

To Higgs or not to Higgs  
vacuum stability beyond the Standard Model



Wolfgang Gregor Hollik

DESY Hamburg, Theory Group

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## The Standard Model (In)Stability

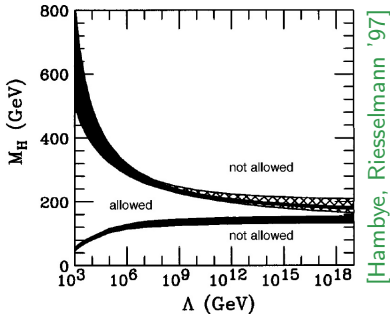
$$V_{\text{SM}} = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

- large field values:  $V \sim \lambda(H^\dagger H)^2$
- RGE:  $\lambda \rightarrow \lambda(Q)$ , where  $Q \sim H$
- $\lambda \rightarrow 0$  around  $Q \sim 10^{10}$  GeV, new minimum beyond  $M_{\text{Planck}}$

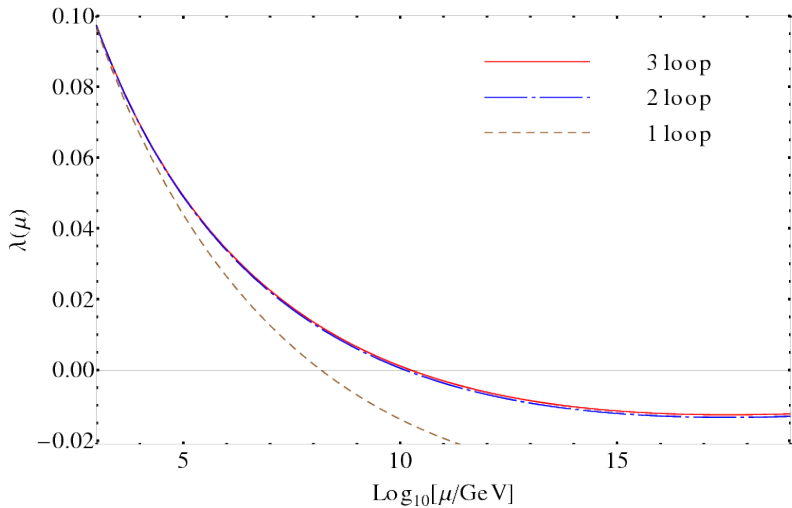
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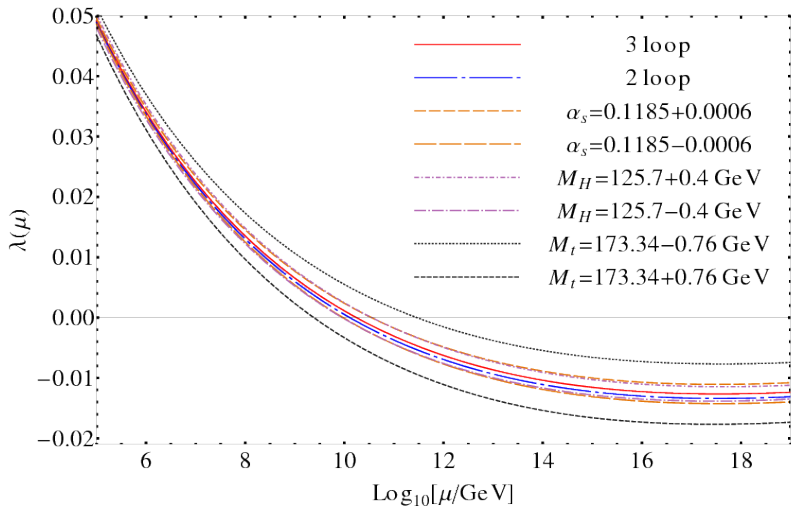


# Precise analysis: up to three loops!



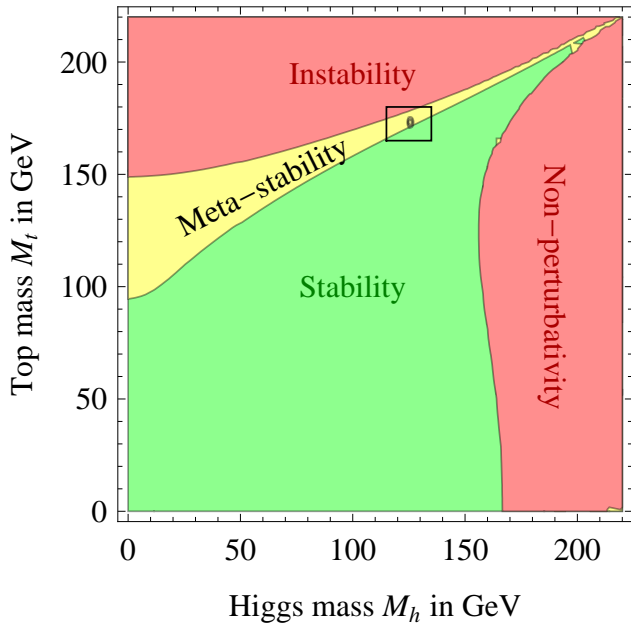
[Zoller 2014]

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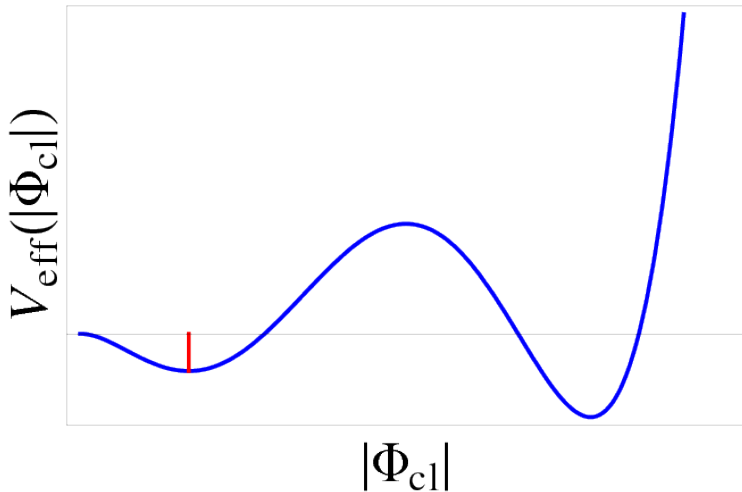
[Zoller 2014]

# The SM phase diagram



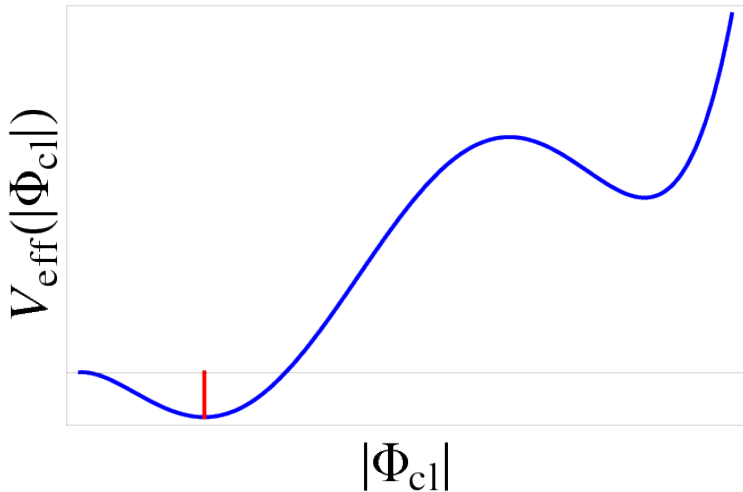
[Degrandi et al. JHEP 1208 (2012) 098]

# Stability, instability or metastability?



[Courtesy of Max Zoller]

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## Metastability of the Standard Model

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Task: Do not introduce further instabilities!  
(generically difficult)



## A multi-scalar theory

- 2 Higgs doublets
- $2 \times 6$  scalar quarks,  $6 + 3$  scalar leptons
- 12 colored and  $18 + 2$  charged directions
- charged Higgs directions “safe”
- SM Higgs potential:  $SO(4)$  symmetry

[Casas et al. 1996]

- large couplings to Higgs doublets ( $y_t$  and  $y_b$  comparably large)
- large stop contribution ( $X_t, A_t$ ) to light Higgs mass needed
- SUSY threshold corrections for  $m_b$  influence  $y_b$

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$$m_{h^0}^2 \simeq m_Z^2 + \frac{3m_t^2}{2\pi^2 v^2} \left[ \ln \left( \frac{M_S^2}{m_t} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{6M_S^2} \right) \right]$$

$$\begin{aligned}
 V_{\tilde{q},h} = & \tilde{t}_L^* (\tilde{m}_L^2 + |y_t h_2|^2) \tilde{t}_L + \tilde{t}_R^* (\tilde{m}_t^2 + |y_t h_2|^2) \tilde{t}_R \\
 & + \tilde{b}_L^* (\tilde{m}_L^2 + |y_b h_1|^2) \tilde{b}_L + \tilde{b}_R^* (\tilde{m}_b^2 + |y_b h_1|^2) \tilde{b}_R \\
 & - [\tilde{t}_L^* (\mu^* y_t h_1^* - A_t h_2) \tilde{t}_R + \text{h.c.}] \\
 & - [\tilde{b}_L^* (\mu^* y_b h_2^* - A_b h_1) \tilde{b}_R + \text{h.c.}] \\
 & + |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \\
 & + \frac{g_1^2}{8} \left( |h_2|^2 - |h_1|^2 + \frac{1}{3} |\tilde{b}_L|^2 + \frac{2}{3} |\tilde{b}_R|^2 + \frac{1}{3} |\tilde{t}_L|^2 - \frac{4}{3} |\tilde{t}_R|^2 \right)^2 \\
 & + \frac{g_2^2}{8} \left( |h_2|^2 - |h_1|^2 + |\tilde{b}_L|^2 - |\tilde{t}_L|^2 \right)^2 \\
 & + \frac{g_3^2}{8} \left( |\tilde{t}_L|^2 - |\tilde{t}_R|^2 + |\tilde{b}_L|^2 - |\tilde{b}_R|^2 \right)^2 \\
 & + (m_{h_2}^2 + |\mu|^2) |h_2|^2 + (m_{h_1}^2 + |\mu|^2) |h_1|^2 - 2 \text{Re}(B_\mu h_1 h_2).
 \end{aligned}$$



# The tree-level scalar potential

$$\begin{aligned}
 V_{\tilde{q},h} = & \tilde{t}_L^* (\tilde{m}_L^2 + |y_t h_2|^2) \tilde{t}_L + \tilde{t}_R^* (\tilde{m}_t^2 + |y_t h_2|^2) \tilde{t}_R \\
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 & - [\tilde{t}_L^* (\mu^* y_t h_1^* - A_t h_2) \tilde{t}_R + \text{h.c.}] \\
 & - [\tilde{b}_L^* (\mu^* y_b h_2^* - A_b h_1) \tilde{b}_R + \text{h.c.}] \\
 & + |y_t|^2 |\tilde{t}_L|^2 |\tilde{t}_R|^2 + |y_b|^2 |\tilde{b}_L|^2 |\tilde{b}_R|^2 \\
 & + \frac{g_1^2}{8} \left( |h_2|^2 - |h_1|^2 + \frac{1}{3} |\tilde{b}_L|^2 + \frac{2}{3} |\tilde{b}_R|^2 + \frac{1}{3} |\tilde{t}_L|^2 - \frac{4}{3} |\tilde{t}_R|^2 \right)^2 \\
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$$|\tilde{t}_L| = |\tilde{t}_R| = |\tilde{t}|, |\tilde{b}_L| = |\tilde{b}_R| = |\tilde{b}|; |\tilde{b}| = |h_1| = |\phi_1|, |\tilde{t}| = |h_2| = |\phi_2|$$

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 & + \phi_1^* (\tilde{m}_L^2 + |y_b \phi_1|^2) \phi_1 + \phi_1^* (\tilde{m}_b^2 + |y_b \phi_1|^2) \phi_1 \\
 & - [\phi_2^* (\mu^* y_t \phi_1^* - A_t \phi_2) \phi_2 + \text{h.c.}] \\
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 & + |y_t|^2 |\phi_2|^2 |\phi_2|^2 + |y_b|^2 |\phi_1|^2 |\phi_1|^2 \\
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Minimize the potential

$$V(\phi) = m^2\phi^2 - A\phi^3 + \lambda\phi^4,$$

with  $m^2 = m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2$ ,  $A = -A_t$  and  $\lambda = 3y_t^2$ .

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Answer:

$$\phi_0 = 0, \quad \phi_{\pm} = \frac{3A \pm \sqrt{9A^2 - 32\lambda m^2}}{8\lambda}.$$

Condition to be safe from non-standard (i.e. non-trivial) minima:

$$V(\phi_{\pm}) > 0 \quad \Leftrightarrow \quad m^2 > \frac{A^2}{4\lambda}$$

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Well-known constraints

[Gunion, Haber, Sher '88]

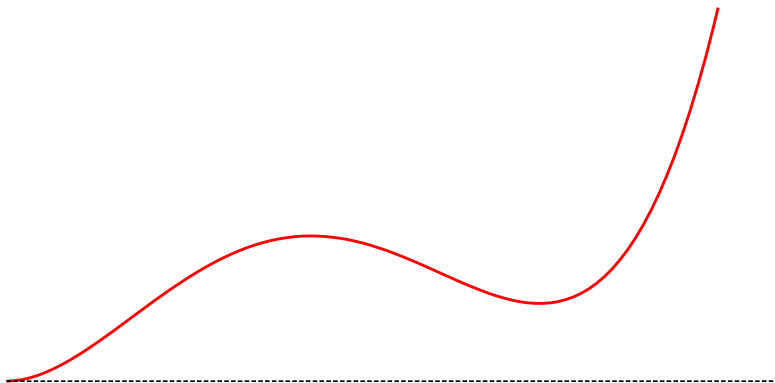
$$|A_t|^2 < 3y_t^2 (m_{h_2}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_t^2)$$

$$|A_b|^2 < 3y_b^2 (m_{h_1}^2 + |\mu|^2 + \tilde{m}_L^2 + \tilde{m}_b^2)$$

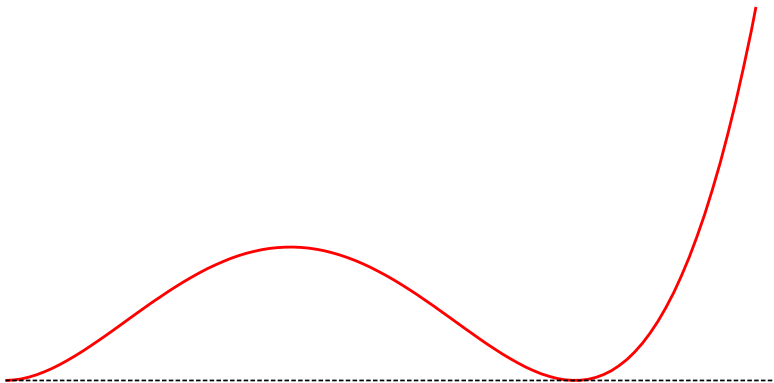
for the limiting cases  $|\tilde{t}_L| = |\tilde{t}_R| = |h_2|$  and  $|\tilde{b}_L| = |\tilde{b}_R| = |h_1|!$



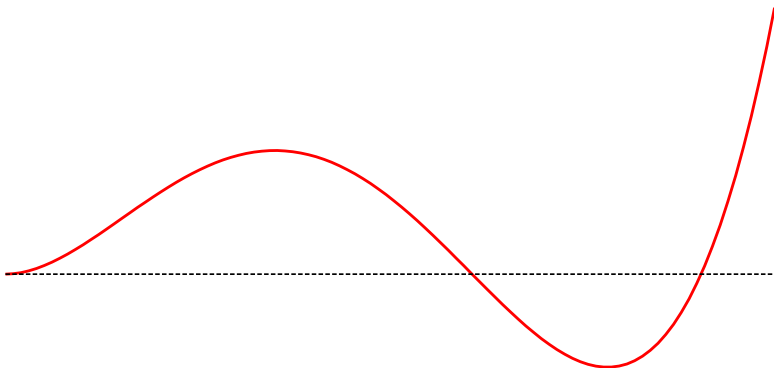
$$A^2 < 4\lambda m^2$$



$$A^2 = 4\lambda m^2$$



$$A^2 > 4\lambda m^2$$



# A simple view of a complicated object

$$h_2 = \phi, \quad |\tilde{t}| = \alpha|\phi|, \quad h_1 = \eta\phi, \quad |\tilde{b}| = \beta|\phi|$$

$$\begin{aligned} V_\phi = & (m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1 + \eta^2)\mu^2 - 2B_\mu\eta \\ & + (\alpha^2 + \beta^2)\tilde{m}_L^2 + \alpha^2\tilde{m}_t^2 + \beta^2\tilde{m}_b^2)\phi^2 \\ & - 2(\alpha^2(\mu y_t \eta - A_t) + \beta^2(\mu y_t - \eta A_b))\phi^3 + (\alpha^2 y_t^2 + \beta^4 y_b^2)\phi^4 \\ & + \left( \frac{g_1^2 + g_2^2}{8} (1 - \eta^2 + \beta^2 - \alpha^2)^2 + 2\alpha^2 y_t^2 + 2\beta^2 y_b^2 \right) \phi^4 \\ \equiv & M^2(\eta, \alpha, \beta)\phi^2 - \mathcal{A}(\eta, \alpha, \beta)\phi^3 + \lambda(\eta, \alpha, \beta)\phi^4, \end{aligned}$$

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with

$$\begin{aligned} M^2 &= m_{h_2}^2 + \eta^2 m_{h_1}^2 + (1 + \eta^2)\mu^2 - 2B_\mu\eta \\ &\quad + (\alpha^2 + \beta^2)\tilde{m}_L^2 + \alpha^2\tilde{m}_t^2 + \beta^2\tilde{m}_b^2, \end{aligned}$$

$$\mathcal{A} = 2\alpha^2\eta\mu y_t - 2\alpha^2 A_t + 2\beta^2\mu y_b - 2\eta\beta^2 A_b,$$

$$\begin{aligned} \lambda &= \frac{g_1^2 + g_2^2}{8} (1 - \eta^2 + \beta^2 - \alpha^2)^2 \\ &\quad + (2 + \alpha^2)\alpha^2 y_t^2 + (2\eta^2 + \beta^2)\beta^2 y_b^2. \end{aligned}$$

[Gunion, Haber, Sher '88; Casas, Lleyda, Muñoz '96]

The same but different (“ $A$ -parameter bounds”)

$$\mathcal{A}^2 < 4\lambda M^2$$

↓

$$4 \min_{\{\eta, \alpha, \beta\}} \lambda(\eta, \alpha, \beta) M^2(\eta, \alpha, \beta) > \max_{\{\eta, \alpha, \beta\}} (\mathcal{A}(\eta, \alpha, \beta))^2$$

$$h_u = \tilde{b}, h_d^0 = 0$$

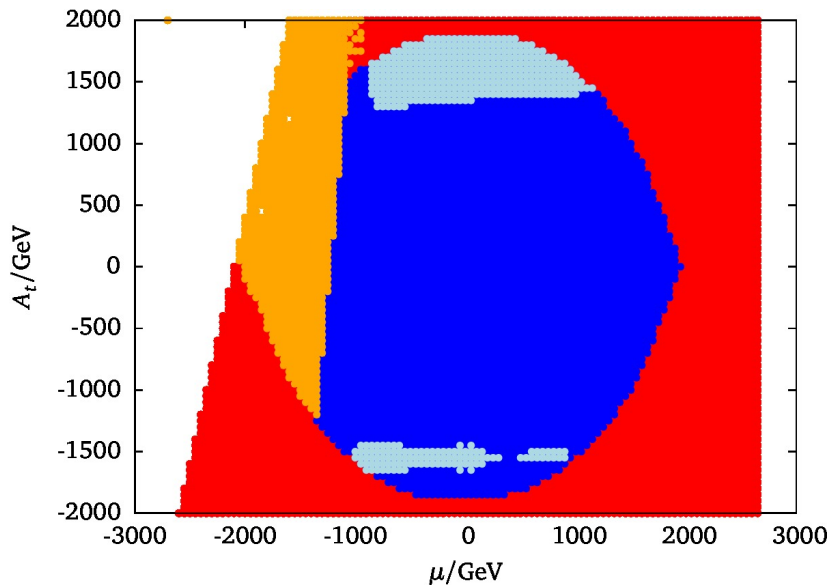
[WGH'15]

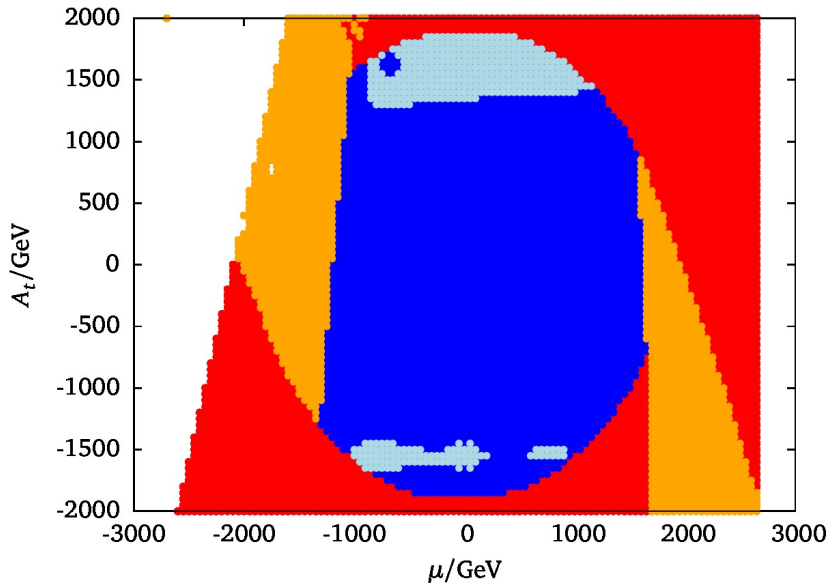
$$m_{H_u}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu y_b)^2}{y_b^2 + (g_1^2 + g_2^2)/2}$$

$$|h_d|^2 = |h_u|^2 + |\tilde{b}|^2, \tilde{b} = \alpha h_u$$

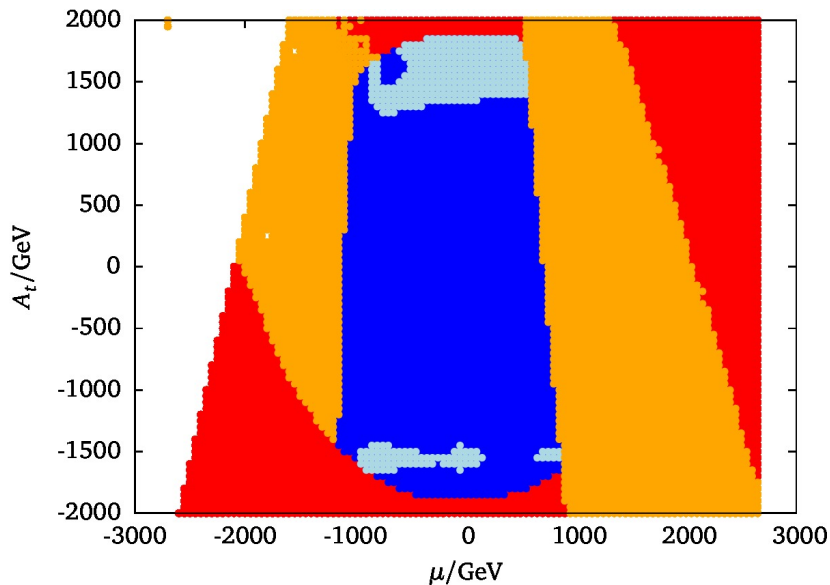
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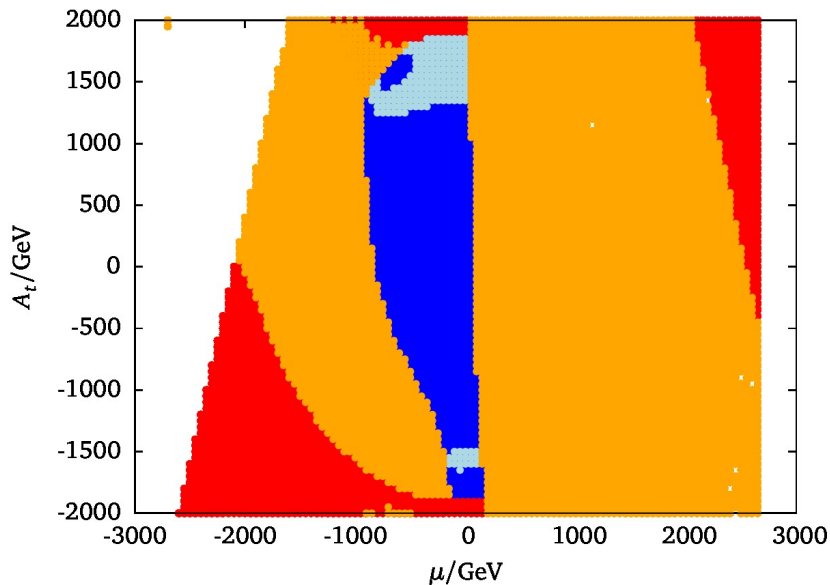
$$m_{11}^2(1 + \alpha^2) + m_{22}^2 \pm 2m_{12}^2\sqrt{1 + \alpha^2} + \alpha^2(\tilde{m}_Q^2 + \tilde{m}_b^2) > \frac{4\mu^2\alpha^2}{2 + 3\alpha^2}$$

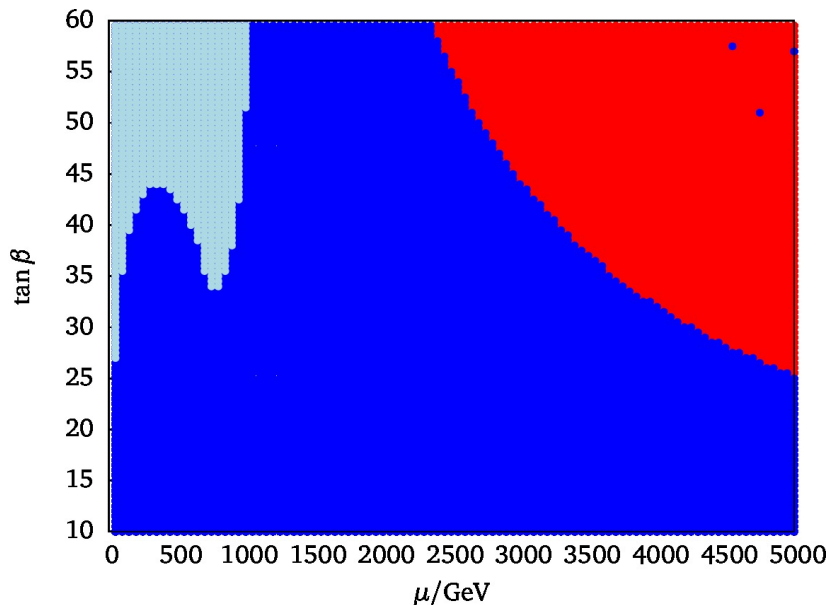


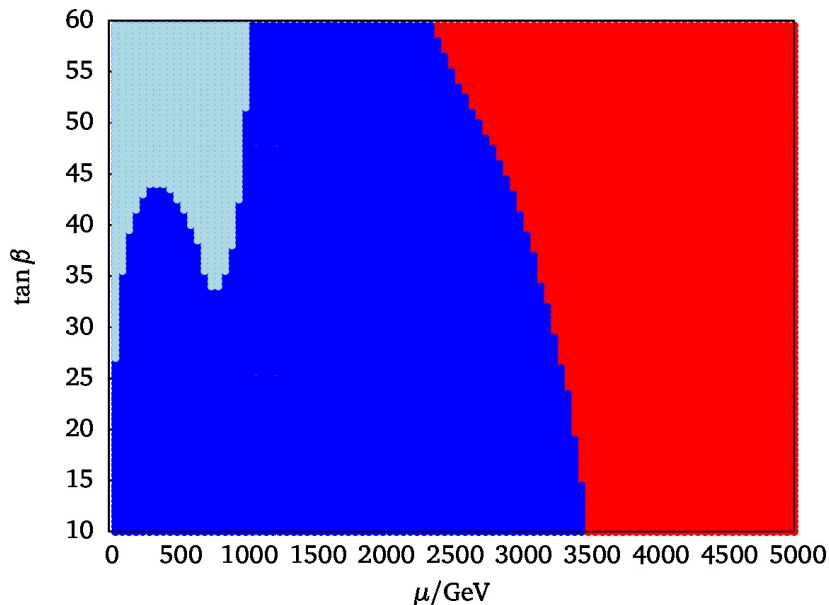


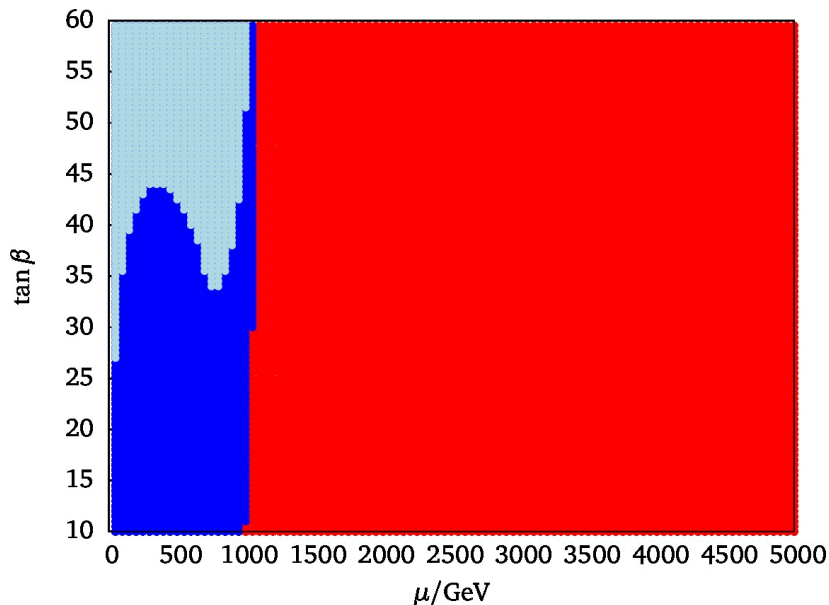


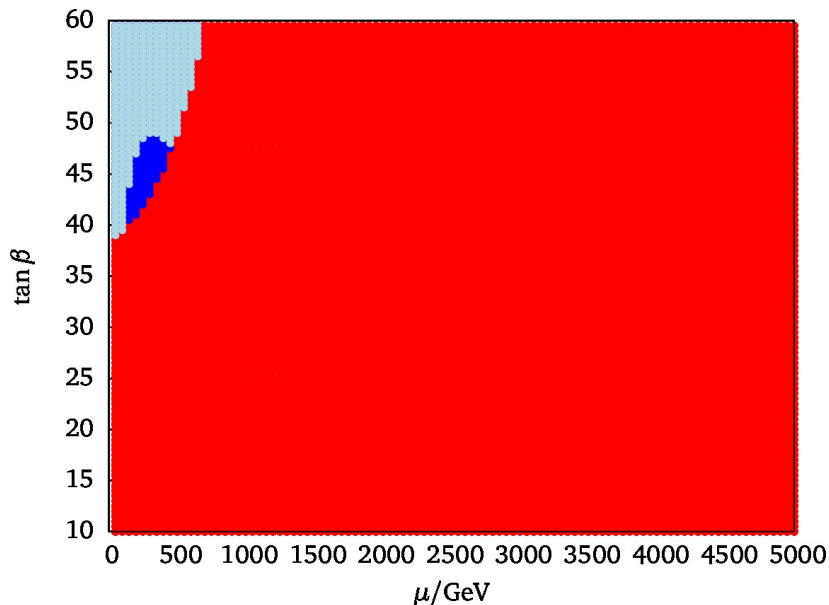




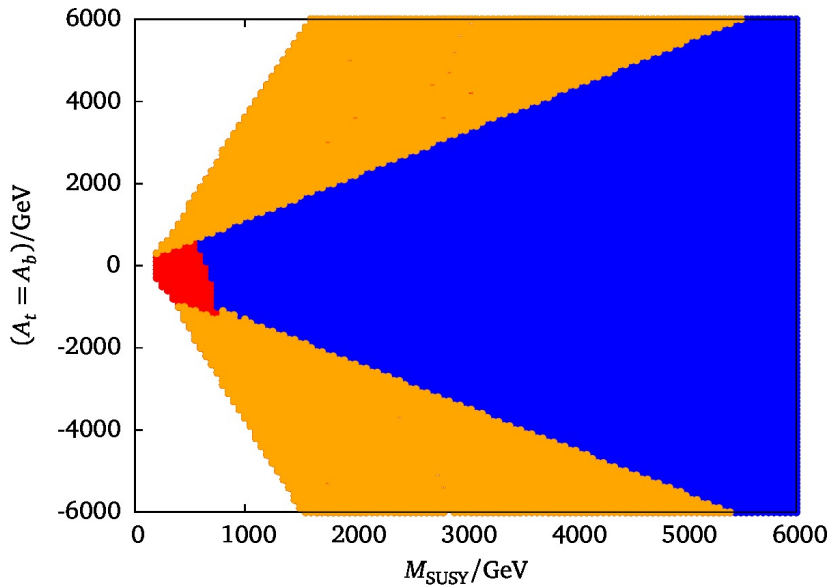


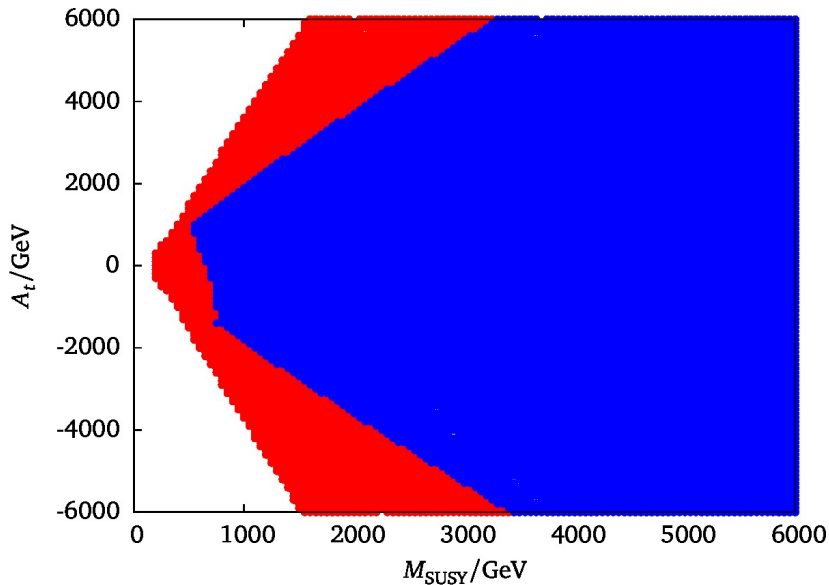




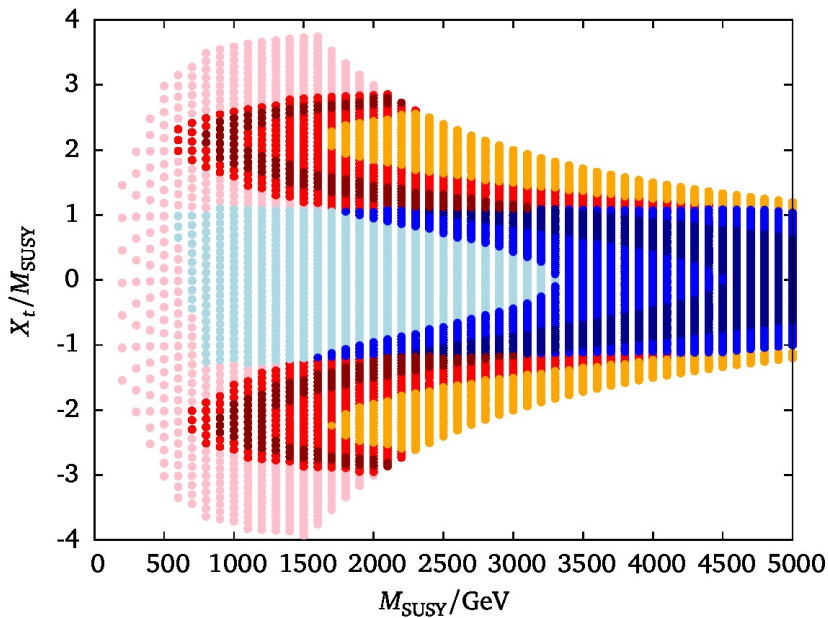


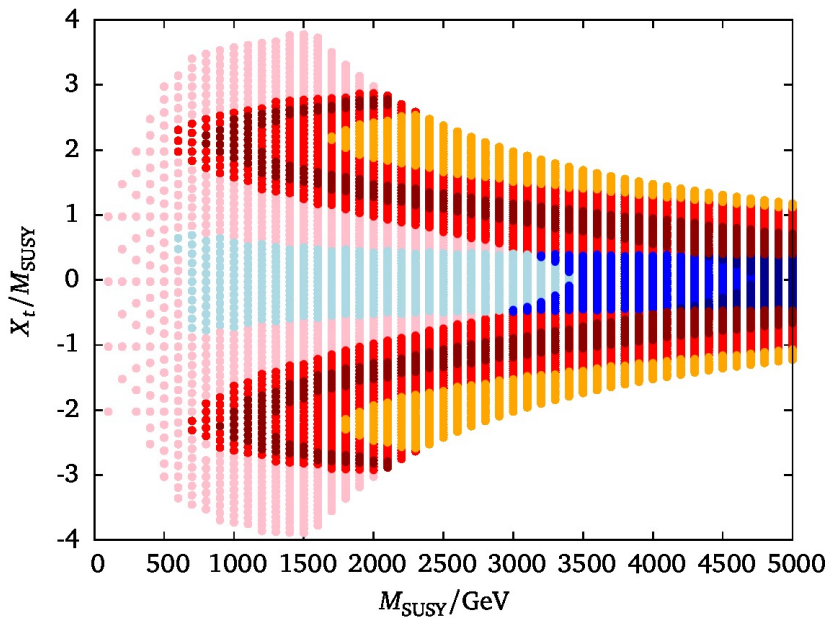
# Resort comes close (increasing $M_{\text{SUSY}}$ )

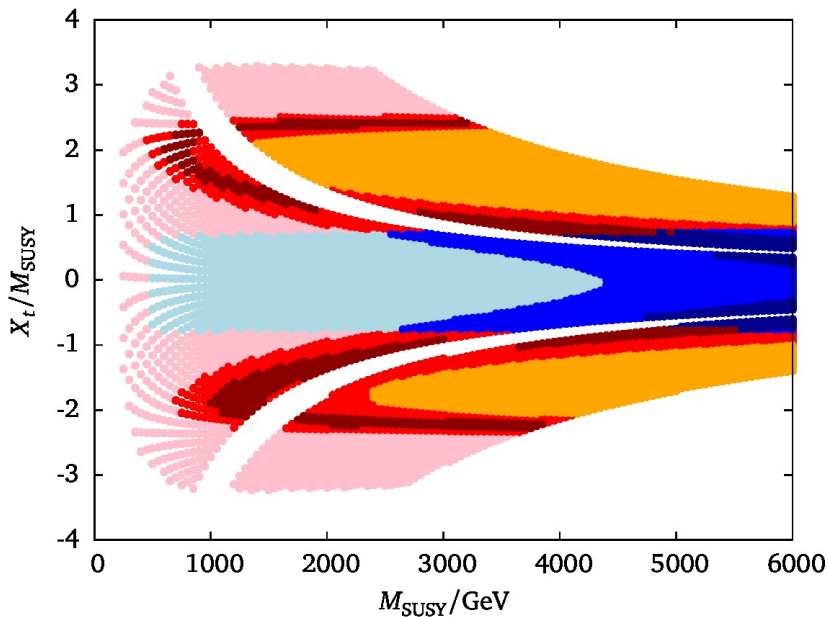


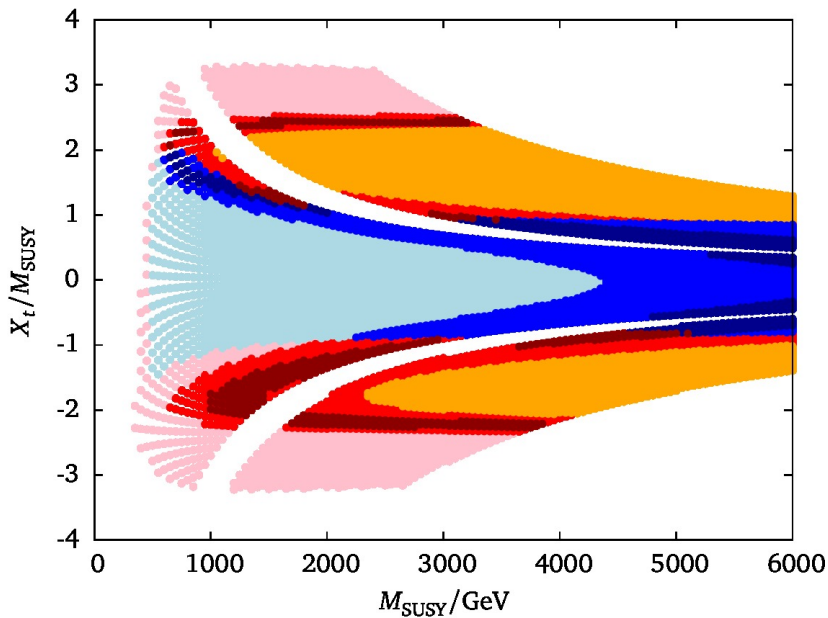






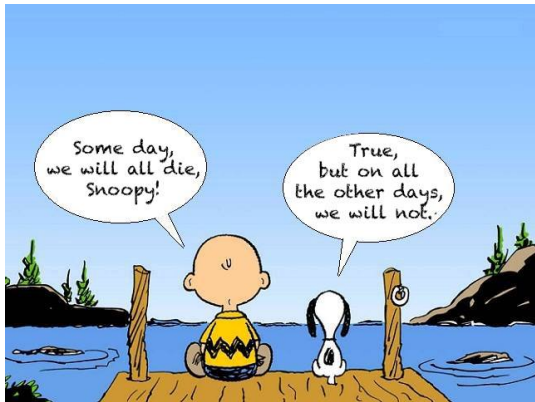






## Short summary

- constraints on model parameters from theoretical consistency: global minimum has to be electroweak minimum
- “heavy” Higgs @ 125 GeV: large SUSY corrections
  - large  $A_{t,b}$  and  $\mu$  induce squark vevs
  - quest for  $m_h^0 = 125$  GeV generically wants heavy SUSY



Backup

Slides

## Cosmological stability

bounce action

$$B \gtrsim 400$$

↔ life-time longer than age of the universe

## Decay probability (per unit volume)

$$\frac{\Gamma}{V} = Ae^{-B/\hbar}$$

[Coleman '77]

## Death and doom

- value of  $B$  crucially depends on field space path
- very different conclusions for different  $\eta, \alpha, \beta$
- *independent* of SUSY parameter choice

