The Anti-proton: How is it known to be the anti-particle of the proton? How is it produced and preserved for laboratory experiments?

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Contents

1 Introduction

1.1 Dirac QM

1.1.1 Negative energy ⇒ Instability of physical system

Let's start with reviewing the theoretical basics of anti-particle from the viewpoint of Dirac RQM. We know that Dirac equation has the both positive as well as negative energy solutions. For example, for free particles

$$E = p^0 = \pm\sqrt{\vec{p}^2 + m^2} \tag{1}$$

There is positive as well as negative energy solutions with a separation of 2m :

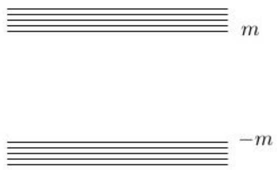

Figure 1: Energy Spectrum for a free Dirac particle

In spite of consistent explanation of probability density, the existence of negative energy solution leads instability of the physical system. Since, physical system has a tendency to go to the lowest energy state which is available, this implies that any such physical system of Dirac particles would make a transition to these negative energy states thereby leading to a collapse of all stable systems(e.g. H-atom). Even if we start with a positive energy solution, any perturbation would cause the energy to lower, destabilize the physical system and leading to an ultimate collapse.

1.1.2 Hole Theory

Dirac postulated an adhoc prescription to save his theory : "Hole Theory". Dirac postulated that the ground state in such a theory is the state where all the negative energy states are filled up completely. And being spin-half particles, Pauli exclusion principle prevents any positive energy particle to come down to this negative energy states. This adhoc postulate explains the stability of Dirac system.

1.1.3 Prediction of anti-particles

However, if enough energy is provided to such a ground state, a negative energy particle can make a transition to a positive energy state and can appear as a positive energy particle. The absence of the negative energy particle can be thought of as a "hole" having exactly the same mass as the particle but otherwise opposite quantum number. This is known as anti-particle. **Thus Dirac theory predicts the existence of anti-particle corresponding to every Dirac particle(1930).**

1.2 Motivations

1.2.1 Discovery of positron in 1932

Though in the beginning peoples thought it's absolutely nonsense to consider a theory with having negative energy solution. But, attitudes changed dramatically in 1932 when a young professor at the California Institute of Technology, Carl David Anderson, working on a project that had originated with his mentor, Robert Millikan, reported the observation of a positively charged electron, which he dubbed the "positron.". Naturally after that peoples asked the question whether proton also does have an anti-proton. After all this is also a spin-half particle obeying Pauli Exclusion principle(Though we should remember the magnetic moment of the proton was not in accord with the theoretical value predicted by DE, while that of the electron was extremely close to it.). According to Dirac theory, the anti-proton should have the properties :
Mass $\Rightarrow$ Same as the proton mass
Electric charge $\Rightarrow$ Negative, opposite to the proton charge, equal to the electron charge
Spin $\Rightarrow$ 1/2 unit, same as the proton.

2 Need of an Accelerator : The Bevatron

- Peoples started thinking what to do to find an anti-proton if it at all exists.
- The creation of an anti-proton would necessitate the simultaneous creation of a proton or a neutron, to respect the conservation laws. For example $p + n = \bar{p} + p + n + p$ to conserve baryon number. Since the energy required to produce a particle is proportional its rest mass, the creation of a proton-antiproton pair would require minimum twice of the proton rest energy, or about 2 billion ev(2 GeV). Given the fixed-target collision technology of the times, the best approach for making 2 GeV available would be to strike a stationary target of neutrons with a beam of protons accelerated to about 6 GeV of energy.
- The accelerator named Bevatron at the university of California, Berkeley was designed to reach upto 6.5 billion ev to search for anti-proton.

3 Things need to be Measured in Bevatron

In the Bevatron accelerator the high-energy proton beam strikes the target and produce a spray of all sorts of secondary particles. Our central AIM is to see :

1. Whether we are getting some particles having the same **mass** as that of proton, but **electrically negative**, and
2. Whether these new particles are annihilated completely with the proton.

4 Separation of Negatively Charged Particles

- Since in the presence of magnetic field, positive and negative charges are deflected in the opposite directions, it is not so difficult to separate out the negatively charged particles with the help of magnetic field.

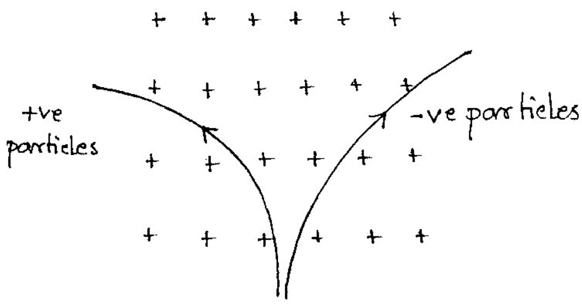

Figure 2: Deflection of oppositely charged particles

- Note that, at this point we only know the separated charged particles are negative but that may be 1-unit, or any other unit. Later we will review this point again namely how to separate out the charges having 1-unit.

5 Measurement of Mass

5.1 Relativistic Relation

Recall the relativistic definition of momentum :

$$p \equiv \gamma m v \equiv \frac{mv}{\sqrt{1-v^2}} \tag{2}$$

$$\Rightarrow m = \frac{p\sqrt{1-v^2}}{v} \tag{3}$$

So, to measure the mass of a particle we have to measure both :

1. **linear-momentum** as well as
2. **speed**

of the particle. So, let's address these questions one by one that how can we measure the linear momentum and speed of a particle?

5.2 Measuring the Momentum

5.2.1 Deflection of Charged Particles in uniform Magnetic Field

After collision the product particles are scattered randomly in all directions. We capture only a small fraction of them and make them pass through a magnetic field. In the presence of a magnetic field $\vec{B}$, the positive and negative charge particles are deflected in the opposite directions. Negative charges within a certain range of linear momentum are deflected by the correct amount which are then passed through an aperture of a collimator. Other negative particles either do not enter into the collimator or are stopped by the walls of the collimator.

If magnetic field acts perpendicular to the velocity of the charged particles and since magnetic field doesn't change the magnitude of the velocity of the particle but direction, so

$$\vec{F}_m = q\vec{v} \times \vec{B} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m \vec{v}) \tag{4}$$

$$= \gamma m \frac{d\vec{v}}{dt} \tag{5}$$

$$\Rightarrow qvB = \gamma m \frac{v^2}{r} \tag{6}$$

$$\Rightarrow r = \frac{\gamma m v}{Bq} = \frac{p}{Bq} \tag{7}$$

So, depending on the initial momentum, the radius of the charged particles differs and consequently they are deflected by different amount. The below figure is showing this:

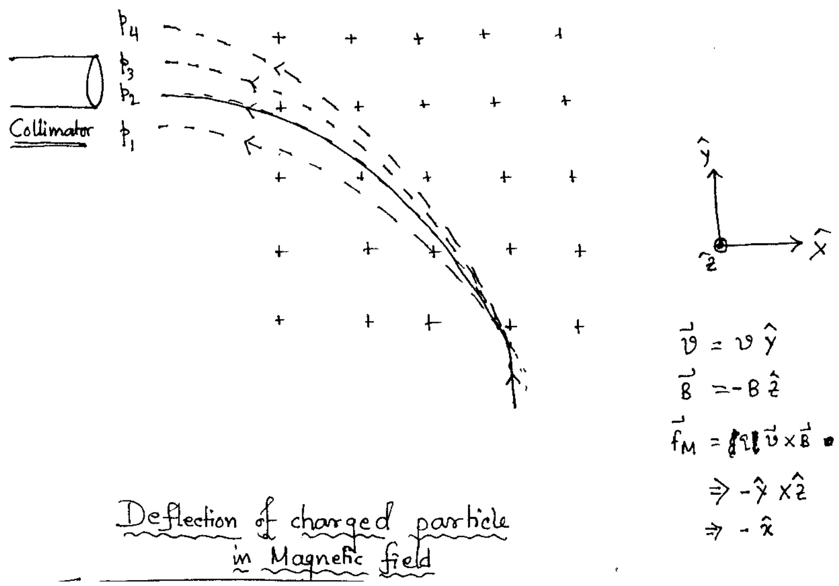

Figure 3: Selection of charged particles within a certain range of momentum

5.2.2 Principle of the Determination of Momentum from Deflection

In the presence of a uniform magnetic field(acting perpendicular to the velocity of the charged particles) we got,

$$r = \frac{p}{qB} \tag{8}$$

Now, frequently with high momentum particles, only an arc corresponding to a small part of the circle is observed. Consider a particle of momentum p passing through a region, of length L, with a magnetic field B. The deviation from a straight line, s, is known as the sagitta of the track(as shown in the figure below).

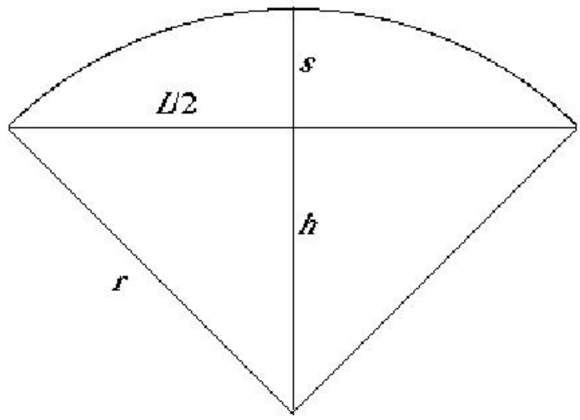

Figure 4: Selection of charged particles within a certain range of momentum

From the figure:

$$h^2 = r^2 - (\frac{L}{2})^2 = r^2(1 - \frac{L^2}{4r^2}) \tag{9}$$

$$\Rightarrow h \approx r(1 - \frac{L^2}{8r^2} + ...) \tag{10}$$

$$s = r - h = r\frac{L^2}{8r^2} = \frac{L^2}{8r} \tag{11}$$

$$\Rightarrow r = \frac{L^2}{8s} \tag{12}$$

Hence,

$$r = \frac{L^2}{8s} = \frac{p}{qB} \tag{13}$$

$$\Rightarrow p = \frac{L^2 qB}{8s} \tag{14}$$

Hence,

$$\boxed{p = \frac{L^2 qB}{8s}} \tag{15}$$

So, observing the deflection one can measure the momentum of the relativistic particle.

5.2.3 Focusing the particles

Now, we may think that particles are travelling solely along the z-direction-that is,straight ahead- but in reality some particles might have non-zero transverse velocity.

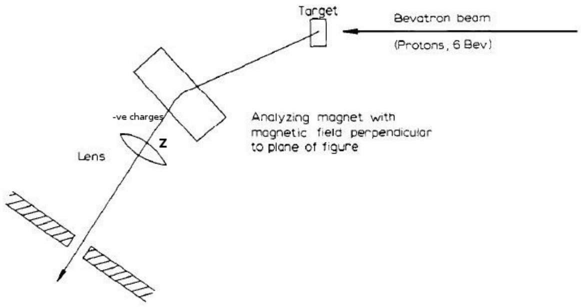

Figure 5: Magnetic spectrograph for producing a beam of charged particles with known momentum

This might happen due to Coulomb force which may try to separate them out. If this separation is not resisted, the bunch of particles will eventually fly apart and particles will collide with beam pipe.**We need to have a method of focusing the particles in the transverse planes.** This is done using a special electromagnet called **quadrupole**. A **cross-sectional** picture of a quadrupole is shown below from the **eye-view of a particle**.

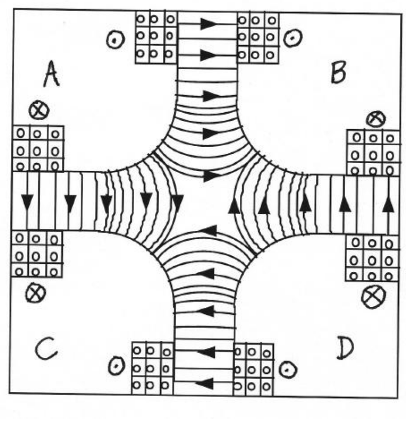

Figure 6: Quadrupole magnet. The four (usually iron) magnetic pole faces are labelled A, B, C, and D. The arrowed lines indicate the magnetic field. The direction of current flow, which is into or out of the page, is labelled near the square bus work.A given quadrupole can only focus the beam in one plane — while defocusing the beam in the other.

Looking at the above diagram, one can see that the quadrupole is made up of four separate loops of wire wrapped around four pieces of iron, called magnetic pole faces. The electric current flowing in four loops creates the magnetic field.

The direction of current flow in these four loops is indicated (A circle with a dot indicates 'out of the page' while a circle with an X indicates 'into the page').

A given quadrupole magnet can only focus the beam in one plane. Applying the right hand rule to the lines of magnetic field shown in the picture, one can see that a positively charged particle travelling into the page will see no net force when passing through the center. When the same particle passes through the quadrupole off-center, net forces are felt. For instance, a positive particle travelling into the page between the A and B pole faces will feel a magnetic force downwards. This force acts to direct the particle back towards the ideal orbit,focusing in the vertical plane. If the same positive particle were to pass through this magnet off-center between the A and C pole faces, a force to the left, away from the ideal orbit, would be felt, defocusing in the horizontal plane. If we rotate the quadrupole above by 90 degrees, we get a magnet that behaves in the opposite way — it defocuses beam in the vertical plane and focuses in the horizontal plane. One can also change the quadrupole's orientation by reversing the flow of electrical current running through the bus work of the magnet.

Alternating the placement of focusing and defocusing quadrupole in an accelerator produces a net focusing effect in both planes.

For better accuracy two successive spectrographs are used in the final experimental set-up. The particles focussed at the second focus point have the same momentum within 2 percent.

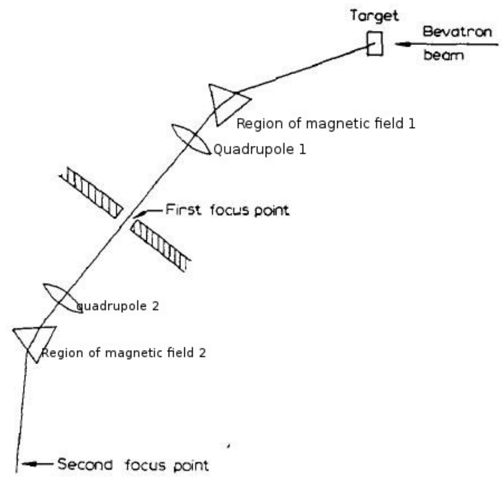

Figure 7: Final arrangement of magnets, represented in optical analogy

5.3 Measuring the Speed

5.3.1 Measuring Speed using Scintillation Counter

The most simplified method of determining speed of a particle is to measure the time of flight between two counters. Two scintillation counters are placed at two focus points.

Scintillation counters constitute one of the major class of particle detectors. In a scintillating material the incident particles excite some atoms to emit pho-

ton, usually visible or near UV. The light was originally detected visually, as in the original Rutherford experiments, but electronic devices are now used. A common arrangement is shown in figure below.The scintillator is optically bonded to a light guide which conveys the light to a photomultiplier tube (PMT). The entire assembly is, of course, shielded to eliminate outside light. The light guide is used when it is necessary to locate the tube in a more favorable area, perhaps away from a large magnetic field or excessive temperature.

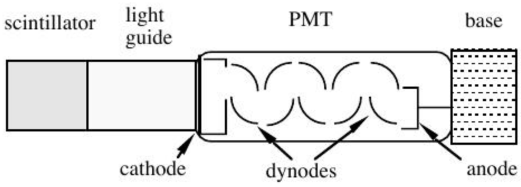

Figure 8: Schematic of a scintillator crystal coupled to a photomultiplier tube

Conversion of the weak burst of scintillation photons to a strong electrical signal is done by the PMT which is extremely sensitive detectors of light in the ultraviolet, visible, and near-infrared ranges of the electromagnetic spectrum. These detectors multiply the current produced by incident light by as much as 100 million times, in multiple dynode stages, enabling (for example) individual photons to be detected when the incident flux of light is very low.

This is done through the following procedure : an applied voltage of 1-2 kV is divided so that the potential between adjacent dynodes is 50-100 V, increasing from the negative cathode end to the anode output. Photons incident on the front plate cause emission of electrons (photoelectric effect) from the cathode. These electrons are accelerated to the closest dynode,where their impact releases additional electrons. This process is repeated at each of the 12-14 dynode stages in the tube, resulting in a pulse of $10^6 - 10^7$ *electrons per incident photon at the anode. Very little noise is added in the process, so it is often possible to detect individual photons incident at the photocathode surface.*

In Bevatron *the two counters were placed at 12 meters apart from each other.The time taken by mesons having momentum 1.19 Bev/c to cross this was 40 millimicroseconds(40 billionth of a sec) whereas for anti-proton of same momentum it was 51 millimicroseconds.*

5.3.2 Measuring Speed using Cherenkov Counter

There is a possibility that two mesons have exactly the right spacing between them to imitate an antiproton, i.e. just 11 millimicroseconds. Therefore, to avoid this there should be an another method to determine the speed. This is done with the help of Cerenkov counter.

A ***Cherenkov detector*** *is a more advanced particle detector which detects particle with the help of Cherenkov Radiation.*

Cherenkov Radiation (1958 Nobel) : *Cherenkov radiation is electromagnetic radiation emitted when a charged particle (such as an electron) passes through a dielectric medium at a speed greater than the* ***phase velocity*** *of light in that medium. The charged particles polarize the molecules of that medium,* *which creates **time-dependent dipole field** resulting electromagnetic dipole radiation.This occurs because immediately after leaving the external charge particle, they turn back rapidly to their ground state, emitting radiation in the process.*

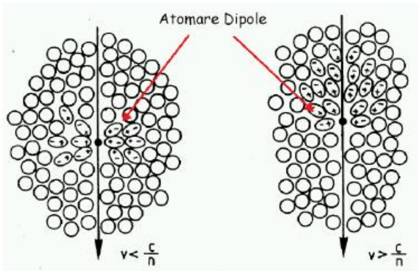

Figure 9: Polarization of material due to charge particle

For $v < \frac{c}{n} \Rightarrow$ *symmetric dipole distribution* $\Rightarrow$ *no net radiation*
but, $v > \frac{c}{n} \Rightarrow$ *asymmetric dipole distribution* $\Rightarrow$ *net radiation*

To understand it more clearly let's consider a static source which emits energy, in the form of waves, in all directions, then that source produces spherical wavefronts as shown in the diagram below :

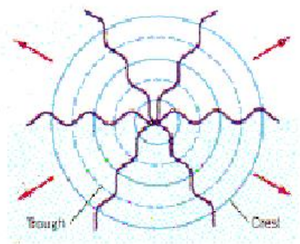

Figure 10: Symmetric distribution of radiation

The waves move at a characteristic speed which is set by their wavelength and the frequency. If the source itself is moving at a speed equal to the speed of the wave, then it nearly keeps pace with its own spherical wave fronts as shown in the figure 11. *But, if the source exceeds the speed of the radiation then the source moves ahead of the waveform(figure* 12*). For the second case: the surface of the cone makes angle* 2θ *and is tangent to all the wavefronts.*

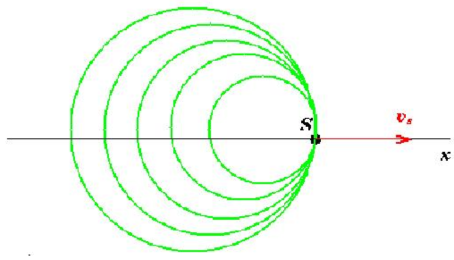

Figure 11: Waveform when source speed = speed of the radiation

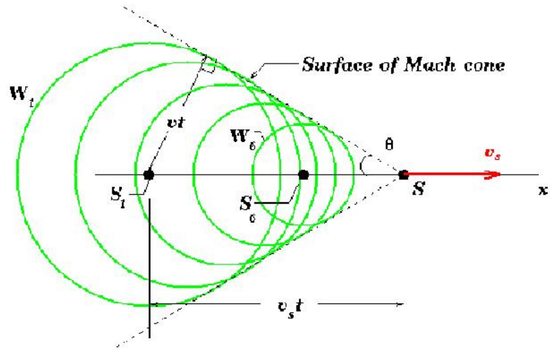

Figure 12: Waveform when source speed > speed of the radiation

Similar *effect takes place in case of Cherenkov Radiation. The excited atoms emit part of their light in the form of a coherent wavefront of radiation at fixed angle with respect to the trajectory of the charged particle :*

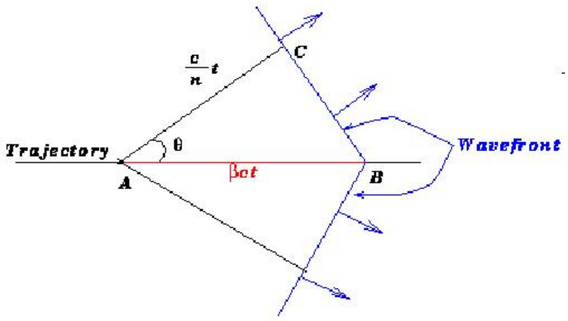

Figure 13: The geometry of the Cherenkov radiation

Defining the ratio of speed of particle and speed of light, $\beta \equiv \frac{v_p}{c}$*; the emitted light wave travel at a speed* $v_{em} = \frac{c}{n}$.
The left corner of the triangle A represents the particle at time $t = 0$*, and the right corner B represents the particle at some later time t. Within this time interval t, the particle travels* $x_p = v_p t = \beta ct$.
Whereas the emitted EM wave travels a distance $x_{em} = v_{em} t = \frac{c}{n} t$*. So,*

$cos\theta = \frac{1}{n\beta} = \frac{c}{nv_p}$.
So, by observing the angle of emission velocity can be determined. In experiment one calculates the angles theoretically at which light would be emitted and places the counters accordingly.

Now, let's return to our original experiment and see how we can use this detector to determine the velocity of the anti-proton.

In Bevatron two Cherenkov detectors were used. Each of which were further focussed to three photomultiplier(PMT) at three different angles(θ) corresponding to electrons, mesons and hoped-for anti-protons. One of them [velocity selector] had Cherenkov radiator **fused quartz(n = 1.46)** where light particles (mesons, electrons) as well as heavy one (hoped-for anti-protons) travel with a velocity greater than that of light in that medium. So, whenever a particle hits the Cherenkov counter, Cherenkov radiation is emitted and we get a signal from PMT. **This detector gives signal for both heavy as well as light particles**.

But, in the other Cherenkov detector[guard counter] it was used **fluorocarbon (n = 1.28)** as radiator. Due to less refractive index, the speed of light in this medium is greater compared to the previous one. **Consequently, particles moving as slowly as our hoped-for antiprotons would produce no Cherenkov light in this medium whereas the lighter ones do**.

As the whole system was assembled in Bevatron, the plan of the apparatus looked as follows(figure 14):

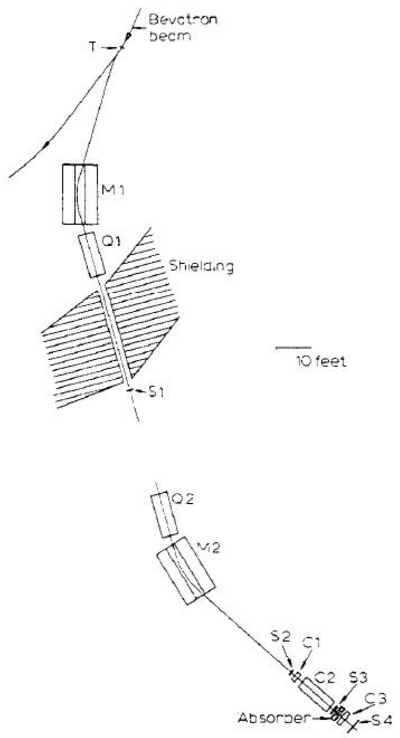

Figure 14: Plan view of the apparatus used in the discovery of antiprotons. T indicates the target, Q1 and Q2 are magnetic lenses, C1 is the guard counter, and C2 is the velocity-selecting Cherenkov counter. The last two counters (C3 and S4) may be ignored.

So, whenever the two scintillation counters S1 & S2 gives signal with a relative time delay of 51 ± 5 millimicroseconds, and also the first Cherenkov counter(guard counter) gives a signal but no signal comes out from the 2nd Cherenkov counter(velocity selector), it means a potential antiproton has been seen in the apparatus.

Since the momentum is accurately known from the magnet arrangement, and since each time measurement determines the velocity of the particle in question, we have in effect a rough mass measurement of each antiproton.

5.4 Some Confirmation Experiments

5.4.1 Confirming Mass

Some confirmation experiments were carried out in Bevatron to determine whether the antiprotons seen were completely real, or whether they might be due to some failure of the apparatus. One effective test involved intentionally tuning system to a somewhat different mass than the proton mass to test whether it was true that the antiprotons were no longer registered. This indicated the apparatus really was sensitive to the mass of the particle.

Another important test entailed lowering the energy of the beam of the Bevatron, to make sure that the supposed antiprotons disappeared when the beam energy was sufficiently low.

5.4.2 Confirming Charge

To get confirmation about their charge, another experiment was carried out in Bevatron using photographic emulsions. Here, indeed annihilation phenomena were seen, in which the antiproton and a proton die simultaneously. In their place about 5 π mesons were emerged.

Hence, from the Bevatron experiment. the existence of antiproton had been established beyond doubt; its charge is of course negative equal to the electrons charge; its mass was established as equal to the mass of the proton, to within 3 percent. Furthermore, the data on production in the Bevatron at different proton beam energies clearly indicated that antiprotons are born in pairs with ordinary protons or neutrons. The emulsion showed that they die in the annihilation process.

6 Preserving Antiproton

6.1 Most Challenging Issue

Now, we will address the most challenging question: how can we produce a stable beam of antiprotons? Since, immediately after getting into contact with matter it gets annihilated, it's a highly non-trivial issue to produce a stable beam.

This question first came in 1966 from a group of scientists at CERN, while they were launching a 25 GeV $p - \bar{p}$ storage ring and some scheme to produce intense $\bar{p}$ was needed.

6.2 AIM

Our central is to **produce an intense $\bar{p}$ beam by compressing same number of particles into a beam of smaller size**.

6.3 Underlying Principle ⇒ Stochastic Cooling

The **principle** consists of using the fact that the particles are points in the phase space with empty space in between. We may push each particle towards the centre of the distribution, squeezing the empty space outwards. The small scale density is strictly conserved (Liouville's theorem), but in a macroscopic scale the particle density increases. This process is called **'cooling' because it reduces the movements of the particles with respect to each other. Kinetic theory of gas** tells us that rms velocity of particles is proportional to the temperature of the particles, so the decreasing the movement of the particles means decreasing the temperature.

One of the most sophisticated methods of cooling is **Stochastic Cooling**, invented by Simon van der Meer, an engineer from Netherlands. He was awarded the Nobel Prize in Physics in 1984. He actually shared his Nobel Prize with Italian Physicist Carlo Rubbia.

6.4 Qualitative description of stochastic cooling

6.4.1 Betatron Oscillation

Under the influence of the focusing fields and the random initial phase, the particles execute betatron oscillations around its central orbit.

6.4.2 Goal

Cooling goal : reduce betatron oscillations.

6.4.3 Pick-up and kicker

Of course, we can only do this if we have the information about the position of the individual particle in phase space and if we can direct the pushing action against the individual particles. Without these two prerequisites, there would be no reason why particles rather than empty space would be pushed. A stochastic cooling system therefore consists of a sensor(pick-up) that acquires electrical signals from the particles, and another sensor(kicker) that pushes the particles. The kicker is excited by the amplified pick-up signals.

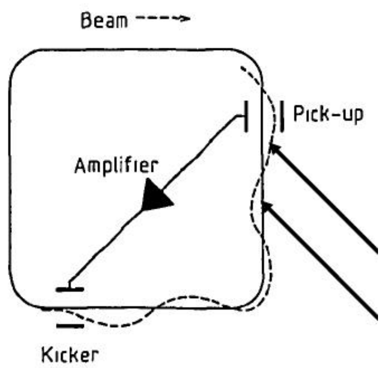

Figure 15: Single particle model for stochastic cooling system

6.4.4 Functioning

At each passage of the particle pick-up provides a short pulse signal that is proportional to the deviation of the particle from the central orbit. This is amplified and applied to the kicker, which will deflect the particle by an angle proportional to its error.

Specifically, consider a horizontal beam pick-up that consists of two plates (usually parallel) and is sensitive to either horizontal motion or equivalently a dipole oscillation. The pick-up is centered on the middle of the beam pipe, with one plate to the left of center and the other to the right. If the particle passes through the pick-up off-center, the plate which the particle passes closest to will have a greater current induced on it. If the signals are combined by measuring the difference between them in a so-called 'delta' or Δ mode, the output will be a measure of the relative particle position with respect to the center of the beam pipe. This signal is then amplified and applied with the most optimal averaged phase (timing) to the kickers. The kicker, like the pick-up, is an arrangement of plates on which a transverse electromagnetic field is created which can deflect the particle.

6.4.5 Distance between PU & Kicker

Since the pick-up detects a position error and the kicker provides a corrective angular kick, their distance apart is chosen to correspond to a quarter of a betatron oscillation (plus a multiple of π wavelengths if more distance is necessary). As shown in figure below, a particle passing the pick-up at the crest of its oscillation will then cross the kicker with zero position error but with an angular deviation that is proportional to the displacement at the pick-up.

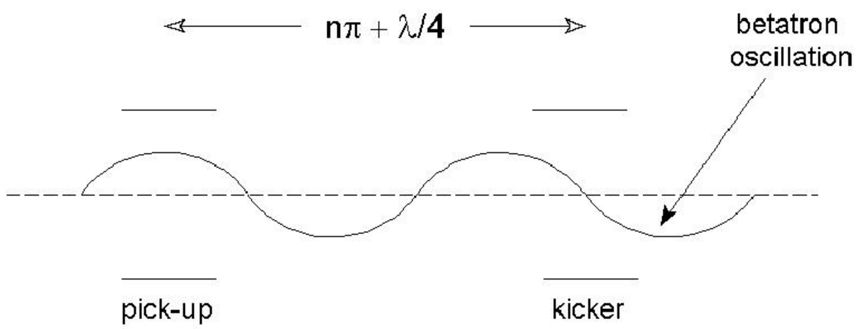

Figure 16: Optimum spacing between pick-up and kicker

Given a perfect kicker response and perfect betatron phasing, the trajectory of the particle would be corrected to that of the central orbit. A particle not crossing the pick-up at the crest of its oscillation would receive only a partial correction and require additional passages to eliminate the oscillation.

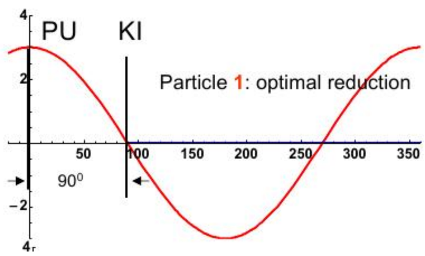

Figure 17: Optimal Reduction

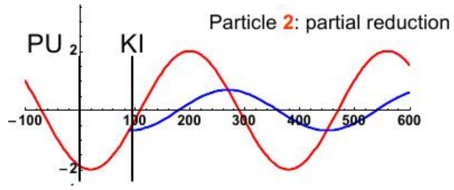

Figure 18: Partial Reduction

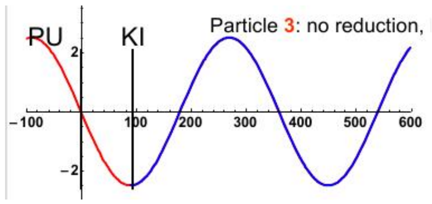

Figure 19: No reduction

6.4.6 Timing of the signal → shortcut path

There is another important aspect of stochastic cooling that this model can illustrate: the correction signal has to arrive at the kicker at the same time as the particle for optimum cooling. Since the signal is delayed in the cables and the amplifier, whereas the particle is moving at close to the speed of light, the path of the correction signal has to take a shortcut across the ring to reach the kicker at the correct time. F

So, the **two conditions for betatron cooling**:

1. Optimal phase advance PU to KI: 90 degrees plus multiples of 180 degrees
2. Correction signal must arrive when the particle passes the KI: Usually, a short cut of the signal path is necessary

6.4.7 Cooling of a beam rather than a single particle

Particle beams, of course, are not composed of just a single particle. Rather, a beam is a distribution of particles around the circumference of the storage ring. Each particle oscillates with a unique amplitude and random initial phase and in this model the cooling system acts on a sample of particles within the beam rather than on a single particle. Each sample receives the same correcting kick during a passage through the system.

6.4.8 Cooling Equation

Due to finite bandwidth W of the cooling system, a pulse produced by a single particle at the pick-up broadened to T_s. So, the sample time

$$T_s = \frac{1}{2W} \tag{16}$$

A particle at time t_0 at the pickup is not only kicked due to its own error signal but also due to other particles(belonging to the sample of the test particle) in the time interval $t_0 – T_s/2 \leq t \leq t_0 + T_s/2$ which causes heating.

In a uniform beam (DC beam) of length T (revolution period) and particle number N, the number of equally spaced samples

$$l_s = \frac{T}{T_s} = 2WT \tag{17}$$

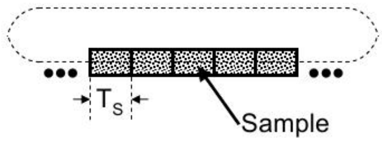

Figure 20: Samples

Number of particles per sample

$$N_s = \frac{N}{l_s} = \frac{N}{2WT} \tag{18}$$

For example: if $N = 10^{10}$, $T = 1\mu s$ & $W = 2\text{GHz}$, then $T_s = 250\text{ps}$, $l_s = 4000$ and $N_s = 2.5 \times 10^6$.

Now, the test particle receives a correction proportional to its error x

$$\Delta x = \lambda x \tag{19}$$

The error x will change x to x_c

$$x_c = x - \Delta x = x - x_c \tag{20}$$

Now, finite bandwidth ⇔ finite sample size

$$x_c = x - \lambda x - \lambda \sum_{others} x_i \tag{21}$$

$$= x - \lambda \sum_{sample} x_i \tag{22}$$

$$= x - (\lambda N_s) \frac{1}{N_s} \sum_{sample} x_i \tag{23}$$

$$\Rightarrow x_c = x - g\langle x \rangle_s \tag{24}$$

So, the test particle receives a correction proportional to the sample average.

After a bit calculation the cooling rate equation comes out to be

$$\boxed{\frac{1}{\tau} = -\frac{2W}{N}(2g - g^2)} \tag{25}$$

6.4.9 Optimal Cooling

Clearly, the cooling process can be looked at as competition between two terms: (a)the coherent term, which is generated by the single particle, and, (b) the incoherent term, which results from disturbances to the single particle from its fellow sample members through the feedback loop. The coherent signal's contribution to the cooling process is linearly proportional to the system gain, while the incoherent heating term is proportional to the square of the system gain. If one plots these two terms as in figure below, it is clear that there is some point at which cooling term is maximized over the heating term. This is known as the optimum gain of the system.

And by measuring and reducing the average sample error, the error of each individual particle on the average slowly decreases.

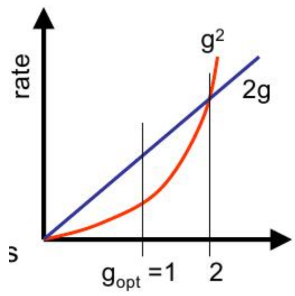

Figure 21: Cooling and heating

So, to get the optimal cooling rate :

(a) need to use large bandwidth

(b) choose optimal gain $g = 1$

Also, it's clear more particle number ⇒ cooling rate decreases.
This is how one accumulates a large number of anti-protons in a beam.

6.4.10 Stability of the beam

I think, this stochastic cooling also does lead to a stable beam of particles. This is because a charged particle executing betatron oscillation radiates synchrotron radiation. Therefore, preventing the charged particles from executing betatron oscillation leads reduction of synchrotron radiation. This eventually results to stable the beam.

7 Bibliography

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