CCB minima in the MSSM

the bottom-up connection

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today $\longrightarrow$

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Minimal Supersymmetric Standard Model

- enhanced scalar sector
- potentially destabilizing electroweak vacuum
- tree-level constraints à la

$$A_{\mathrm{t}}^2 < 3(m_{22}^2 + \tilde{m}_Q^2 + \tilde{m}_t^2)$$

- further: replace $t \to b$, $2 \to 1$
- relating soft SUSY breaking masses and trilinear couplings

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- Higgs + 3rd generation squarks: already too complicated
- look for promising "directions in field space"
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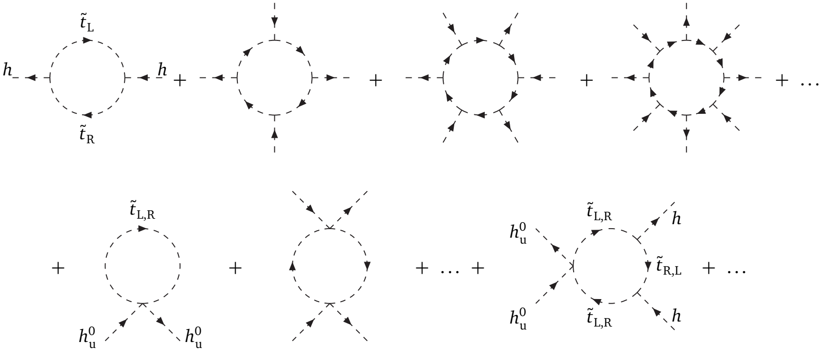
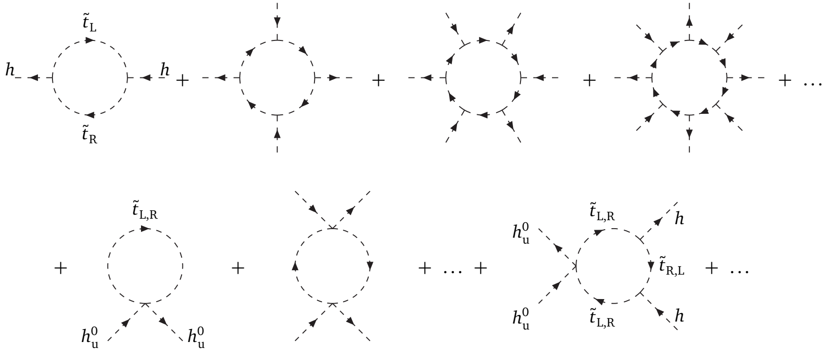
- problem solved (numerically) [Vevacious]



- dominant contribution from third generation squarks ($\tilde{t}$ and $\tilde{b}$)
- quadrilinear couplings ($\sim |Y_\mathsf{t}|^2, |Y_\mathsf{b}|^2$)
- trilinear coupling to a linear combination ($\mu^* Y_\mathsf{t} h_\mathsf{d}^\dagger - A_\mathsf{t} h_\mathsf{u}^0$)
- series summable to an infinite number of external legs

The 1-loop effective Higgs potential and CCB minima



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- **Do not stop after renormalizable / dim 4 terms!**

Features of the resummed series

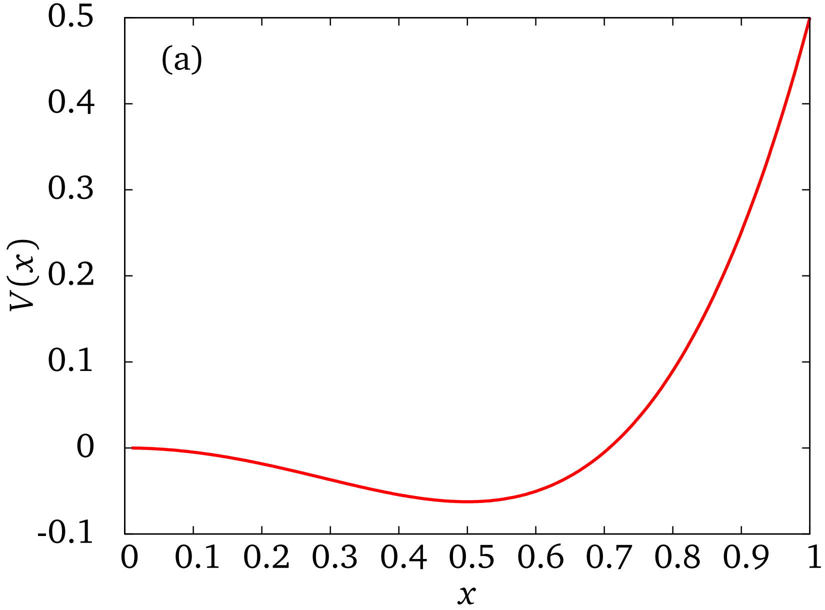
[Bobrowski, Chalons, WGH, Nierste: PRD90.035025]

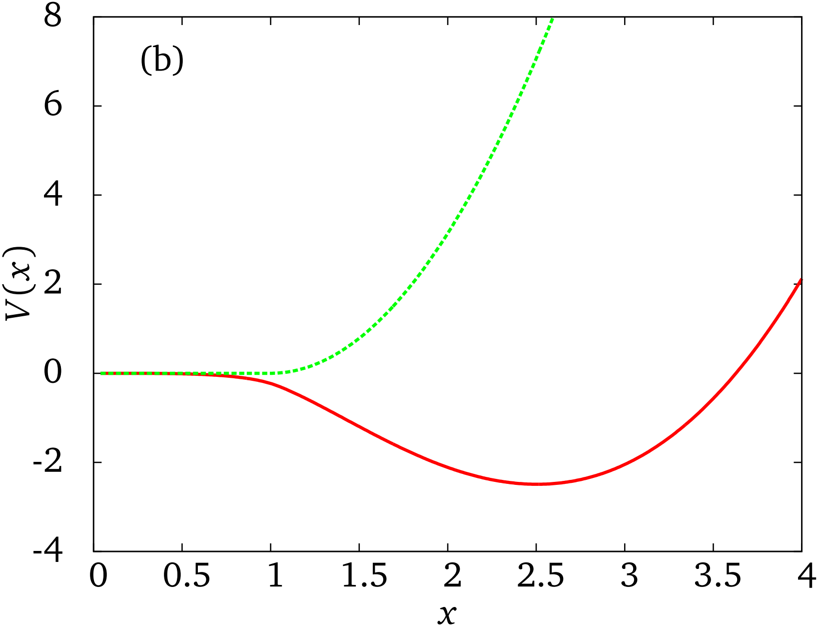
$$V_1(h_u^0, h_d^0) = \frac{N_c M^4}{32\pi^2}\Big[(1+y+x)^2 \log(1+y+x) + (1+y-x)^2 \log(1+y-x) - 3(x^2+y^2+2y)\Big]$$

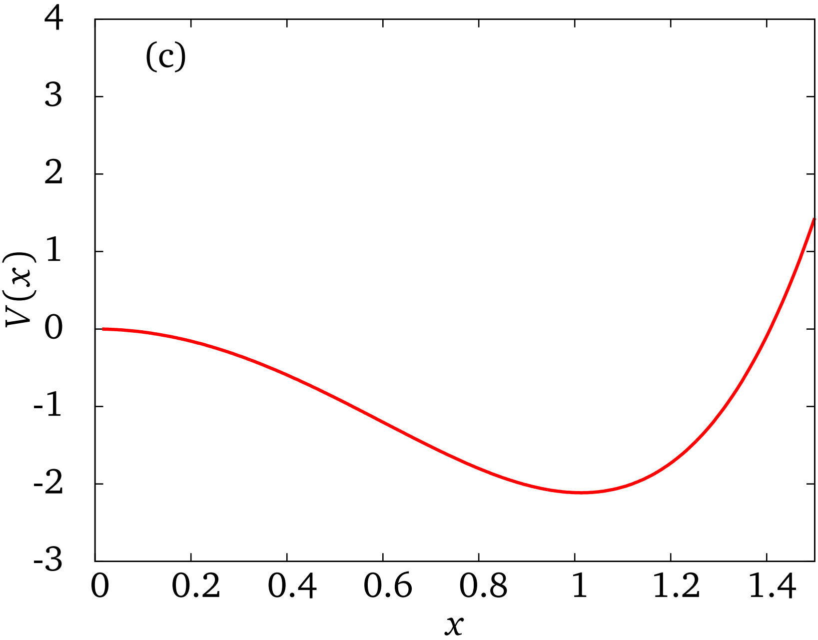
$$x^2 = \frac{|\mu Y_t|^2 h^\dagger h}{M^4},\ h = h_d^{0*} - \frac{A_t}{\mu^* Y_t} h_u^0, \qquad y = \frac{|Y_t h_u^0|^2}{M^2}$$

- branch cut at $x - y = 1$: take real part (analytic continuation)
- ignore imaginary part? $\log(1+y-x) = \frac{1}{2}\log\left((1+y-x)^2\right)$
- minimum independent of Higgs parameters from tree potential
- minimum sensitive to New Physics in the loop

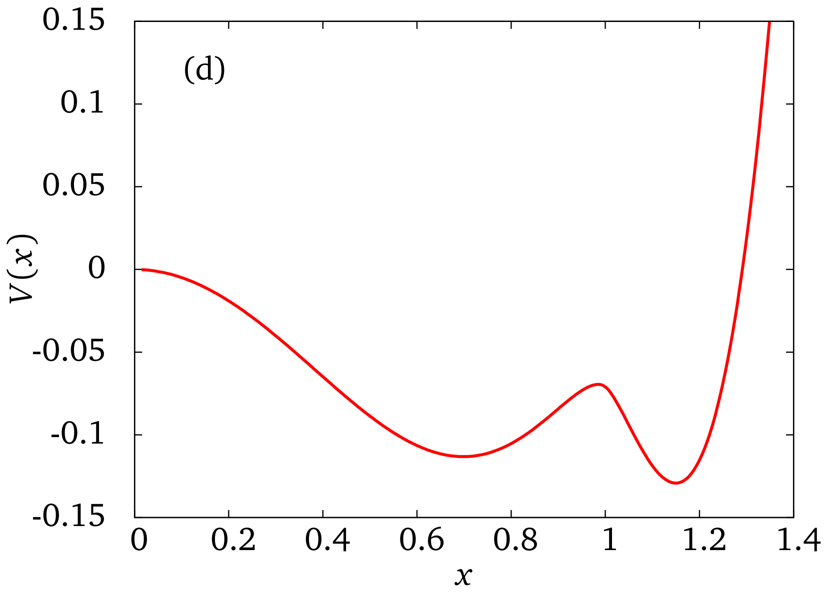
Tree, loop and tree + loop



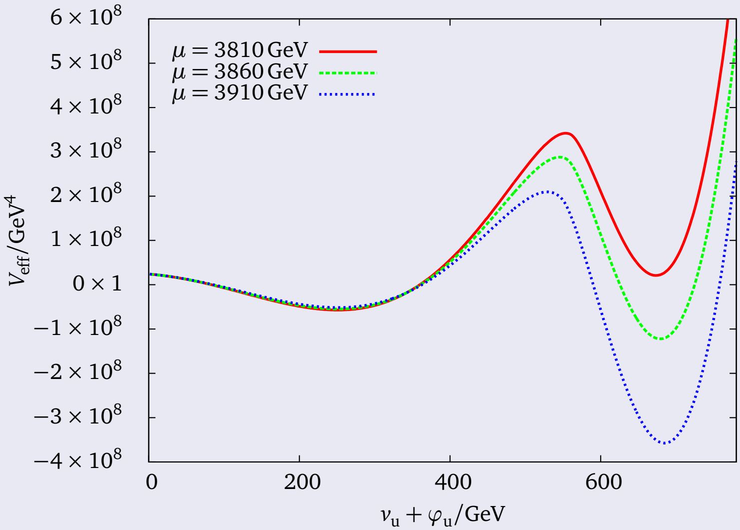

(b)
8
6
4
2
0
-2
-4
V(x)
0
0.5
1
1.5
2
2.5
3
3.5
4
x


(c)
V(x)
x
4
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Tree, loop and tree + loop



Access to Charge and Color breaking minima



Access to Charge and Color breaking minima

$$\mathcal{M}^2_{\tilde{t}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_t h_u^0|^2 & A_t h_u^0 - \mu^* Y_t h_d^{0*} \\ A_t^* h_u^{0*} - \mu Y_t^* h_d^0 & \tilde{m}_t^2 + |Y_t h_u^0|^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}}(h_u^0, h_d^0) = \begin{pmatrix} \tilde{m}_Q^2 + |Y_b h_d^0|^2 & A_b h_d^0 - \mu^* Y_b h_u^{0*} \\ A_b^* h_d^{0*} - \mu Y_b^* h_u^0 & \tilde{m}_b^2 + |Y_b h_d^0|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev

Access to Charge and Color breaking minima

$$\mathcal{M}^2_{\tilde{t}}(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}^2_Q + |Y_t h^0_u|^2 & A_t h^0_u - \mu^* Y_t h^{0*}_d \\ A^*_t h^{0*}_u - \mu Y^*_t h^0_d & \tilde{m}^2_t + |Y_t h^0_u|^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{b}}(h^0_u, h^0_d) = \begin{pmatrix} \tilde{m}^2_Q + |Y_b h^0_d|^2 & A_b h^0_d - \mu^* Y_b h^{0*}_u \\ A^*_b h^{0*}_d - \mu Y^*_b h^0_u & \tilde{m}^2_b + |Y_b h^0_d|^2 \end{pmatrix}$$

- non-trivial behaviour of sfermions masses with Higgs vev:

$$m^2_{\tilde{b}_{1,2}}(h^0_u, h^0_d) = \frac{\tilde{m}^2_Q + \tilde{m}^2_b}{2} + |Y_b h^0_d|^2 \pm \frac{1}{2}\sqrt{(\tilde{m}^2_Q - \tilde{m}^2_b)^2 + 4|A_b h^0_d - \mu^* Y_b h^{0*}_u|^2}$$

- expand theory around new minimum: $m^2_{\tilde{b}_2} < 0$

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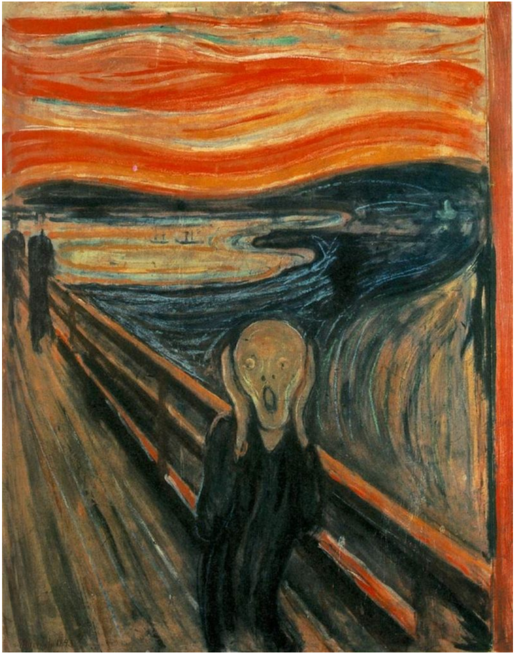
- **tachyonic squark mass!**



[commons.wikimedia.org]

What does a tachyonic mass mean?

- mass ⇔ second derivative: $m_\phi^2 = \partial^2 V / \partial \phi^2$
- $m_\phi^2 < 0 \quad \Leftrightarrow \quad$ negative curvature
- non-convex potential: imaginary part
- $\log(1 + y - x) \sim \log(m_\phi^2)$

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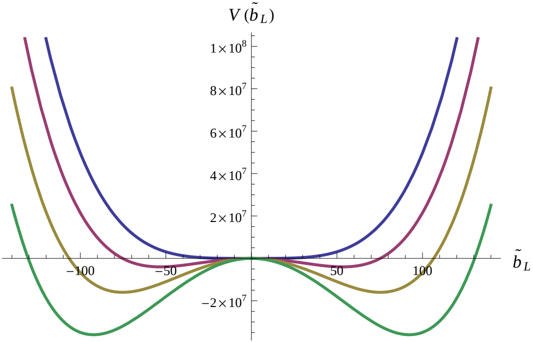

A more complete picture

Including colored directions

$$V_{\tilde{b}}^{\text{tree}} = \tilde{b}_{\mathrm{L}}^{*}(M_{\tilde{Q}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{L}} + \tilde{b}_{\mathrm{R}}^{*}(M_{\tilde{\mathrm{b}}}^{2} + |Y_{\mathrm{b}} v_{\mathrm{d}}|^{2})\tilde{b}_{\mathrm{R}}$$

$$- \left[\tilde{b}_{\mathrm{L}}^{*}(\mu^{*} Y_{\mathrm{b}} h_{\mathrm{u}}^{0\dagger} - A_{\mathrm{b}} v_{\mathrm{d}})\tilde{b}_{\mathrm{R}} + \text{h. c.}\right] + |Y_{\mathrm{b}}|^{2}|\tilde{b}_{\mathrm{L}}|^{2}|\tilde{b}_{\mathrm{R}}|^{2}$$

$$+ \; D\text{-terms.}$$

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D-flat direction

- D-terms: $g^2\phi^4$

$$V_D = \frac{g_1^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 + \frac{1}{3}|\tilde{b}_L|^2 + \frac{2}{3}|\tilde{b}_R|^2\right)^2 + \frac{g_2^2}{8}\left(|h_{\mathrm{u}}^0|^2 - |h_{\mathrm{d}}^0|^2 + |\tilde{b}_L|^2\right)^2 + \frac{g_3^2}{6}\left(|\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2.$$

- will always take over unless they vanish
- either take $\tilde{b}_L = \tilde{b}_R = \tilde{b}$, $|h_{\mathrm{d}}^0|^2 = |h_{\mathrm{u}}^0|^2 + |\tilde{b}|^2$

Including colored directions

$$V_{\tilde{b}}^{\text{tree}} = \tilde{b}_{\rm L}^*(M_{\tilde{Q}}^2 + |Y_{\rm b}v_{\rm d}|^2)\tilde{b}_{\rm L} + \tilde{b}_{\rm R}^*(M_{\tilde{\rm b}}^2 + |Y_{\rm b}v_{\rm d}|^2)\tilde{b}_{\rm R}$$
$$- \left[\tilde{b}_{\rm L}^*(\mu^* Y_{\rm b} h_{\rm u}^{0\dagger} - A_{\rm b}v_{\rm d})\tilde{b}_{\rm R} + \text{h. c.}\right] + |Y_{\rm b}|^2|\tilde{b}_{\rm L}|^2|\tilde{b}_{\rm R}|^2$$
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$$+ \frac{g_2^2}{8}\left(|h_{\rm u}^0|^2 - |h_{\rm d}^0|^2 + |\tilde{b}_L|^2\right)^2 + \frac{g_3^2}{6}\left(|\tilde{b}_L|^2 - |\tilde{b}_R|^2\right)^2.$$

- will always take over unless they vanish
- or $h_{\rm d}^0 \approx 0$ and $h_{\rm u}^0 = \tilde{b}$

Analytic bounds at the tree-level

1. choose appropriate direction $\hookrightarrow$ one-field problem

$$V_\phi^{\text{tree}} = \bar{m}^2\phi^2 - A\phi^3 + \lambda\phi^4$$

$$h_\mathrm{u}^0 = \tilde{b},\ h_\mathrm{d}^0 = 0$$

2. identify parameters, e.g. $\bar{m}^2 = \tilde{m}_Q^2 + \tilde{m}_b^2 + m_{H_\mathrm{u}}^2 + \mu^2$, $\lambda = Y_\mathrm{b}^2 + \frac{g_1^2+g_2^2}{2}$, $A = 2\mu Y_\mathrm{b}$
3. necessary condition: $\bar{m}^2 > \frac{A^2}{4\lambda}^2$ $\hookrightarrow$ second minimum not below first (trivial) one)

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$h_\mathrm{u}^0 = \tilde{b}$, $h_\mathrm{d}^0 = 0$

$$m_{H_\mathrm{u}}^2 + \mu^2 + \tilde{m}_Q^2 + \tilde{m}_b^2 > \frac{(\mu Y_\mathrm{b})^2}{Y_\mathrm{b}^2 + (g_1^2 + g_2^2)/2}$$

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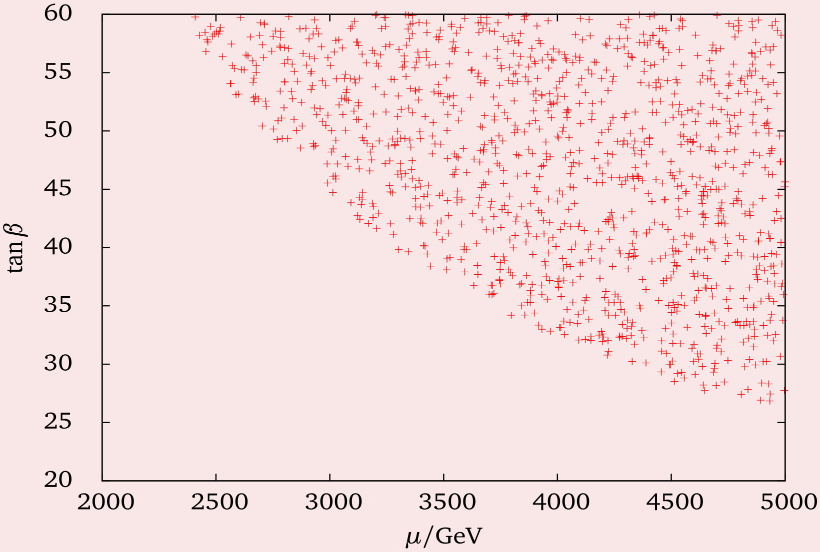
$|h_\mathrm{d}^0|^2 = |h_\mathrm{u}^0|^2 + |\tilde{b}|^2$, $\tilde{b} = \alpha h_\mathrm{u}^0$

$$m_{11}^2(1+\alpha^2)+m_{22}^2\pm 2m_{12}^2\sqrt{1+\alpha^2}+\alpha^2(\tilde{m}_Q^2+\tilde{m}_b^2) > \frac{4\mu^2\alpha^4}{2+2\alpha^2+\alpha^4}$$

$h_u^0 = \tilde{b}$ and $h_d^0 = 0$ $M_{\mathsf{SUSY}} = 1\,\text{TeV}$



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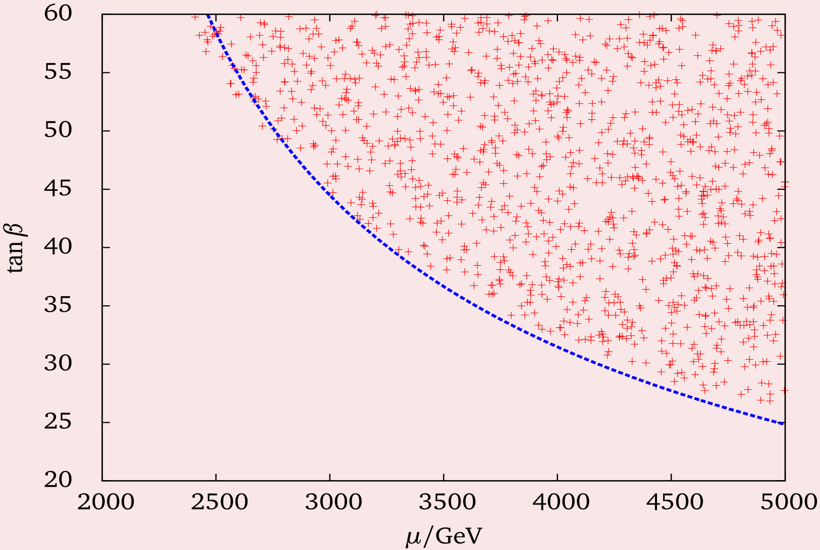

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  - $|h_\mathrm{d}^0|^2 = |h_\mathrm{u}^0|^2 + |\tilde{b}|^2$, $\tilde{b} = h_\mathrm{u}^0$

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- update coming soon [arXiv:1508.07201]