Radiative Corrections and Degenerate Neutrinos

Loop-induced Neutrino Mixing

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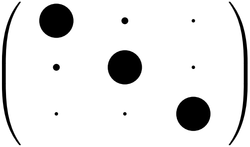
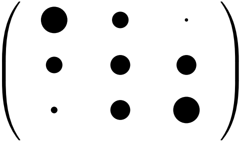
July 1st, 2015 | Flasy 2015 (Manzanillo)

[mascot of the 1997 World Championships of Athletics, Athens]

Quark and Neutrino Mixing

$|V_{\mathrm{CKM}}| =$ 

$|U_{\mathrm{PMNS}}| =$ 

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- close to unit matrix
- remnants of new Physics?
- get mixings from loops?

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- no hierarchy
- not close to trivial mixing?
- tree-level symmetries?

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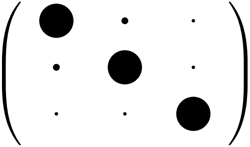
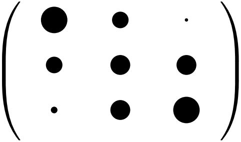
$|V_{\mathrm{CKM}}| =$ 

$|U_{\mathrm{PMNS}}| =$ 

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different or similar?

Let's see. . .

$$|V_{\text{CKM}}| = \begin{pmatrix} \huge\bullet & \bullet & \cdot \\ \bullet & \huge\bullet & \cdot \\ \cdot & \cdot & \huge\bullet \end{pmatrix} \qquad |U_{\text{PMNS}}| = \begin{pmatrix} \Large\bullet & \large\bullet & \cdot \\ \large\bullet & \large\bullet & \large\bullet \\ \bullet & \large\bullet & \Large\bullet \end{pmatrix}$$

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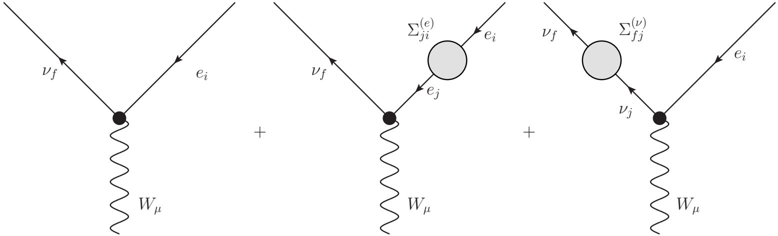
Task: Find out whether radiative corrections can change any tree-level mixing pattern in the PMNS case.



[Denner & Sack 1990]

mixing matrix renormalization

$$i\frac{g}{\sqrt{2}}\gamma^\mu P_L U^\dagger_{\text{PMNS}} \to i\frac{g}{\sqrt{2}}\gamma^\mu P_L \left(U^{(0)\dagger} + \Delta U^e U^{(0)\dagger} + \Delta U^\nu U^{(0)\dagger}\right),$$

sensitivity to neutrino mass

$$\Delta U^\nu_{fi} \sim \frac{m_{\nu_f}\Sigma_{fi}}{\Delta m^2_{fi}}$$

Enhanced corrections for quasi-degenerate neutrinos

[WGH, arxiv: 1411.2946]

enhancement by degeneracy of neutrino mass spectrum

$$\Delta U^{\nu}_{fi} \sim \frac{m_{\nu_f} \Sigma_{fi}}{\Delta m^2_{\nu}} \sim \frac{m_{\nu_f} m_{\nu_i}}{\Delta m^2_{fi}} \leq 5 \times 10^3$$

for $m^{(0)}_{\nu} \sim 0.35$ eV and $f, i = 1, 2$

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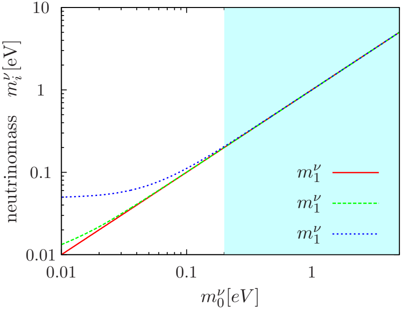

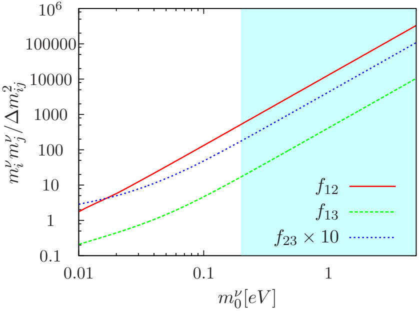

$$f_{ij} = m_{\nu_i} m_{\nu_j} / \Delta m^2_{ij}$$

The neutrino mass spectrum (normal hierarchy)

$$m_1 = m_0\,, \quad m_2 = \sqrt{m_0^2 + \Delta m_{21}^2}\,, \quad m_3 = \sqrt{m_0^2 + \Delta m_{31}^2}$$

- Oscillations: [νfit: www.nu-fit.org]
  - $\Delta m_{21}^2 = 7.50^{+0.19}_{-0.17} \times 10^{-5}\,\mathrm{eV}^2$
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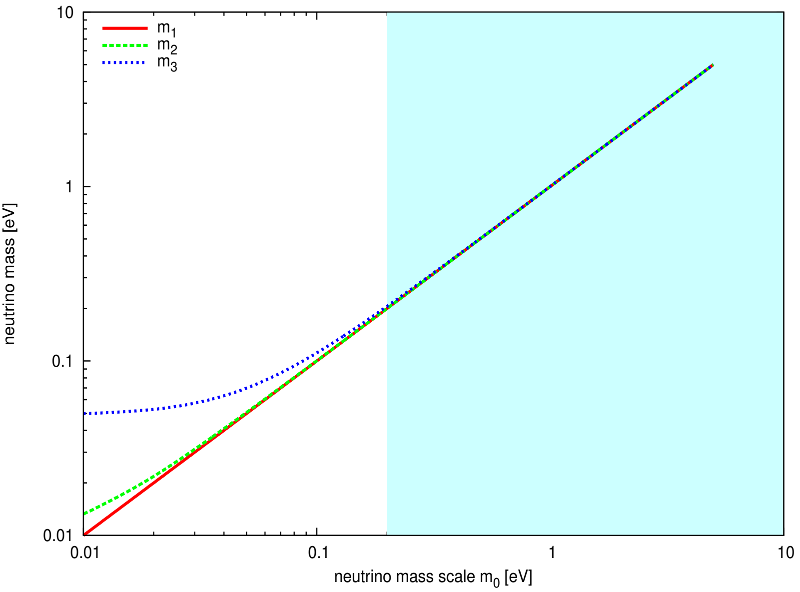
Quasi-Degeneration



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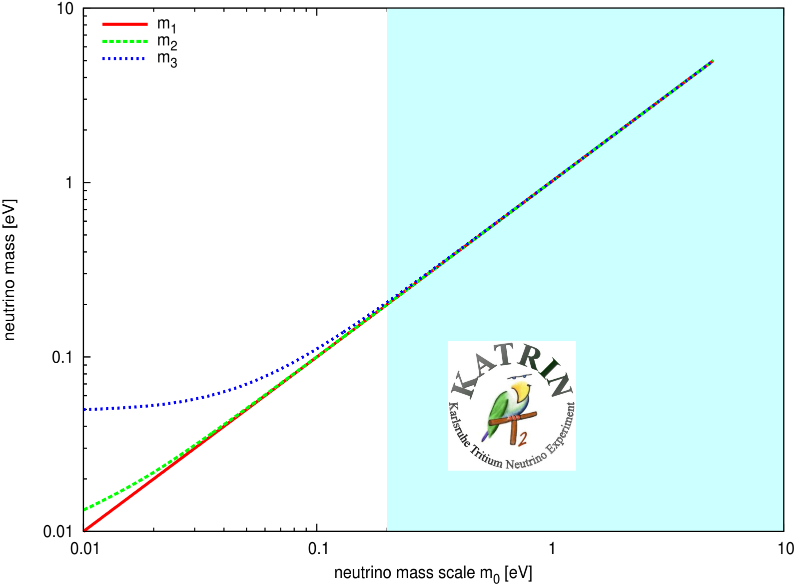
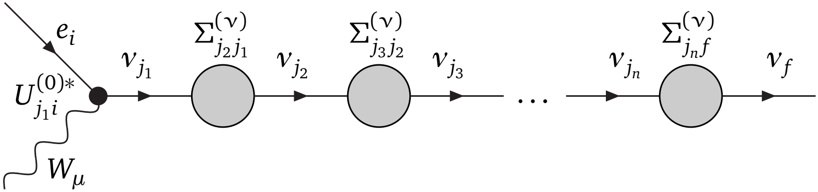
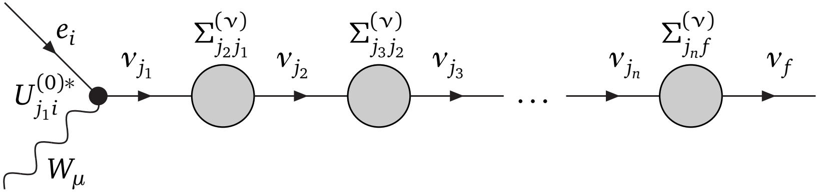



e_i
$U^{(0)*}_{j_1 i}$
W_μ
ν_{j_1}
$\Sigma^{(\nu)}_{j_2 j_1}$
ν_{j_2}
$\Sigma^{(\nu)}_{j_3 j_2}$
ν_{j_3}
ν_{j_n}
$\Sigma^{(\nu)}_{j_n f}$
ν_f

Resummation of enhanced Corrections



Dyson resummed propagator

- Matrix in Dirac and Flavor space [Kniehl et al. 2012–2014]
- complicated expression, especially when renormalized

Symbolically

$$\mathrm{i}\,\boldsymbol{S}(p) = \frac{\mathrm{i}}{\not p - \boldsymbol{m}^{(0)} - \boldsymbol{\Sigma}}$$

Renormalized mass matrix

$$\boldsymbol{m} = \boldsymbol{m}^{(0)} + \boldsymbol{\Sigma}$$

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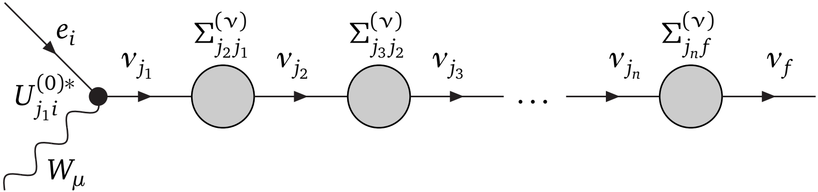

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Renormalized mixing matrix

$$\boldsymbol{U}^{(0)\mathsf{T}}\boldsymbol{m}^{(0)}\boldsymbol{U}^{(0)} \to \boldsymbol{U}\boldsymbol{m}\boldsymbol{U}$$

Generic treatment

$$m^{\nu}_{AB} = m^{(0)}_{AB} + m^{(0)}_{AC} I_{CB} + I_{AC} m^{(0)}_{CB}$$

I: threshold correction $I \sim \frac{y^2}{16\pi^2} f\left(\ln(M^2/Q^2)\right)$ (in the SM diagonal)

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Mass basis

$$m^\nu_{ab} = m^{(0)}_a \delta_{ab} + \left(m^{(0)}_a + m^{(0)}_b\right) I_{ab}$$

$I_{ab} = \sum_{AB} I_{AB} U^{(0)}_{Aa} U^{(0)}_{Bb}$

Threshold Corrections

[e.g. Chun and Pokorski '99]

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- assumption: degenerate tree-level masses, $|m^{(0)}_1| = |m^{(0)}_2| = |m^{(0)}_3|$

[WGH: PRD 91, 033001(2015)]

Trivial mixing @ tree-level

$$\boldsymbol{m}^\nu = m_0\,\mathbb{1} + m_0 \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{pmatrix}$$

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Generate e.g. tri-bimaximal mixing:

- requirements for I_{ij}:
  1. $\theta_{23} \approx \pi/4$

$$\boldsymbol{U}_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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  3. $\theta_{12} = \frac{1}{2}\arctan\left(\frac{2\sqrt{2}I_{12}}{2I_{22}-I_{11}}\right)$
- get $m_{1,2}$ in terms of I_{ij}, $m_3 = m_0$

Deviations

- $\theta_{13} \neq 0$ $\hookrightarrow I_{13} \neq I_{12}$
- $\theta_{23} \lesssim \frac{\pi}{4}$

$$I_{33} = I_{22} + \varepsilon$$
$$I_{13} = I_{12} + \delta$$

$$\begin{pmatrix} m_0 & & \\ & \sqrt{m_0^2 + \Delta m_{21}^2} & \\ & & \sqrt{m_0^2 + \Delta m_{31}^2} \end{pmatrix}$$

$$= m\,\boldsymbol{U}(\theta_{12}, \theta_{13}, \theta_{23})^T \begin{pmatrix} 1 + I_{11} & I_{12} & I_{12} + \delta \\ I_{12} & 1 + I_{22} & I_{22} \\ I_{12} + \delta & I_{22} & 1 + I_{22} + \varepsilon \end{pmatrix} \boldsymbol{U}(\theta_{12}, \theta_{13}, \theta_{23})$$

[WGH: PRD 91, 033001(2015)]

Numerical example

Inputs (central values), $\boldsymbol{m_0 = 0.35\,\mathrm{eV}}$

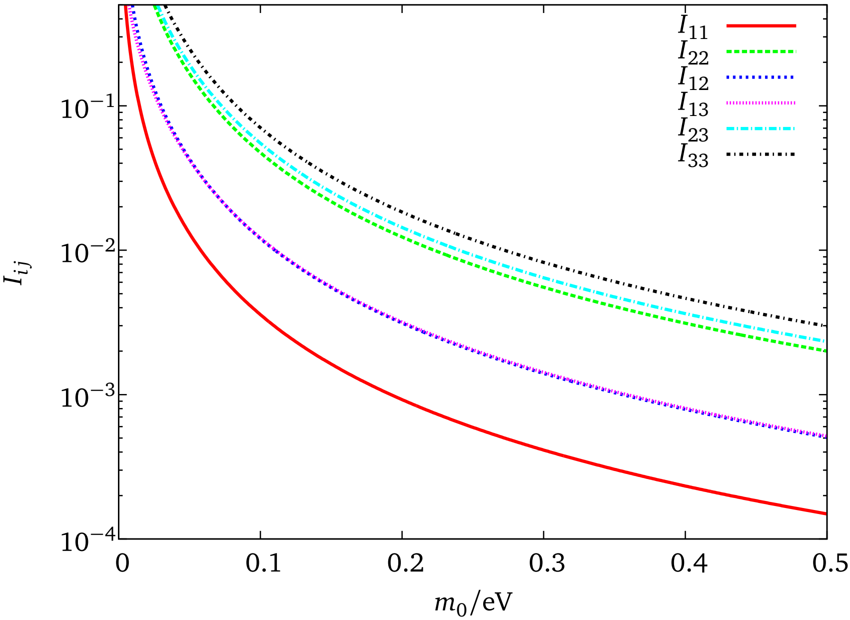
$$\theta_{12} \approx 31.8^\circ \;,\; \theta_{13} \approx 8.5^\circ \;,\; \theta_{23} \approx 39.2^\circ \;,$$
$$\Delta m^2_{21} \approx 7.5 \times 10^{-5}\,\mathrm{eV}^2 \;,\; \Delta m^2_{31} \approx 2.458 \times 10^{-3}\,\mathrm{eV}^2$$

Outputs

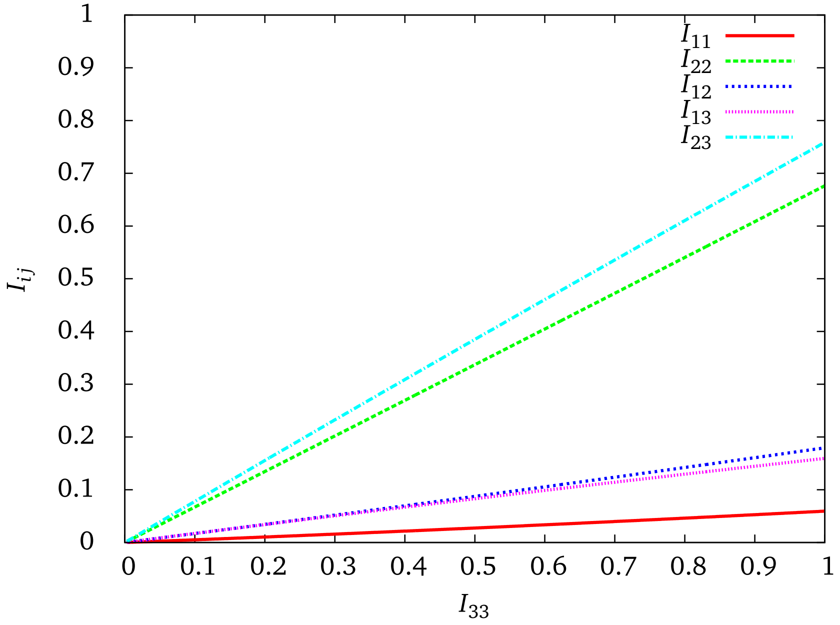
$$I_{11} \approx 3.00 \times 10^{-4} \;, \quad I_{22} \approx 4.01 \times 10^{-3} \;, \quad I_{12} \approx 1.02 \times 10^{-3} \;,$$
$$\delta \approx 1.56 \times 10^{-5} \;, \quad \varepsilon \approx 1.96 \times 10^{-3}$$

$$\boldsymbol{I} = \begin{pmatrix} 0.30 & 1.02 & 1.03 \\ 1.02 & 4.01 & 4.67 \\ 1.03 & 4.67 & 5.97 \end{pmatrix} \times 10^{-3}$$

[WGH: PRD 91, 033001(2015)]



I_{11}
I_{22}
I_{12}
I_{13}
I_{23}
I_{33}
I_{ij}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
0.1
0.2
0.3
0.4
0.5
m_0/eV


I_{11}
I_{22}
I_{12}
I_{13}
I_{23}
I_{ij}
I_{33}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

MSSM with righthanded neutrinos

$$\mathcal{W} \supset \mu H_\mathrm{d} \cdot H_\mathrm{u} + Y^\nu_{ij}\, H_\mathrm{u} \cdot L_{L,i} N_{R,j} - Y^\ell_{ij}\, H_\mathrm{d} \cdot L_{L,i} E_{R,j} + \frac{1}{2} M^R_{ij} N_{R,i} N_{R,j}$$

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New soft SUSY breaking terms

$$V^{\tilde{\nu}}_\mathrm{soft} = \left(\boldsymbol{m}^2_{\tilde{L}}\right)_{ij} \tilde{\nu}^*_{L,i}\tilde{\nu}_{L,j} + \left(\boldsymbol{m}^2_{\tilde{R}}\right)_{ij} \tilde{\nu}_{R,i}\tilde{\nu}^*_{R,j} + \left(A^\nu_{ij}\, h^0_\mathrm{u}\, \tilde{\nu}_{L,i}\tilde{\nu}^*_{R,j} + \left(\boldsymbol{B}^2\right)_{ij} \tilde{\nu}^*_{R,i}\tilde{\nu}^*_{R,j} + \text{h.c.}\right)$$

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$$+ \left(A^\nu_{ij}\, h^0_\mathrm{u}\, \tilde\nu_{L,i}\tilde\nu^*_{R,j} + \left(\boldsymbol{B}^2\right)_{ij} \tilde\nu^*_{R,i}\tilde\nu^*_{R,j} + \text{h.c.} \right)$$

- seesaw type I:

$$\boldsymbol{m}^{(0)}_\nu = -v_\mathrm{u}^2 \boldsymbol{Y}^T_\nu \boldsymbol{M}^{-1}_\mathrm{R} \boldsymbol{Y}_\nu + \mathcal{O}(v_\mathrm{u}^4/M^3_\mathrm{R})$$

- adding SUSY 1-loop [Dedes, Haber, Rosiek 2007]

$$\left(\boldsymbol{m}^{1-\mathrm{loop}}_\nu\right)_{ij} = (\boldsymbol{m}_\nu)_{ij} + \mathrm{Re}\left[\Sigma^{(\nu),S}_{ij} + \frac{m_{\nu_i}}{2}\Sigma^{(\nu),V}_{ij} + \frac{m_{\nu_j}}{2}\Sigma^{(\nu),V}_{ji}\right]$$

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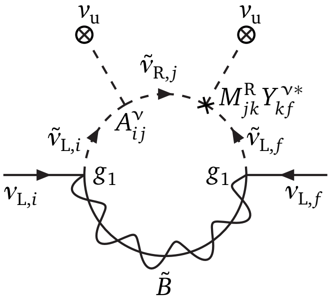
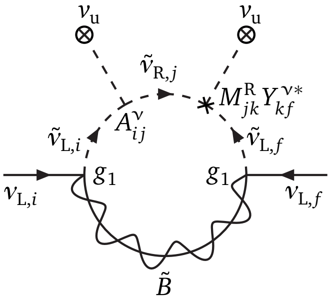
$$\begin{aligned} \Sigma^{(\nu)}_{ij}(p) =& \Sigma^{(\nu),S}_{ij}(p^2) P_\mathrm{L} + \Sigma^{(\nu),S^*}_{ij}(p^2) P_\mathrm{R} + \\ & \not{p} \left[\Sigma^{(\nu),V}_{ij}(p^2) P_\mathrm{L} + \Sigma^{(\nu),V^*}_{ij}(p^2) P_\mathrm{R} \right]. \end{aligned}$$

[WGH, arXiv:1505.07764]



$$\boldsymbol{\Sigma} \sim \boldsymbol{A}^\nu \frac{1}{M_R^2} \boldsymbol{M}_R \boldsymbol{Y}^\nu$$

[WGH, arXiv:1505.07764]



$$\boldsymbol{\Sigma} \sim \boldsymbol{A}^{\nu} \frac{1}{M_{\mathrm{R}}^2} \boldsymbol{M}_{\mathrm{R}} \boldsymbol{Y}^{\nu} = y_{\nu} \boldsymbol{A}^{\nu} / M_{\mathrm{R}}$$

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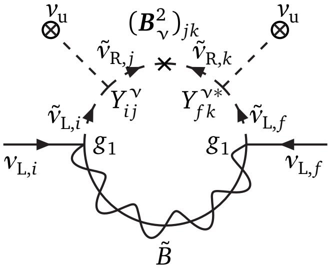

$$\boldsymbol{\Sigma} \sim (\boldsymbol{Y}^\nu)^\mathsf{T} \boldsymbol{M}_\mathrm{R} \frac{1}{M_\mathrm{R}^2} \boldsymbol{B}_\nu^2 \frac{1}{M_\mathrm{R}^2} \boldsymbol{M}_\mathrm{R} \boldsymbol{Y}^\nu$$

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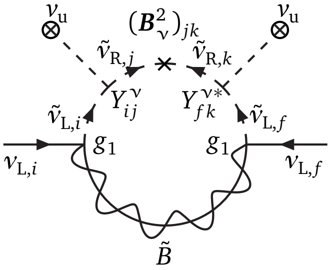

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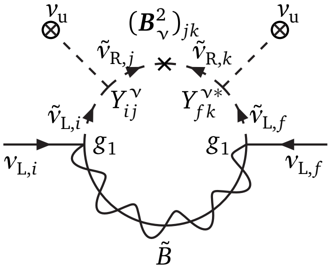

$$\boldsymbol{\Sigma} \sim (\boldsymbol{Y}^\nu)^\mathsf{T} \boldsymbol{M}_\mathrm{R} \frac{1}{M_\mathrm{R}^2} \boldsymbol{B}_\nu^2 \frac{1}{M_\mathrm{R}^2} \boldsymbol{M}_\mathrm{R} \boldsymbol{Y}^\nu = y_\nu^2 \boldsymbol{B}_\nu^2 / M_\mathrm{R}^2 = y_\nu^2 \boldsymbol{b}_\nu / M_\mathrm{R}$$

$\boldsymbol{Y}^\nu = y_\nu \mathbf{1}$, $\boldsymbol{M}_\mathrm{R} = M_\mathrm{R} \mathbf{1}$, $\boldsymbol{B}_\nu^2 = \boldsymbol{b}_\nu M_\mathrm{R}$

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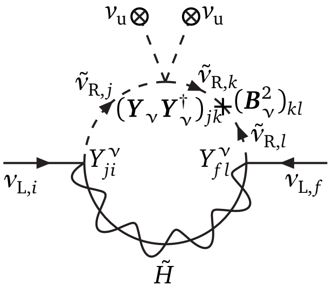

$$\boldsymbol{\Sigma} \sim (\boldsymbol{Y}^\nu)^\mathsf{T} \frac{1}{M_\mathrm{R}^2} \boldsymbol{Y}^\nu (\boldsymbol{Y}^\nu)^\dagger \frac{1}{M_\mathrm{R}^2} \boldsymbol{B}_\nu^2 \frac{1}{M_\mathrm{R}^2} \boldsymbol{Y}^\nu$$

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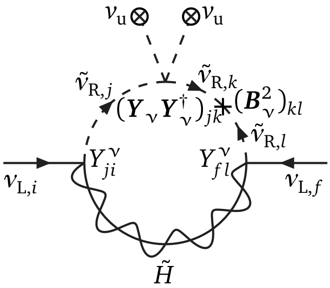

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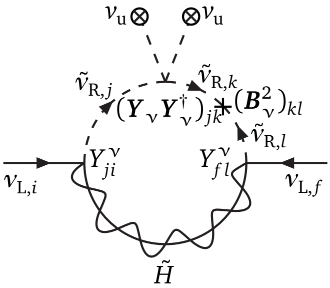

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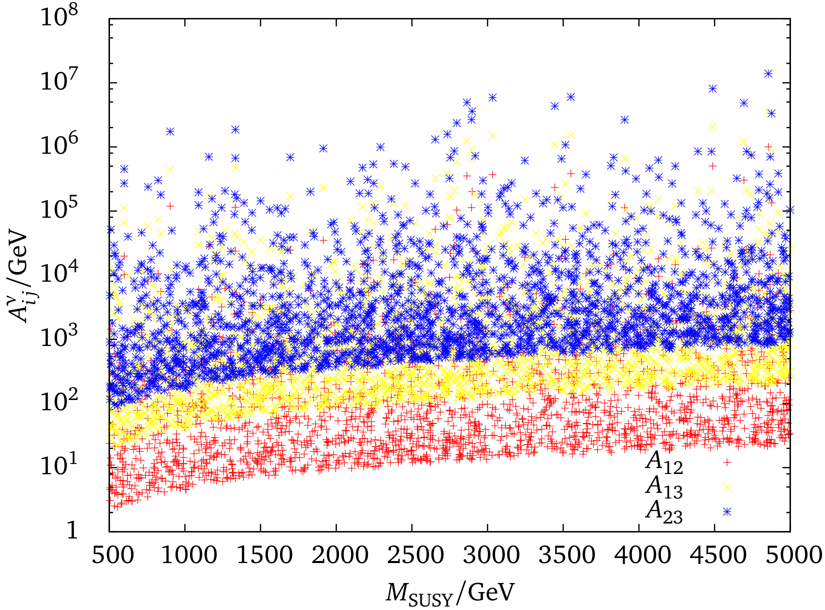
$\boldsymbol{Y}^\nu = y_\nu \mathbf{1}$, $\boldsymbol{M}_\mathrm{R} = M_\mathrm{R}\mathbf{1}$, $\boldsymbol{B}_\nu^2 = \boldsymbol{b}_\nu M_\mathrm{R}$

A non-decoupling contribution



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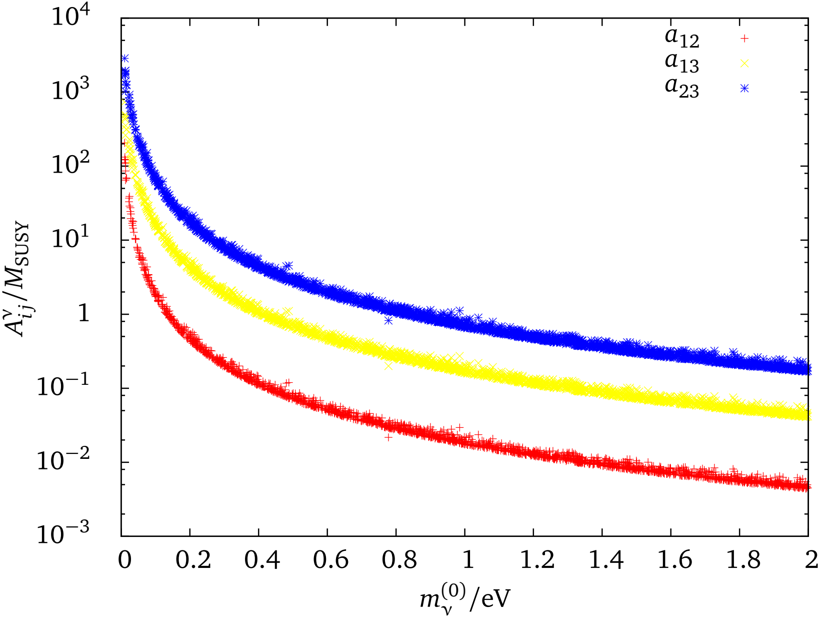

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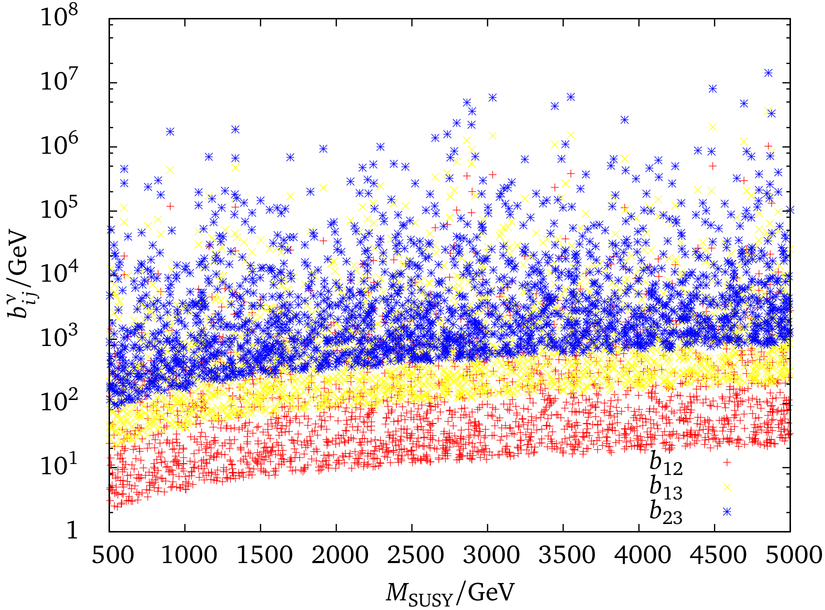

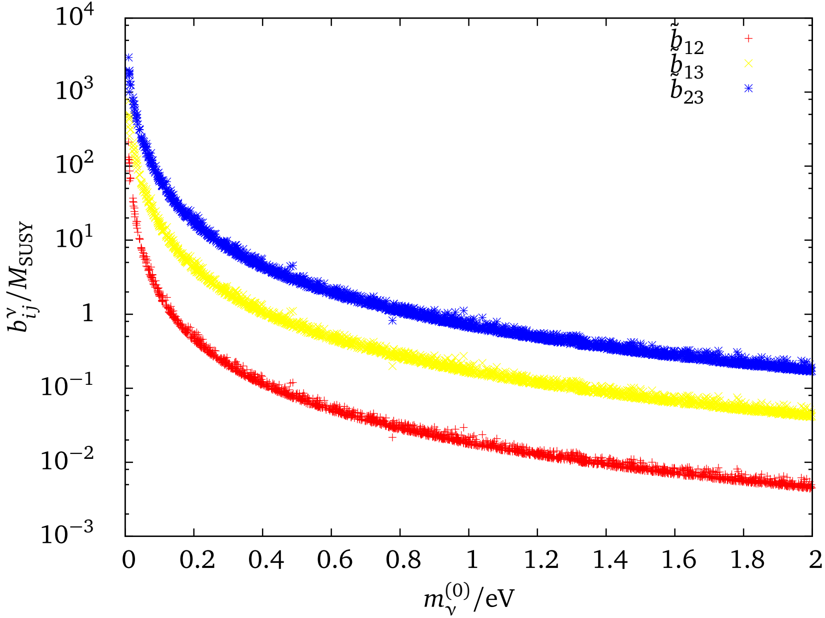
10^4
10^3
10^2
10^1
1
10^{-1}
10^{-2}
10^{-3}
$b^\nu_{ij}/M_{\mathrm{SUSY}}$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$m_\nu^{(0)}/\mathrm{eV}$
$\tilde{b}_{12}$
$\tilde{b}_{13}$
$\tilde{b}_{23}$

Side Remark

Altering the mixing pattern without A- or B-terms

- requires hierarchical right-handed neutrinos
- changes tree-level mixing pattern

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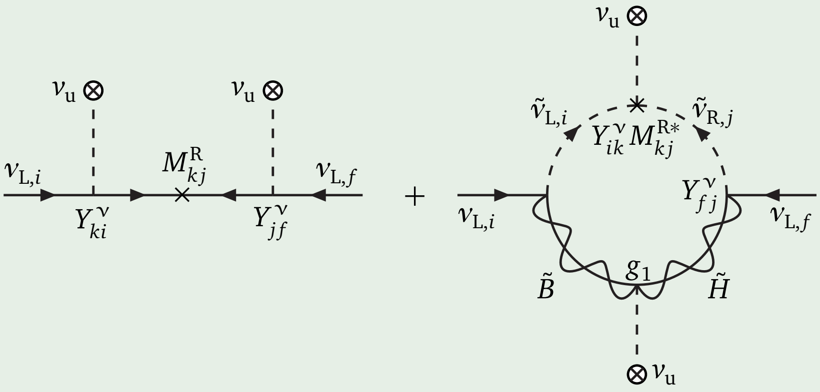

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- changes tree-level mixing pattern

$$\boldsymbol{m}_\nu = v_\mathrm{u}^2(\boldsymbol{\kappa}_\nu + \Delta\boldsymbol{\kappa}_\nu)$$
$$= v_\mathrm{u}^2 \boldsymbol{Y}_\nu \operatorname{diag}\left(\frac{1}{m_{\tilde{\nu}_{\mathrm{R}_k}}} + \frac{g_1^2}{64\pi^2}\frac{\log\left(m_\mathrm{S}^2/m_{\tilde{\nu}_{\mathrm{R},k}}^2\right)}{m_{\tilde{\nu}_{\mathrm{R}_k}}}\right)\boldsymbol{Y}_\nu^\mathsf{T}$$

- rediagonalization of the seesaw operator:

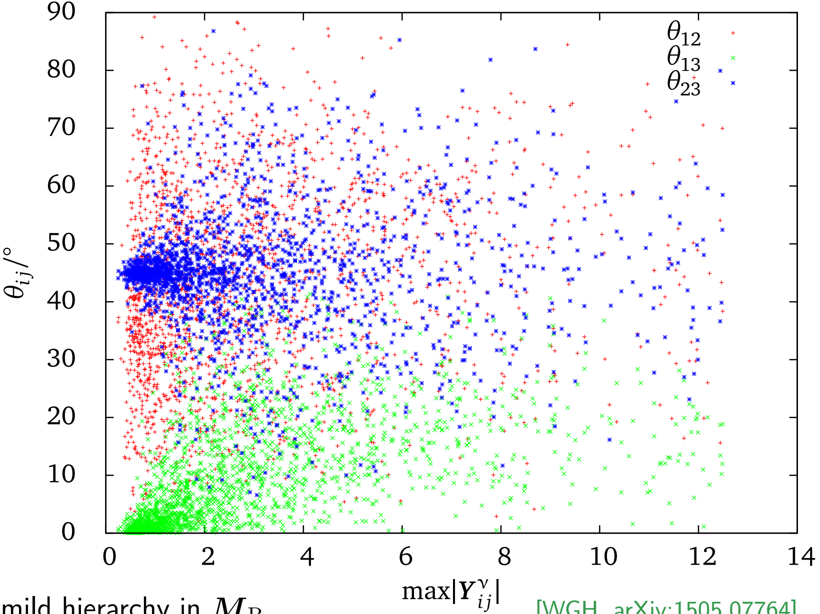
$$\boldsymbol{U}^*\boldsymbol{m}_\nu\boldsymbol{U}^\dagger \sim \boldsymbol{U}^*\left(\boldsymbol{\kappa}_\nu + \Delta\boldsymbol{\kappa}_\nu\right)\boldsymbol{U}^\dagger$$

- $\boldsymbol{U} \neq \boldsymbol{U}^{(0)}$ also for $\boldsymbol{M}_\mathrm{R}$ = diagonal

similar phenomenon in type-I SM: $\Delta\kappa_\nu^\mathrm{SM} \sim \log(M_W/M_R)$

[Grimus, Lavoura 2002; Aristizabal Sierra, Yaguna 2011]

Influence on previously set mixing angles, e.g. TBM



mild hierarchy in M_R

[WGH, arXiv:1505.07764]

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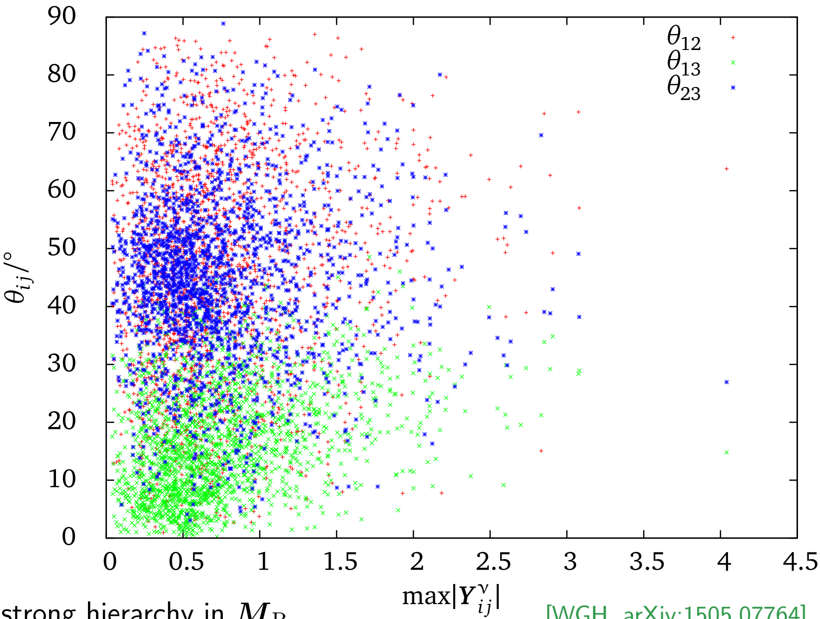

strong hierarchy in M_{R}

[WGH, arXiv:1505.07764]

- Radiative Flavour Violation in the lepton sector: loop-induced mixing from SUSY breaking terms
- large contribution, if neutrino mass spectrum is quasi-degenerate ($m_\nu^0 > 0.1\,\mathrm{eV}$)
- exact degeneracy: $\mathrm{SO}(3)$ or $\mathrm{SU}(3)$ @ tree-level
- radiative breaking via (SUSY) threshold corrections
- trilinear coupling matrix $\boldsymbol{A}^\nu$ carries flavor information
- neutrino mixing via SUSY breaking: potential flavor symmetries in the soft breaking sector

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Backup

Slides

Renormalization Group Equation for ν masses and mixing

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{C} = -K\boldsymbol{C} - \kappa\left[\left(\boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e\right)^T \boldsymbol{C} + \boldsymbol{C}\left(\boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e\right)\right]$$

$$t = \frac{1}{16\pi^2}\ln\left(\frac{Q}{M_Z}\right)$$

SM: $\kappa = -\frac{3}{2}$ and $K = -3g_2^2 + 2\,\mathrm{Tr}\left(3\boldsymbol{Y}_u^\dagger \boldsymbol{Y}_u + 3\boldsymbol{Y}_d^\dagger \boldsymbol{Y}_d + \boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e\right) + 2\lambda$

MSSM: $\kappa = +1$ and $K = -6g_2^2 - 2g_Y^2 + 2\,\mathrm{Tr}\left(3\boldsymbol{Y}_u^\dagger \boldsymbol{Y}_u\right)$

Solving the RGE

$\boldsymbol{C}(t) = I_K \mathcal{I}\boldsymbol{C}(0)\mathcal{I}$, where $\mathcal{I} = \mathrm{diag}(I_e, I_\mu, I_\tau)$ and

$$I_K = \exp\left(-\int_0^t K(t')\mathrm{d}t'\right), \quad I_{e_A} = \exp\left(-\kappa\int_0^t y_{e_A}^2(t')\mathrm{d}t'\right).$$

Comments on degenerate masses

- if $\boldsymbol{m}^{(0)} = m_0\,\mathbb{1}$: ${\boldsymbol{U}^{(0)}}^T\ \boldsymbol{m}^{(0)}\ \boldsymbol{U}^{(0)} = m_0\,\mathbb{1}$ for any (real) $\boldsymbol{U}^{(0)}$
- if e.g. $\boldsymbol{m}^{(0)} = \mathrm{diag}(1,-1,1)$ this is not true
- in general: Majorana phases!
  - phase matrix $\boldsymbol{U}^{(0)} \to \boldsymbol{U}^{(0)}\boldsymbol{P}$ with $\boldsymbol{P} = \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$
  - $\boldsymbol{m}^{(0)} \to \boldsymbol{P}^T {\boldsymbol{U}^{(0)}}^T \boldsymbol{m}^{(0)} \boldsymbol{U}^{(0)} \boldsymbol{P} = m_0\,\mathrm{diag}(e^{2i\alpha_1} e^{2i\alpha_2}, 1)$
  - redefine masses
    - $m_1 = e^{2i\alpha_1} m_0$,
    - $m_2 = e^{2i\alpha_2} m_0$,
    - $m_3 = m_0$.
  - taking CP as good symmetry: $\alpha_{1,2} \in \{0, \pm\frac{\pi}{2}\}$
- choice: $m_1 = -m_2 = m_3$:

$$\boldsymbol{m}^\nu = m_0 \begin{pmatrix} 1 + 2U_{\alpha 1}U_{\beta 1}I_{\alpha\beta} & 0 & 2U_{\alpha 1}U_{\beta 3}I_{\alpha\beta} \\ 0 & -1 - 2U_{\alpha 2}U_{\beta 2}I_{\alpha\beta} & 0 \\ 2U_{\alpha 1}U_{\beta 3}I_{\alpha\beta} & 0 & 1 + 2U_{\alpha 3}U_{\beta 3}I_{\alpha\beta} \end{pmatrix}$$

update of [Chankowski, Pokorski 2002]

Brief review of [Chankowski, Pokorski 2002]

- degeneracy leaves freedom of rotation $U^{(0)} \to U^{(0)} R_{13}$

$$\sum_{\alpha\beta} U^{(0)}_{\alpha 1} U^{(0)}_{\beta 3} I_{\alpha\beta} = 0$$

- flavour diagonal corrections: $I_{\alpha\beta} = I_\alpha \delta_{\alpha\beta}$
- explain deviation from (tri-)bi-maximal mixing:
  $s_{13} = \sin\theta_{13} \approx 0$

$$s_{13} = -\frac{s_{12}}{c_{12}} s_{23} c_{23} \frac{I_\tau}{I_e}, \qquad \text{where } I_\mu = 0 \text{ and } I_e \gg I_\tau$$

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$I_\mu \neq 0$

try to accommodate $s_{13} \approx 0.15$ and $\Delta m^2_{31}/\Delta m^2_{21} \approx 33$

$$s_{13} = c_{23} s_{23} \frac{s_{12}}{c_{12}} \frac{I_\mu - I_\tau}{I_e - s^2_{23} I_\mu - c^2_{23} I_\tau}$$

$$\Delta m^2_{ab} = m^2 \left([1 + 2U^2_{\alpha a} I_\alpha]^2 - [1 + 2U^2_{\alpha b} I_\alpha]^2\right)$$

- m^2 overall scale
- use relation for s_{13} to get correlation between I_e and I_μ, I_τ
- try to fit

$$\Delta m^2_{31} / \Delta^2_{m21} = \frac{\left([1 + 2U^2_{\alpha 3} I_\alpha]^2 - [1 + 2U^2_{\alpha 1} I_\alpha]^2\right)}{\left([1 + 2U^2_{\alpha 2} I_\alpha]^2 - [1 + 2U^2_{\alpha 1} I_\alpha]^2\right)}$$

the same follows from a special tree-level mass matrix

$$\boldsymbol{m}^{\nu}_{\text{tree}} = x \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + z \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} x & y & y \\ y & x+z & z \\ y & z & x+z \end{pmatrix},$$

which can be diagonalized by

$$\boldsymbol{U}_{\text{tree}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

with $s_{12} = \sin\theta_{12}, c_{12} = \cos\theta_{12}$,
$\tan 2\theta_{12} = \sqrt{2}\frac{y}{z}$

can be inverted

$$\boldsymbol{m}^{\nu}_{\text{tree}} =$$

$$\begin{pmatrix} m_1 & \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} & \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} \\ \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} & \frac{m_2+m_3}{2} & \frac{1}{2}\left(\sum_i m_i - 3m_1\right) \\ \pm\frac{\mathrm{i}}{\sqrt{2}}\sqrt{\frac{\Delta m^2_{31}}{m1+m3}\frac{\Delta m^2_{21}}{m1+m2}} & \frac{1}{2}\left(\sum_i m_i - 3m_1\right) & \frac{m_2+m_3}{2} \end{pmatrix}$$

The philosophy behind threshold corrections

- exact degeneracy @ tree-level: trivial mass matrix
- $m^{(1)} = m^{(0)} + m^{(0)} I, \qquad I \sim \frac{1}{16\pi^2} \approx \frac{1}{100}$
- small perturbation