Slides

Higgs self-couplings

$$\lambda_{111} = -\frac{3}{2}(g_1^2+g_2^2)c_\beta v \qquad \lambda_{112} = \frac{1}{2}(g_1^2+g_2^2)s_\beta v - 2\lambda^2 s_\beta v \qquad (1)$$

$$\lambda_{113} = -2\lambda(\mu_{\text{eff}}+\mu) \qquad \lambda_{122} = \frac{1}{2}(g_1^2+g_2^2)c_\beta v - 2\lambda^2 c_\beta v \qquad (2)$$

$$\lambda_{123} = A_\lambda \lambda + 2\kappa\mu_{\text{eff}} \qquad \lambda_{133} = -2\lambda^2 c_\beta v + 2\kappa\lambda s_\beta v \qquad (3)$$

$$\lambda_{222} = -\frac{3}{2}(g_1^2+g_2^2)s_\beta v \qquad \lambda_{223} = -2\lambda(\mu_{\text{eff}}+\mu) \qquad (4)$$

$$\lambda_{233} = -2\lambda^2 s_\beta v + 2\kappa\lambda c_\beta v \qquad \lambda_{333} = -2A_\kappa\kappa - 12\frac{\kappa}{\lambda}\mu_{\text{eff}} \qquad (5)$$

$$\lambda_{144} = -\frac{1}{2}(g_1^2+g_2^2)c_\beta v \qquad \lambda_{244} = \frac{1}{2}(g_1^2+g_2^2)s_\beta v - 2\lambda^2 s_\beta v \qquad (6)$$

$$\lambda_{344} = -2\lambda(\mu_{\text{eff}}+\mu) \qquad \lambda_{345} = -\lambda A_\lambda - 2\kappa\mu_{\text{eff}} \qquad (7)$$

$$\lambda_{155} = \frac{1}{2}(g_1^2+g_2^2)c_\beta v - 2\lambda c_\beta v \qquad \lambda_{255} = -\frac{1}{2}(g_1^2+g_2^2)s_\beta v \qquad (8)$$

$$\lambda_{355} = -2\lambda(\mu_{\text{eff}}+\mu) \qquad (9)$$