Threshold Corrections To DY and Higgs at N^{3}LO QCD

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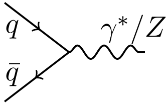
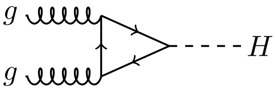
April 12, 2016

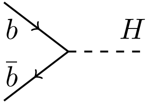
- Drell-Yan (DY) $pp \to \gamma^*/Z$: Xsection and rapidity distributions.



- Higgs boson production through gg : Rapidity distributions.



- Higgs boson production through $b\bar{b}$: Xsection and rapidity distributions.



Such processes are very important for precision studies in SM.

Prologue : State of the Art in Early 2014

DY

- Xsection : NNLO QCD Hamberg, Matsuura, Van Neerven ('91)
  N^3LO_{pSV} Moch, Vogt (2005); Ravindran ('06)
- Rapidity: NNLO QCD Anastasiou, Dixon, Melnikov, Petriello ('03); Melnikov, Petriello ('06)
  N^3LO_{pSV} Ravindran, Smith, van Neerven ('07)

Higgs in gg

- Xsection : NNLO QCD Harlander, Kilgore ('02); Anastasiou, Melnikov ('02); Ravindran, Smith, van Neerven ('03)
  N^3LO_{pSV} Moch, Vogt ('05); Ravindran ('06)
- Rapidity : NNLO QCD Anastasiou, Melnikov, Petriello ('04, '05)
  N^3LO_{pSV} Ravindran, Smith, van Neerven ('07)

Higgs in $b\bar{b}$

- Xsection : NNLO QCD Harlander, Kilgore ('03)
  N^3LO_{pSV} Ravindran ('06)
- Rapidity : NNLO QCD Buehler, Herzog, Lazopoulos, Mueller ('12)
  N^3LO_{pSV} Ravindran, Smith, van Neerven ('07)

Can we push the theoretical boundary little more?

- Testing SM with more accuracy to answer many unanswered questions
- Reducing the unphysical scale uncertainties

First crucial step towards this direction

- Higgs production through gg fusion at threshold N^{3}LO by

  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger ('14)

Prologue : What next?

Can we push the theoretical boundary little more?

- Testing SM with more accuracy to answer many unanswered questions
- Reducing the unphysical scale uncertainties

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Prologue : Spin-offs from Higgs Production at Threshold N^{3}LO

Recent results on threshold N^{3}LO QCD corrections

TA, Mahakhud, Mandal, Rana, Ravindran

- $d\sigma/dQ^2$ of DY

  Phys. Rev. Lett. 113 (2014) 112002

- $d\sigma/dY$ of Higgs in gg fusion and $d^2\sigma/dQ^2/dY$ of DY

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- $\sigma_{\rm tot}$ and $d\sigma/dY$ of Higgs in $b\bar{b}$ annihilation

  JHEP 1410 (2014) 139

  JHEP 1502 (2015) 131

Prologue : Spin-offs from Higgs Production at Threshold N^3LO

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- $d\sigma/dQ^2$ of DY

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- σ_{tot} and $d\sigma/dY$ of Higgs in $b\bar{b}$ annihilation

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Plan of the Talk

◇ What is threshold corrections?

◇ Prescription of Computation

- Factorization of Soft Gluons
- Overall Operator Renormalization
- Form Factor
- Soft-Collinear Distribution
- Mass Factorization

◇ Master Formula

◇ Result of DY SV Xsection at N^3LO

◇ Conclusions

Why threshold corrections?

Basic calculational framework for inclusive production : QCD factorization theorem

$$\sigma_X\left(\tau,q^2\right)=\frac{1}{S}\sum_{ab}\int_\tau^1\frac{dx}{x}\Phi_{ab}\left(x\right)\Delta_{ab}\left(\frac{\tau}{x},q^2\right)$$

$\tau\equiv\frac{q^2}{S},\; q^2\equiv m^2_{l^+l^-}$ for DY and m_H^2 for Higgs

- Perturbatively calculable partonic cross section (CS)

$$\Delta_{ab}\equiv\hat{s}\,\hat{\sigma}_{ab}$$

- Non-perturbative partonic flux

$$\Phi_{ab}\left(x\right)=\int_x^1\frac{du}{u}f_a\left(u\right)f_b\left(\frac{x}{u}\right)$$

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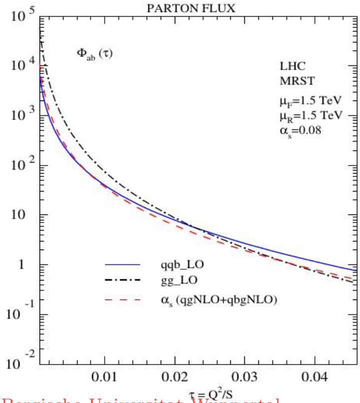
Why Threshold ...

- $\Phi_{ab}(x)$ becomes large when $x \to x_{\min} = \tau \Rightarrow$ enhances σ_X
- $x \to \tau \Rightarrow \hat{s} \to q^2 \Rightarrow$ partonic COM energy is just enough to produce final state particle $\Rightarrow$ all the radiations are soft $\rightsquigarrow$ soft limit.

- In this region, Dominant contributions arise from Virtual and Soft gluon emission processes (SV).
- For Higgs and DY productions, substantial contribution come from this region. e.g. for DY 91% - 95% at NNLO



Threshold Expan : Soft-Plus-Virtual (SV)

Splitting the partonic CS into singular & regular parts around $z \equiv \frac{\tau}{x} = \frac{q^2}{\hat{s}} = 1$

$$\Delta\left(z\right) = \boxed{\Delta^{\text{sing}}(z)} + \Delta^{\text{hard}}(z)$$

$$\boxed{\Delta^{\text{sing}}(z) \equiv \Delta^{\text{SV}}(z) = \Delta^{\text{SV}}_{\delta}\,\delta(1-z) + \sum_{j=0}^{\infty} \Delta^{\text{SV}}_{j} \mathcal{D}_j} \quad \mathcal{D}_j \equiv \left(\frac{\ln^j(1-z)}{1-z}\right)_+$$

- These two are the most singular terms when $z \to 1$, so dominant contribution comes from these.
- $\Delta^{\text{hard}}(z)$: subleading and polynomial in $\ln(1-z)$.

Plus distribution $+$ is defined by its action on test function $f(z)$

$$\int_0^1 dz \mathcal{D}_j(z) f(z) \equiv \int_0^1 dz \frac{\ln^j(1-z)}{1-z}\left[f(z) - f(1)\right]$$

Goal : Computation of $\Delta^{\mathrm{SV}}(z)$

- Matrix elements square and phase space integrals in the soft limit.

Catani et al; Harlander and Kilgore

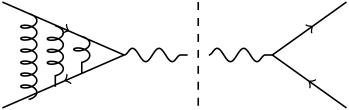
- Form factors and DGLAP kernels :
  - Factorization theorem
  - Renormalization group invariance
  - Sudakov resummation of form factors

Moch and Vogt; Ravindran, Smith and Van Neerven

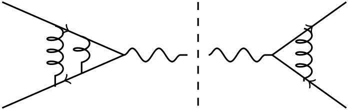
- Soft Collinear effective theory

Becher and Neubert

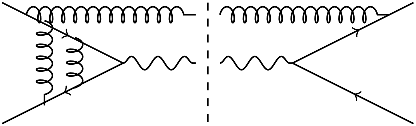
DIAGRAMS AT $\mathcal{O}(a_s^3)$ FOR DY



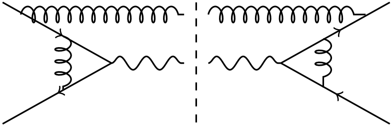
Triple virtual : 244



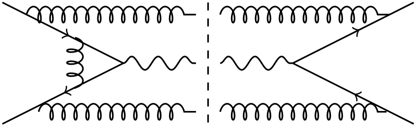
Double virtual : 13 × 1



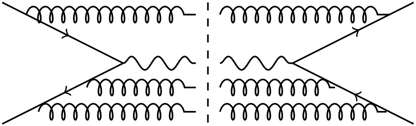
Double virtual real : 229 × 2



Real virtual squared : 13 × 13



Double real virtual : 134 × 8



Triple real squared: 50 × 50

Various Methods to Compute SV

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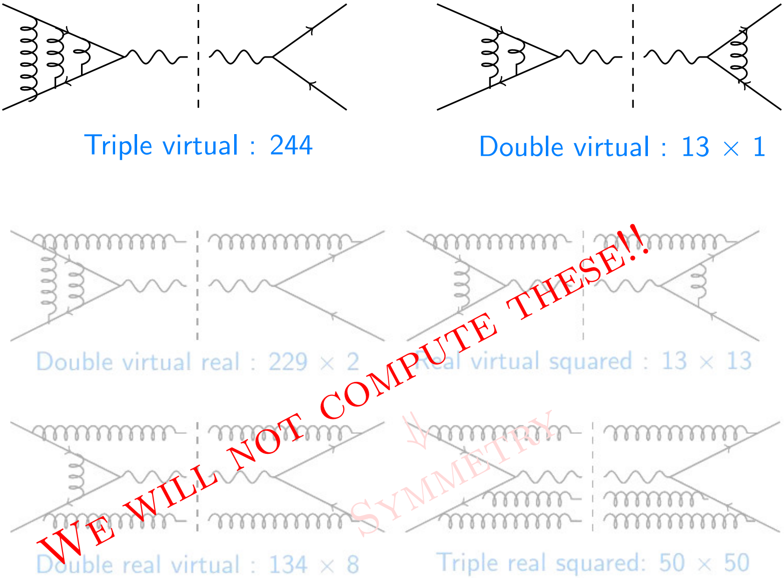
Diagrams at $\mathcal{O}(a_s^3)$ for DY



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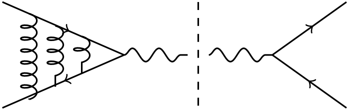
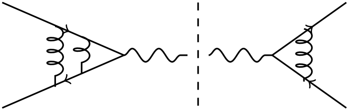
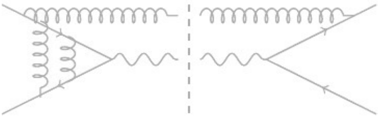


Triple virtual : 244

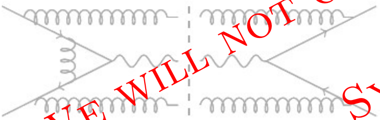


Double virtual : 13×1

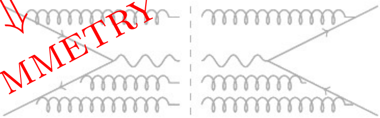


Double virtual real : 229×2

Real virtual squared : 13×13



Double real virtual : 134×8



Triple real squared: 50×50

We will not compute these!!

$\Downarrow$

Symmetry

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The Prescription

$$\Downarrow$$

Master Formula

General structure of Δ upon taking radiative corrections :

$$\Delta(z,q^2) = \delta(1-z)$$
$$+ a_s\left(q^2\right)\left[c_1^{(1)}\delta\left(1-z\right) + c_2^{(1)}\left(\frac{\ln\left(1-z\right)}{\left(1-z\right)}\right)_+ + R_1(z,q^2)\right]$$
$$+ a_s^2\left(q^2\right)\left[\cdots + \cdots + R_2(z,q^2)\right] + \cdots$$

$z = \frac{q^2}{\hat{s}}$, $R_i(z,q^2)$ are remaining terms.

- Virtual $\rightsquigarrow \delta(1-z)$
- Real emission $\rightsquigarrow \delta(1-z), \mathcal{D}_j, R_k$
- Soft approximation $z \to 1$

Defn.: $a_s \equiv \alpha_s/4\pi$

Structure of Partonic XSection

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Soft distribution factorizes as exponentiation

$$\Delta\left(z,q^2\right) = S\left(z,q^2,\mu_R^2\right) \otimes \left[\delta\left(1-z\right) + a_s \tilde{R}_1(z,q^2) + a_s^2 \tilde{R}_2(z,q^2)\right]$$

and

$$S\left(z,q^2,\mu_R^2\right) = \mathcal{C}\exp\left(\Phi(z,q^2,\mu_R^2)\right),$$ Φ is a finite distribution

Defn:

$$\mathcal{C}e^{f(z)} = \delta(1-z) + \frac{1}{1!}f(z) + \frac{1}{2!}f(z) \otimes f(z) + \cdots$$

$\otimes$ is Mellin convolution.

Drop all regular functions.

Factorization of Soft Gluons

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SV Cross Section : Conjecture

We propose UV renormalized

$$\hat{\Delta}^{\mathrm{SV},I}\left(z, q^2, \mu_R^2\right) = \mathcal{C} e^{2\Phi^I\left(\hat{a}_s, q^2, \mu^2, z\right)}$$
$$\otimes \left(Z^I\left(\hat{a}_s, \mu_R^2, \mu^2\right)\right)^2 |\hat{F}^I\left(\hat{a}_s, Q^2, \mu^2\right)|^2 \delta(1-z)$$

$I = q, g, b$ for DY, Higgs through gg and Higgs in $b\bar{b}$.

- Soft contribution
- Form factor factorizes due to Born kinematics.
- Overall operator ren. const. required for
  1. effective Lagrangian of Higgs in gg
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  and $Z^I = 1$ for DY.

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- $\hat{a}_s$ renormalization and Z^I makes partonic CS UV finite.
- Soft divergences from real radiation and virtual correction cancel upon summing over all the final states (KLN).
- Final state collinear singularities also cancel upon summing over all the final states (KLN)
- Still XSection is NOT fully finite due to initial state collinear singularities
- This can arise from virtual as well as real corrections
- Mass factorization to make collinear finite.

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- $\hat{a}_s$ renormalization and Z^I makes partonic CS UV finite.
- Soft divergences from real radiation and virtual correction cancel upon summing over all the final states (KLN).
- Final state collinear singularities also cancel upon summing over all the final states (KLN)
- Still XSection is NOT fully finite due to initial state collinear singularities
- This can arise from virtual as well as real corrections
- Mass factorization to make collinear finite.

$$\hat{\Delta}^{\mathrm{SV},I}\left(z,q^2,\mu_R^2\right) = \Gamma^{\mathrm{T}}(z,\mu_F^2)\otimes \boxed{\Delta^{\mathrm{SV},I}\left(z,q^2,\mu_R^2,\mu_F^2\right)} \otimes \Gamma(z,\mu_F^2)$$

UV, Soft & Collinear finite

- Kernels Γ absorb all the initial state collinear singularities.
- Only diagonal elements contribute to threshold limit.
- Substituting this we arrive at the master formula to compute SV CS.

$$\hat{\Delta}^{\mathrm{SV},I}\left(z,q^2,\mu_R^2\right) = \Gamma^{\mathrm{T}}(z,\mu_F^2) \otimes \boxed{\Delta^{\mathrm{SV},I}\left(z,q^2,\mu_R^2,\mu_F^2\right)} \otimes \Gamma(z,\mu_F^2)$$

$\nearrow$

UV, Soft & Collinear finite

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Master Formula for SV CS

$$\boxed{\Delta^{\mathrm{SV},I}\left(z,q^2,\mu_R^2,\mu_F^2\right)=\mathcal{C}\exp\left(\Psi^I\left(z,q^2,\mu_R^2,\mu_F^2,\epsilon\right)\right)|_{\epsilon=0}}$$

where

$$\boxed{\begin{aligned}\Psi^I = &\left(\ln\left[Z^I(\hat{a}_s,\mu_R^2,\mu^2,\epsilon)\right]^2+\ln\left|\hat{F}^I(\hat{a}_s,Q^2,\mu^2,\epsilon)\right|^2\right)\delta(1-z)\\ &+2\Phi^I\left(\hat{a}_s,q^2,\mu^2,z,\epsilon\right)-2\mathcal{C}\ln\Gamma_{II}\left(\hat{a}_s,\mu^2,\mu_F^2,z,\epsilon\right)\end{aligned}}$$

Ravindran

1. Logarithm of the overall operator renormalization $\ln Z^I$
2. Logarithm of the form factor $\ln \hat{F}^I$
3. Soft distribution Φ^I
4. Logarithm of the mass factorization kernel $\ln \Gamma_{II}$

Goal boils down to compute these quantities.

Ingredients for SV CS

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2. Logarithm of the form factor $\ln \hat{F}^I$
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Goal boils down to compute these quantities.

THE PRESCRIPTION

$$\Downarrow$$

MASTER FORMULA

$$\Downarrow$$

HOW DO WE COMPUTE THESE?

Ingredient 1 : Overall Operator Renormalization $\rightsquigarrow \ln Z^I$

It satisfies the RGE

$$\boxed{\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \gamma_{i-1}^I}$$

- γ_i^I's are anomalous dimensions.
- γ^I are available up to $\mathcal{O}(a_s^3)$ for $I = b, g$ and $\gamma^q = 0$ $(q \neq b)$.
- Solution of this RGE provides $\ln Z^I$.
- OOR is not required for DY$\rightsquigarrow Z^I = 1$.

Ingredient 2 : Form factor $\rightsquigarrow \ln \hat{F}^I$

- $\ln|\hat{F}^I|^2$ can be computed from available results up to 3-loop.
- Instead, we have followed an alternative way. The reason would be clear soon.
- Gauge and RG invariance implies KG eqn

$$\boxed{Q^2 \frac{d}{dQ^2} \ln \hat{F}^I = \frac{1}{2}\left[K^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon\right) + G^I\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon\right)\right]}$$

Ashoke Sen, Mueller, Collins, Magnea

  - K^I contains all the poles in ϵ where, ST dimensions $d = 4 + \epsilon$.
  - G^I collects finite terms at $\epsilon \to 0$.

RG invariance of $\hat{F}$ implies

$$\mu_R^2 \frac{d}{d\mu_R^2} K^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon \right) = -\mu_R^2 \frac{d}{d\mu_R^2} G^I \left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon \right) = -A^I \left(a_s(\mu_R^2) \right)$$

- A^I's are finite, called cusp anomalous dimensions.
- A^I's are maximally non-Abelian

$$A^g = \frac{C_A}{C_F} A^q$$

$C_A = N$, $C_F = (N^2 - 1)/2N$

- Solve by expanding K^I, G^I and A^I in powers of $a_s\left(\mu_R^2\right)$.
- Substitute these K^I & G^I in KG eqn to solve for $\ln \hat{F}^I$ to order by order.

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The solution up to 3 loops are:

$$\ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \, \hat{\mathcal{L}}_F^{I,(i)}(\epsilon)$$

Moch, Vogt, Vermaseren; Ravindran; Magnea

with

$$\hat{\mathcal{L}}_F^{I,(1)}(\epsilon) = \frac{1}{\epsilon^2}\Big\{-2A_1^I\Big\} + \frac{1}{\epsilon}\Big\{G_1^I(\epsilon)\Big\},$$

$$\hat{\mathcal{L}}_F^{I,(2)}(\epsilon) = \frac{1}{\epsilon^3}\Big\{\beta_0 A_1^I\Big\} + \frac{1}{\epsilon^2}\Big\{-\frac{1}{2}A_2^I - \beta_0 G_1^I(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{2}G_2^I(\epsilon)\Big\},$$

$$\begin{aligned}\hat{\mathcal{L}}_F^{I,(3)}(\epsilon) =& \frac{1}{\epsilon^4}\Big\{-\frac{8}{9}\beta_0^2 A_1^I\Big\} + \frac{1}{\epsilon^3}\Big\{\frac{2}{9}\beta_1 A_1^I + \frac{8}{9}\beta_0 A_2^I + \frac{4}{3}\beta_0^2 G_1^I(\epsilon)\Big\} \\ &+ \frac{1}{\epsilon^2}\Big\{-\frac{2}{9}A_3^I - \frac{1}{3}\beta_1 G_1^I(\epsilon) - \frac{4}{3}\beta_0 G_2^I(\epsilon)\Big\} + \frac{1}{\epsilon}\Big\{\frac{1}{3}G_3^I(\epsilon)\Big\}\end{aligned}$$

At every order, all the poles except $\frac{1}{\epsilon}$ can be predicted from previous order.

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Ravindran, Smith, van Neerven

2-loop results for $\hat{F}^q$ & $\hat{F}^g$ in SU(N) dictates

$$G_1^I(\epsilon) = 2\left(B_1^I - \gamma_1^I\right) + f_1^I + \sum_{k=1}^{\infty} \epsilon^k {g_1}^{I,k}$$

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with

$$f_3^g = \frac{C_A}{C_F} f_3^q$$

- f^I's are universal.
- Explicit computation of FF gives ϵ dependent $g_i^{I,k}$.
- This understanding is crucial to get soft part.

Ingredient 2 . . . Single Pole Also Predictable

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Ingredient 3 : Mass Factorization $\rightsquigarrow \ln \Gamma$

DGLAP kernel Γ satisfies the RGE

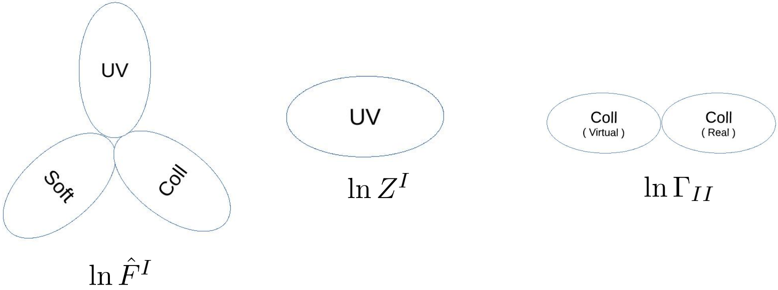
$$\boxed{\mu_F^2 \frac{d}{d\mu_F^2} \Gamma(z, \mu_F^2, \epsilon) = \frac{1}{2} P\left(z, \mu_F^2\right) \otimes \Gamma\left(z, \mu_F^2, \epsilon\right)}$$

- P's are Altarelli Parisi splitting functions (matrix valued).
- Diagonal parts of the kernel ONLY contribute to SV limit.
- By solving we get $\Gamma \rightsquigarrow \ln \Gamma$
- Γ is available up to $\mathcal{O}(a_s^4)$.

Where are we?

- ✓ $\ln[Z^I]^2$
- ✓ $\ln|\hat{F}^I|^2$
- ✓ $\ln\Gamma_{II}$
- ?? Φ^I

Revisiting cancellation of poles



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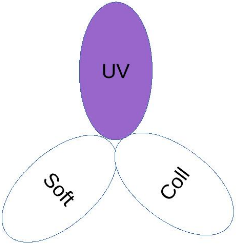
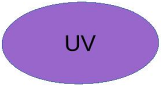
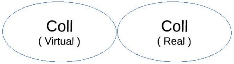


$\ln \hat{F}^I$

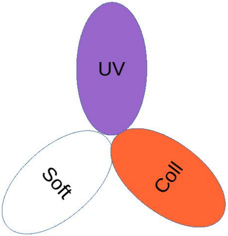


$\ln Z^I$



$\ln \Gamma_{II}$

Revisiting cancellation of poles



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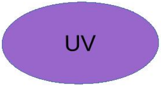

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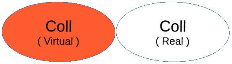

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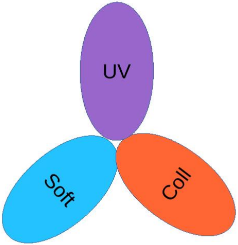

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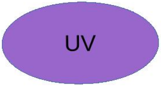

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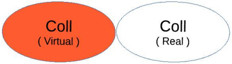

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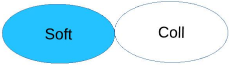
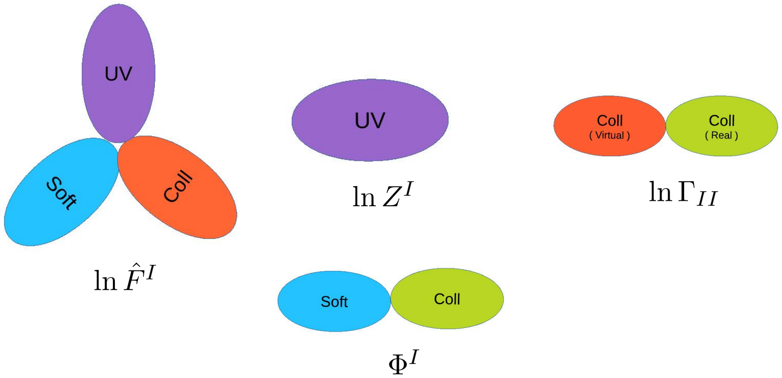


Φ^I

Revisiting cancellation of poles



- Demanding finiteness of partonic CS $\Rightarrow$ Φ^I must have soft as well as collinear contributions.
- Now onward we will call Φ^I as Soft-Collinear distribution.

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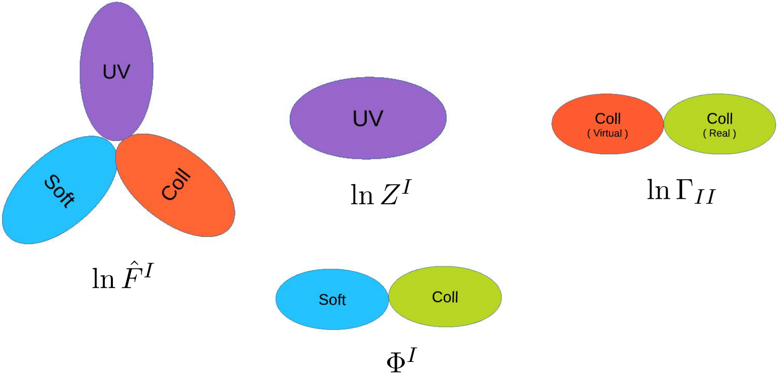

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Demand of

1. finiteness of $\Delta^{\mathrm{SV},I}(z)$ at $\epsilon \to 0$ and
2. the RGE

$$\mu_R^2 \frac{d}{d\mu_R^2} \Phi^I \left(\hat{a}_s, q^2, \mu^2, z, \epsilon\right) = 0$$

can be achieved if we make an ansatz that Φ^I satisfies KG type DE

$$\boxed{q^2 \frac{d}{dq^2} \Phi^I = \frac{1}{2} \left[\overline{K}^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \epsilon \right) + \overline{G}^I \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \epsilon \right) \right]}$$

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- The solutions of $\overline{KG}$ consistent with our demands

$$\Phi^I(\hat{a}_s, q^2, \mu^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2} \right)^{i\frac{\epsilon}{2}} S_\epsilon^i \left(\frac{i\epsilon}{1-z} \right) \hat{\phi}^{I,(i)}(\epsilon)$$

where

$$\hat{\phi}^{I,(i)}(\epsilon) = \hat{\mathcal{L}}_F^{I,(i)}(\epsilon) \Big(A^I \to -A^I, G^I(\epsilon) \to \overline{\mathcal{G}}^I(\epsilon) \Big)$$

This implies

$$\overline{\mathcal{G}}_1^I(\epsilon) = -f_1^I + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}_1^{I,k}$$

$$\overline{\mathcal{G}}_2^I(\epsilon) = -f_2^I - 2\beta_0 \overline{\mathcal{G}}_1^{I,1} + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}_2^{I,k}$$

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Explicit computation is required to get ϵ dependent cof $\bar{\mathcal{G}}_i^{I,k}$.

$$\Phi^I(\hat{a}_s, q^2, \mu^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \left(\frac{i\epsilon}{1-z}\right) \hat{\phi}^{I,(i)}(\epsilon)$$

where

$$\hat{\phi}^{I,(i)}(\epsilon) = \hat{\mathcal{L}}_F^{I,(i)}(\epsilon)\left(A^I \to -A^I, G^I(\epsilon) \to \overline{\mathcal{G}}^I(\epsilon)\right)$$

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$$\overline{\mathcal{G}}_1^I(\epsilon) = -f_1^I + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}_1^{I,k}$$

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Ingredient 4 . . . Crucial Property of Φ^I

Explicit computation shows

$$\Phi^q = \frac{C_F}{C_A}\Phi^g \quad \text{and} \quad \bar{\mathcal{G}}_i^{q,k} = \frac{C_F}{C_A}\bar{\mathcal{G}}_i^{g,k}$$

up to $\mathcal{O}(a_s^2)$.

$\rightsquigarrow$ soft-collinear distributions are universal

$$\Phi^I = C_I \Phi$$

with

$$C_I = \begin{cases} C_A & \text{for Higgs in } gg \\ C_F & \text{for DY and for Higgs in } b\bar{b} \end{cases}$$

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Where are we?

- ✓ $\ln[Z^I]^2$
- ✓ $\ln|\hat{F}^I|^2$
- ✓ $\ln \Gamma_{II}$
- ✓ Φ^I

THE PRESCRIPTION

$\Downarrow$

MASTER FORMULA

$\Downarrow$

HOW DO WE COMPUTE THESE?

$\Downarrow$

WHAT ABOUT DY AT N^3LO?

$$\Delta^{\mathrm{SV},(3)} = \Delta^{\mathrm{SV},(3)}|_{\delta}\delta(1-z) + \sum_{j=0}^{5} \Delta^{\mathrm{SV},(3)}|_{\mathcal{D}_j}\mathcal{D}_j$$

- All the $\mathcal{D}_j$, $j = 0, 1, \ldots, 5$ and partial $\delta(1-z)$ contributions were known for a decade.

Moch and Vogt

- We completed the full computation of $\delta(1-z)$ part.

TA, Mahakhud, Rana, Ravindran

- All the required quantities were available except $\bar{\mathcal{G}}_3^{q,1}$ from $\Phi^{q,(3)}$.

✓ $\ln[Z^q]^2$ up to $\mathcal{O}(a_s^3)$

✓ $\ln|\hat{F}^q|^2$ up to $\mathcal{O}(a_s^3)$

✓ $\ln\Gamma_{qq}$ up to $\mathcal{O}(a_s^3)$

✓ Φ^q up to $\mathcal{O}(a_s^2)$ BUT ? $\mathcal{O}(a_s^3)$

Results : SV XSection for DY at N^{3}LO

$$\Delta^{\mathrm{SV},(3)} = \Delta^{\mathrm{SV},(3)}|_\delta \delta(1-z) + \sum_{j=0}^{5} \Delta^{\mathrm{SV},(3)}|_{\mathcal{D}_j} \mathcal{D}_j$$

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- From which we extracted Φ^g at $\mathcal{O}(a_s^3)$.
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- Using this we got $\Phi^q \rightsquigarrow \bar{\mathcal{G}}_3^{q,1}$.

- This gives the ONLY missing part to achieve $\delta(1-z)$ part.
- In addition, now $\mathcal{D}_7, \ldots \mathcal{D}_1$ are available at N^{4}LO exactly.

  TA, Mahakhud, Rana, Ravindran; de Florian, Mazzitelli, Moch, Vogt
- Later the result was reconfirmed by two groups : *Catani, Cieri, de Florian, Ferrera, Frazzini; Li, von Manteuffel, Schabinger, Zhu*
- This in turn establishes our conjecture at $\mathcal{O}(a_s^3)$.

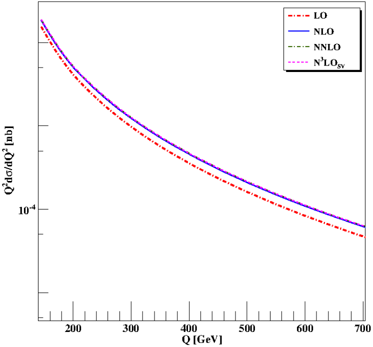
At 14 TeV LHC with $\mu_F = \mu_R = Q = m_{l^+l^-}$

- $\left[Q^2 \frac{d\sigma}{dQ^2}\right]_\delta \approx -\sum_j \left[Q^2 \frac{d\sigma}{dQ^2}\right]_{\mathcal{D}_j} \Rightarrow$ Most of the contributions from $\mathcal{D}_j$'s are cancelled against $\delta(1-z)$ making $\mathrm{N^3LO_{SV}}$ more subleading.

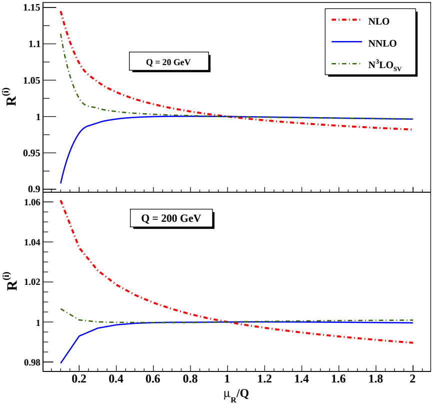
Q (GeV)	50	90	200	400	600	1000
δ (nb)	2.561 10^{-3}	140.114 10^{-3}	4.567 10^{-5}	3.153 10^{-6}	6.473 10^{-7}	7.755 10^{-8}
$\mathcal{D}$ (nb)	-2.053 10^{-3}	-124.493 10^{-3}	-4.421 10^{-5}	-3.368 10^{-6}	-7.455 10^{-7}	-9.959 10^{-8}

- Correction at N^3LO_{SV} is very very small $\rightsquigarrow$ Good newz!

Q (GeV)	50	90	200	400	600	1000
NNLO (nb)	0.158	11.296	$5.233\ 10^{-3}$	$4.694\ 10^{-4}$	$1.116\ 10^{-4}$	$1.607\ 10^{-5}$
N^3LO_{SV} (nb)	0.158	11.311	$5.234\ 10^{-3}$	$4.692\ 10^{-4}$	$1.116\ 10^{-4}$	$1.605\ 10^{-5}$



- Scale dependence reduces.



$$R^{(i)} \equiv \left[Q^2 \frac{d}{dQ^2}\sigma^{(i)}(\mu_R^2)\right] \Bigg/ \left[Q^2 \frac{d}{dQ^2}\sigma^{(i)}(Q^2)\right]$$ and $\mu_F = Q$

Summary

- A systematic way of computing threshold corrections to inclusive Higgs and DY productions in pQCD is prescribed.

- Underlying principles : factorization of soft and collinear divergences, RG invariance, gauge invariance and Sudakov resummation.

- $\delta(1-z)$ part of N^{3}LO$_{\rm SV}$ DY Xsection is computed for the first time.

- Importantly, the calculation is done even without performing the explicit computation of the real emission diagrams for DY at $\mathcal{O}(a_s^3)$.

- Necessary information arising from real emission processes is extracted from the computation of Higgs boson in gg at N^{3}LO threshold.

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- We find that the impact of the $\delta(1-z)$ contribution is quite large to the pure $\mathrm{N^3LO_{SV}}$ correction.
- This method has been later employed to compute some other inclusive and exclusive observables at threshold by us in

Phys. Rev. Lett. 113 (2014) 212003

JHEP 1410 (2014) 139

JHEP 1502 (2015) 131

RG Improved Higgs Boson Production to N^{3}LO in QCD

TA, Das, Kumar, Rana, Ravindran

arXiv:1505.07422

State of the art : The production cross section of Higgs boson through gg at N^{3}LO by Anastasiou et. al.

Consequence

(a) Spectacular accuracy

(b) Significant reduction in unphysical renormalization & factorization scales.

Points of concern

(a) Significant increase in scale uncertainties upon increasing the range of scale variation : $\mu_R < m_H/4$

(b) Total cross section may become negative in this region.

Reason : The presence of large logarithms of the scales at every order $\Rightarrow a_s^n(\mu_R^2)\ln^k(\mu_R^2/m_H^2)$.

Remedy : (a) Go beyond N^{3}LO, (b) Perform all order resummations

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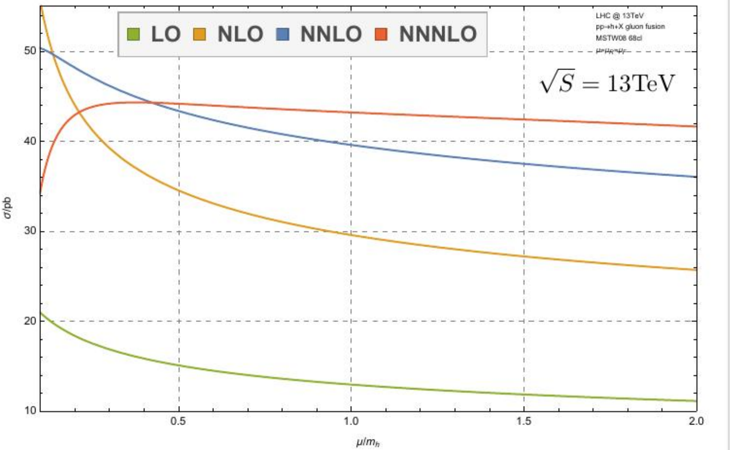

Figure: Higgs boson production in gg fusion

Duhr, Talk at RADCOR '15

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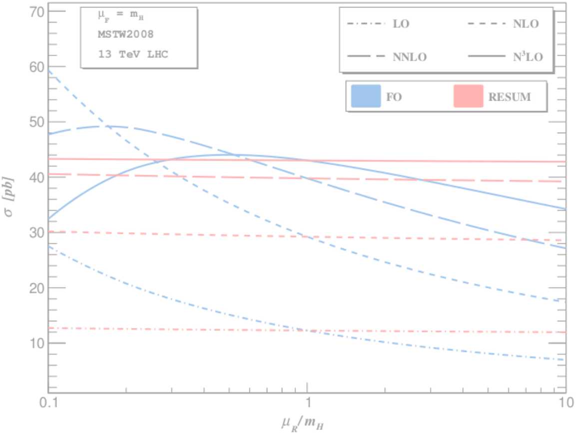
Our proposal : Perform resummations by including RG accessible μ_R dependent logarithms of all orders.

Consequence : Quite remarkable!



What are we resumming?

The inclusive cross section of Higgs production

$$\sigma^{\mathrm{H}}(S, m_{\mathrm{H}}^2) = a_s^2(\mu_R^2)\ \overline{\sigma}(S, m_{\mathrm{H}}^2, \mu_R^2)$$

Structure of perturbation theory :

$$\begin{aligned}
\overline{\sigma} &= \overline{\sigma}_0^{(0)} \\
&+ a_s\left(\mu_R^2\right)\left\{\overline{\sigma}_0^{(1)} + \overline{\sigma}_1^{(1)} L_R\right\} \\
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&+ a_s^3\left(\mu_R^2\right)\left\{\overline{\sigma}_0^{(3)} + \overline{\sigma}_1^{(3)} L_R + \overline{\sigma}_2^{(3)} L_R^2 + \overline{\sigma}_3^{(3)} L_R^3\right\} \\
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where

$$L_R \equiv \ln\left(\frac{\mu_R^2}{m_H^2}\right)$$

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What are we resumming? ...

Collecting the highest logs to all orders

$$\overline{\sigma}_{\Sigma}^{(0)}(S, m_{\mathrm{H}}^2, \mu_R^2) \equiv \sum_{n=0}^{\infty} \overline{\sigma}_n^{(n)} \left(a_s L_R\right)^n$$

Collecting the next-to-highest logs to all orders

$$\overline{\sigma}_{\Sigma}^{(1)}(S, m_{\mathrm{H}}^2, \mu_R^2) \equiv \sum_{n=1}^{\infty} \overline{\sigma}_{n-1}^{(n)} \left(a_s L_R\right)^{n-1}$$

and so on. We re-write the Xsection in terms of these as

$$\begin{aligned}\sigma^{\mathrm{H}}(S, m_{\mathrm{H}}^2) &= a_s^2(\mu_R^2)\ \overline{\sigma}(S, m_{\mathrm{H}}^2, \mu_R^2) \\ &= a_s^2(\mu_R^2)\ \sum_{i=0}^{\infty} a_s^i(\mu_R^2) \overline{\sigma}_{\Sigma}^{(i)}(S, m_{\mathrm{H}}^2, \mu_R^2)\end{aligned}$$

Goal : Compute $\overline{\sigma}_{\Sigma}^{(i)}$ for $i = 0, 1, 2, 3$ for $gg \to H$.

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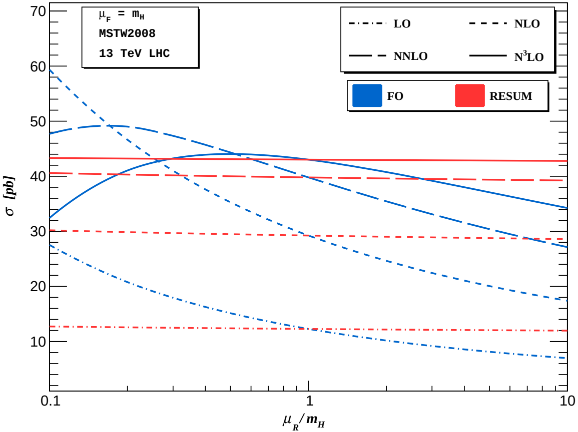

OUR PROPOSAL TO IMPROVE μ_R DEPENDENCE ...

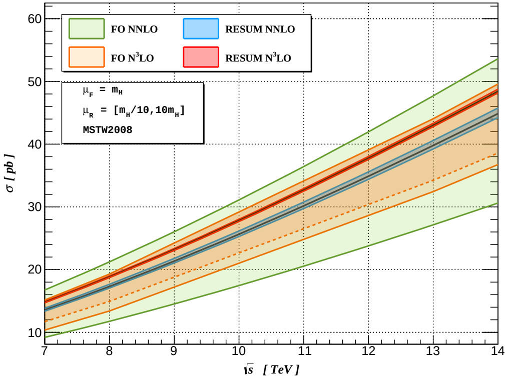

Figure: Dependence of μ_R in both the fixed order and resummed Xsections on $\sqrt{S}$

Demerit : Method fails to improve the central value of Xsection, unlike other resummation.

Two-loop QCD corrections to Higgs $\rightarrow b + \bar{b} + g$ amplitude

TA, Mahakhud, Mathews, Rana, Ravindran

JHEP 1408 (2014) 075

- Theory : SM.
- Bottom quark mass $\rightsquigarrow$ VFS scheme.
- No of diagrams : 251
- Mostly in-house codes written in FORM and Mathematica.
- For IBP, LI identities $\rightsquigarrow$ LiteRed by Lee.
- Agrees with the universal IR-pole structures of QCD.
- Result will be used for Higgs + jet production through $b\bar{b}$ at NNLO QCD in hadron colliders.
- Analytically computed.

Two-Loop QCD Correction to massive spin-2 resonance $\rightarrow$ 3 gluons

TA, Mahakhud, Mathews, Rana, Ravindran

JHEP 1405 (2014) 107

- Spin-2 couples to SM minimally through SM energy-momentum tensor.
- No of diagrams : 2362
- Mostly in-house codes written in FORM and Mathematica.
- For IBP, LI identities $\rightsquigarrow$ LiteRed by Lee.
- Explicit verification of the universal IR-pole structure of QCD when spin-2 presents.
- Result will be used for spin-2 + jet production at NNLO QCD in hadron colliders.
- Analytically computed.

Thank You!

Any Questions?

Extra Slides

Extra 1 : RG Resum in Details

The inclusive cross section of Higgs production

$$\sigma^{\mathrm{H}}(S, m_{\mathrm{H}}^2) = \sigma^0 a_s^2(\mu_R^2) \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2)$$

$$\times\, \mathcal{C}_{\mathrm{H}}^2 \left(a_s(\mu_R^2)\right) \Delta_{ab}^{\mathrm{H}} \left(\frac{\tau}{x_1 x_2}, m_{\mathrm{H}}^2, \mu_R^2, \mu_F^2\right)$$

$$\equiv a_s^2(\mu_R^2)\, \overline{\sigma}(S, m_{\mathrm{H}}^2, \mu_R^2)$$

- RG invariance wrt μ_R of the observable $\mu_R^2 \frac{d}{d\mu_R^2} \sigma^H = 0$
- The Solution

$$\overline{\sigma}(\mu_R^2) = \overline{\sigma}(\mu_0^2) \exp\left[-\int_{\mu_0^2}^{\mu_R^2} \frac{d\mu^2}{\mu^2} \frac{2\ \beta(a_s(\mu^2))}{a_s(\mu^2)}\right]$$

The inclusive cross section of Higgs production

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$$\equiv a_s^2(\mu_R^2)\, \overline{\sigma}(S, m_{\mathrm{H}}^2, \mu_R^2)$$

- RG invariance wrt μ_R of the observable $\mu_R^2 \frac{d}{d\mu_R^2} \sigma^H = 0$
- The Solution

$$\overline{\sigma}(\mu_R^2) = \overline{\sigma}(\mu_0^2) \exp\left[-\int_{\mu_0^2}^{\mu_R^2} \frac{d\mu^2}{\mu^2} \frac{2\ \beta(a_s(\mu^2))}{a_s(\mu^2)}\right]$$

Understanding The Source Of The Problem

The inclusive cross section of Higgs production

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- Considering μ_0 as central scale m_H and using naive evolution of a_s, the solution

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where, $L_R \equiv \ln\left(\frac{\mu_R^2}{m_H^2}\right)$.

- The RG invariance dictates

$$\mathcal{R}_{n,n-m} = \frac{1}{(n-m)}\sum_{i=0}^{m}(n-i+1)\beta_i\mathcal{R}_{n-i-1,n-m-1}$$

i.e. the coefficients of the logarithms $\mathcal{R}_{n,k}(0 < k \leq n)$ can be expressed in terms of the lower order ones $\mathcal{R}_{n-1,0}$.

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UNDERSTANDING THE SOURCE OF THE PROBLEM

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- RG invariance can be used to resume these large logarithms to all orders.

We extend the approach by Ahmady et. al. to resum these large contributions.

- We rewrite the solution $\overline{\sigma}(\mu_R^2)$ as

$$\overline{\sigma}(\mu_R^2) = \sum_{m=0}^{\infty} a_s^m(\mu_R^2) \sum_{n=m}^{\infty} \mathcal{R}_{n,n-m}(a_s L_R)^{n-m}$$
$$\equiv \sum_{m=0}^{\infty} a_s^m(\mu_R^2)\, \overline{\sigma}_{\Sigma}^{(m)} \left(a_s(\mu_R^2) L_R\right),$$

$\overline{\sigma}_{\Sigma}^{(m)}$ resums $a_s(\mu_R^2) L_R$ to all orders.

- Our Goal : Determine $\overline{\sigma}_{\Sigma}^{(m)}$'s.

The Remedy : Our Prescription

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Using the recursion relation we show $\overline{\sigma}_{\Sigma}^{(m)}$ satisfies

$$\boxed{\begin{aligned}&\left[\omega\frac{d}{d\omega}+(m+2)\right]\overline{\sigma}_{\Sigma}^{(m)}\\&=\Theta_{m-1}\sum_{i=1}^{m}\eta_i\left[(1-\omega)\frac{d}{d\omega}-(m-i+2)\right]\overline{\sigma}_{\Sigma}^{(m-i)}\end{aligned}}$$

$\eta_i \equiv \beta_i/\beta_0$ and $\omega \equiv 1-\beta_0 a_s(\mu_R^2)L_R$

$$\overline{\sigma}_{\Sigma}^{(0)} = \frac{1}{\omega^2}\Big\{\mathcal{R}_{0,0}\Big\}, \quad \overline{\sigma}_{\Sigma}^{(1)} = \frac{1}{\omega^3}\Big\{\mathcal{R}_{1,0} - 2\eta_1\mathcal{R}_{0,0}\ln(\omega)\Big\},$$

$$\overline{\sigma}_{\Sigma}^{(2)} = \frac{1}{\omega^3}\Big\{2\mathcal{R}_{0,0}\left({\eta_1}^2 - \eta_2\right)\Big\} + \frac{1}{\omega^4}\Big\{\mathcal{R}_{2,0} + 2\mathcal{R}_{0,0}\left(\eta_2 - {\eta_1}^2\right) + \ln(\omega)\Big(-2{\eta_1}^2\mathcal{R}_{0,0} - 3\eta_1\mathcal{R}_{1,0}\Big)$$
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$$+ \ln(\omega)\Big(\mathcal{R}_{0,0}\left(6{\eta_1}^3 - 8\eta_1\eta_2\right) - 3{\eta_1}^2\mathcal{R}_{1,0} - 4\eta_1\mathcal{R}_{2,0}\Big) + \ln^2(\omega)\Big(7{\eta_1}^3\mathcal{R}_{0,0} + 6{\eta_1}^2\mathcal{R}_{1,0}\Big)$$
$$- 4{\eta_1}^3\mathcal{R}_{0,0}\ln^3(\omega)\Big\},$$

$$\overline{\sigma}_{\Sigma}^{(4)} = \textit{Big expression}$$

- We have resummed only μ_R dependent logarithms.
- μ_F has been chosen to some specific value $m_H \rightsquigarrow \mu_F$ dependence remains unchanged.

The Remedy : The Solutions

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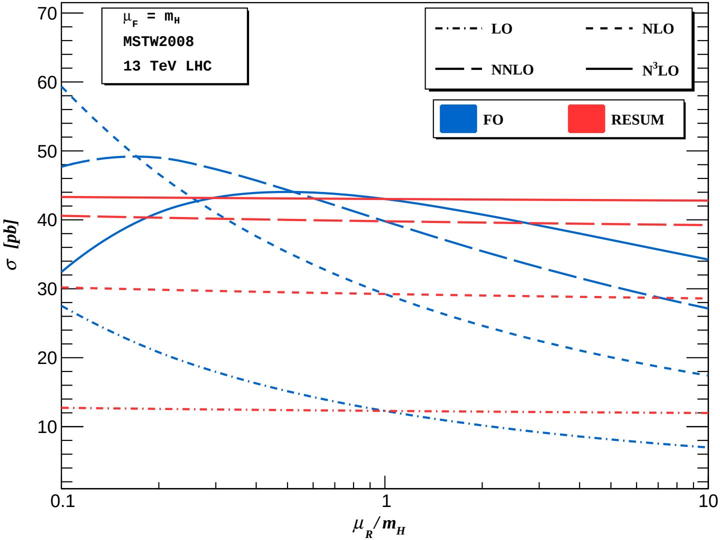
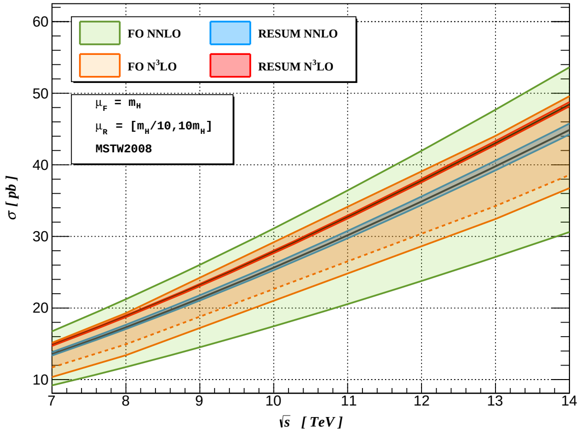
(a) Result is almost μ_R independent for a wide range of $\mu_R \in [0.1 m_{\mathrm{H}}, 10 m_{\mathrm{H}}]$

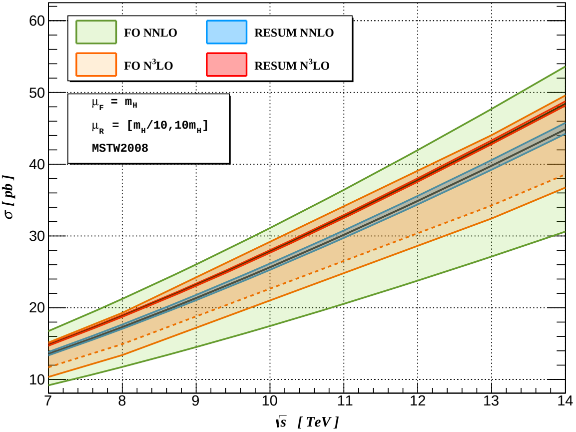
	LO	NLO	NNLO	N^{3}LO
FO (%)	167.26	143.40	54.99	27.01
RESUM (%)	6.11	5.47	3.39	1.23


$\mu_F = m_H$
MSTW2008
13 TeV LHC
LO
NLO
NNLO
N^3LO
FO
RESUM
70
60
50
40
30
20
10
σ [pb]
0.1
1
10
μ_R/m_H

The Remedy : Numerical Implications



(b) Cross section is always positive and reliable.



(b) Cross section is always positive and reliable.

Extra 2 : What are we resumming?

The inclusive cross section of Higgs production

$$\sigma^{\mathrm{H}}(S, m_{\mathrm{H}}^2) = a_s^2(\mu_R^2)\ \overline{\sigma}(S, m_{\mathrm{H}}^2, \mu_R^2)$$

Structure of perturbation theory :

$$\begin{aligned}
\overline{\sigma} &= \overline{\sigma}_0^{(0)} \\
&\quad + a_s\left(\mu_R^2\right)\left\{\overline{\sigma}_0^{(1)} + \overline{\sigma}_1^{(1)} L_R\right\} \\
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What are we resumming? ...

Collecting the highest logs to all orders

$$\overline{\sigma}_{\Sigma}^{(0)}(S, m_{\mathrm{H}}^2, \mu_R^2) \equiv \sum_{n=0}^{\infty} \overline{\sigma}_n^{(n)} \left(a_s L_R\right)^n$$

Collecting the next-to-highest logs to all orders

$$\overline{\sigma}_{\Sigma}^{(1)}(S, m_{\mathrm{H}}^2, \mu_R^2) \equiv \sum_{n=1}^{\infty} \overline{\sigma}_{n-1}^{(n)} \left(a_s L_R\right)^{n-1}$$

and so on. We re-write the Xsection in terms of these as

$$\begin{aligned}\sigma^{\mathrm{H}}(S, m_{\mathrm{H}}^2) &= a_s^2(\mu_R^2)\ \overline{\sigma}(S, m_{\mathrm{H}}^2, \mu_R^2) \\ &= a_s^2(\mu_R^2)\ \sum_{i=0}^{\infty} a_s^i(\mu_R^2) \overline{\sigma}_{\Sigma}^{(i)}(S, m_{\mathrm{H}}^2, \mu_R^2)\end{aligned}$$

Goal : Compute $\overline{\sigma}_{\Sigma}^{(i)}$ for $i = 0, 1, 2, 3$ for $gg \to H$.

Extra 3 : SV Xsection of Higgs in $b\bar{b}$

$\Delta^{\mathrm{SV},(3)}$ for Higgs production in $b\bar{b}$ annihilation

- **All the** $\mathcal{D}_i$ and **partial** $\delta(1-z)$ contributions were **known**.
- We completed the full computation of $\delta(1-z)$ part.
- All the required quantities were available except $\bar{\mathcal{G}}_3^{b,1}$ from Φ^b and $g_3^{b,1}$ from 3-loop form factor $\hat{F}^b$.
- Being flavor independent

$$\boxed{\bar{\mathcal{G}}_3^{b,1} = \bar{\mathcal{G}}_3^{q,1}}$$

  which is available from our DY result.
- $g_3^{b,1}$ is extracted from recent result of *3-loop $Hb\bar{b}$ form factor* by Gehrmann et. al.
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Extra 4 : Fixing soft-collinear distribution Φ^I

Fixing Forms of Φ^I

$$\Delta^{\mathrm{SV},I} = \mathcal{C}\exp(\Psi^I) \text{ with}$$

$$\Psi^I = \left[\ln\left(Z^I\right)^2 + \ln|\hat{F}^I|^2\right]\delta(1-z) + 2\Phi^I - 2\mathcal{C}\ln\Gamma_{II}$$

Considering only poles at $\mathcal{O}(a_s)$ with $\mu_R = \mu_F$ in $d = 4+\epsilon$

$$\ln\left(Z^{I,(1)}\right)^2 = a_s(\mu_F^2)\frac{4\gamma_0^I}{\epsilon}$$

$$\ln|\hat{F}^{I,(1)}|^2 = a_s(\mu_F^2)\left(\frac{q^2}{\mu_F^2}\right)^{\epsilon/2}\left[-\frac{4A_1^I}{\epsilon^2} + \frac{1}{\epsilon}\left(2f_1^I + 4B_1^I - 4\gamma_0^I\right)\right]$$

$$2\mathcal{C}\ln\Gamma_{II,(1)} = 2a_s(\mu_F^2)\left[\frac{2B_1^I}{\epsilon}\delta(1-z) + \frac{2A_1^I}{\epsilon}\mathcal{D}_0\right]$$

Collecting the coefficient of $a_s(\mu_F^2)$ $\left[\text{neglecting } \ln\left(q^2/\mu_F^2\right) \text{ terms}\right]$

$$\Psi^{I,(1)} = a_s(\mu_F^2)\left[\left\{\cancel{\frac{4\gamma_0^I}{\epsilon}} - \frac{4A_1^I}{\epsilon^2} + \frac{1}{\epsilon}\left(2f_1^I + \cancel{4B_1^I} - \cancel{4\gamma_0^I}\right)\right\}\delta(1-z) - \left\{\cancel{\frac{4B_1^I}{\epsilon}}\delta(1-z) + \frac{4A_1^I}{\epsilon}\mathcal{D}_0\right\} + 2$$

$$= a_s(\mu_F^2)\left[\left\{-\frac{4A_1^I}{\epsilon^2} + \frac{2f_1^I}{\epsilon}\right\}\delta(1-z) - \frac{4A_1^I}{\epsilon}\mathcal{D}_0\right] + 2\Phi^I$$

Demand 1 : Hence to cancel all the poles, we must have at $\mathcal{O}(a_s)$

$$\boxed{2\Phi^{I,(1)}|_{\mathrm{poles}} = a_s(\mu_F^2)\left[\left\{\frac{4A_1^I}{\epsilon^2} - \frac{2f_1^I}{\epsilon}\right\}\delta(1-z) + \frac{4A_1^I}{\epsilon}\mathcal{D}_0\right]}$$

Fixing Forms of Φ^I

$$\Delta^{\mathrm{SV},I} = \mathcal{C}\exp(\Psi^I) \text{ with}$$

$$\Psi^I = \left[\ln\left(Z^I\right)^2 + \ln|\hat{F}^I|^2\right]\delta(1-z) + 2\Phi^I - 2\mathcal{C}\ln\Gamma_{II}$$

Considering only poles at $\mathcal{O}(a_s)$ with $\mu_R = \mu_F$ in $d = 4+\epsilon$

$$\ln\left(Z^{I,(1)}\right)^2 = a_s(\mu_F^2)\frac{4\gamma_0^I}{\epsilon}$$

$$\ln|\hat{F}^{I,(1)}|^2 = a_s(\mu_F^2)\left(\frac{q^2}{\mu_F^2}\right)^{\epsilon/2}\left[-\frac{4A_1^I}{\epsilon^2} + \frac{1}{\epsilon}\left(2f_1^I + 4B_1^I - 4\gamma_0^I\right)\right]$$

$$2\mathcal{C}\ln\Gamma_{II,(1)} = 2a_s(\mu_F^2)\left[\frac{2B_1^I}{\epsilon}\delta(1-z) + \frac{2A_1^I}{\epsilon}\mathcal{D}_0\right]$$

Collecting the coefficient of $a_s(\mu_F^2)$ $\left[\text{neglecting } \ln\left(q^2/\mu_F^2\right) \text{ terms}\right]$

$$\Psi^{I,(1)} = a_s(\mu_F^2)\left[\left\{\cancel{\frac{4\gamma_0^I}{\epsilon}} - \frac{4A_1^I}{\epsilon^2} + \frac{1}{\epsilon}\left(2f_1^I + \cancel{4B_1^I} - \cancel{4\gamma_0^I}\right)\right\}\delta(1-z) - \left\{\cancel{\frac{4B_1^I}{\epsilon}\delta(1-z)} + \frac{4A_1^I}{\epsilon}\mathcal{D}_0\right\} + 2\right.$$

$$= a_s(\mu_F^2)\left[\left\{-\frac{4A_1^I}{\epsilon^2} + \frac{2f_1^I}{\epsilon}\right\}\delta(1-z) - \frac{4A_1^I}{\epsilon}\mathcal{D}_0\right] + 2\Phi^I$$

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Fixing Form of Φ^I ...

Demand 2 : Also, Φ^I has to be RG-invariant i.e. $\mu_R^2 \frac{d}{d\mu_R^2} \Phi^I = 0$.

We make an ansatz that if Φ^I satisfies KG-type DE, then we can accomplish this

$$q^2 \frac{d}{dq^2} \Phi^I \left(\hat{a}_s, q^2, \mu^2, z, \epsilon \right) = \frac{1}{2} \left[\overline{K}^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \epsilon \right) + \overline{G}^I \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \epsilon \right) \right]$$

using Demand 2

$$\mu_R^2 \frac{d}{d\mu_R^2} \overline{K}^I = -\mu_R^2 \frac{d}{d\mu_R^2} \overline{G}^I \equiv -X^I$$

The solution

$$\Phi^I = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2} \right)^{i\epsilon/2} S_\epsilon^i \Phi^{I,(i)}(z, \epsilon)$$

with

$$\Phi^{I,(i)}(z, \epsilon) = \hat{\mathcal{L}}^{I,(i)} \left(A^I \to X^I, G^I \to \overline{G}^I(z, \epsilon) \right).$$

So

$$\boxed{2\Phi^{I,(1)}(z, \epsilon) = \frac{1}{\epsilon^2} \left(-4X_1^I \right) + \frac{2}{\epsilon} \overline{G}_1^I (z, \epsilon))}$$

Fixing Form of Φ^I ...

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$$\Phi^{I,(i)}(z,\epsilon) = \hat{\mathcal{L}}^{I,(i)}\left(A^I \to X^I, G^I \to \overline{G}^I(z,\epsilon)\right).$$

So

$$\boxed{2\Phi^{I,(1)}(z,\epsilon) = \frac{1}{\epsilon^2}\left(-4X_1^I\right) + \frac{2}{\epsilon}\overline{G}_1^I(z,\epsilon))}$$

Fixing Form of Φ^I ...

where,

$$\Phi^I = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{i\epsilon/2} S_\epsilon^i \Phi^{I,(i)}(z,\epsilon)$$
$$= \sum_{i=1}^{\infty} a_s^i \left(\mu_F^2\right) \left(\frac{q^2}{\mu^2}\right)^{i\epsilon/2} Z_{a_s}^i \Phi^{I,(i)}(z,\epsilon)$$
$$\equiv \sum_{i=1}^{\infty} a_s^i \Phi_R^{I,(i)}(z,\epsilon)$$

At $\mathcal{O}(a_s(\mu_F^2)) \Rightarrow \Phi^{I,(1)}(z,\epsilon) = \Phi_R^{I,(1)}(z,\epsilon)$

$$X^I = \sum_{i=1}^{\infty} a_s^i(\mu_F^2) X_i^I$$
$$\overline{G}^I(z,\epsilon)) = \sum_{i=1}^{\infty} a_s^i(\mu_F^2) \overline{G}_i^I(z,\epsilon))$$

Hence,

$$\boxed{\begin{aligned} X_1^I &= -A_1^I \delta(1-z) \\ \overline{G}_1^I(z,\epsilon)) &= -f_1^I \delta(1-z) + 2A_1^I \mathcal{D}_0 + \sum_{k=1}^{\infty} \epsilon^k \overline{g}_1^{I,k}(z) \end{aligned}}$$

Explicit computation is required to determine $\mathcal{O}(\epsilon)$ terms $\overline{g}_1^{I,k}(z)$.

Fixing Form of Φ^I ...

Alternative Method : We say our demands can be fulfilled if we assume the solution of $\overline{KG}$ Eq.

$$\Phi^I = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{i\epsilon/2} S_\epsilon^i \Phi^{I,(i)}(z,\epsilon)$$

with

$$\Phi^{I,(i)}(z,\epsilon) \equiv \left\{ i\epsilon \frac{1}{1-z} \left[(1-z)^2\right]^{i\epsilon/2} \right\} \phi^{I,(i)}(\epsilon)$$

$$= \left\{ \delta(1-z) + \sum_{j=0}^{\infty} \frac{(i\epsilon)^{j+1}}{j!} \mathcal{D}_j \right\} \phi^{I,(i)}(\epsilon)$$

RGE invarinace of Φ^I

$$\mu_R^2 \frac{d}{d\mu_R^2} \overline{K}^I = -\mu_R^2 \frac{d}{d\mu_R^2} \overline{G}^I \equiv -Y^I$$

The solution

$$\phi^{I,(i)}(\epsilon) = \hat{\mathcal{L}}^{I,(i)} \left(A^I \to Y^I, G^I \to \mathcal{G}^I(\epsilon) \right).$$

So

$$\boxed{2\Phi^{I,(1)}(z,\epsilon) = \left\{ \frac{1}{\epsilon^2} \left(-4Y_1^I\right) + \frac{2}{\epsilon} \overline{\mathcal{G}}_1^I(\epsilon)) \right\} \delta(1-z) + \left\{ -\frac{4Y_1^I}{\epsilon^2} + \frac{2}{\epsilon} \overline{\mathcal{G}}_1^I(\epsilon) \right\} \sum_{j=0}^{\infty} \frac{\epsilon^{j+1}}{j!} \mathcal{D}_j}$$

where

$$Y^I = \sum_{i=1}^{\infty} a_s^i(\mu_F^2) X_i^I$$

$$\overline{\mathcal{G}}^I(\epsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_F^2) \overline{\mathcal{G}}_i^I(\epsilon)$$

Hence,

$$\boxed{\begin{aligned} Y_1^I &= -A_1^I \\ \overline{\mathcal{G}}_1^I(\epsilon) &= -f_1^I + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}_1^{I,k} \end{aligned}}$$

Explicit computation is required to determine the $\mathcal{O}(\epsilon)$ terms $\overline{\mathcal{G}}_1^{I,k}$.

- The methodology holds at every order.
- Both methods are equivalent. We have followed the 2nd one to perform our computations.

Extra 5 : Higgs N^{3}LO

Band Plot for Higgs Boson

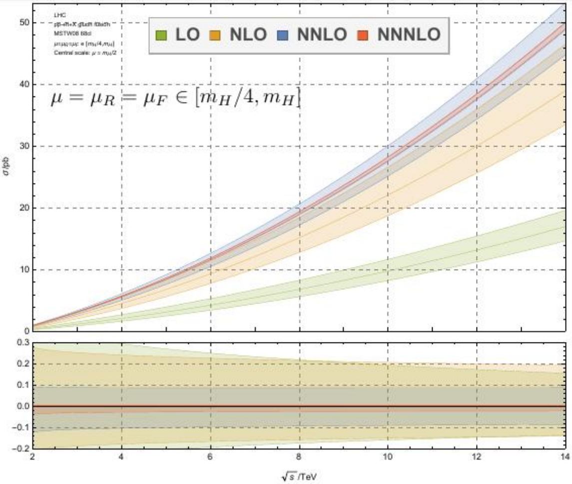

Figure: Higgs boson production in gg fusion

Anastasiou, Duhr, Dulat, Herzog, Mistlberger

N^3LL Threshold Resum for Higgs Boson

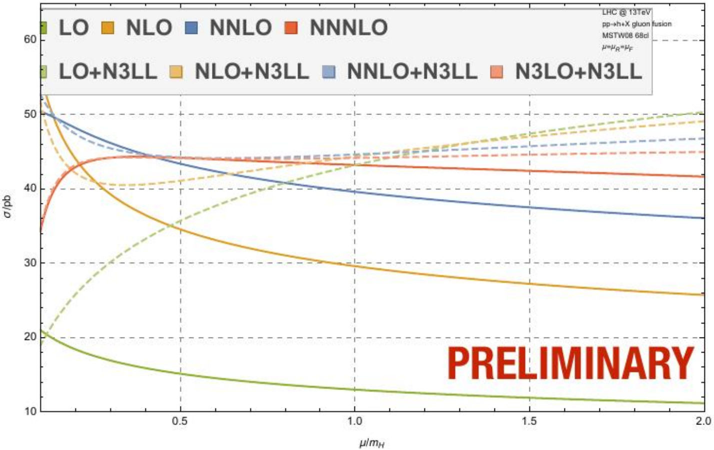

Figure: Higgs boson production in gg fusion

Duhr, Talk at RadCor '15