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Neutrino Mixing from SUSY breaking

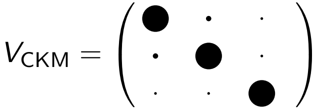
in collaboration with Ulrich Nierste

Wolfgang Gregor Hollik | March 5, 2013

INSTITUT FÜR THEORETISCHE TEILCHENPHYSIK KIT

CKM vs. PMNS matrix

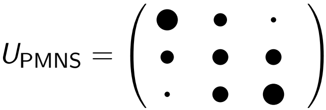
- CKM matrix close to unity



- small off-diagonal: generate mixings radiatively ?

[Weinberg 1972]

- different pattern for the leptonic mixing matrix:



- large mixings
- non-vanishing θ_{13}: possible CP violation in ν oscillations

[T2K, DoubleChooz, Reno, DayaBay]

- try to model quark and lepton mixing using the same mechanism?

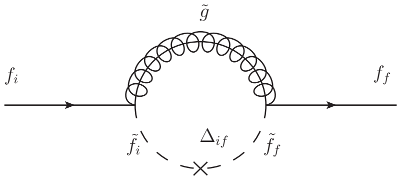
Radiative Flavour Violation in the MSSM

Theories with Additional Sources of Flavour Violation

- non-minimal flavour violating MSSM
- arbitrary flavour structure in the soft breaking terms?

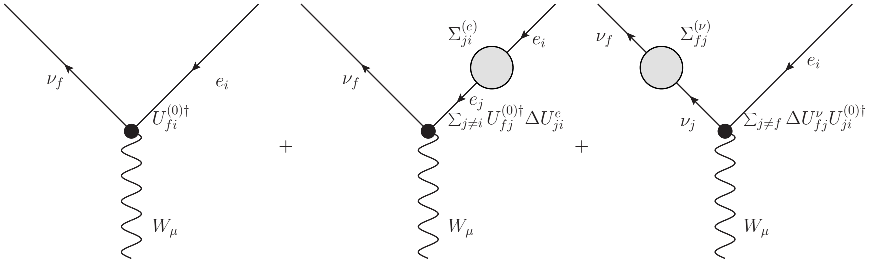
$$\mathcal{M}^2_{\tilde{Q}}, \mathcal{M}^2_{\tilde{u}}, \mathcal{M}^2_{\tilde{d}}, \mathcal{M}^2_{\tilde{\ell}}, \mathcal{M}^2_{\tilde{e}}, \qquad A^u, A^d, A^e$$

- additional flavour mixing in fermion–sfermion–gaugino interaction
- especially non-CKM-like: e.g. quark–squark–gluino and lepton–slepton–neutralino



[Crivellin, Nierste 2009]

radiative lepton flavour violation



PMNS matrix renormalization

$$i\frac{g}{\sqrt{2}}\gamma^\mu P_L U_{\text{PMNS}}^\dagger \to i\frac{g}{\sqrt{2}}\gamma^\mu P_L \left(U^{(0)\dagger} + \Delta U^e U^{(0\dagger)\dagger} + \Delta U^\nu U^{(0\dagger)\dagger}\right),$$

flavour changing self energies and sensitivity to neutrino mass

$$\Delta U_{fi}^\nu \sim \frac{m_{\nu_f}\Sigma_{fi}}{\Delta m_\nu^2}$$

Neutrino masses and seesaw

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Standard Model + righthanded Neutrinos = Seesaw Type I

$$-\mathcal{L}_{\nu,\text{mass}} = \underbrace{\bar{\nu}_L m_D \nu_R}_{\text{Dirac mass}} + \frac{1}{2} \underbrace{\overline{\nu_L^c} m_R \nu_R}_{\text{Majorana mass}} + \text{h. c.}$$

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Neutrino mass matrix:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix}.$$

What about m_R?

- righthanded neutrinos are SM singlets $\rightarrow$ no constraint for mass
- seesaw : $m_\nu = -m_D m_R^{-1} m_D \approx \mathcal{O}(0.1\text{ eV})$
- assumption: Dirac mass of order EW scale ($\mathcal{O}(10\dots100\text{ GeV})$):
  $m_R \sim \mathcal{O}(10^{13\dots14}\text{ GeV})$

The MSSM with righthanded neutrinos

Superpotential of the νMSSM

$$\mathcal{W}^\ell = \mu H_d \cdot H_u - Y_\ell^{IJ} H_d \cdot L_L^I E_R^J + Y_\nu^{IJ} H_u \cdot L_L^I N_R^J + \frac{1}{2} m_R^{IJ} N_R^I N_R^J,$$

with $L_L = (\ell_L, \tilde{\ell}_L) \in SU(2)_L$ and $E_R = (e_L^c, \tilde{e}_R^*)$, $N_R = (\nu_L^c, \tilde{\nu}_R^*)$.

Soft-breaking terms

$$\begin{aligned}\mathcal{V}_{\text{soft}} =& (\mathcal{M}_{\tilde{\ell}}^2)^{IJ} \tilde{L}_L^{I*} \tilde{L}_L^J + (\mathcal{M}_{\tilde{e}}^2)^{IJ} \tilde{e}_R^I \tilde{e}_R^{J*} + (\mathcal{M}_{\tilde{\nu}}^2)^{IJ} \tilde{\nu}_R^I \tilde{\nu}_R^{J*} \\ & - \left[(B_\nu)^{IJ} \tilde{\nu}_R^{I*} \tilde{\nu}_R^{J*} + A_e^{IJ} H_1 \cdot \tilde{L}_L^I \tilde{e}_R^{J*} - A_\nu^{IJ} H_2 \cdot \tilde{L}_L^I \tilde{\nu}_R^{J*} + \text{h.c.} \right],\end{aligned}$$

One Numerical Example

- try to generate PMNS mixing completely radiatively
- "νMSSM": Y^ν in general arbitrary (for simplicity taken diagonal)
- all soft breaking masses assumed flavour blind
- only source of flavour mixing: soft trilinear couplings
- large off-diagonal values of A^e ruled out by $\ell_j \to \ell_i \gamma$
- toy numbers for m_0, M_1, M_2, μ, $\tan\beta$, ...

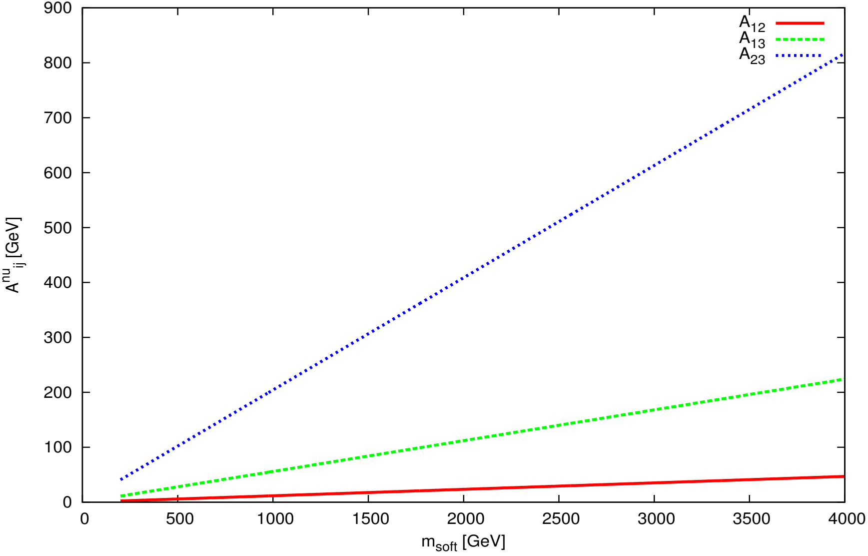
Some remarks...

- corrections to U_{ij}^{PMNS} more or less linear in A_{ij}^ν (for $i \neq j$)
- no decoupling, if all SUSY parameters shifted uniformly
- size of corrections correlated to Δm_{ij}^2 and, of course, U_{ij}^{phys}

One Numerical Example

- scan over uniform soft mass (assuming $\mathcal{M}^2_{\tilde{Q}}, \mathcal{M}^2_{\tilde{u}}, \mathcal{M}^2_{\tilde{d}}, \mathcal{M}^2_{\tilde{\ell}}, \mathcal{M}^2_{\tilde{e}} = m^2_{\text{soft}} \mathbb{1}$)
- display values of off-diagonal A parameters needed to generate corresponding mixing matrix element fully radiatively
- since corrections are linear in A, those grow linearly with m_{soft}

One Numerical Example

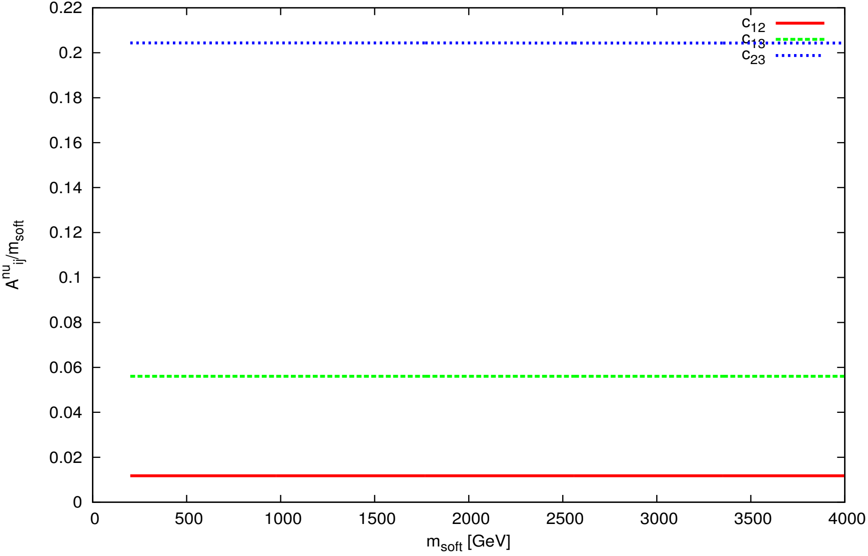


One Numerical Example

- dividing by $m_{\text{soft}}\ldots$

$$c_{ij} = \frac{A^{\nu}_{ij}(m_{\text{soft}})}{m_{\text{soft}}}$$

One Numerical Example



One Numerical Example

- corrections crucially depend on the neutrino mass spectrum:

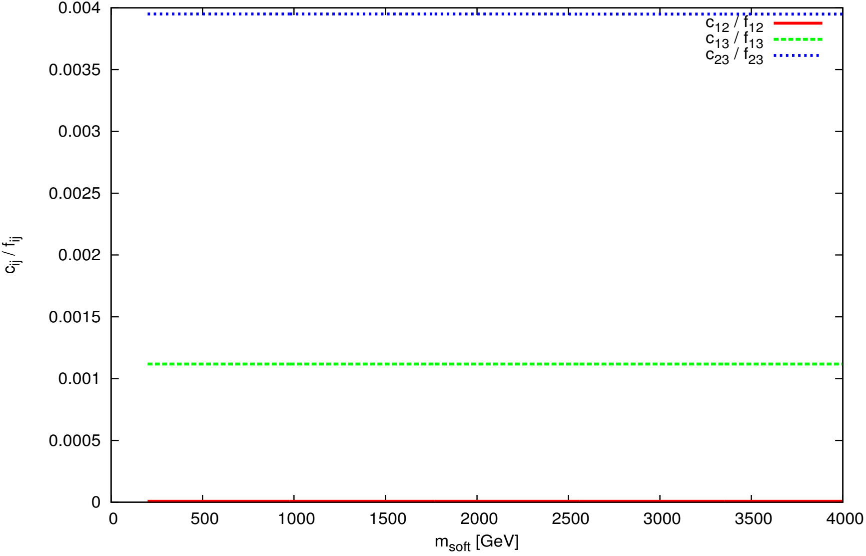
$$\Delta U^{\nu}_{fi} \sim \frac{m_{\nu_f} \Sigma_{fi}}{\Delta m^2_{\nu}}$$

- rough estimate: self energy $\Sigma_{fi} \sim m_{\nu_{i,f}}$
- devide by that:

$$f_{ij} = \frac{m_{\nu_f} m_{\nu_i}}{\Delta m^2_{fi}}$$

$$c_{ij} = \frac{A^{\nu}_{ij}(m_{\text{soft}})}{m_{\text{soft}}} \qquad \hookrightarrow \qquad \frac{c_{ij}}{f_{ij}}$$

One Numerical Example



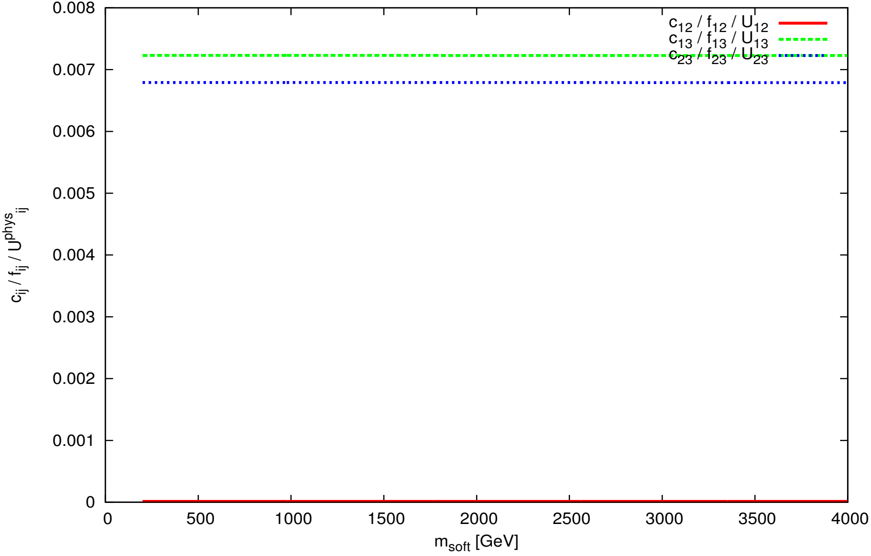
One Numerical Example

generate large PMNS elements

- size of correction larger for large U_{ij}
- correct for that

$$\tilde{c}_{ij} = \frac{c_{ij}}{f_{ij}} \bigg/ U_{ij}^{\text{phys}}$$

One Numerical Example



One Numerical Example

For completeness: numerical input values used for those plots

- $M_R = 10^{12}$ GeV
- SUSY scale: 2 TeV, $m_{\text{soft}} = 200, \ldots, 4000$ GeV
- $\tan\beta = 10$, $\mu = -3600$ GeV
- $m_\nu^{(0)} = 0.35$ eV (potential KATRIN discovery)
- $\Delta m^2_{12} = 7.54 \times 10^{-23}$ GeV^2, $\left|\Delta m^2_{13}\right| = 2.47 \times 10^{-21}$ GeV^2
- $|U_{12}| = 0.53$, $|U_{13}| = 0.15$, $|U_{23}| = 0.58$
- $M_1 = M_2 = m_{\text{soft}}$
- all other A values set to zero
- neutrino B term set to zero

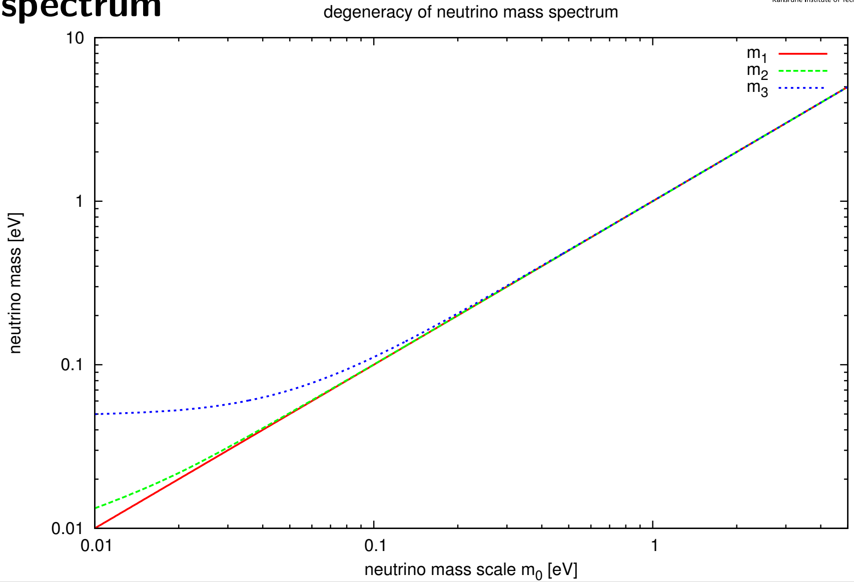
Conclusion

- described corrections very general to theories with new flavour structures
- can completely spoil tree-level mixing patterns
- simple extension of MSSM to incorporate ν masses can lead to lepton mixing from SUSY breaking in *sneutrino* sector
- numer(olog)ical example: at least for quasi-degenerate neutrino masses potential size of corrections for $(1,3)$ and $(2,3)$ mixing rather the same

Backup

Slides

Splitting of the neutrino mass spectrum



enhanced corrections to PMNS mixing

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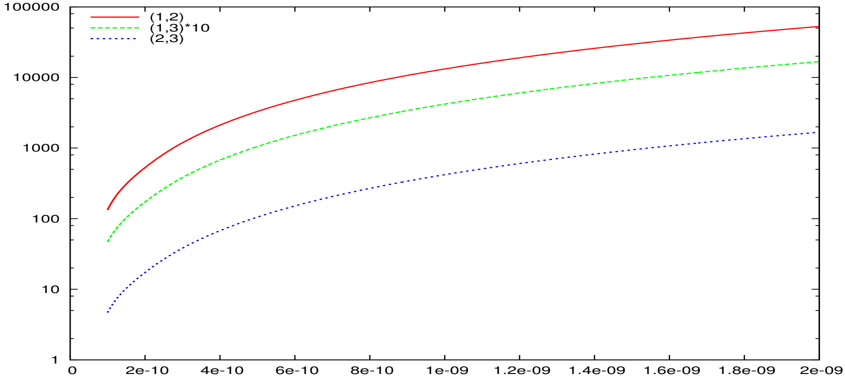
flavour changing self energies and sensitivity to neutrino mass

$$\Delta U^{\nu}_{fi} \sim \frac{m_{\nu_f} \Sigma_{fi}}{\Delta m^2_{\nu}} \sim \frac{m_{\nu_f} m_{\nu_i}}{\Delta m^2_{fi}} \leq 5 \times 10^3 \text{ for } m^0_{\nu} \sim 0.35 \text{ eV and } f, i = 1, 2$$



see-saw extended MSSM

Superpotential of the νMSSM

$$\mathcal{W}^\ell = \mu H_d \cdot H_u - Y_\ell^{IJ} H_d \cdot L_L^I E_R^J + Y_\nu^{IJ} H_u \cdot L_L^I N_R^J + \frac{1}{2} m_R^{IJ} N_R^I N_R^J,$$

where the chiral superfields are $L_L = (\ell_L, \tilde{\ell}_L) \in SU(2)_L$ and $E_R = (e_R^c \equiv (e_R)^c, \tilde{e}_R^*)$, $N_R = (\nu_L^c, \tilde{\nu}_R^*) \in SU(2)_R$, but leftchiral.

Soft-breaking terms

$$\begin{aligned}\mathcal{V}_{\text{soft}} =& (\mathcal{M}_{\tilde{\ell}}^2)^{IJ} \tilde{L}_L^{I*} \tilde{L}_L^J + (\mathcal{M}_{\tilde{e}}^2)^{IJ} \tilde{e}_R^I \tilde{e}_R^{J*} + (\mathcal{M}_{\tilde{\nu}}^2)^{IJ} \tilde{\nu}_R^I \tilde{\nu}_R^{J*} \\ & - \left[(B_\nu)^{IJ} \tilde{\nu}_R^{I*} \tilde{\nu}_R^{J*} + A_e^{IJ} H_1 \cdot \tilde{L}_L^I \tilde{e}_R^{J*} - A_\nu^{IJ} H_2 \cdot \tilde{L}_L^I \tilde{\nu}_R^{J*} + \text{h.c.} \right],\end{aligned}$$

effects on sneutrino mass matrix

- charged slepton mass matrix as in the MSSM
- sneutrino mass matrix in the MSSM: simple

$$\mathcal{M}^2_{\tilde{\nu}} = \begin{pmatrix} \mathcal{M}^2_{\tilde{\ell}} + M_Z^2 T_{3L}^{\tilde{\nu}} \cos 2\beta \mathbb{1} & \mathbb{1} \\ \mathbb{1} & \mathbb{0} \end{pmatrix}$$

- Majorana mass term $\nu_R^T m_R \nu_R$ inflates sneutrino mass matrix: additional terms $\sim \tilde{\nu}_R \tilde{\nu}_R, \tilde{\nu}_R^* \tilde{\nu}_R^*$

$$\mathcal{M}^2_{\tilde{\nu}} = \begin{pmatrix} \mathcal{M}^2_{L^*L} & \mathcal{M}^2_{L^*L^*} & \mathcal{M}^2_{L^*R^*} & \mathcal{M}^2_{L^*R} \\ \mathcal{M}^2_{LL} & \mathcal{M}^2_{LL^*} & \mathcal{M}^2_{LR^*} & \mathcal{M}^2_{LR} \\ \mathcal{M}^2_{RL} & \mathcal{M}^2_{RL^*} & \mathcal{M}^2_{RR^*} & \mathcal{M}^2_{RR} \\ \mathcal{M}^2_{R^*L} & \mathcal{M}^2_{R^*L^*} & \mathcal{M}^2_{R^*R^*} & \mathcal{M}^2_{R^*R} \end{pmatrix}$$

12×12-Matrix

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$$\mathcal{M}^2_{\tilde{\nu}} = \begin{pmatrix} \mathcal{M}^2_{LL} & \mathcal{M}^2_{LR} \\ (\mathcal{M}^2_{LR})^\dagger & \mathcal{M}^2_{RR} \end{pmatrix}$$

12×12-Matrix

full sneutrino squared mass matrix in the νMSSM

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$$\mathcal{M}^2_{\tilde{\nu}} = \frac{1}{2} \begin{pmatrix} \mathcal{M}^2_{LL} & \mathcal{M}^2_{LR} \\ (\mathcal{M}^2_{LR})^\dagger & \mathcal{M}^2_{RR} \end{pmatrix}$$

$$\mathcal{M}^2_{LL} = \begin{pmatrix} \mathcal{M}^2_{\tilde{\ell}} + \frac{1}{2} M_Z^2 \cos 2\beta \mathbf{1} + \mathbf{m}_\nu \mathbf{m}_\nu^\dagger & \mathbf{0} \\ \mathbf{0} & (\searrow)^* \end{pmatrix},$$

$$\mathcal{M}^2_{RL} = \begin{pmatrix} \frac{1}{2}\mathbf{m}_\nu \mathbf{m}_R & -\mu \cot\beta \mathbf{m}_\nu - v_2 \mathbf{A}_\nu^* \\ -\mu^* \cot\beta \mathbf{m}_\nu^* - v_2 \mathbf{A}_\nu & \frac{1}{2}\mathbf{m}_\nu^* \mathbf{m}_R^* \end{pmatrix},$$

$$\mathcal{M}^2_{RR} = \begin{pmatrix} (\mathcal{M}^2_{\tilde{\nu}})^T + \mathbf{m}_\nu^T \mathbf{m}_\nu^* + \frac{1}{2}\mathbf{m}_R^* \mathbf{m}_R & -2\mathbf{B}^* \\ -2\mathbf{B} & \mathcal{M}^2_{\tilde{\nu}} + \mathbf{m}_\nu^\dagger \mathbf{m}_\nu + \frac{1}{2}\mathbf{m}_R \mathbf{m}_R^* \end{pmatrix}.$$

effective sneutrino mass matrix

$$\mathcal{M}^2_{\tilde{\nu}\ell} = \begin{pmatrix} \mathbf{m}^2_{\Delta L=0} & (\mathbf{m}^2_{\Delta L=2})^* \\ \mathbf{m}^2_{\Delta L=2} & (\mathbf{m}^2_{\Delta L=0})^* \end{pmatrix} + \mathcal{O}\left(M^2_{\mathsf{SUSY}} m_R^{-2}\right),$$

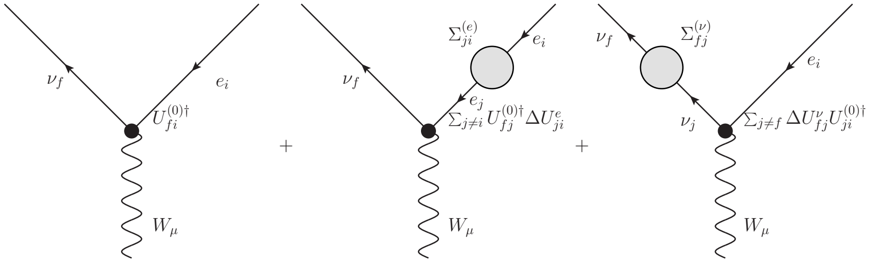
$$\mathbf{m}^2_{\Delta L=0} = \mathsf{MSSM} + \mathbf{m}^D_\nu \mathbf{m}^{D\dagger}_\nu - \mathbf{m}^D_\nu \mathbf{m}_R \left(\mathbf{m}^2_R + \mathcal{M}^2_{\tilde{\nu}}\right)^{-1} \mathbf{m}_R \mathbf{m}^D_\nu,$$

$$\mathbf{m}^2_{\Delta L=2} = X_\nu \mathbf{m}^D_\nu \left(\mathbf{m}^2_R + \mathcal{M}^2_{\tilde{\nu}}\right)^{-1} \mathbf{m}_R \mathbf{m}^{DT}_\nu + (\rightarrowtail)^T$$

$$- 2\mathbf{m}^{D*}_\nu \mathbf{m}_R \left[\mathbf{m}^2_R + (\mathcal{M}^2_{\tilde{\nu}})^T\right]^{-1} \mathbf{B} \left(\mathbf{m}^2_R + \mathcal{M}^2_{\tilde{\nu}}\right)^{-1} \mathbf{m}_R \mathbf{m}^{D\dagger}_\nu.$$

$$\boxed{X_\nu \mathbf{m}^D_\nu = -\mu^* \cot\beta \mathbf{m}^{D*}_\nu - v_2 \mathbf{A}_\nu}$$

radiative flavour violation in the lepton sector



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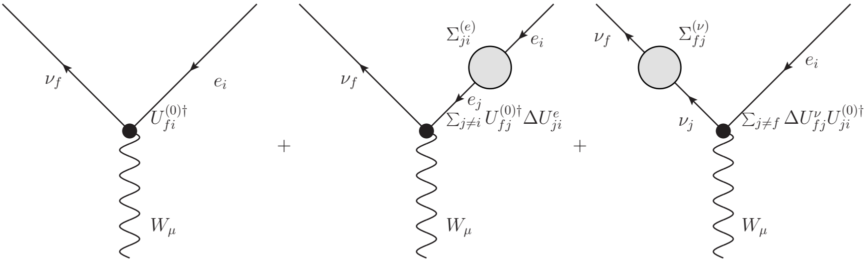


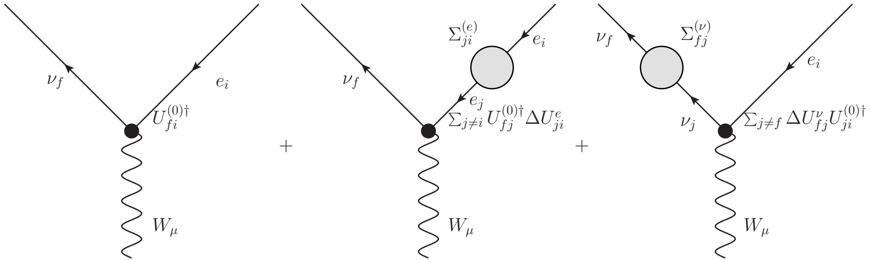
flavour changing self energies

$$\Sigma_{fi}^{\ell}(p) = \Sigma_{fi}^{\ell RL}(p^2) P_L + \Sigma_{fi}^{\ell LR}(p^2) P_R + \not{p}\left[\Sigma_{fi}^{\ell LL}(p^2) P_L + \Sigma_{fi}^{\ell RR}(p^2) P_R\right]$$

PMNS matrix renormalization

$$i\frac{g}{\sqrt{2}}\gamma^\mu P_L U^\dagger \to i\frac{g}{\sqrt{2}}\gamma^\mu P_L \left(\mathbb{1} + D_{L,fi} + D_{R,fi}\right),$$

radiative flavour violation in the lepton sector

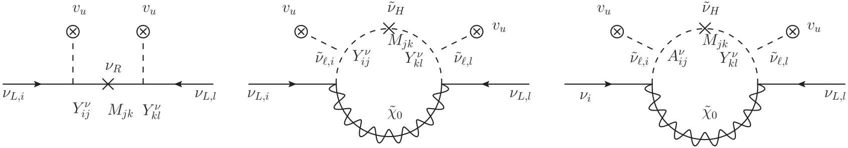


PMNS matrix renormalization

$$D_{L,fi} = \sum_{j\neq f} \frac{m_{\nu_f}\left(\Sigma^{(\nu)LR}_{fj} + m_{\nu_f}\Sigma^{(\nu)RR}_{fj}\right) + m_{\nu_j}\left(\Sigma^{(\nu)RL}_{fj} + m_{\nu_f}\Sigma^{(\nu)LL}_{fi}\right)}{m^2_{\nu_j} - m^2_{\nu_f}} U^{(0)\dagger}_{ji}$$

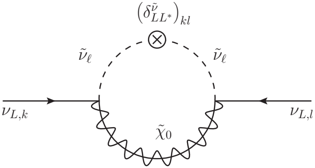
$$\equiv \sum_{j=1}^{3} [\Delta U^\nu_L]_{fj}\, U^{(0)\dagger}_{ji}$$

Majorana mass renormalization



effects of righthanded Neutrinos

- trilinear couplings A_ν
- see-saw-like terms in sneutrino mass matrix



$$\delta^{\tilde\nu}_{LL^*} \sim X_\nu \mathbf{m}^D_\nu \left(\mathbf{m}^2_R + \mathcal{M}^2_{\tilde\nu}\right)^{-1} \mathbf{m}_R \mathbf{m}^{DT}_\nu$$

$$\sim \frac{v_u A_\nu}{v_R^2} \qquad \text{with} \quad \mathbf{m}_R = v_R \mathbf{h}_R$$