Alternative vevs in the NMSSM

Wolfgang Gregor Hollik

DESY Hamburg

Theory Group

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Alternative facts...

Alternative facts. . .

Desired = constructed

$$V = (m_{H_u}^2 + |\mu|^2)|H_u|^2 + (m_{H_d}^2 + |\mu|^2)|H_d|^2 - 2\,\mathrm{Re}\,(B_\mu H_u \cdot H_d)$$
$$+ \frac{g_1^2 + g_2^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g_2^2}{2}|H_d^\dagger H_u|^2$$

with

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix},$$

and $v_u^2 + v_d^2 = (174\,\mathrm{GeV})^2$, $v_u/v_d = \tan\beta$.

How to?

$$\left.\frac{\partial V}{\partial h_u^0}\right|_{h_{u,d} \to v_{u,d}} = 2(\boldsymbol{m_{H_u}^2} + |\mu|^2)v_u - 2\,\mathrm{Re}\,B_\mu v_d + \frac{g_1^2+g_2^2}{4}(v_u^2 - v_d^2)v_u$$
$$\left.\frac{\partial V}{\partial h_d^0}\right|_{h_{u,d} \to v_{u,d}} = 2(\boldsymbol{m_{H_d}^2} + |\mu|^2)v_d - 2\,\mathrm{Re}\,B_\mu v_u - \frac{g_1^2+g_2^2}{4}(v_u^2 - v_d^2)v_d$$

Colored scalars

$$V_{\text{MSSM}} = V_F + V_{\text{soft}} + V_D$$

with (only 3rd generation squarks and Higgses; intro to MSSM omitted)

$$\begin{aligned} V_{\text{soft}} = {} & m_{H_{\rm d}}^2 |h_{\rm d}|^2 + m_{H_{\rm u}}^2 |h_{\rm u}|^2 - (B_\mu h_{\rm d} \cdot h_{\rm u} + \text{h. c.}) \\ & + \tilde{t}_{\rm L}^* \tilde{m}_Q^2 \tilde{t}_{\rm L} + \tilde{t}_{\rm R}^* \tilde{m}_t^2 \tilde{t}_{\rm R} + \tilde{b}_{\rm L}^* \tilde{m}_Q^2 \tilde{b}_{\rm L} + \tilde{b}_{\rm R}^* \tilde{m}_b \tilde{b}_{\rm R} \\ & + \left(A_{\rm t}\ h_{\rm u} \tilde{t}_{\rm L}^* \tilde{t}_{\rm R} + A_{\rm b}\ h_{\rm d} \tilde{b}_{\rm L}^* \tilde{b}_{\rm R} + \text{h. c.} \right) \end{aligned}$$

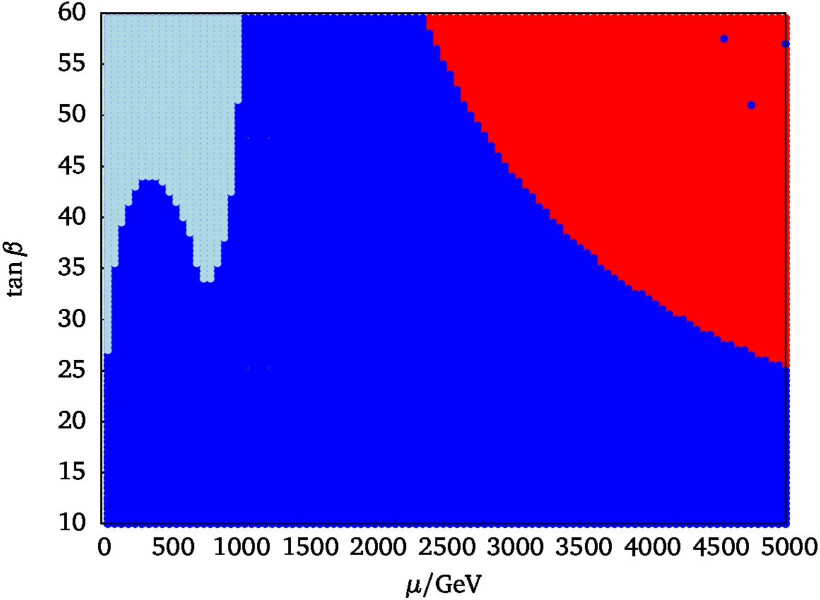
$$V_F \supset -\mu y_t\ h_{\rm d} \tilde{t}_R^* \tilde{t}_L - \mu y_b\ h_{\rm u} \tilde{b}_R^* \tilde{b}_L$$

Existing analytic constraints

- define certain directions in field space: great simplification
- e.g. D-terms absent: $|\tilde{Q}_L| = |\tilde{t}_R| = |h_2|$ (possibly miss sth.)
  [Frère et al. '83, Gunion et al. '88, Casas et al. '96]

$$A_{\rm t}^2 < 3(m_{H_u}^2 + |\mu|^2 + \tilde{m}_Q^2 + \tilde{m}_t^2)$$

Advertising JHEP**08**(2016)126 [WGH 2016].



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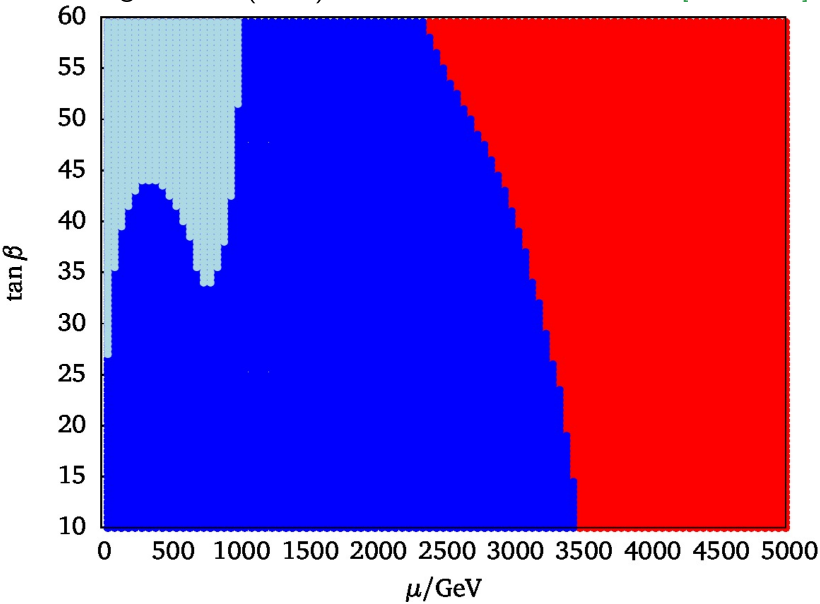

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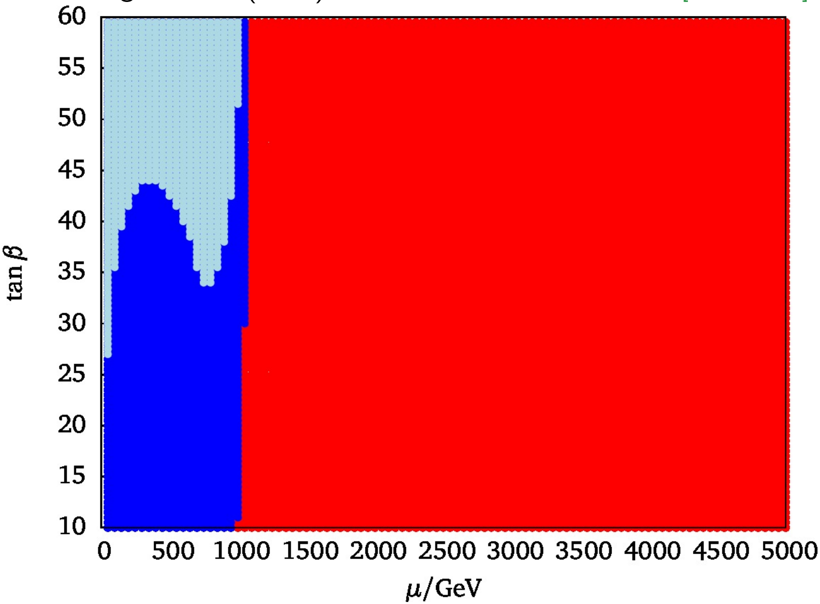

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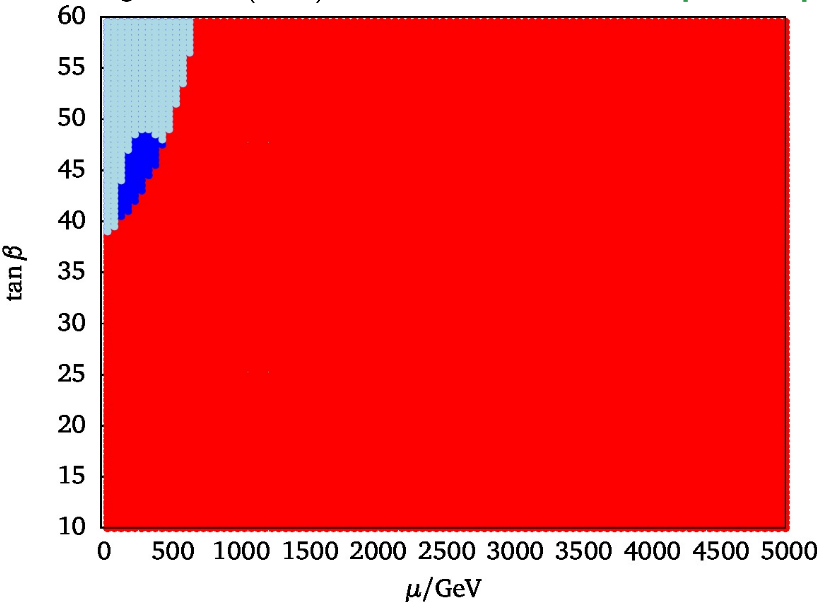

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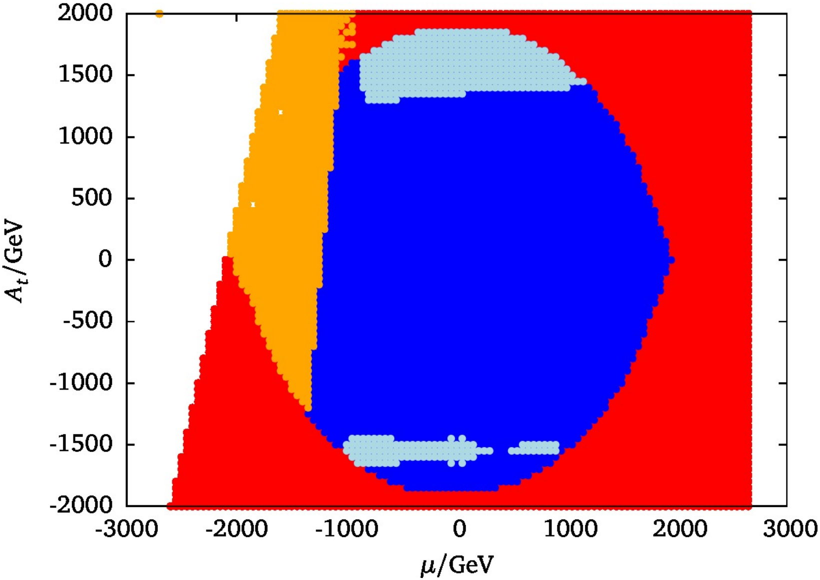

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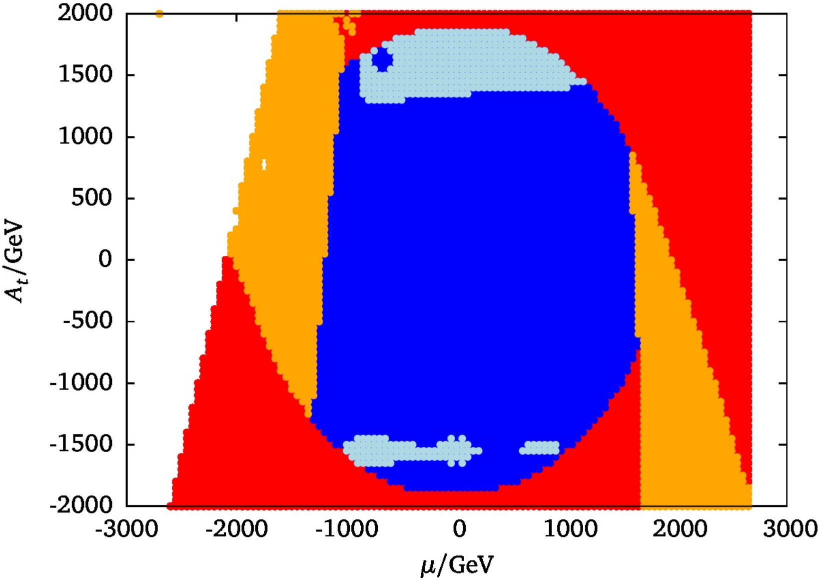

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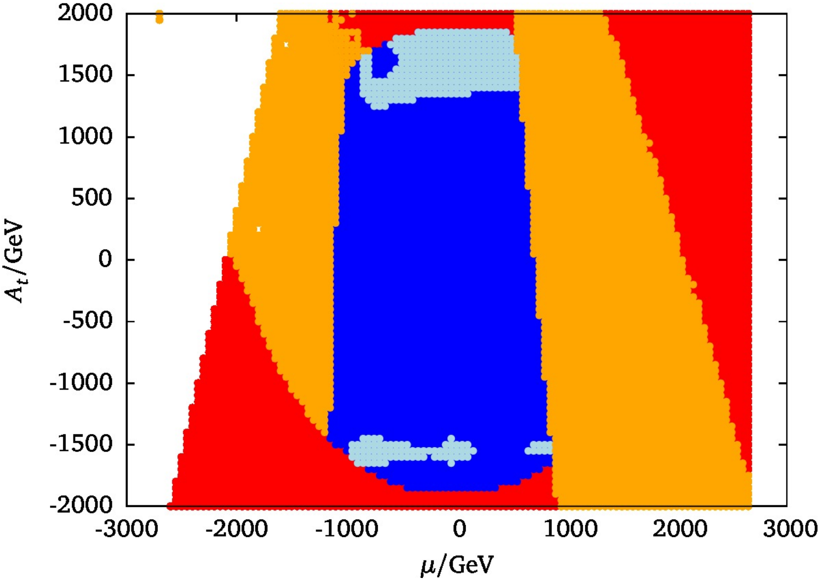

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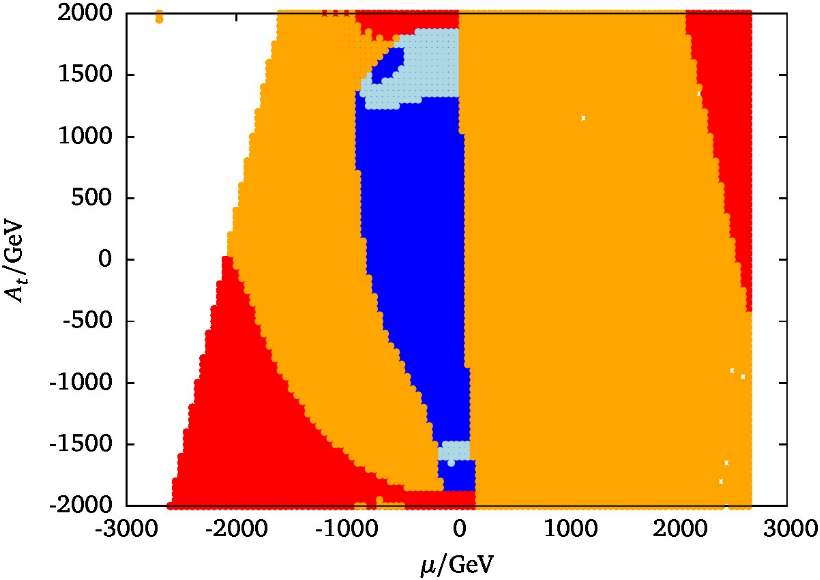


Why next?

Why next?

- Why not?
- Add a SM singlet superfield.
- Richer phenomenology (Higgs and neutralino sector)

The NMSSM solves the "μ-problem"

$$\mathcal{W}_{\text{MSSM}} = \mu\, H_u \cdot H_d + \text{Yukawa}$$

only dimensionful parameter μ has to be $\sim$ electroweak scale

$$\mathcal{W}_{\text{NMSSM}} = \lambda\, S H_u \cdot H_d + \frac{\kappa}{3} S^3$$

dynamical μ-term: $\lambda\langle S\rangle = \mu_{\text{eff}}$

$\mathbb{Z}_3$ symmetry forbids dimensionful couplings (bilinear, tadpole terms)

Trilinear terms in the Higgs sector

$$V_{\mathsf{soft}} = m_{H_u}^2 |h_u|^2 + m_{H_d}^2 |h_d|^2 + m_S^2 |s|^2 + \left(A_\lambda \lambda \, s h_u \cdot h_d + \frac{1}{3} A_\kappa \kappa \, s^3 + \text{h. c.} \right)$$

V_{Higgs} is a *multivariate* polynomial to order 4

$$V_{\mathsf{Higgs}} \subset \{h_u^2, h_d^2, s^2, s h_u h_d, s^2, s^2 h_u h_d, h_u^2 h_d^2, h_u^4, h_d^4, s^4\}$$

Not only one minimum! V_{eq} not necessarily the global minimum!

Minimisation conditions are in general misleading!

$$\frac{\partial V}{\partial h_u}\Big|_{\mathsf{vev}} = 2 m_{H_u}^2 v_u + \dots$$
$$\frac{\partial V}{\partial h_d}\Big|_{\mathsf{vev}} = 2 m_{H_d}^2 v_d + \dots$$
$$\frac{\partial V}{\partial h_u}\Big|_{\mathsf{vev}} = 2 m_S^2 v_s + \dots$$

linear equations for soft SUSY breaking masses $m_{H_u}^2$, $m_{H_d}^2$, m_S^2, can be solved uniquely; determine numerical values for those

Simplification: h_u^0, h_d^0, s^0 only (three fields, many vacua), real fields and parameters

$$V_{\text{Higgs}} = \boldsymbol{m^2_{H_u}}\, h_u^2 + \boldsymbol{m^2_{H_d}}\, h_d^2 + \boldsymbol{m^2_S}\, s^2 + \tfrac{2}{3}\kappa A_\kappa\, s^3 + 2\lambda A_\lambda\, s h_u h_d + \left(\kappa\, s^2 - \lambda\, h_u h_d\right)^2 + \lambda^2\, s^2 \left(h_u^2 + h_d^2\right) + \tfrac{g_1^2+g_2^2}{8}\left(h_u^2 - h_d^2\right)^2$$

However

Solutions for minimization equations with $\langle h_u\rangle \neq v_u$, $\langle h_d\rangle \neq v_d$ and $\langle s\rangle \neq \mu_{\text{eff}}/\lambda$ possible, viable, existing *and* leading to a true vacuum.

Potential value at the minimum to be compared with

$$V_{\text{min}}^{\text{des}} = -\tfrac{g_1^2+g_2^2}{8} v^4 \cos^2(2\beta) - \tfrac{\lambda^2}{4} v^4 \sin^2(2\beta) - \tfrac{\kappa^2}{\lambda^4}\mu_{\text{eff}}^4 - v^2\mu_{\text{eff}}^2\left(1 - \tfrac{\kappa^2}{\lambda^2}\sin(2\beta)\right) - \tfrac{1}{3}\frac{\kappa A_\kappa}{\lambda^3}\mu_{\text{eff}}^3 + \tfrac{v^2}{2} A_\lambda \mu_{\text{eff}} \sin(2\beta)$$

Constraint on A_κ

$$A_\kappa^2 > 9m_S^2$$

$A_\kappa^2 < 8m_S^2$: no $\langle s \rangle \neq 0$. [Derendinger, Savoy '84; Ellwanger et al. '97]

"Tachyonic" Higgs masses

- "problem" of tachyonic masses well known
- one mass eigenvalue of $\mathcal{M}_S^2$, $\mathcal{M}_P^2$ or charged Higgs mass $m_{H^\pm}^2$ negative
- tachyonic mass = negative curvature = alternative vev (!)

However

Careful analysis shows that tachyonic masses are not enough!

[see e.g. Kanehata, Kobayashi, Konishi, Seto, Shimomura '11]

A_λ from $m^2_{H^\pm}$

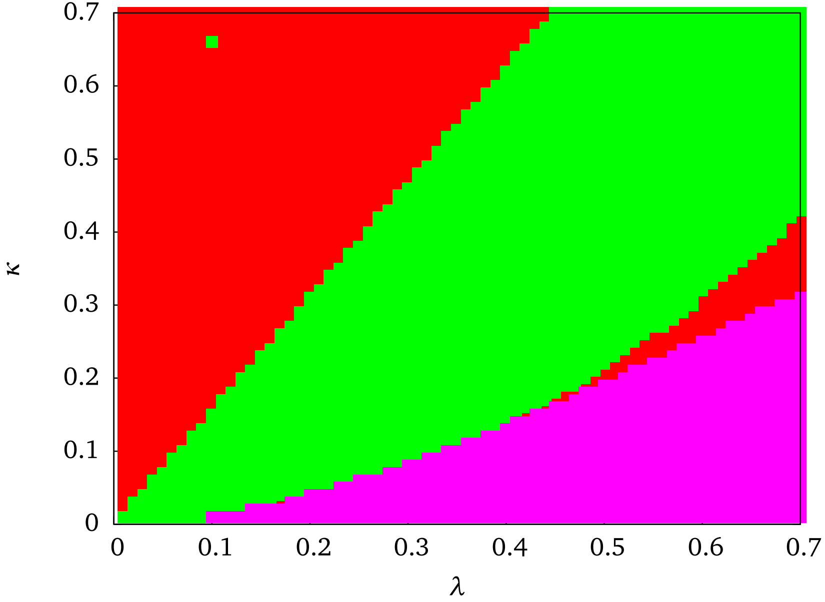
$$m^2_{H^\pm} = \frac{2\mu_{\text{eff}}}{\sin 2\beta}\left(A_\lambda + \mu_{\text{eff}}\frac{\kappa}{\lambda}\right) + v^2\left(\frac{g_2^2}{2} - \lambda^2\right)$$

- invert: $A_\lambda = A_\lambda(m^2_{H^\pm})$
- advantage: no tachyonic charged Higgs by construction
- select benchmark points:
  "small" $(300\,\text{GeV})$ and "large" $(800\,\text{GeV})$ $m_{H^\pm}$

free parameters

$$\lambda, \kappa, \tan\beta, \mu_{\text{eff}}, A_\kappa (, m_{H^\pm})$$

- $\tan\beta$ smallish: $\tan\beta = 3$ (fixed for all cases)
- strong constraints on λ (not too large), similar for μ_{eff} and $A_\kappa(\mu_{\text{eff}}$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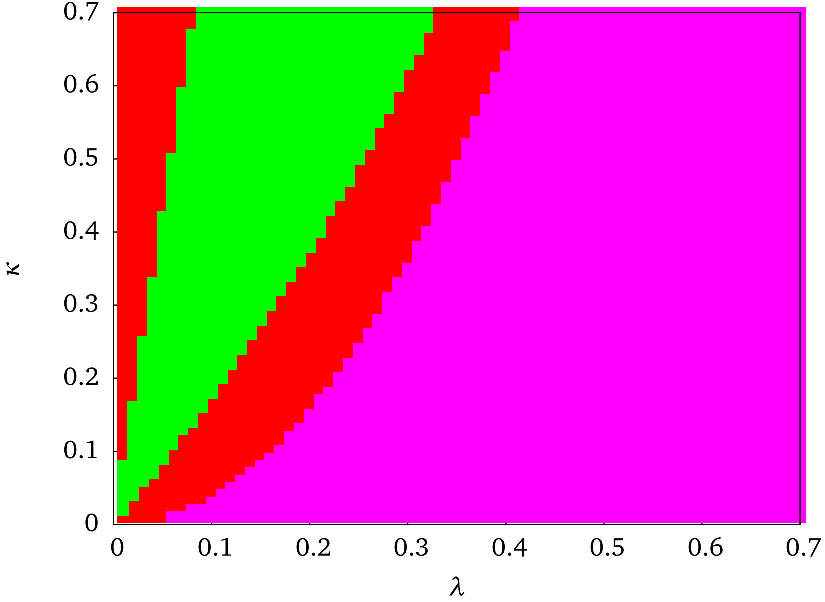

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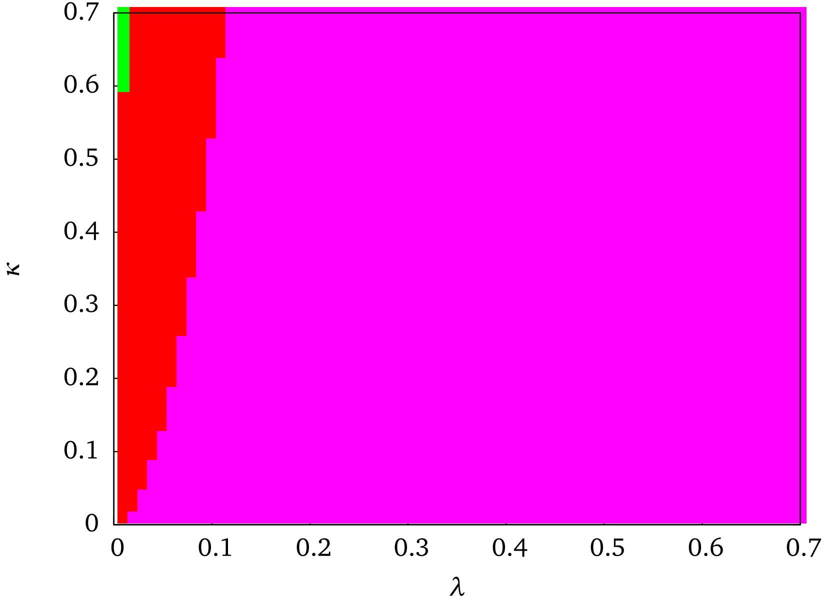

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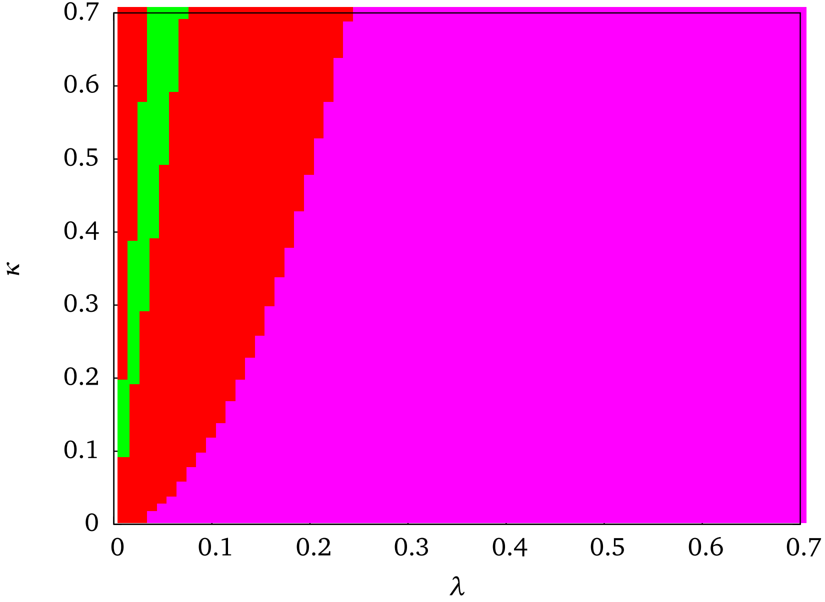
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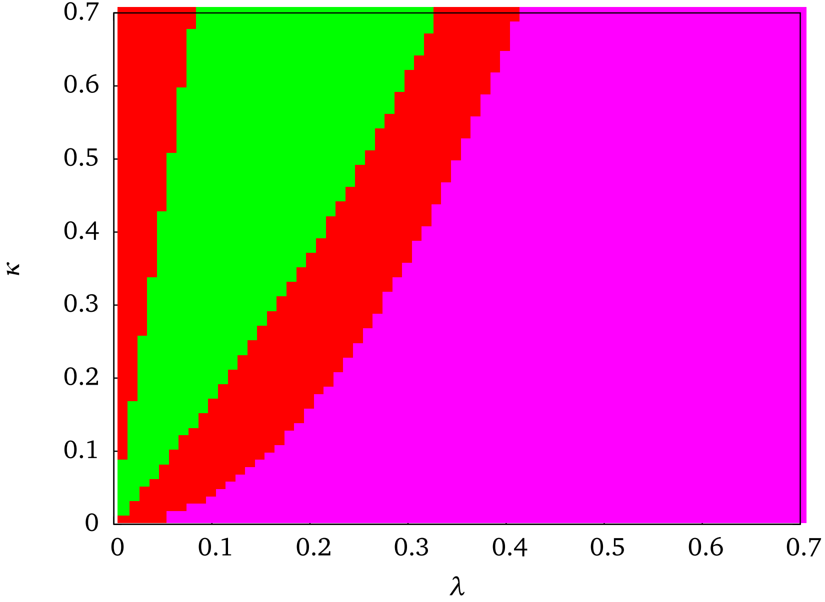
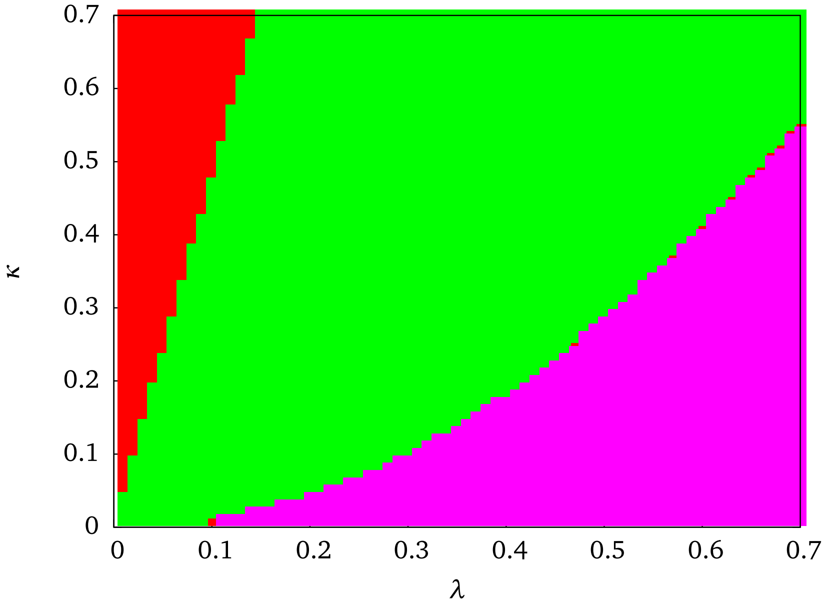
Funny results

$A_\kappa = 0\,\mathrm{GeV}\ \ \mu_{\mathrm{eff}} = 150\,\mathrm{GeV}$

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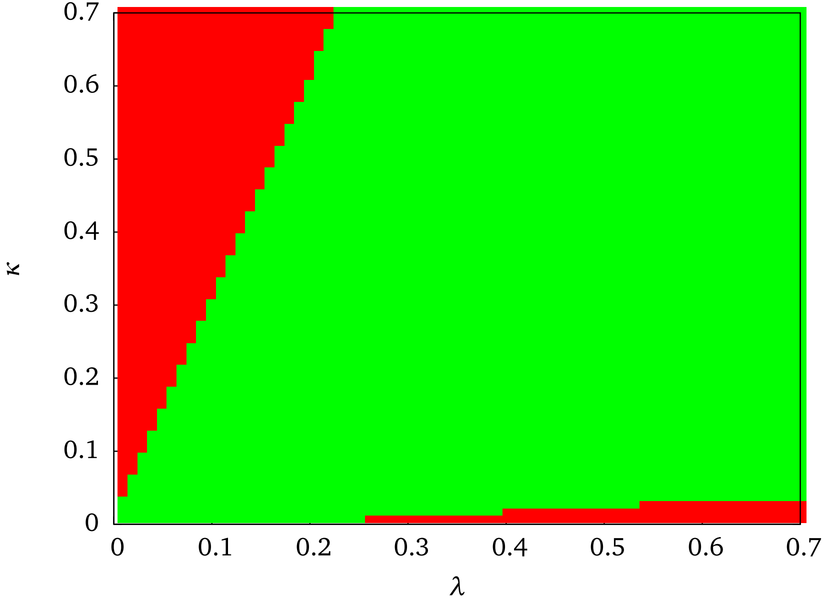
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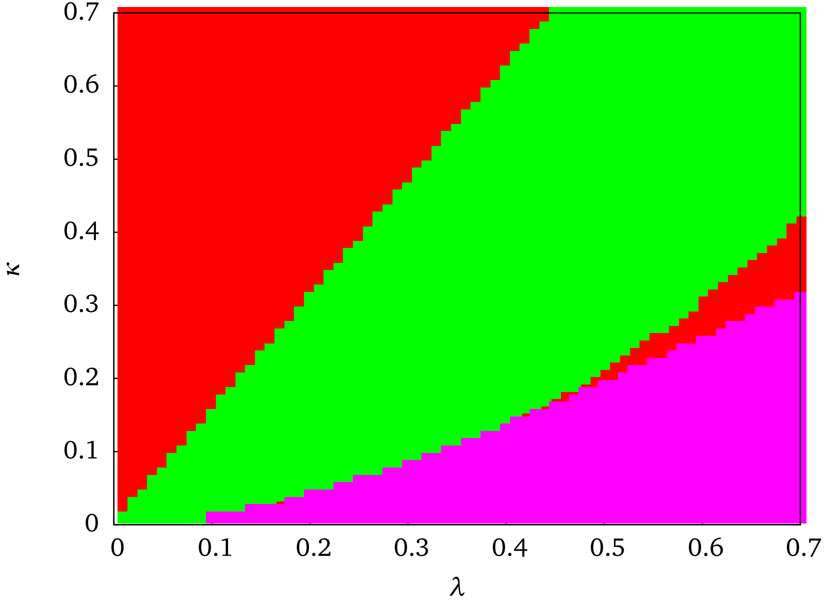
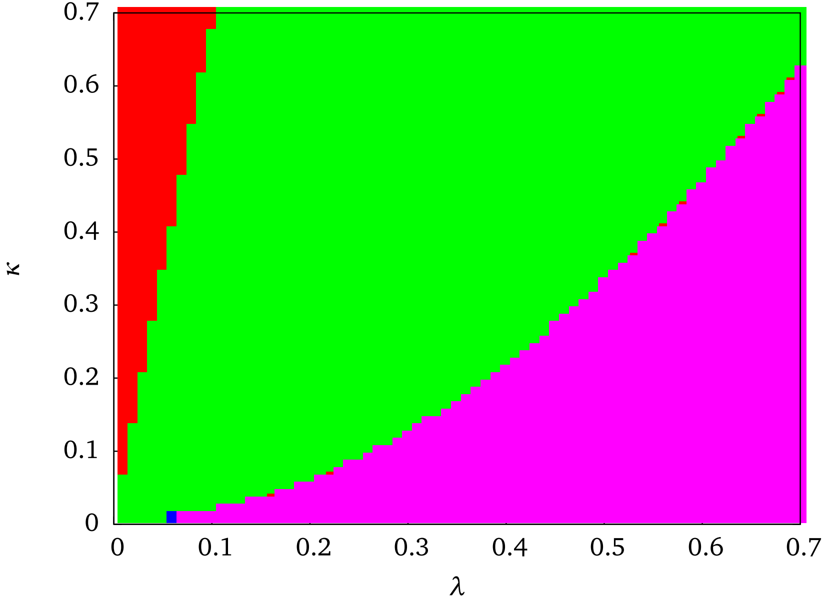
Funny results $A_\kappa = 0\,\text{GeV}\ \ \mu_{eff} = 350\,\text{GeV}$

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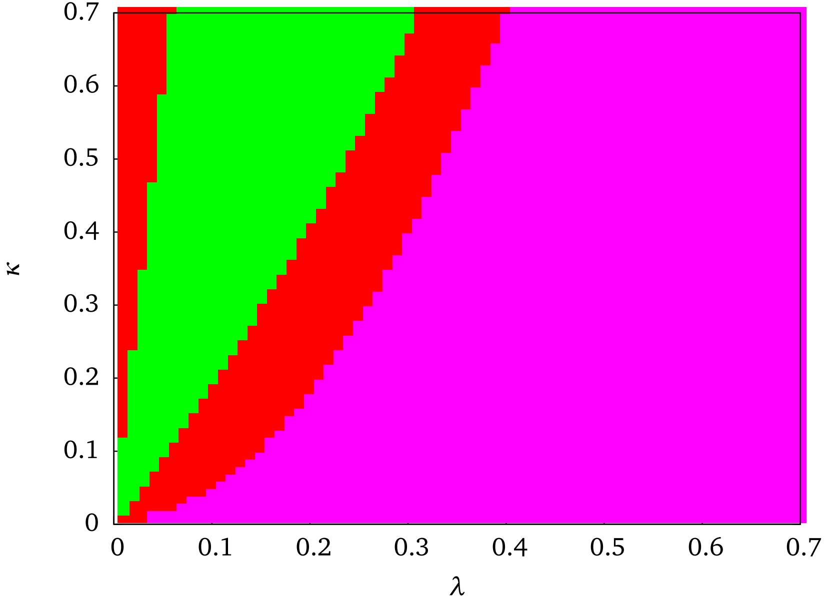
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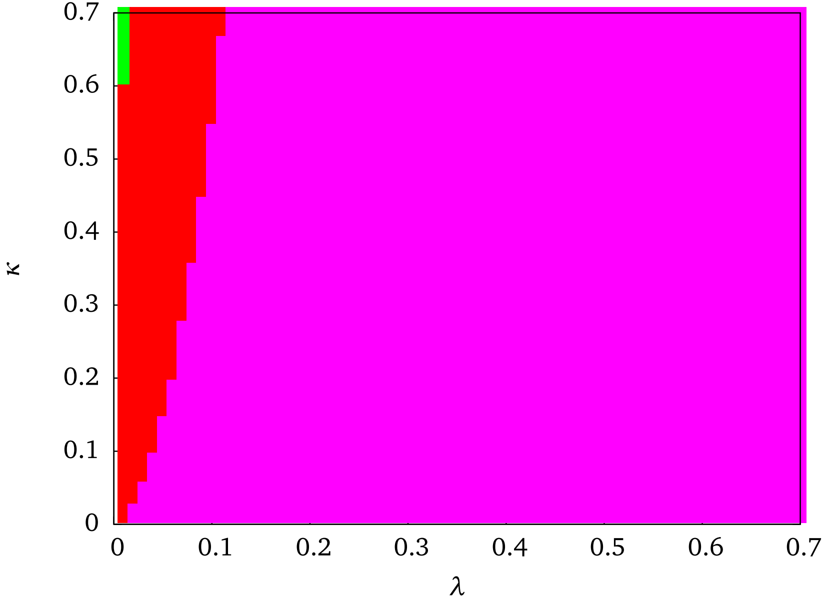
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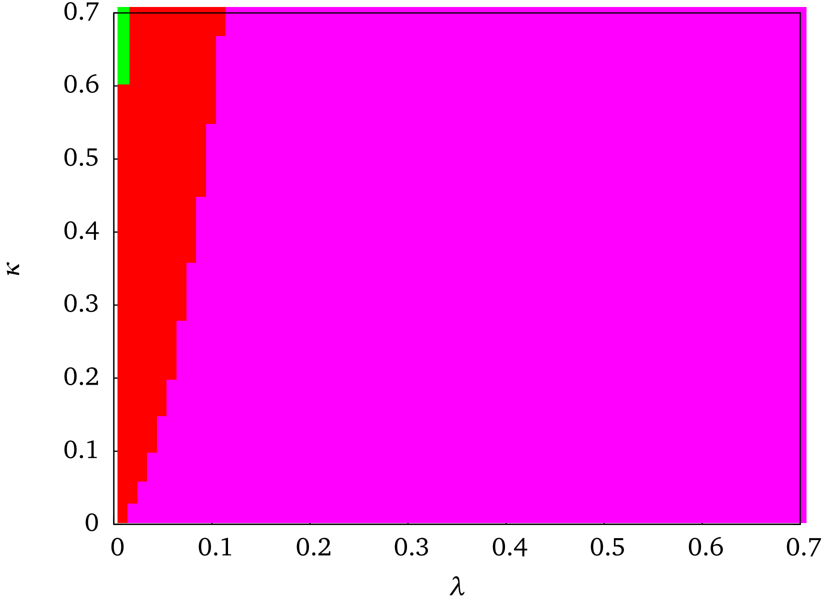
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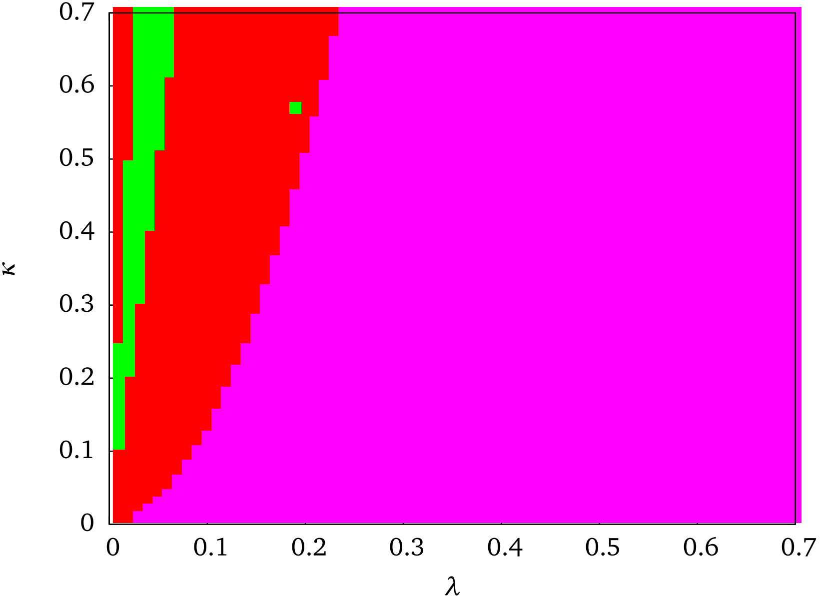
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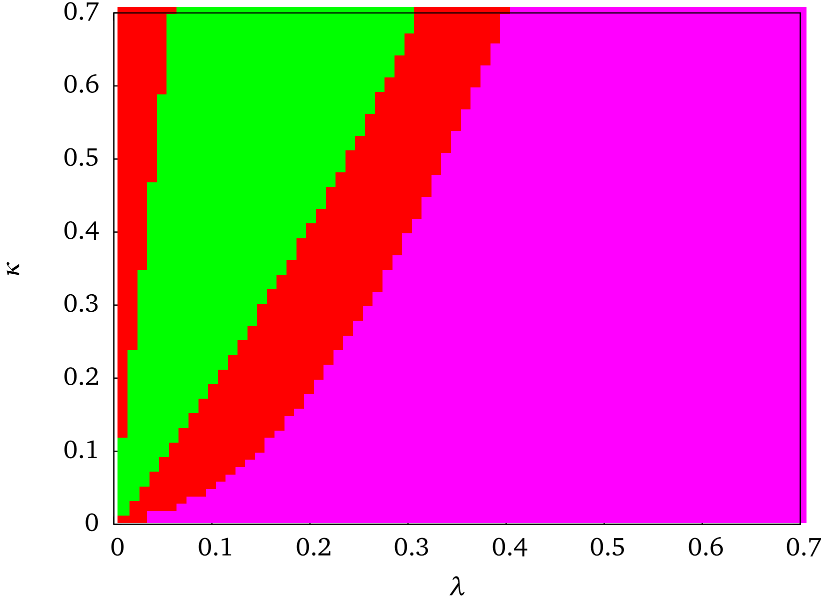
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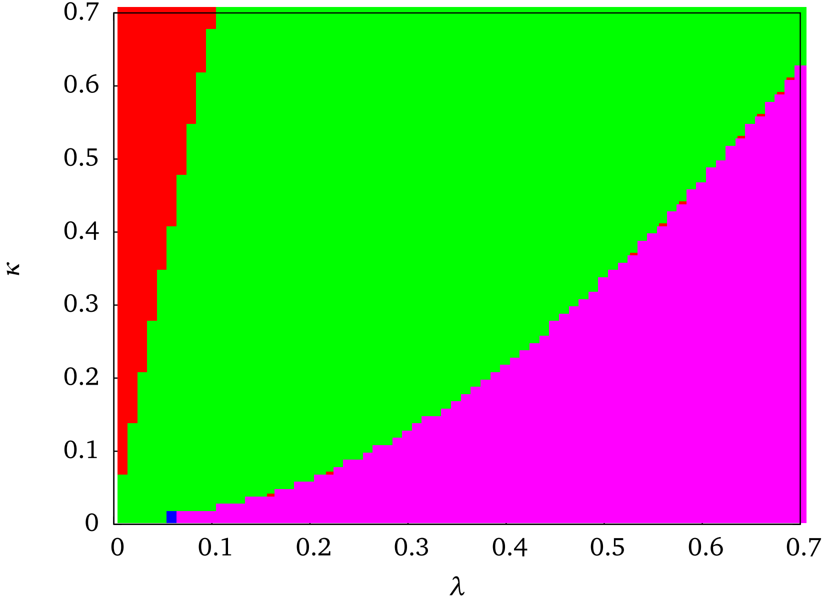
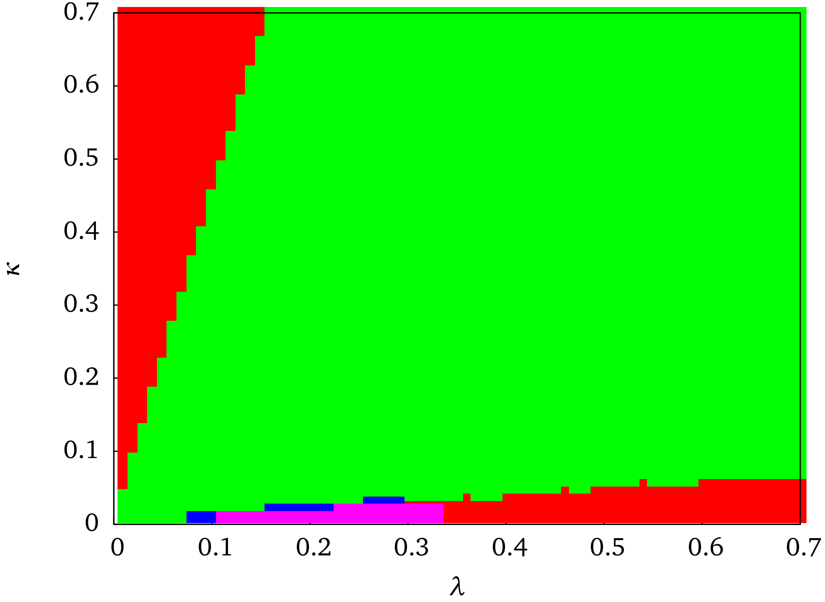
Funny results $A_\kappa = -100\,\mathrm{GeV}$ $\mu_{eff} = 200\,\mathrm{GeV}$

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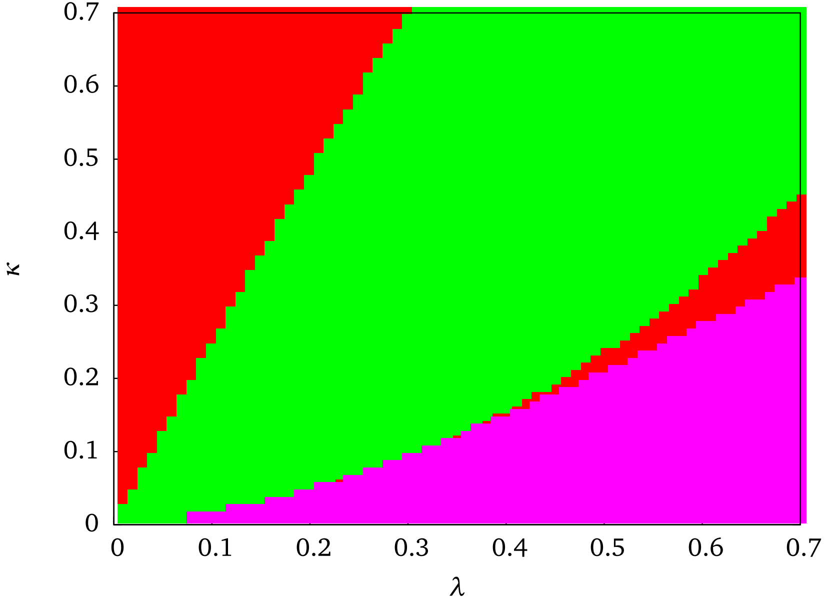
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Problems

- minimization of multivariate problems in general tedious, delicate and demanding
- numerical approaches
  - get stuck in wrong local minimum
  - do not find all minima
  - time consuming

Available tools

- try to define analytic boundaries (difficult to impossible)
- rely on some machinery [Vevacious]
- still not without problems. . .

An easy pedestrian way to check constraints

1. take NMSSM Higgs potential
2. determine $m_{H_u}^2$, $m_{H_d}^2$, m_S^2
3. reprocess potential: e.g. look for stationary points

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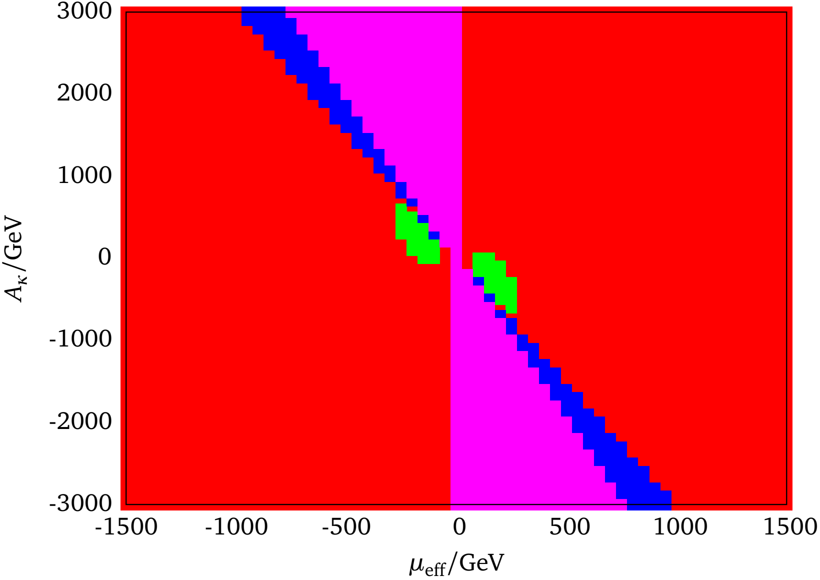
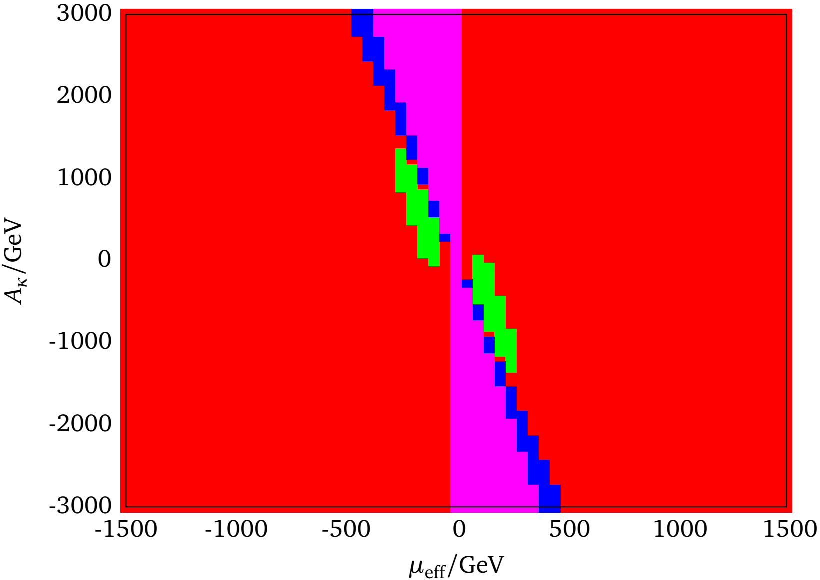
1. take NMSSM Higgs potential
2. determine $m_{H_u}^2$, $m_{H_d}^2$, m_S^2
3. reprocess potential: e.g. look for stationary points

$$\begin{aligned} V_{\text{Higgs}} = {} & \boldsymbol{m_{H_u}^2}\ h_u^2 + \boldsymbol{m_{H_d}^2}\ h_d^2 + \boldsymbol{m_S^2}\ s^2 \\ & + \tfrac{2}{3}\kappa A_\kappa\ s^3 + 2\lambda A_\lambda\ s h_u h_d \\ & + \left(\kappa\ s^2 - \lambda\ h_u h_d\right)^2 + \lambda^2\ s^2 \left(h_u^2 + h_d^2\right) \\ & + \tfrac{g_1^2+g_2^2}{8}\left(h_u^2 - h_d^2\right)^2 \end{aligned}$$

$$\begin{aligned} m_{H_u}^2 &= -\mu_{\text{eff}}^2 + \tfrac{g_1^2+g_2^2}{4} v^2 c_{2\beta} - \tfrac{\lambda^2 v^2}{4}\left(1 + c_{2\beta}\right) + \tfrac{\kappa}{\lambda}\mu_{\text{eff}}^2/t_\beta + A_\lambda \mu_{\text{eff}}/t_\beta, \\ m_{H_d}^2 &= -\mu_{\text{eff}}^2 - \tfrac{g_1^2+g_2^2}{4} v^2 c_{2\beta} - \tfrac{\lambda^2 v^2}{4}\left(1 - c_{2\beta}\right) + \tfrac{\kappa}{\lambda}\mu_{\text{eff}}^2 t_\beta + A_\lambda \mu_{\text{eff}} t_\beta, \\ m_S^2 &= -\lambda^2 v^2 - 2\tfrac{\kappa^2}{\lambda^2}\mu_{\text{eff}}^2 + v^2\kappa\lambda s_{2\beta} + A_\lambda \lambda^2 v^2 \tfrac{s_{2\beta}}{2\mu_{\text{eff}}} - \tfrac{\kappa}{\lambda} A_\kappa \mu_{\text{eff}} \end{aligned}$$

Another view: A_κ vs. μ_{eff} $\lambda = 0.1$ $\kappa = 0.1$, $m_{H^\pm} = 300\,\mathrm{GeV}$

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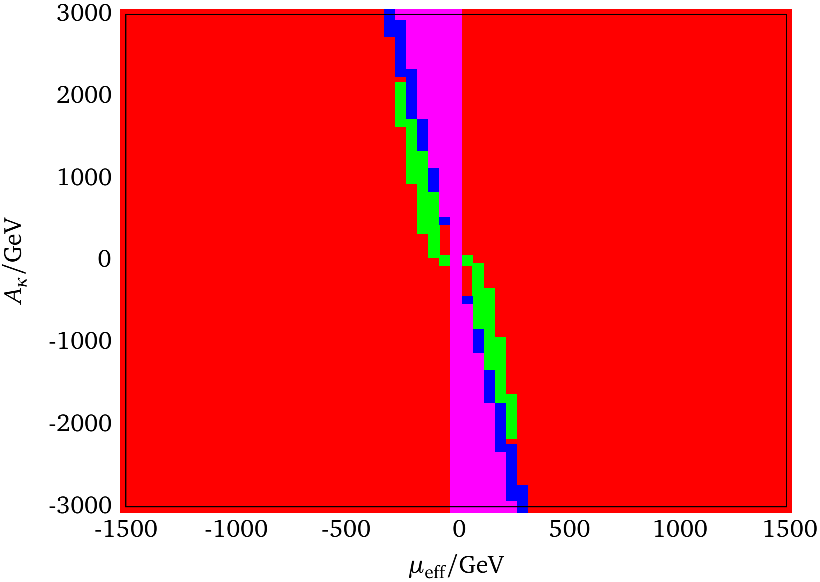
$\mu_{\rm eff}$/GeV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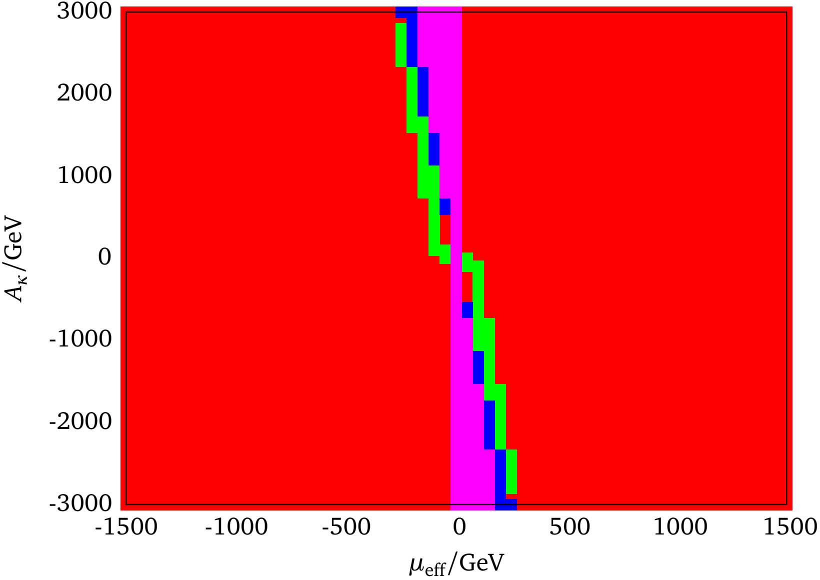

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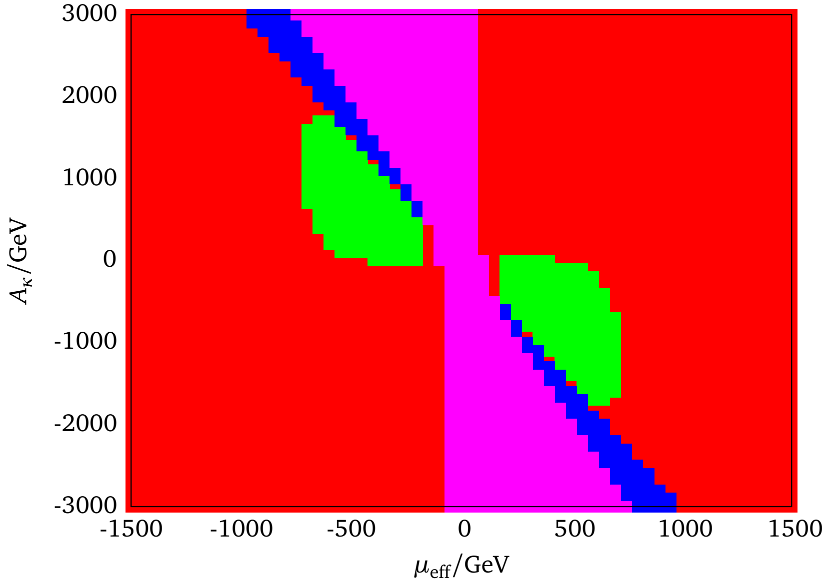
μ_{eff}/GeV



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-3000
A_κ/GeV
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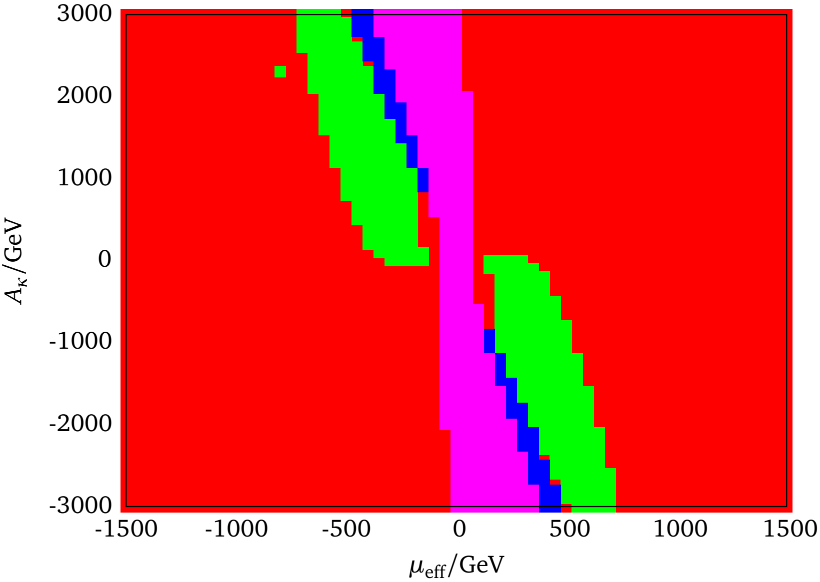
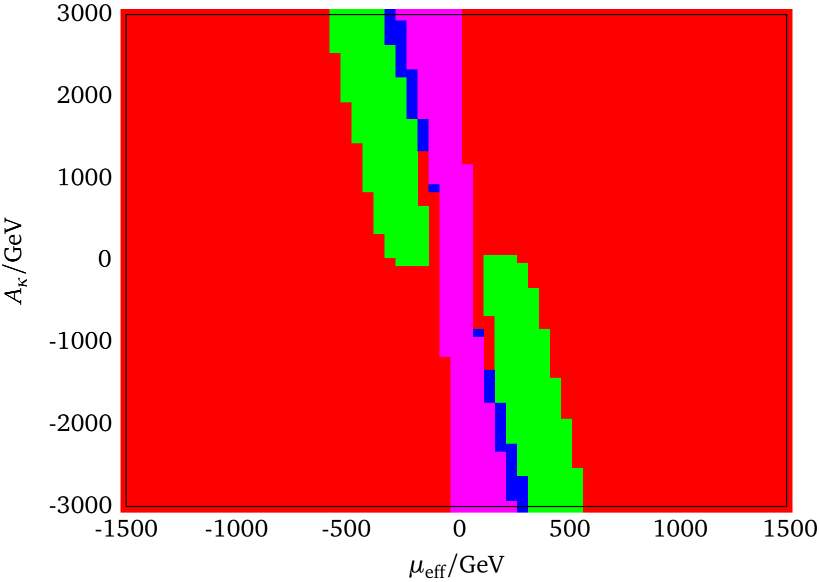
$\mu_{\rm eff}$/GeV

Another view: A_κ vs. μ_{eff} $\lambda = 0.1$ $\kappa = 0.2$, $m_{H^\pm} = 800\,\mathrm{GeV}$

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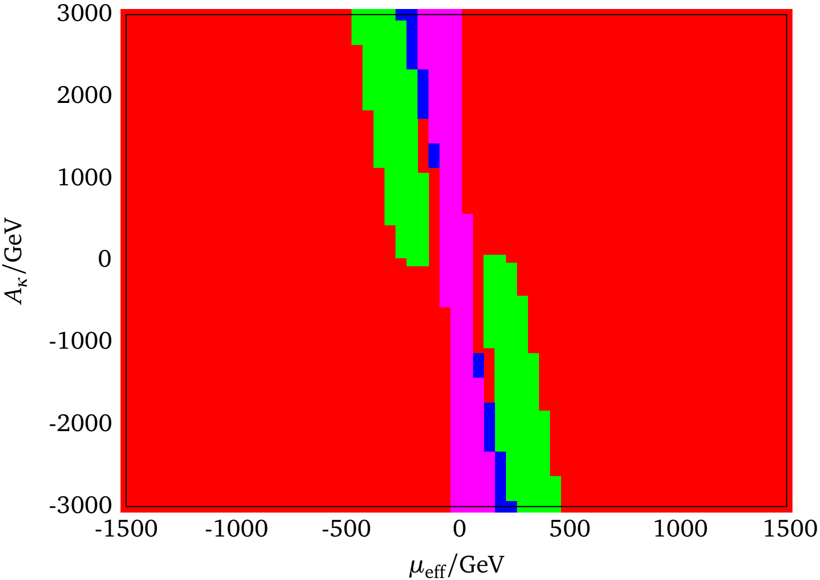
1500
$\mu_{\rm eff}$/GeV


3000
2000
1000
0
-1000
-2000
-3000
A_κ/GeV
-1500
-1000
-500
0
500
1000
1500
μ_{eff}/GeV

- mind alternative views
- non-standard Higgs vevs in the NMSSM
- global minimum: severe constraints on model parameters