Relating masses and mixing angles

a model-independent model

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[Nuclear Physics B 892 (2015) 364–389]

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29. Februar 2016| DPG Frühjahrstagung, Hamburg

Counting free (?) parameters

In the Standard Model: 19

gauge sector:	g_1, g_2, g_3
Higgs sector:	m_H, v
lepton masses:	m_e, m_μ, m_τ
quark masses:	m_u, m_d, m_s, m_c, m_b, m_t
CKM angles:	θ_{12}^{CKM}, θ_{23}^{CKM}, θ_{13}^{CKM}
CKM phase:	δ^{CKM}
θ-parameter:	θ

Neutrino oscillations? At least + 6

PMNS angles:	$\theta_{12}^{\text{PMNS}}$, $\theta_{23}^{\text{PMNS}}$, $\theta_{13}^{\text{PMNS}}$
PMNS phase:	δ^{PMNS}
neutrino masses:	m_0?, Δm_{12}^2, Δm_{23}^2

Most of the SM parameters are in the masses/mixings-sector.

Observation: $m_t \gg m_c \gg m_u$, $m_b \gg m_s \gg m_d$, $m_\tau \gg m_\mu \gg m_e$

Empirical relation [Gatto, Sartori, Tonin 1968]

Cabbibo angle:

$$\theta_C \approx \sqrt{\frac{m_d}{m_s}}$$

+ small correction from $\frac{m_u}{m_c}$
approximation: large hierarchy $m_d \ll m_s$

GST-like mixing angles follow from mass matrices with a structure

$$|\mathbf{M}| = \begin{pmatrix} 0 & \sqrt{m_1 m_2} \\ \sqrt{m_1 m_2} & m_2 - m_1 \end{pmatrix}$$

Hierarchy: $\sqrt{\frac{m_1}{m_2}} = \varepsilon \ll 1$, most general mass matrix

$$|\mathbf{M}| \sim \begin{pmatrix} \mathcal{O}(\varepsilon^2) & \mathcal{O}(\varepsilon) \\ \mathcal{O}(\varepsilon) & 1 + \mathcal{O}(\varepsilon^2) \end{pmatrix}, |\mathbf{M}\mathbf{M}^\dagger| \sim \begin{pmatrix} \mathcal{O}(\varepsilon^2) & \mathcal{O}(\varepsilon) \\ \mathcal{O}(\varepsilon) & 1 + \mathcal{O}(\varepsilon^2) \end{pmatrix}$$

Singular Value Decomposition

$$-\mathcal{L}_Y \supset Y_{ij}\bar{L}_i \cdot \Phi\ R_j + \text{h.c.}$$

diagonalize $\mathbf{Y}$ as $\mathbf{S}_L\mathbf{Y}\ \mathbf{S}_R^\dagger = \mathbf{\Sigma}$

Large hierarchy in singular values: $\Sigma_{11} \ll \Sigma_{22} \ll \Sigma_{33}$

Schmidt–Eckart–Young–Mirsky theorem

lower-rank approximation, take $\mathbf{S}_{L/R} = [\vec{s}_{L/R,1}, \vec{s}_{L/R,2}, \vec{s}_{L/R,3}]$:

$$\mathbf{M} = m_3\left[\left(\vec{s}_{L,1}\frac{m_1}{m_2}\vec{s}_{R,1}^{\,\dagger} + \vec{s}_{L,2}\vec{s}_{R,2}^{\,\dagger}\right)\frac{m_2}{m_3} + \vec{s}_{L,3}\vec{s}_{R,3}^{\,\dagger}\right]$$

Singular Value Decomposition

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Rank one approximation

$$\hat{\mathbf{M}} = \vec{s}_{L,3} \vec{s}_{R,3}^{\,\dagger} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Singular Value Decomposition

$$-\mathcal{L}_Y \supset Y_{ij}\bar{L}_i \cdot \Phi\, R_j + \text{h.c.}$$

diagonalize $\mathbf{Y}$ as $\mathbf{S}_L \mathbf{Y}\, \mathbf{S}_R^\dagger = \mathbf{\Sigma}$

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Rank one approximation

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The philosphy behind rank-one matrices

Minimal breaking of maximal flavor symmetry

SM gauge interactions w/o Yukawa couplings (+rh νs):

$$\left[U(3)\right]^6$$
$$\downarrow$$
$$\left[U(2)\right]^6$$
$$\downarrow$$
$$\left[U(1)\right]^6$$
$$\downarrow$$
$$U(1)_B \times U(1)_L$$

$\mathrm{rank} = 0 \to \mathrm{rank} = 3$

The "flavor blind principle" [Saldaña-Salazar 2016]

Yukawa couplings are "flavor blind"

$$\mathbf{Y}_1 \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

and/or obey special symmetries

$$\mathbf{Y}_2 = \begin{pmatrix} \alpha & \alpha & \beta_1 \\ \alpha & \alpha & \beta_1 \\ \beta_2 & \beta_2 & \gamma \end{pmatrix}$$

$S_3^L \times S_3^R \to S_2^L \times S_2^R \to S_2^S + S_2^A$

no masses

flavor symmetry: $U(3)_{123}$

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$m_3 \neq 0$$

remnant flavor symmetry: $U(2)_{12}$

$$\mathbf{M}_{r=1} = \begin{pmatrix} m_3 & m_3 & m_3 \\ m_3 & m_3 & m_3 \\ m_3 & m_3 & m_3 \end{pmatrix} \rightarrow m_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$m_3 \gg m_2 \neq 0$$

remnant flavor symmetry: $U(1)_1$

$$\mathbf{M}_{r=3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{m_2 m_3} \\ 0 & \sqrt{m_2 m_3} & m_3 - m_2 \end{pmatrix}$$

Physical mixing angle

$$\tan\theta_{23}^{(0)} = \sqrt{m_2/m_3}$$

$$m_3 \gg m_2 \gg m_1 \neq 0$$

global symmetry: $U(1)_F$, where $F = B, L$

$$\tilde{\mathbf{M}} = \begin{pmatrix} * & * & * \\ * & m_2 & * \\ * & * & m_3 \end{pmatrix}$$

corrections $\mathcal{O}(m_1)$ everywhere

Chain of successive rotations

$$\begin{aligned} R = & R_{12}\left(\frac{m_1}{m_2}\right) R_{13}\left(\frac{m_1 m_2}{m_3^2}\right) R_{13}\left(\frac{m_2^2}{m_3^2}\right) R_{13}\left(\frac{m_1}{m_3}\right) \\ & \times R_{23}\left(\frac{m_1 m_2}{m_3^2}\right) R_{23}\left(\frac{m_1}{m_3}\right) R_{23}\left(\frac{m_2}{m_3}\right) \end{aligned}$$

A guess for the mixing matrix

So far:

- hierarchical masses
- expectation: $\tan\theta_{ij} = \sqrt{m_i/m_j}$ with $i < j$
- be aware that mass matrices are arbitrary complex matrices
- finally only one complex phase remains

$$R\left(\frac{m_i}{m_j}, \delta_{ij}\right) = \frac{1}{\sqrt{1+\frac{m_i}{m_j}}} \begin{pmatrix} 1 & \sqrt{\frac{m_i}{m_j}} e^{-\mathrm{i}\,\delta_{ij}} \\ -\sqrt{\frac{m_i}{m_j}} e^{\mathrm{i}\,\delta_{ij}} & 1 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta_{ij} & \sin\theta_{ij} e^{\mathrm{i}\,\delta_{ij}} \\ -\sin\theta_{ij} e^{-\mathrm{i}\,\delta_{ij}} & \cos\theta_{ij} \end{pmatrix}$$

puzzle:

- What to do with the complex phases?
- possible interferences

The origin of the CKM matrix

$$\mathcal{L}_{\mathsf{CC}} = -\frac{\mathrm{i}g_2}{\sqrt{2}} W_\mu^+ \bar{u}_L \gamma^\mu d_L + \text{h.c.} \to -\frac{\mathrm{i}g_2}{\sqrt{2}} W_\mu^+ \bar{u}'_L \boldsymbol{S}_L^u \gamma^\mu \boldsymbol{S}_L^{d\,\dagger} d_L + \text{h.c.}$$

V_{CKM}

$$\boldsymbol{V}_{\mathsf{CKM}} = \boldsymbol{S}_L^u\, \boldsymbol{S}_L^{d\,\dagger}$$

$$\boldsymbol{V}_{\mathsf{CKM}} = \boldsymbol{V}_{23}(\theta_{23}^{\mathsf{CKM}}) \boldsymbol{V}_{13}(\theta_{13}^{\mathsf{CKM}}, \delta_{\mathsf{CKM}}) \boldsymbol{V}_{12}(\theta_{12}^{\mathsf{CKM}})$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{-\mathrm{i}\,\delta_{\mathsf{CKM}}} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{\mathrm{i}\,\delta_{\mathsf{CKM}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

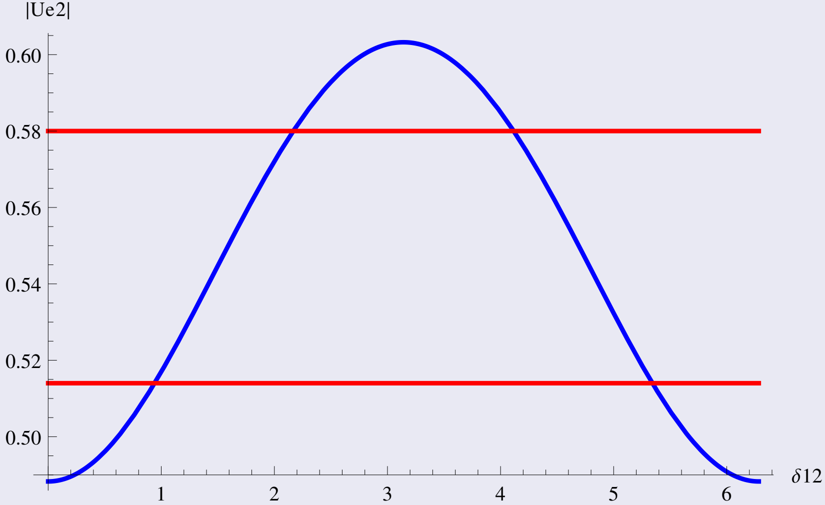
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-\mathrm{i}\,\delta_{\mathsf{CKM}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathsf{CKM}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathsf{CKM}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathsf{CKM}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\mathrm{i}\,\delta_{\mathsf{CKM}}} & c_{23}c_{13} \end{pmatrix}$$

We deconstruct the CKM matrix as $\boldsymbol{V}_{\mathsf{CKM}} = \boldsymbol{S}^u_{\mathrm{L}} \left(\boldsymbol{S}^d_{\mathrm{L}}\right)^\dagger$ with

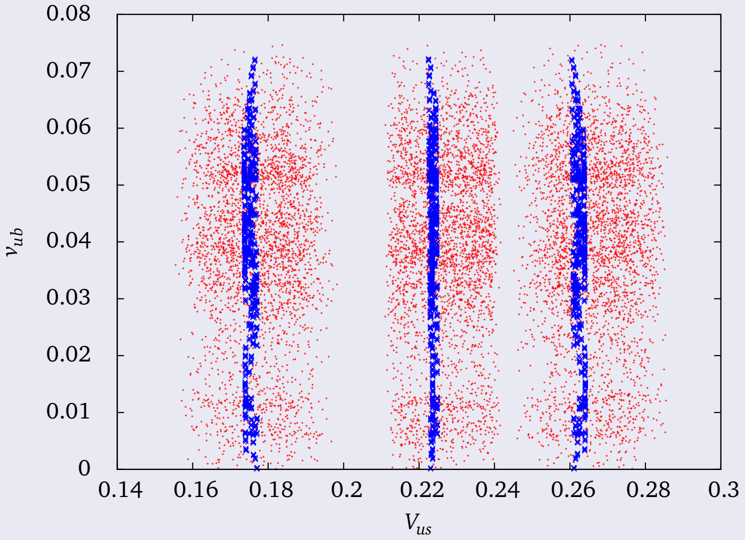
$$\begin{aligned}
\boldsymbol{S}^u_{\mathrm{L}} = {} & \boldsymbol{S}^{\mathrm{L},\,u}_{12}\left(\frac{m_{\mathrm{u}}}{m_{\mathrm{c}}}\right)\boldsymbol{S}^{\mathrm{L},\,u}_{13}\left(\frac{m_{\mathrm{u}}m_{\mathrm{c}}}{m_{\mathrm{t}}^2}\right)\boldsymbol{S}^{\mathrm{L},\,u}_{13}\left(\frac{m_{\mathrm{c}}^2}{m_{\mathrm{t}}^2}\right)\boldsymbol{S}^{\mathrm{L},\,u}_{13}\left(\frac{m_{\mathrm{u}}}{m_{\mathrm{t}}}\right) \\
& \times \boldsymbol{S}^{\mathrm{L},\,u}_{23}\left(\frac{m_{\mathrm{u}}m_{\mathrm{c}}}{m_{\mathrm{t}}^2}\right)\boldsymbol{S}^{\mathrm{L},\,u}_{23}\left(\frac{m_{\mathrm{u}}}{m_{\mathrm{t}}}\right)\boldsymbol{S}^{\mathrm{L},\,u}_{23}\left(\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\right), \\
\boldsymbol{S}^{d\,\dagger}_{\mathrm{L}} = {} & \boldsymbol{S}^{\mathrm{L},\,d\dagger}_{23}\left(\frac{m_{\mathrm{s}}}{m_{\mathrm{b}}},\delta^{(0)}_{23}\right)\boldsymbol{S}^{\mathrm{L},\,d\dagger}_{23}\left(\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}},\delta^{(1)}_{23}\right)\boldsymbol{S}^{\mathrm{L},\,d\dagger}_{23}\left(\frac{m_{\mathrm{d}}m_{\mathrm{s}}}{m_{\mathrm{b}}^2},\delta^{(2)}_{23}\right) \\
& \times \boldsymbol{S}^{\mathrm{L},\,d\dagger}_{13}\left(\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}},\delta^{(0)}_{13}\right)\boldsymbol{S}^{\mathrm{L},\,d\dagger}_{13}\left(\frac{m_{\mathrm{s}}^2}{m_{\mathrm{b}}^2},\delta^{(1)}_{13}\right)\boldsymbol{S}^{\mathrm{L},\,d\dagger}_{13}\left(\frac{m_{\mathrm{d}}m_{\mathrm{s}}}{m_{\mathrm{b}}^2},\delta^{(2)}_{13}\right) \\
& \times \boldsymbol{S}^{\mathrm{L},\,d\dagger}_{12}\left(\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}},\delta_{12}\right),
\end{aligned}$$

Final mixing matrix elements depent on the phases

e.g. PMNS mixing, $U_{e2} = 0.514 \ldots 0.580$ @ 3σ



Finally only *one* out of 2187 combinations allowed



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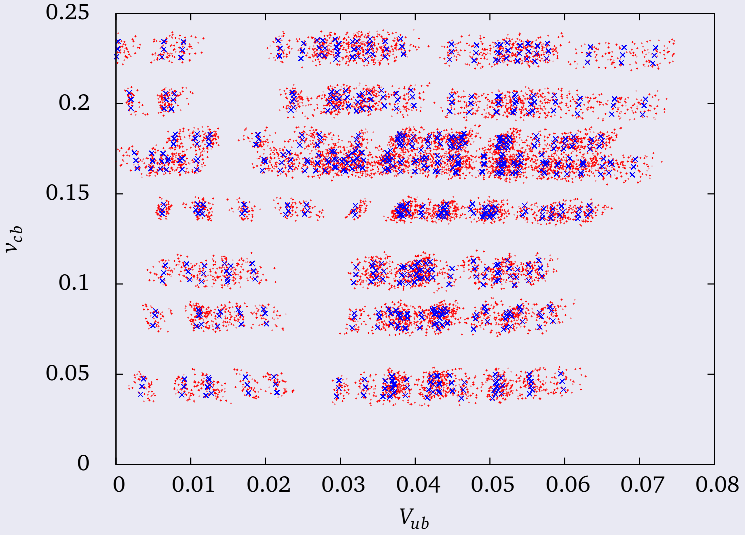


Comments on the phase choice

- on one hand, we can (nearly) predict everything
- on the other hand: fixing the phases by a look into data sets viable patterns/textures for the mass matrices
- "fitting" the phase combinations point towards the underlying flavor symmetry

Either minimal (i.e. no) or maximal CP violation

choose $\delta_{ij}^{a(x)} \in \{0, \frac{\pi}{2}, \pi\}$

	δ_{12}	$\delta_{13}^{(0)}$	$\delta_{13}^{(1)}$	$\delta_{13}^{(2)}$	$\delta_{23}^{(0)}$	$\delta_{23}^{(1)}$	$\delta_{23}^{(2)}$
CKM	$\frac{\pi}{2}$	0	π	π	0	π	π
PMNS	$\frac{\pi}{2}$	0	π	π	π	π	0

- only one non-vanishing CP-phase: $\delta_{12} = \frac{\pi}{2}$

[see also Masina, Savoy 2006]

Final result: Our guess for the mixing matrices

CKM matrix (our values)

$$|V_{\mathsf{CKM}}^{\mathsf{th}}| = \begin{pmatrix} 0.974^{+0.004}_{-0.003} & 0.225^{+0.016}_{-0.011} & 0.0031^{+0.0018}_{-0.0015} \\ 0.225^{+0.016}_{-0.011} & 0.974^{+0.004}_{-0.003} & 0.039^{+0.005}_{-0.004} \\ 0.0087^{+0.0010}_{-0.0008} & 0.038^{+0.004}_{-0.004} & 0.9992^{+0.0002}_{-0.0001} \end{pmatrix}$$

Jarlskog invariant: $J_q = \mathrm{Im}(V_{us}V_{cb}V_{ub}^*V_{cs}^*) = (2.6^{+1.3}_{-1.0}) \times 10^{-5}$

PMNS matrix (our values)

$$|U_{\mathsf{PMNS}}^{\mathsf{th}}| = \begin{pmatrix} 0.83^{+0.04}_{-0.05} & 0.54^{+0.06}_{-0.09} & 0.14 \pm 0.03 \\ 0.38^{+0.04}_{-0.06} & 0.57^{+0.03}_{-0.04} & 0.73 \pm 0.02 \\ 0.41^{+0.04}_{-0.06} & 0.61^{+0.03}_{-0.04} & 0.67 \pm 0.02 \end{pmatrix}$$

$$J_\ell = \mathrm{Im}(U_{e2}U_{\mu 3}U_{e3}^*U_{\mu 2}^*) = 0.031^{+0.006}_{-0.007}$$

$\Rightarrow$ our prediction: $|\delta_{\mathsf{Dirac}}^{\mathsf{PMNS}}| = 90^\circ \pm 20^\circ$

Final result: Our guess for the mixing matrices

CKM matrix (PDG)

$$|V_{\mathsf{CKM}}| = \begin{pmatrix} 0.97427^{+0.00014}_{-0.00014} & 0.22536^{+0.00061}_{-0.00061} & 0.00355^{+0.00015}_{-0.00015} \\ 0.22522^{+0.00061}_{-0.00061} & 0.97343^{+0.00015}_{-0.00015} & 0.0414^{+0.0012}_{-0.0012} \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914^{+0.00005}_{-0.00005} \end{pmatrix}$$

Jarlskog invariant: $J_q = \mathrm{Im}(V_{us} V_{cb} V_{ub}^{*} V_{cs}^{*}) = (3.06^{+0.21}_{-0.20}) \times 10^{-5}$

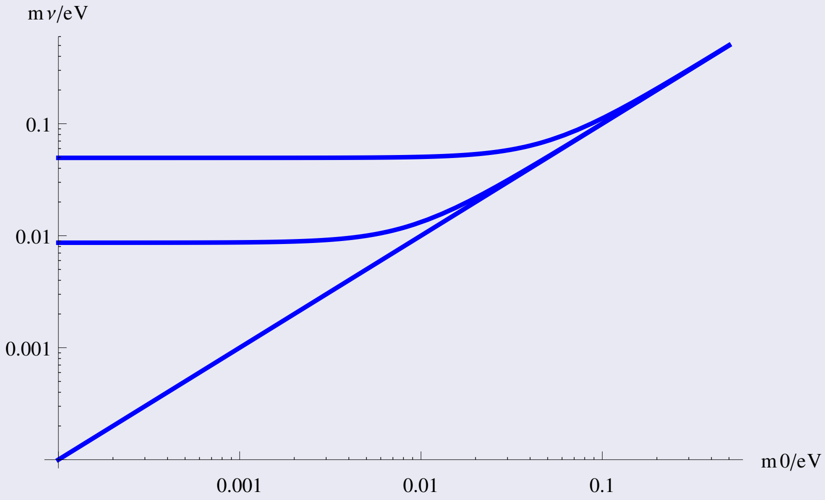
PMNS matrix (`nu-fit.org`, $3\,\sigma$)

$$|U_{\mathsf{PMNS}}| = \begin{pmatrix} 0.801 \to 0.845 & 0.514 \to 0.580 & 0.137 \to 0.158 \\ 0.225 \to 0.517 & 0.441 \to 0.699 & 0.614 \to 0.793 \\ 0.246 \to 0.529 & 0.464 \to 0.713 & 0.590 \to 0.776 \end{pmatrix}$$

$J_\ell^{\mathsf{max}} = 0.033 \pm 0.010$

$\Rightarrow$ our prediction: $|\delta_{\mathsf{Dirac}}^{\mathsf{PMNS}}| = 90^\circ \pm 20^\circ$

Neutrino mass hierarchy unknown



Inverting the procedure

$$|U_{e2}| \approx \sqrt{\frac{\hat{m}_{e\mu} + \hat{m}_{\nu 12} - 2\sqrt{\hat{m}_{e\mu}\hat{m}_{\nu 12}}\cos(\delta^e_{12} - \delta^\nu_{12})}{(1+\hat{m}_{e\mu})(1+\hat{m}_{\nu 12})}}$$

with $\delta^e_{12} - \delta^\nu_{12} = \frac{\pi}{2}$, we get

$$\hat{m}_{\nu 12} = \frac{|U_{e2}|^2(1+\hat{m}_{e\mu}) - \hat{m}_{e\mu}}{1 - |U_{e2}|^2(1+m_{e\mu})} = 0.41 \ldots 0.45$$

Predicting the neutrino mass spectrum

Δm^2_{21} and Δm^2_{31} from `nu-fit`

$$\begin{aligned} m_{\nu_1} &= (0.0041 \pm 0.0015)\,\mathrm{eV} \\ m_{\nu_2} &= (0.0096 \pm 0.0005)\,\mathrm{eV} \\ m_{\nu_3} &= (0.050 \pm 0.001)\,\mathrm{eV} \end{aligned}$$

KATRIN-mass: $\sqrt{\sum_i |U_{ei}|^2 m^2_{\nu_i}} \approx 0.01\,\mathrm{eV} < 0.2\,\mathrm{eV}$

Minimal breaking of maximal flavor symmetry
$\to$ The flavor blind principle [Saldaña-Salazar 2016]
$=$ a discrete version

the flavor blind principle
flavor symmetry: $S_3^L \times S_3^R$

$$\mathbf{M}_1 = m \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Minimal breaking of maximal flavor symmetry
$\rightarrow$ The flavor blind principle [Saldaña-Salazar 2016]
$=$ a discrete version

successive breaking: rank 1 $\rightarrow$ rank 2
remnant flavor symmetry: $S_2^L \times S_2^R$

$$\mathbf{M}_2 = \begin{pmatrix} \alpha & \alpha & \beta_1 \\ \alpha & \alpha & \beta_1 \\ \beta_2 & \beta_2 & \gamma \end{pmatrix}$$

Things to do

Minimal breaking of maximal flavor symmetry
$\rightarrow$ The flavor blind principle [Saldaña-Salazar 2016]
$=$ a discrete version

last step: minimal breaking
remnant flavor symmetry: $S_2^S + S_2^A$

$$\mathbf{M}_3 = \begin{pmatrix} \zeta & \eta & \kappa_1 \\ \eta & \zeta & \kappa_1 \\ \kappa_2 & \kappa_2 & \lambda \end{pmatrix} + \begin{pmatrix} \mu & \nu & \xi_1 \\ -\nu & -\mu & -\xi_1 \\ \xi_2 & -\xi_2 & 0 \end{pmatrix}$$

Interpreting the symmetrical structures

different bases: democratic vs. heavy

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} : \ \mathcal{O}_3 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

different bases: democratic vs. heavy

$$\begin{pmatrix} \alpha & \alpha & \beta_1 \\ \alpha & \alpha & \beta_1 \\ \beta_2 & \beta_2 & \gamma \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{m_2 m_3} \\ 0 & -\sqrt{m_2 m_3} & m_3 - m_2 \end{pmatrix}$$

different bases: democratic vs. heavy

$$\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} : \mathcal{O}_3 = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}}\end{pmatrix} \quad \rightarrow \quad \begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$$

Mass matrices as linear combinations of Yukawa couplings:

$$\mathbf{M} = v_1\, y \begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + v_2 \begin{pmatrix}\alpha&\alpha&\beta_1\\\alpha&\alpha&\beta_1\\\beta_2&\beta_2&\gamma\end{pmatrix}$$
$$+ v_3 \left[\begin{pmatrix}\zeta&\eta&\kappa_1\\\eta&\zeta&\kappa_1\\\kappa_2&\kappa_2&\lambda\end{pmatrix} + \begin{pmatrix}\mu&\nu&\xi_1\\-\nu&-\mu&-\xi_1\\\xi_2&-\xi_2&0\end{pmatrix}\right]$$

motivates a flavored Higgs multi-doublet model, e.g. $S_3^L \times S_3^R \times S_3^H$

- hierarchical masses $\hookrightarrow$ minimal breaking of maximal symmetry
- reparameterization of mixing angles in terms of mass ratios reduces the number of free parameters
- "prediction" where CP violation shall be located
- PMNS case: predicts low m_{ν_1}, approx. maximal Dirac phase
- mass matrices decomposable into linear combinations of lower symmetric matrices
- applicability in flavored multi-Higgs models explored

Conclusions

- hierarchical masses $\hookrightarrow$ minimal breaking of maximal symmetry
- reparameterization of mixing angles in terms of mass ratios reduces the number of free parameters
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Special thanks to Ulises Jésus Saldaña-Salazar who brought this idea!