Charting the Vacuum Landscape in SUSY Benchmark Scenarios

relies on: JHEP 1903 (2019) 109 with J. Wittbrodt and G. Weiglein
and JHEP 08 (2016) 126 and PLB 752 (2016) 7–12

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The Standard Model (In)Stability

$$V_{\mathrm{SM}} = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2$$

- large field values: $V \sim \lambda(H^\dagger H)^2$
- RGE: $\lambda \to \lambda(Q)$, where $Q \sim H$
- $\lambda \to 0$ around $Q \sim 10^{10}\,\mathrm{GeV}$, new minimum beyond M_{Planck}

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The MSSM: less simple

$$V_{\mathrm{MSSM}} = V_F + V_{\mathrm{soft}} + V_D$$

with (only 3rd generation squarks and Higgses)

$$\begin{aligned} V_{\mathrm{soft}} = m_{H_{\mathrm{d}}}^2 |h_{\mathrm{d}}|^2 + m_{H_{\mathrm{u}}}^2 |h_{\mathrm{u}}|^2 - \left(B_\mu h_{\mathrm{d}} \cdot h_{\mathrm{u}} + \mathrm{h.\,c.}\right) \\ + \tilde{t}_{\mathrm{L}}^* \tilde{m}_Q^2 \tilde{t}_{\mathrm{L}} + \tilde{t}_{\mathrm{R}}^* \tilde{m}_t^2 \tilde{t}_{\mathrm{R}} + \tilde{b}_{\mathrm{L}}^* \tilde{m}_Q^2 \tilde{b}_{\mathrm{L}} + \tilde{b}_{\mathrm{R}}^* \tilde{m}_b \tilde{b}_{\mathrm{R}} \\ + \left(A_{\mathrm{t}} h_{\mathrm{u}} \tilde{t}_{\mathrm{L}}^* \tilde{t}_{\mathrm{R}} + A_{\mathrm{b}} h_{\mathrm{d}} \tilde{b}_{\mathrm{L}}^* \tilde{b}_{\mathrm{R}} + \mathrm{h.\,c.}\right) \end{aligned}$$

A multi-scalar theory

- 2 Higgs doublets
- 2×6 scalar quarks, $6 + 3$ scalar leptons
- 12 colored and 18 + 2 charged directions
- charged Higgs directions "safe" [Casas et al. 1996]
- SM Higgs potential: SO(4) symmetry

- large couplings to Higgs doublets (y_t and y_b comparably large)
- large stop contribution (X_t, A_t) to light Higgs mass needed
- SUSY threshold corrections for m_b influence y_b

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impossible!

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An analytic solution?

impossible! *only approximative*

$$
\begin{aligned}
V_{\tilde{q},h} = {} & \tilde{t}_L{}^* \left(\tilde{m}_L^2 + |y_t h_2|^2\right) \tilde{t}_L + \tilde{t}_R{}^* \left(\tilde{m}_t^2 + |y_t h_2|^2\right) \tilde{t}_R \\
& + \tilde{b}_L{}^* \left(\tilde{m}_L^2 + |y_b h_1|^2\right) \tilde{b}_L + \tilde{b}_R{}^* \left(\tilde{m}_b^2 + |y_b h_1|^2\right) \tilde{b}_R \\
& - \left[\tilde{t}_L{}^* (\mu^* y_t\, h_1{}^* - A_t h_2)\, \tilde{t}_R + \text{h.c.}\right] \\
& - \left[\tilde{b}_L{}^* (\mu^* y_b\, h_2{}^* - A_b h_1)\, \tilde{b}_R + \text{h.c.}\right] \\
& + |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \\
& + \frac{g_1^2}{8} \left(|h_2|^2 - |h_1|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2 + \frac{1}{3}|\tilde{t}_L|^2 - \frac{4}{3}|\tilde{t}_R|^2\right)^2 \\
& + \frac{g_2^2}{8} \left(|h_2|^2 - |h_1|^2 + |\tilde{b}_L|^2 - |\tilde{t}_L|^2\right)^2 \\
& + \frac{g_3^2}{8} \left(|\tilde{t}_L|^2 - |\tilde{t}_R|^2 + |\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2 \\
& + (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2\,\mathrm{Re}(B_\mu\, h_1 h_2).
\end{aligned}
$$

$$\begin{aligned}
V_{\tilde{q},h} = {} & \tilde{t}_L{}^* \left(\tilde{m}_L^2 + |y_t h_2|^2\right) \tilde{t}_L + \tilde{t}_R{}^* \left(\tilde{m}_t^2 + |y_t h_2|^2\right) \tilde{t}_R \\
& + \tilde{b}_L{}^* \left(\tilde{m}_L^2 + |y_b h_1|^2\right) \tilde{b}_L + \tilde{b}_R{}^* \left(\tilde{m}_b^2 + |y_b h_1|^2\right) \tilde{b}_R \\
& - \left[\tilde{t}_L{}^* (\mu^* y_t\, h_1{}^* - A_t h_2)\, \tilde{t}_R + \text{h.c.}\right] \\
& - \left[\tilde{b}_L{}^* (\mu^* y_b\, h_2{}^* - A_b h_1)\, \tilde{b}_R + \text{h.c.}\right] \\
& + |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \\
& + \frac{g_1^2}{8} \left(|h_2|^2 - |h_1|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2 + \frac{1}{3}|\tilde{t}_L|^2 - \frac{4}{3}|\tilde{t}_R|^2\right)^2 \\
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\end{aligned}$$

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|$, $|\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$

$$
\begin{aligned}
V_{\tilde{q},h} = {} & \tilde{t}^{\,*}\left(\tilde{m}_L^2 + |y_t h_2|^2\right)\tilde{t} + \tilde{t}^{\,*}\left(\tilde{m}_t^2 + |y_t h_2|^2\right)\tilde{t} \\
& + \tilde{b}^{\,*}\left(\tilde{m}_L^2 + |y_b h_1|^2\right)\tilde{b} + \tilde{b}^{\,*}\left(\tilde{m}_b^2 + |y_b h_1|^2\right)\tilde{b} \\
& - \left[\tilde{t}^{\,*}(\mu^* y_t\, h_1{}^* - A_t h_2)\tilde{t} + \text{h.c.}\right] \\
& - \left[\tilde{b}^{\,*}(\mu^* y_b\, h_2{}^* - A_b h_1)\tilde{b} + \text{h.c.}\right] \\
& + |y_t|^2|\tilde{t}|^2|\tilde{t}|^2 + |y_b|^2|\tilde{b}|^2|\tilde{b}|^2 \\
& + \frac{g_1^2}{8}\left(|h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2\right)^2 \\
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\end{aligned}
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& - \left[\tilde{t}^{\,*}(\mu^* y_t\, h_1{}^* - A_t h_2)\tilde{t} + \text{h.c.}\right] \\
& - \left[\tilde{b}^{\,*}(\mu^* y_b\, h_2{}^* - A_b h_1)\tilde{b} + \text{h.c.}\right] \\
& + |y_t|^2 |\tilde{t}|^2 |\tilde{t}|^2 + |y_b|^2 |\tilde{b}|^2 |\tilde{b}|^2 \\
& + \frac{g_1^2}{8}\left(|h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2\right)^2 \\
& + \frac{g_2^2}{8}\left(|h_2|^2 - |h_1|^2 + |\tilde{b}|^2 - |\tilde{t}|^2\right)^2 \\
& + (m_{h_2}^2 + |\mu|^2)|h_2|^2 + (m_{h_1}^2 + |\mu|^2)|h_1|^2 - 2\,\mathrm{Re}(B_\mu\, h_1 h_2).
\end{aligned}
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$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|$, $|\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$; $|\tilde{b}| = |h_1| = |\phi_1|$, $|\tilde{t}| = |h_2| = |\phi_2|$

$$
\begin{aligned}
V_{\tilde{q},h} = {} & \phi_2{}^* \left(\tilde{m}_L^2 + |y_t \phi_2|^2\right) \phi_2 + \phi_2{}^* \left(\tilde{m}_t^2 + |y_t \phi_2|^2\right) \phi_2 \\
& + \phi_1{}^* \left(\tilde{m}_L^2 + |y_b \phi_1|^2\right) \phi_1 + \phi_1{}^* \left(\tilde{m}_b^2 + |y_b \phi_1|^2\right) \phi_1 \\
& - \left[\phi_2{}^* (\mu^* y_t \, \phi_1{}^* - A_t \phi_2) \, \phi_2 + \text{h.c.} \right] \\
& - \left[\phi_1{}^* (\mu^* y_b \, \phi_2{}^* - A_b \phi_1) \, \phi_1 + \text{h.c.} \right] \\
& + |y_t|^2 |\phi_2|^2 |\phi_2|^2 + |y_b|^2 |\phi_1|^2 |\phi_1|^2
\end{aligned}
$$

$$
+ (m_{h_2}^2 + |\mu|^2)|\phi_2|^2 + (m_{h_1}^2 + |\mu|^2)|\phi_1|^2 - 2\,\mathrm{Re}(B_\mu \, \phi_1 \phi_2).
$$

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|$, $|\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$; $|\tilde{b}| = |h_1| = |\phi_1|$, $|\tilde{t}| = |h_2| = |\phi_2|$

$$V_{\tilde{q},h} = \phi_2{}^* \left(\tilde{m}_L^2 + |y_t \phi_2|^2\right) \phi_2 + \phi_2{}^* \left(\tilde{m}_t^2 + |y_t \phi_2|^2\right) \phi_2$$

$$-\left[\phi_2{}^* (\qquad\qquad -A_t \phi_2)\, \phi_2 + \text{h.c.}\right]$$

$$+ |y_t|^2 |\phi_2|^2 |\phi_2|^2$$

$$+ (m_{h_2}^2 + |\mu|^2) |\phi_2|^2 \qquad\qquad .$$

$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|$, $|\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|$; $\cancel{|\tilde{b}| = |h_1| = |\phi_1|}$, $|\tilde{t}| = |h_2| = |\phi_2|$

Minimize the potential

$$V(\phi) = m^2\phi^2 - A\phi^3 + \lambda\phi^4,$$

with $m^2 = m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2$, $A = -A_t$ and $\lambda = 3y_t^2$.

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Answer:

$$\phi_0 = 0, \qquad \phi_\pm = \frac{3A \pm \sqrt{9A^2 32\lambda m^2}}{8\lambda}.$$

Condition to be safe from non-standard (i.e. non-trivial) minima:

$$V(\phi_\pm) > 0 \quad \hookrightarrow \quad m^2 > \frac{A^2}{4\lambda}$$

Mathematics for the kindergarden

Minimize the potential

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$$V(\phi_\pm) > 0 \quad \hookrightarrow \quad m^2 > \frac{A^2}{4\lambda}$$

Well-known constraints [Gunion, Haber, Sher '88]

$$|A_t|^2 < 3y_t^2 \left(m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2\right)$$

$$|A_b|^2 < 3y_b^2 \left(m_{h_1}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_b^2\right)$$

for the limiting cases $|\tilde{t}_L| = |\tilde{t}_R| = |h_2|$ and $|\tilde{b}_L| = |\tilde{b}_R| = |h_1|$!

Problem solved?

Problem already known for a while

- problem noticed [Frere, Jones, Raby '83]
- "A-parameter bounds" [Gunion, Haber, Sher '87]
- classification of dangerous directions [Casas, Lleyda, Muñoz '96]
- including flavor violation [Casas and Dimopuolos '96]

Stability $\neq$ no Instability $\Rightarrow$ Metastability

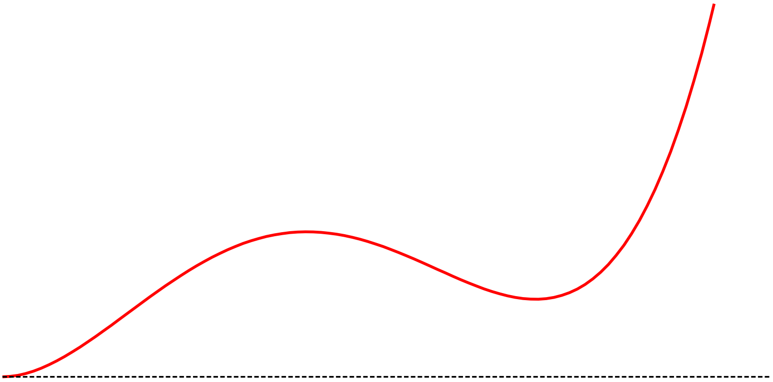
Vacuum tunneling [Kusenko, Langacker '96; Blinov, Morissey '13]

A tool

VeVacious [Camargo-Molina, O'Leary, Porod, Staub '13]

- finds all (?) tree-level minima
- minimizes scalar potential in the vicinity at one loop
- calculates bounce action / tunneling times [CosmoTransitions]

$A^2 < 4\lambda m^2$



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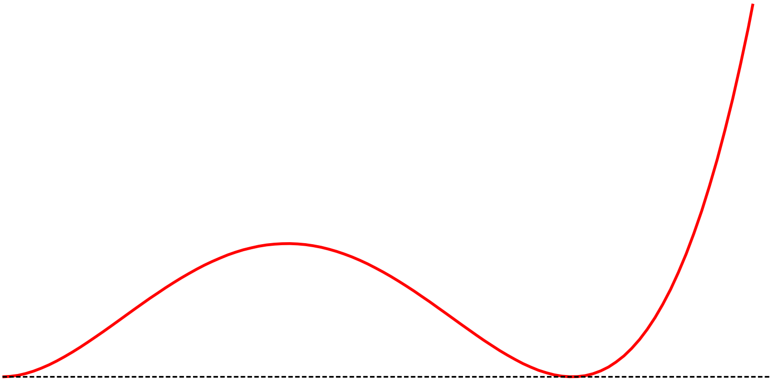

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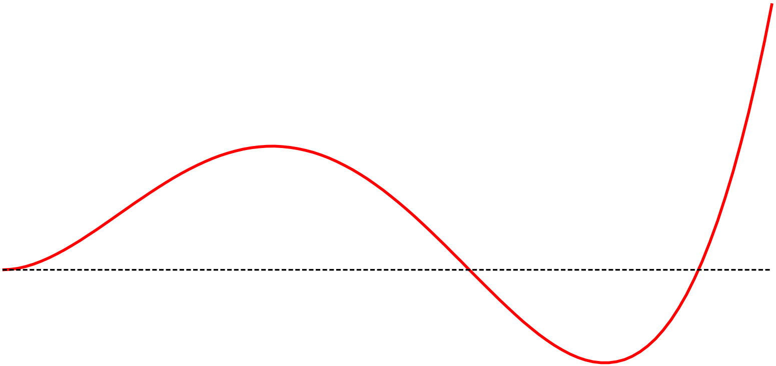

$$h_2 = \phi, \qquad |\tilde{t}| = \alpha|\phi|, \qquad h_1 = \eta\phi, \qquad |\tilde{b}| = \beta|\phi|$$

$$\begin{aligned}
V_\phi =& \Big(m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1+\eta^2)\mu^2 - 2B_\mu \eta \\
&\qquad + (\alpha^2 + \beta^2)\tilde{m}_L^2 + \alpha^2 \tilde{m}_t^2 + \beta^2 \tilde{m}_b^2\Big)\phi^2 \\
&- 2\left(\alpha^2(\mu y_t \eta - A_t) + \beta^2(\mu y_t - \eta A_b)\right)\phi^3 + (\alpha^2 y_t^2 + \beta^4 y_b^2)\phi^4 \\
&+ \left(\frac{g_1^2 + g_2^2}{8}(1 - \eta^2 + \beta^2 - \alpha^2)^2 + 2\alpha^2 y_t^2 + 2\beta^2 y_b^2\right)\phi^4 \\
\equiv& \boldsymbol{M^2(\eta, \alpha, \beta)\phi^2 - \mathcal{A}(\eta, \alpha, \beta)\phi^3 + \lambda(\eta, \alpha, \beta)\phi^4},
\end{aligned}$$

$$h_2 = \phi, \qquad |\tilde{t}| = \alpha|\phi|, \qquad h_1 = \eta\phi, \qquad |\tilde{b}| = \beta|\phi|$$

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&- 2\left(\alpha^2(\mu y_t\eta - A_t) + \beta^2(\mu y_t - \eta A_b)\right)\phi^3 + (\alpha^2 y_t^2 + \beta^4 y_b^2)\phi^4 \\
&+ \left(\frac{g_1^2+g_2^2}{8}(1-\eta^2+\beta^2-\alpha^2)^2 + 2\alpha^2 y_t^2 + 2\beta^2 y_b^2\right)\phi^4 \\
\equiv& \boldsymbol{M^2(\eta,\alpha,\beta)\phi^2 - \mathcal{A}(\eta,\alpha,\beta)\phi^3 + \lambda(\eta,\alpha,\beta)\phi^4},
\end{aligned}$$

with

$$\begin{aligned}
M^2 =&\ m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1+\eta^2)\mu^2 - 2B_\mu\eta \\
&+ (\alpha^2+\beta^2)\tilde{m}_L^2 + \alpha^2\tilde{m}_t^2 + \beta^2\tilde{m}_b^2\,, \\
\mathcal{A} =&\ 2\alpha^2\eta\mu y_t - 2\alpha^2 A_t + 2\beta^2\mu y_b - 2\eta\beta^2 A_b\,, \\
\lambda =&\ \frac{g_1^2+g_2^2}{8}(1-\eta^2+\beta^2-\alpha^2)^2 \\
&+ (2+\alpha^2)\alpha^2 y_t^2 + (2\eta^2+\beta^2)\beta^2 y_b^2\,.
\end{aligned}$$

[Gunion, Haber, Sher '88; Casas, Lleyda, Muñoz '96]

The same but different ("A-parameter bounds")

$$\mathcal{A}^2 < 4\lambda M^2$$

$$\downarrow$$

$$4 \min_{\{\eta,\alpha,\beta\}} \lambda(\eta,\alpha,\beta) M^2(\eta,\alpha,\beta) > \max_{\{\eta,\alpha,\beta\}} (\mathcal{A}(\eta,\alpha,\beta))^2$$

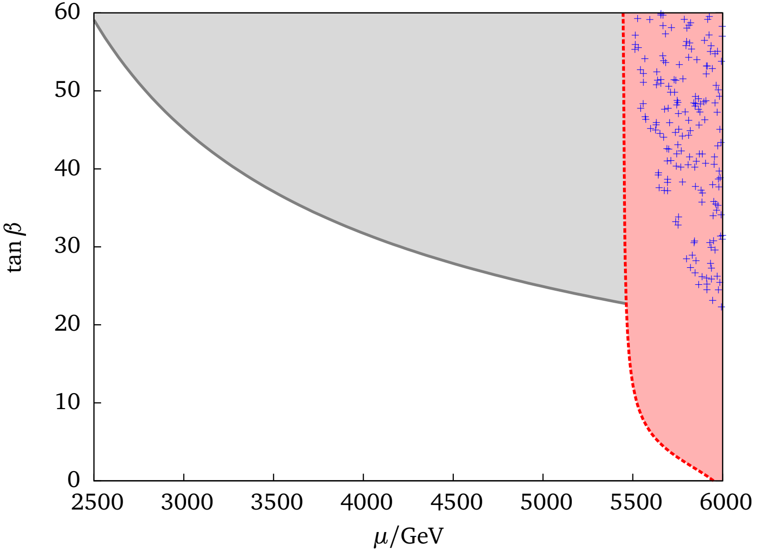
$h_u = \tilde{b}$, $h_d^0 = 0$ [WGH'15]

$$m_{H_u}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu y_b)^2}{y_b^2 + (g_1^2 + g_2^2)/2}$$

$|h_d|^2 = |h_u|^2 + |\tilde{b}|^2$, $\tilde{b} = \alpha h_u$ [WGH'15]

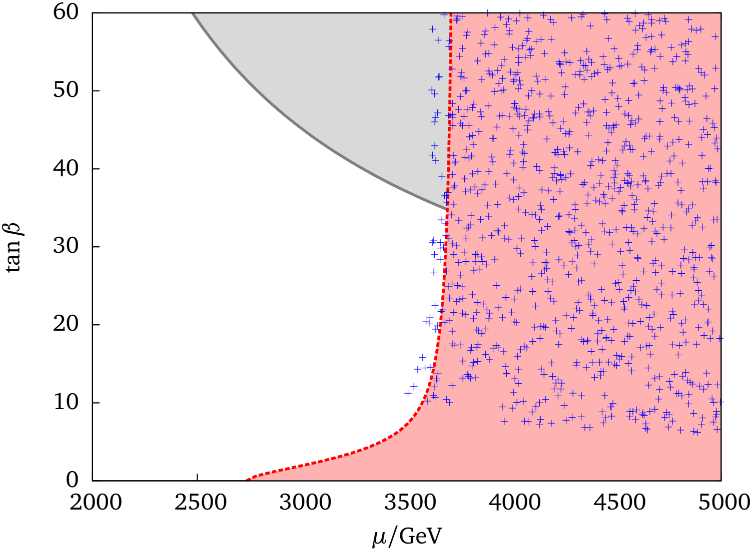
$$m_{11}^2(1+\alpha^2) + m_{22}^2 \pm 2m_{12}^2\sqrt{1+\alpha^2} + \alpha^2(\tilde{m}_Q^2 + \tilde{m}_b^2) > \frac{4\mu^2\alpha^2}{2+3\alpha^2}$$

[WGH: PLB752 7 (2016)]



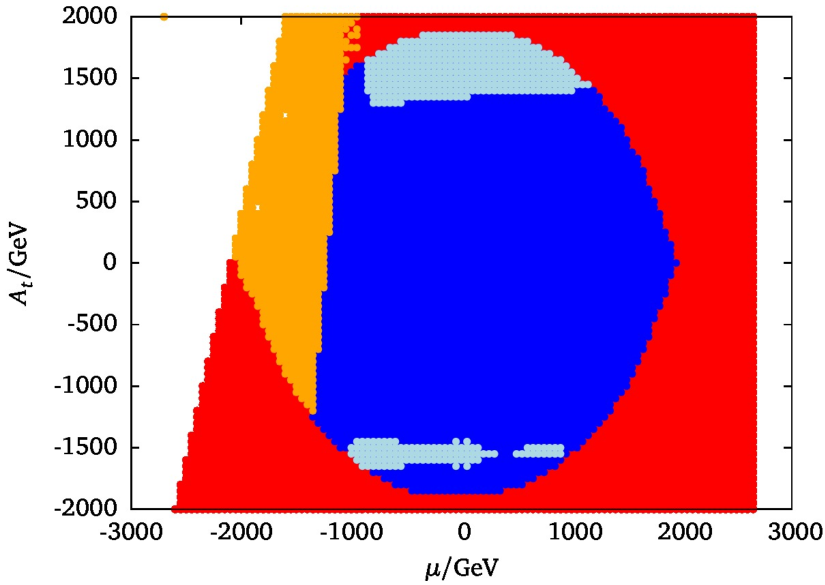
CCB sbottom vev, $h_d = -\sqrt{|h_u|^2 + |\tilde{b}|^2}$

[WGH: PLB752 7 (2016)]

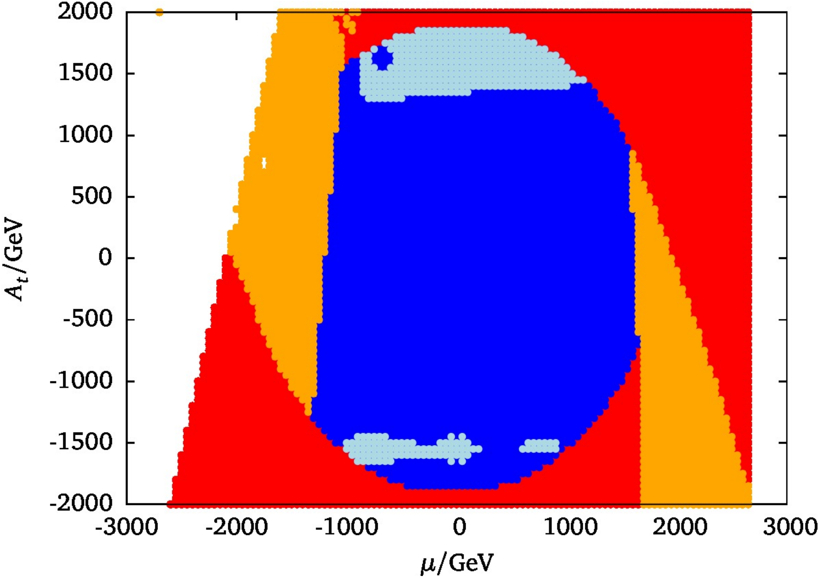


CCB sbottom vev, $h_d = +\sqrt{|h_u|^2 + |\tilde{b}|^2}$

- one field direction is not enough
- one type of vevs is not enough
- choice of trilinear couplings (both μ and $A_{t,b}$) crucial
- numerical minimization
- Don't underestimate bottom Yukawa corrections!

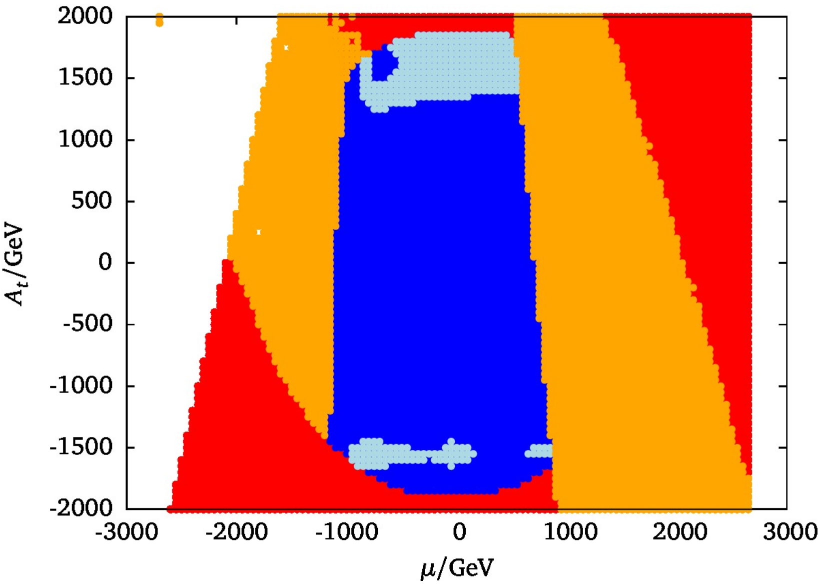

2000
1500
1000
500
0
-500
-1000
-1500
-2000
A_t/GeV
-3000
-2000
-1000
0
1000
2000
3000
μ/GeV

[WGH 2016]

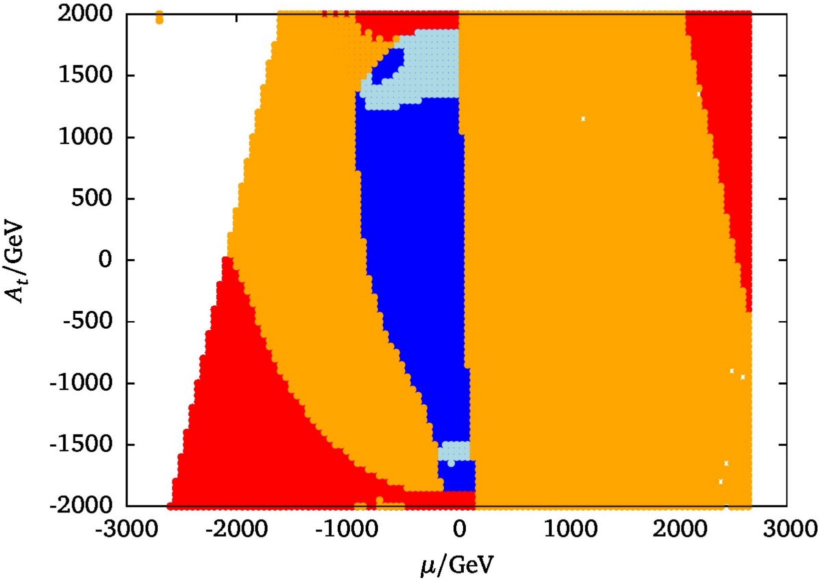

2000
1500
1000
500
0
-500
-1000
-1500
-2000
A_t/GeV
-3000
-2000
-1000
0
1000
2000
3000
μ/GeV

[WGH 2016]

Closing in on the parameter space $A_b = 1000\,\mathrm{GeV}$



[WGH 2016]



2000
1500
1000
500
0
-500
-1000
-1500
-2000
A_t/GeV
-3000
-2000
-1000
0
1000
2000
3000
μ/GeV

[WGH 2016]

simple considerations (not stress tested!)

- assume $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$
- needs certainly large A_t to get m_{h^0} right
- near criticality

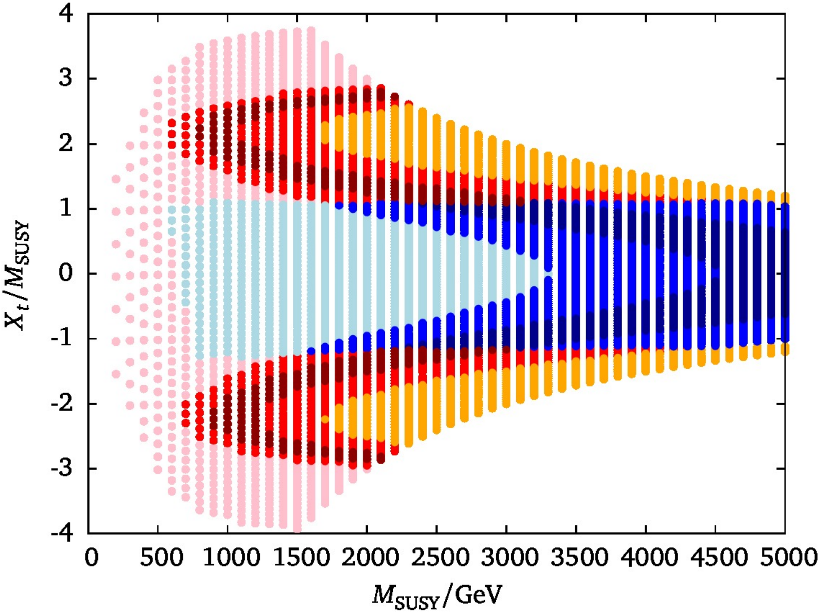
The MSSM is still alive! status quo 2016

Some observations:

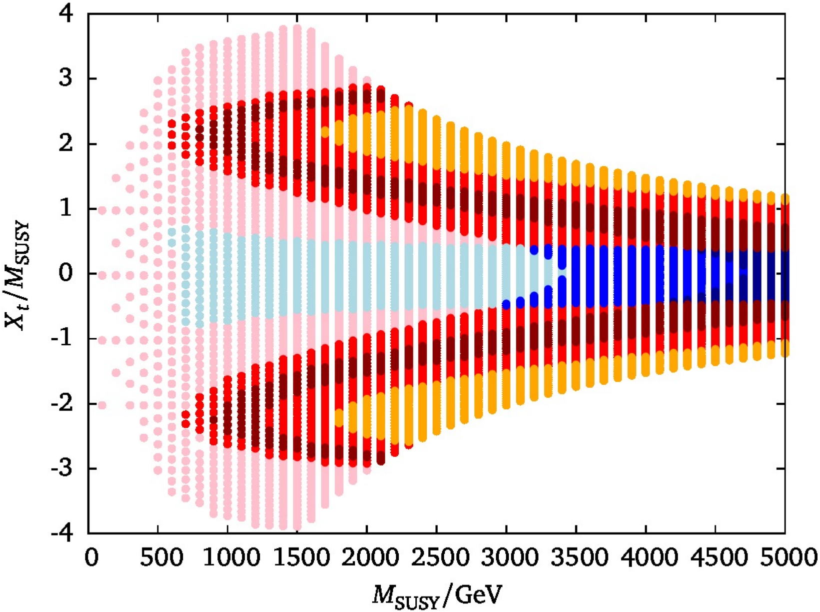
- constraints get weaker with increasing M_{SUSY}
- crucial: X_t/M_{SUSY} $\quad X_t = A_t/y_t - \mu \cot\beta$
- estimate: $X_t/M_{\mathrm{SUSY}} \lesssim 1$ (modifications possible, see next)
- μ negative opens up space: reduces X_t compared to A_t

Higgs mass band easier to catch for $M_{\mathrm{SUSY}} \gtrsim 2\,\mathrm{TeV}$

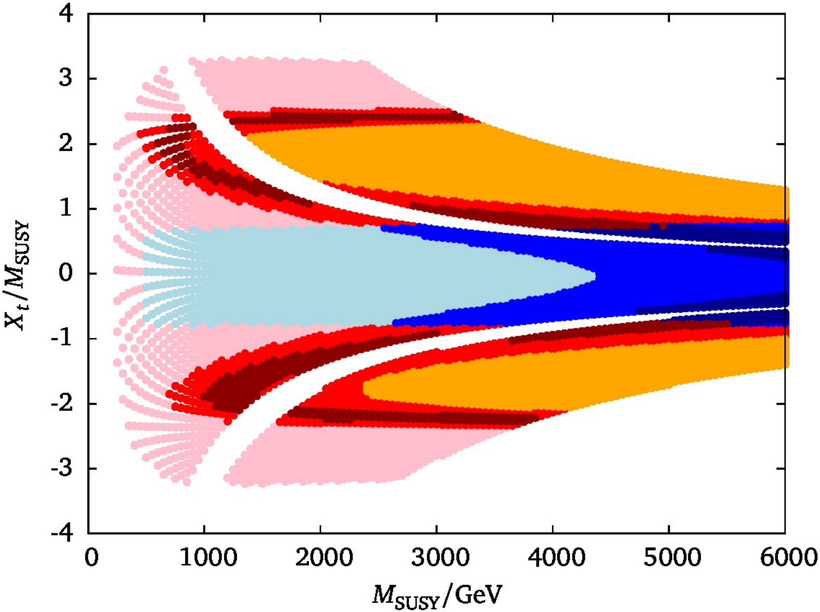
see next slides, compilation with SPheno and FeynHiggs



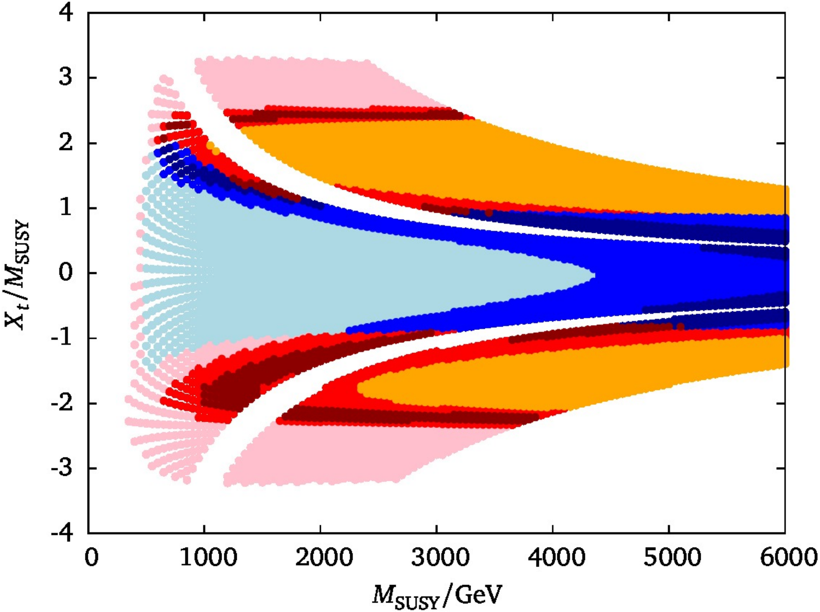
4
3
2
1
0
-1
-2
-3
-4
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
X_t/M_{SUSY}
$M_{\mathrm{SUSY}}/\mathrm{GeV}$



X_t/M_{SUSY}
$M_{\text{SUSY}}/\text{GeV}$
4
3
2
1
0
-1
-2
-3
-4
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000



4
3
2
1
0
-1
-2
-3
-4
X_t/M_{SUSY}
0
1000
2000
3000
4000
5000
6000
$M_{\mathrm{SUSY}}/\mathrm{GeV}$



4
3
2
1
0
-1
-2
-3
-4
X_t/M_{SUSY}
0
1000
2000
3000
4000
5000
6000
$M_{\mathrm{SUSY}}/\mathrm{GeV}$

A comment on metastability and quantum tunneling

Cosmological stability

bounce action

$$B \gtrsim 400$$

$\hookrightarrow$ life-time longer than age of the universe

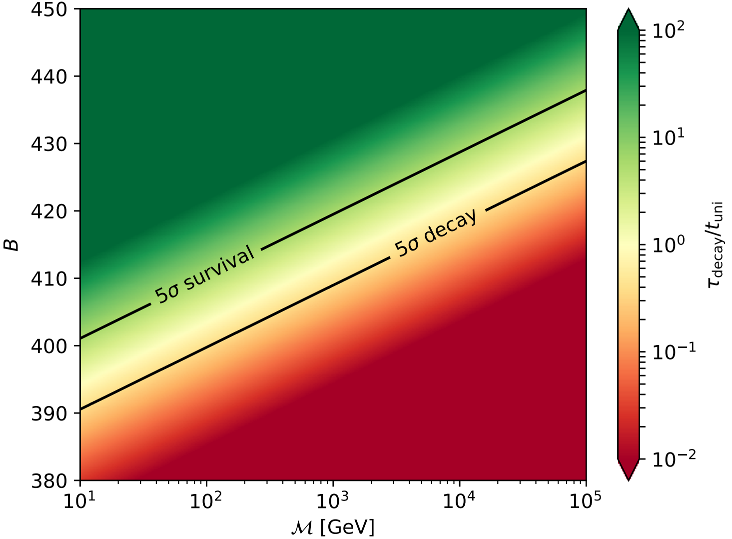
Decay probability (per unit volume)

$$\frac{\Gamma}{V} = A e^{-B/\hbar}$$

[Coleman '77]

Death and doom

- value of B crucially depends on field space path
- multifield spaces: reduction to single field space (!)
- *independent* of SUSY parameter choice


450
440
430
420
410
400
390
380
B
10^1
10^2
10^3
10^4
10^5
$\mathcal{M}$ [GeV]
5σ survival
5σ decay
10^2
10^1
10^0
10^{-1}
10^{-2}
$\tau_{\mathrm{decay}}/t_{\mathrm{uni}}$

A general n scalar potential

$$V(\vec{\phi}) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c$$

- includes up to 3^n stationary points
- initial vacuum at $\vec{\phi} = \vec{\phi}_v$

$$\left.\frac{\partial V}{\partial \phi_a}\right|_{\vec{\phi}=\vec{\phi}_v} = 0$$

Expanding around the vacuum

- $\vec{\phi} = \vec{\phi}_v + \vec{\varphi}$, with $\vec{\varphi} = (\varphi_1, \ldots, \varphi_n)^{\mathrm{T}}$

$$V(\vec{\varphi}) = \lambda'_{abcd}\varphi_a\varphi_b\varphi_c\varphi_d + A'_{abc}\varphi_a\varphi_b\varphi_c + m'^2_{ab}\varphi_a\varphi_b$$

- rewrite $\vec{\varphi} \to \varphi\hat{\varphi}$ with unit vector $\hat{\varphi}$, $\varphi = \sqrt{\varphi_1^2 + \ldots + \varphi_n^2}$

$$V(\varphi, \hat{\varphi}) = \lambda(\hat{\varphi})\varphi^4 - A(\hat{\varphi})\varphi^3 + m^2(\hat{\varphi})\varphi^2$$

A general n scalar potential

$$V(\vec{\phi}) = \lambda_{abcd}\phi_a\phi_b\phi_c\phi_d + A_{abc}\phi_a\phi_b\phi_c + m^2_{ab}\phi_a\phi_b + t_a\phi_a + c$$

- includes up to 3^n stationary points
- initial vacuum at $\vec{\phi} = \vec{\phi}_v$

$$\left.\frac{\partial V}{\partial \phi_a}\right|_{\vec{\phi}=\vec{\phi}_v} = 0$$

A semi-analytic approximation

A quartic potential: $V(\phi) = \lambda\,\phi^4 - A^2\,\phi^3 + m^2\,\phi^2$

$$B = \frac{\pi^2}{3\lambda}(2-\delta)^{-3}\left(13.832\,\delta - 10.819\,\delta^2 + 2.0765\,\delta^3\right)$$

with

$$\delta = \frac{8\lambda m^2}{A^2}$$

[Adams 1993]

Reduction to a single real scalar field

$$\phi \to \frac{1}{\sqrt{2}} \mathrm{Re}(\phi) + \frac{i}{\sqrt{2}} \mathrm{Im}(\phi)$$

- φ is canonically normalised after expanding $\vec{\varphi} = \varphi \hat{\varphi}$
- EW vacuum is given by

$$\mathrm{Re}(h_u^0) = v \sin\beta \,, \quad \mathrm{Re}(h_d^0) = v \cos\beta$$

where $v = \sqrt{v_u^2 + v_d^2} \approx 246\,\mathrm{GeV}$ is the SM Higgs vev

Unfeasible to vary all real scalar degrees of freedom simultaneously $\hookrightarrow$ selection of fields

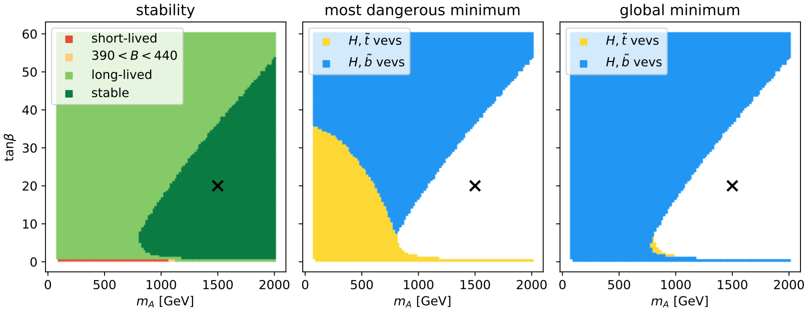
$$\{\mathrm{Re}(h_u^0), \mathrm{Re}(h_d^0), \mathrm{Re}(\tilde{t}_L), \mathrm{Re}(\tilde{t}_R), \mathrm{Re}(\tilde{b}_L), \mathrm{Re}(\tilde{b}_R)\}$$
$$\{\mathrm{Re}(h_u^0), \mathrm{Re}(h_d^0), \mathrm{Re}(\tilde{t}_L), \mathrm{Re}(\tilde{t}_R), \mathrm{Re}(\tilde{\tau}_L), \mathrm{Re}(\tilde{\tau}_R)\}$$
$$\{\mathrm{Re}(h_u^0), \mathrm{Re}(h_d^0), \mathrm{Re}(\tilde{b}_L), \mathrm{Re}(\tilde{b}_R), \mathrm{Re}(\tilde{\tau}_L), \mathrm{Re}(\tilde{\tau}_R)\}$$

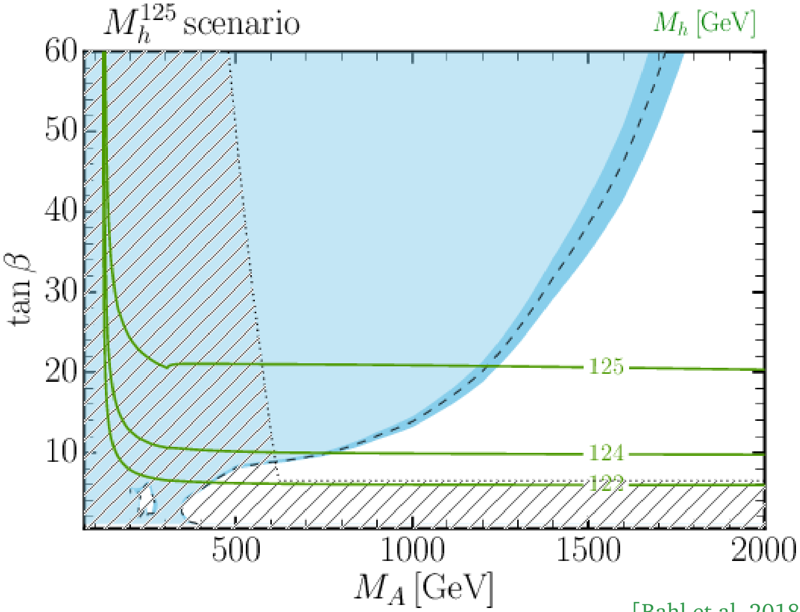
Benchmark scenario M_h^{125} [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 1.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,, \quad \mu = 1\,\text{TeV}\,,$$

$$X_t = A_t - \frac{\mu}{\tan\beta} = 2.8\,\text{TeV}\,, \quad A_b = A_\tau = A_t\,,$$

$$M_1 = M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$



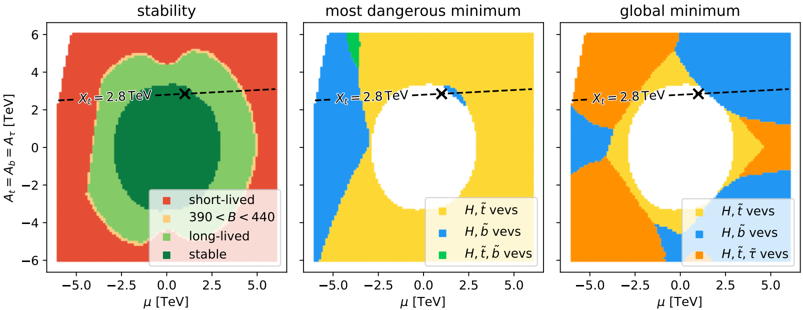


[Bahl et al. 2018]

An absolutely stable and experimentally allowed point

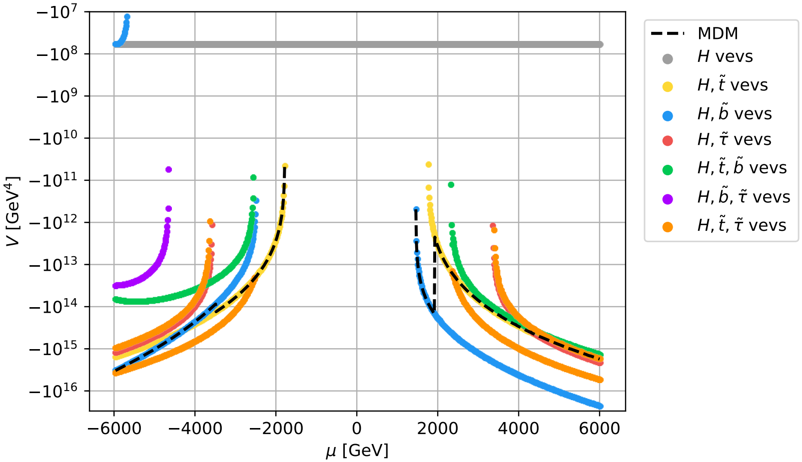
$$\tan\beta = 20 \qquad m_A = 1500\,\text{GeV}$$

$$A \equiv A_t = A_b = A_\tau$$



- tachyonic sbottom masses upper left corner
- caveat: still limited numbers of fields included!

How is the "most dangerous minimum" (MDM) defined?
Go along the dashed line with $X_t = 2.8\,\mathrm{TeV}$



A vast set of constraints

$$A_t^2 + 3\mu^2 < (m_{\tilde{t}_R}^2 + m_{\tilde{t}_L}^2) \cdot \begin{cases} 3 & \text{stable,} \\ 7.5 & \text{long-lived.} \end{cases}$$

[Casas, Lleyda, Muñoz 1996, Kusenko, Langacker, Segre 1996]

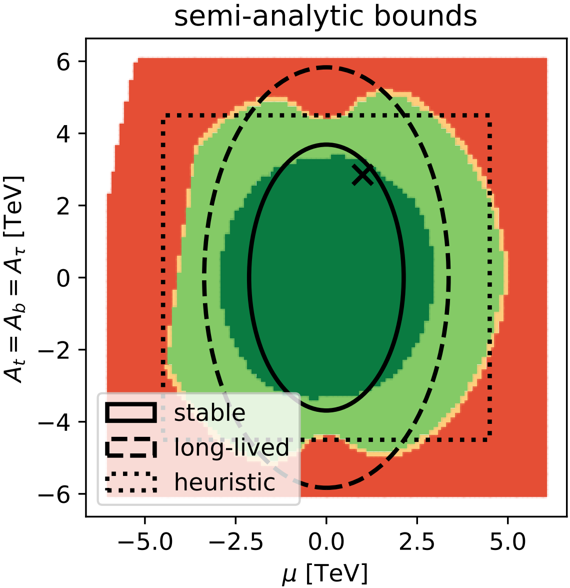
Furthermore, a "heuristic" bound of

$$\frac{\max(A_{\tilde{t},\tilde{b}}, \mu)}{\min(m_{Q_3,U_3})} \lesssim 3$$

exists. [Bechtle, Haber, Heinemeyer, Stefaniak, Stål, Weiglein, Zeune 2016]

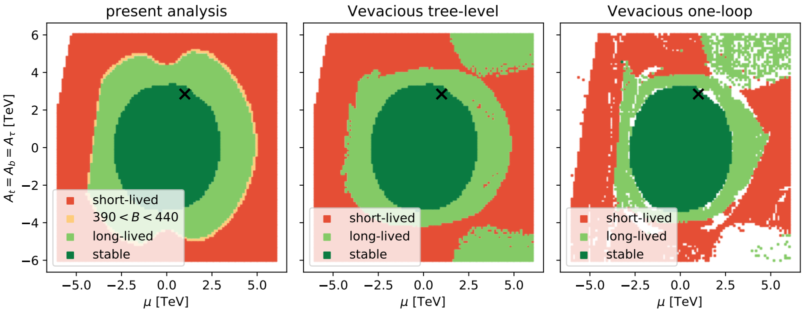
- vacuum tunneling weakens the "traditional" constraint
- metastable vacuum vs. absolute minimum
- quick and dirty versus sophisticated and precise (i. e. slow)

  $\hookrightarrow$ needs numerical evaluation!


semi-analytic bounds
$A_t = A_b = A_\tau$ [TeV]
6
4
2
0
−2
−4
−6
−5.0
−2.5
0.0
2.5
5.0
μ [TeV]
stable
long-lived
heuristic

- inclusion of one-loop effective potential
- thermal corrections
- quantum tunneling by `CosmoTransitions` [Wainwright 2011]

`Vevacious` [Camargo-Molina, O'Leary, Porod, Staub 2013]



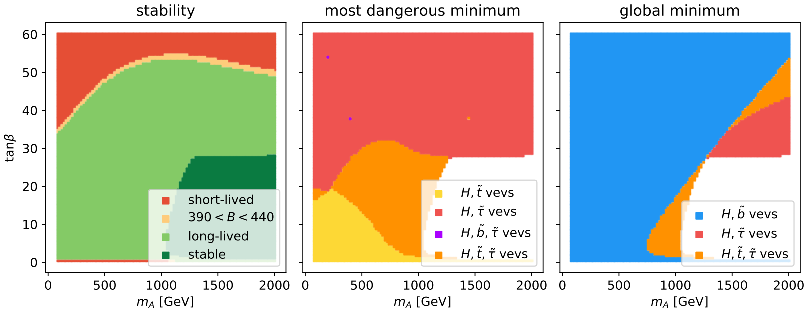
[WGH, Weiglein, Wittbrodt 2019]

Benchmark scenario $M_h^{125}(\tilde{\tau})$: light stau [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 1.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 350\,\text{GeV}\,, \quad \mu = 1\,\text{TeV}\,,$$

$$X_t = A_t - \frac{\mu}{\tan\beta} = 2.8\,\text{TeV}\,, \quad A_b = A_t\,, \quad A_\tau = 800\,\text{GeV}\,,$$

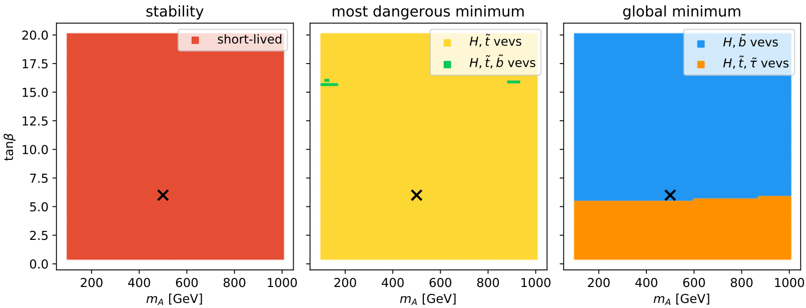
$$M_1 = M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$



large $\tan\beta$ values: short-lived

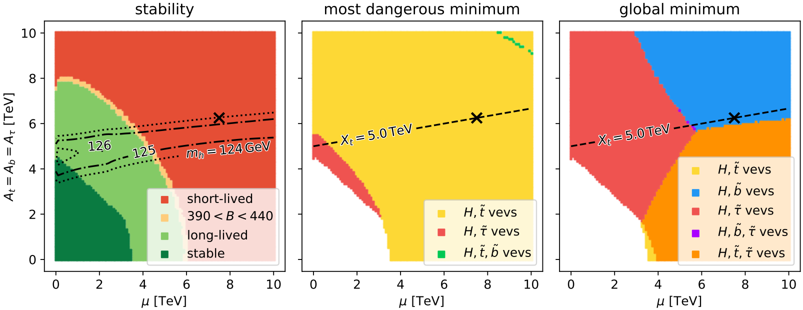
Benchmark scenario M_h^{125}(alignment) [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 2.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,,$$
$$\mu = 7.5\,\text{TeV}\,, \quad A_t = A_b = A_\tau = 6.25\,\text{TeV}\,,$$
$$M_1 = 500\,\text{GeV}\,, \quad M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$



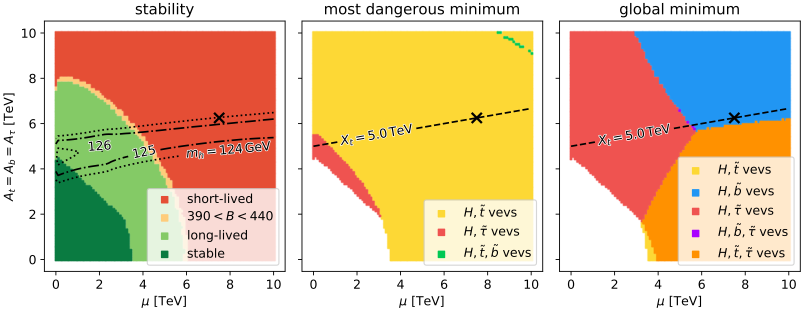
Benchmark scenario M_h^{125}(alignment) [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 2.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,,$$
$$\mu = 7.5\,\text{TeV}\,, \quad A_t = A_b = A_\tau = 6.25\,\text{TeV}\,,$$
$$M_1 = 500\,\text{GeV}\,, \quad M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$

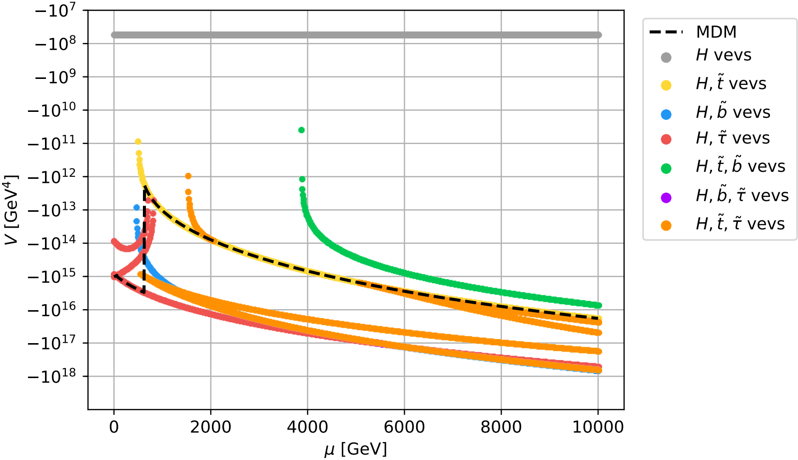


Benchmark scenario M_h^{125}(alignment) [Bahl et al. 2018]

$$m_{Q_3} = m_{U_3} = m_{D_3} = 2.5\,\text{TeV}\,, \quad m_{L_3} = m_{E_3} = 2\,\text{TeV}\,,$$
$$\mu = 7.5\,\text{TeV}\,, \quad A_t = A_b = A_\tau = 6.25\,\text{TeV}\,,$$
$$M_1 = 500\,\text{GeV}\,, \quad M_2 = 1\,\text{TeV}\,, \quad M_3 = 2.5\,\text{TeV}$$



$$\mu = 7.5\,\text{TeV} \rightarrow 4\,\text{TeV} \quad \text{and} \quad A = 6.25\,\text{TeV} \rightarrow 5\,\text{TeV}$$


-10^7
-10^8
-10^9
-10^{10}
-10^{11}
-10^{12}
-10^{13}
-10^{14}
-10^{15}
-10^{16}
-10^{17}
-10^{18}
V [GeV^4]
0
2000
4000
6000
8000
10000
μ [GeV]
MDM
H vevs
$H, \tilde{t}$ vevs
$H, \tilde{b}$ vevs
$H, \tilde{\tau}$ vevs
$H, \tilde{t}, \tilde{b}$ vevs
$H, \tilde{b}, \tilde{\tau}$ vevs
$H, \tilde{t}, \tilde{\tau}$ vevs

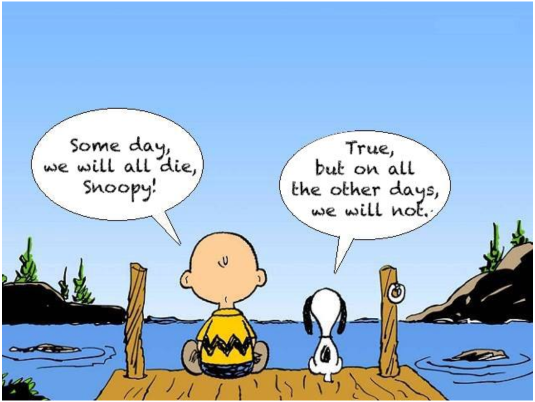
A new tool

[J. Wittbrodt, https://gitlab.com/jonaswittbrodt/EVADE]

- c++-11 library
- finds *all* minima via homotopy continuation: HOM4PS and/or Bertini (for details see documentation)
- up to six different fields/vevs: tree-level only
- new models via Mathematica
- vacuum tunneling approximation: tree-level only
- crucial(!): degenerate minima may have different lifetime
- finds "*most dangerous*" minimum

- tested against VeVacious: numerically more stable
- no significant difference between tree level and one loop
- "only" tree-level implementation in EVADE (sufficient...)
- one-loop tunneling inconsistent [Andreassen et al. 2016/17]

- severe constraints from vacuum (meta)stability in SUSY
- fast and numerically stable approach (EVADE)
- global minimum not the “most dangerous” one
- tree-level analysis sufficient (in comparison with 1-loop)



Backup

Slides

False vacuum decay

The bounce solution [Coleman 1977]

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2}+\frac{3}{\phi}\frac{\mathrm{d}\phi}{\mathrm{d}\rho}=\frac{\partial U}{\partial\phi}$$

with boundary conditions

$$\phi(\infty)=\phi_v\,,\qquad \left.\frac{\mathrm{d}\phi}{\mathrm{d}\rho}\right|_{\rho=0}=0$$

U is the euclidean scalar potential, ρ is a spacetime variable and ϕ_v is the location of the metastable minimum.

The bounce action B is the stationary point of the euclidean action given by the integral

$$B=2\pi^2\int_0^\infty\rho^3\,\mathrm{d}\rho\left[\frac{1}{2}\left(\frac{\mathrm{d}}{\mathrm{d}\rho}\phi_B(\rho)\right)^2+U(\phi_B(\rho))\right]$$

Decay rate of the metastable vacuum

$$\frac{\Gamma}{V_S} = K\, e^{-B} \qquad \text{per spatial volume } V_S$$

What does it mean?

Coefficient K barely calculable. Rough estimate: dimensionful parameter, $[K] = \mathrm{GeV}^4$, typical scale $\mathcal{M}$ of the theory

$$K = \mathcal{M}^4$$

- compare vacuum decay time τ_{decay} with age of universe t_{uni}

$$\frac{\tau_{\mathrm{decay}}}{t_{\mathrm{uni}}} = \left(\frac{\Gamma}{V_S}\right)^{-\frac{1}{4}} \frac{1}{t_{\mathrm{uni}}} = \frac{1}{t_{\mathrm{uni}}\mathcal{M}} e^{B/4}$$

- highly sensitive to B, only mildly sensitive to $\mathcal{M}$

$$P = \exp\left(-\frac{\Gamma}{V_S}\tilde{V}_{\text{light-cone}}\right) = \exp\left(-\mathcal{M}^4\,\tilde{V}_{\text{light-cone}}\,e^{-B}\right)$$

- (spacetime) volume of the past light-cone

$$\tilde{V}_{\text{light-cone}} \sim 0.15/H_0^4$$

- H_0 is the current value of the Hubble parameter

Error estimate

Vary scale $\mathcal{M} \in [10\,\text{GeV}, 100\,\text{TeV}]$: shift less than 10% in B.

- $B > 440$ long-lived
- $B < 390$ short-lived
- $390 < B < 440$ uncertainty on the stability threshold

Yukawa coupling not given directly by the mass

$$y_b = \frac{m_b}{v_d(1+\Delta_b)}$$

$$\Delta_b^{\text{gluino}} = \frac{2\alpha_s}{3\pi}\mu M_{\tilde{G}} \tan\beta \, C_0(\tilde{m}_{\tilde{b}_1}, \tilde{m}_{\tilde{b}_2}, M_{\tilde{G}}),$$
$$\Delta_b^{\text{higgsino}} = \frac{Y_t^2}{16\pi^2}\mu A_t \tan\beta \, C_0(\tilde{m}_{\tilde{t}_1}, \tilde{m}_{\tilde{t}_2}, \mu).$$