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Aufgabe 15: The extraction of α_S

One way to extract α_S is from the decay of charmonium ($c\bar{c}$ bound states) and bottomonium ($b\bar{b}$ bound states) decays. In first approximation, the decay rate for a $q\bar{q}$ bound state in a ${}^{2S+1}S_J$-state (J is the total spin) is obtained using Pirenne-Wheeler formula (found in 1946):

$$\Gamma\left({}^{2S+1}S_J\left(q\bar{q}\right)\to X\right)=\frac{1}{2J+1}\left|\phi_0\right|^2\left(4v_{rel}\sigma\left(q\bar{q}\to X\right)\right)_{v_{rel}\to 0}\ , \tag{1}$$

with v_{rel} the $q\bar{q}$ relative velocity in their center-of-mass frame, and ϕ_0 the Schrödinger wavefunction in configuration space at zero separation. One can understand this model as a replacement of the scattering cross section initial flux factor by the probability of contact in the bound state, i.e. the wavefunction at zero. Using this formula, the decay rates are, neglecting the masses of the decay products:

η_c, η_b *decays* :

$$\Gamma({}^1S_0\to\gamma\gamma)=48\pi\alpha^2e_q^4\frac{\left|\phi_0\right|^2}{M^2}\ ,$$
$$\Gamma({}^1S_0\to gg)=\frac{32}{3}\pi\alpha_S^2\frac{\left|\phi_0\right|^2}{M^2}\ ,$$

$J/\psi, \Upsilon$ *decays* :

$$\Gamma({}^3S_1\to\gamma^*\to f\bar{f})=16\pi\left(e_q^2e_f^2\alpha^2\right)\frac{\left|\phi_0\right|^2}{M^2}\ ,$$
$$\Gamma({}^3S_1\to\gamma\gamma\gamma)=\frac{64\left(\pi^2-9\right)}{3}\left(e_q^6\alpha^3\right)\frac{\left|\phi_0\right|^2}{M^2}\ ,$$
$$\Gamma({}^3S_1\to ggg)=\frac{160\left(\pi^2-9\right)}{81}\alpha_S^3\frac{\left|\phi_0\right|^2}{M^2}\ ,$$
$$\Gamma({}^3S_1\to\gamma gg)=\frac{128\left(\pi^2-9\right)}{9}e_q^2\alpha\alpha_S^2\frac{\left|\phi_0\right|^2}{M^2}\ , \tag{2}$$

where e_q is the electric charge of the heavy quark ($e_c = 2/3$ and $e_b = -1/3$), e_f is that of the final state fermion, and M the mass of the bound state. In this exercise, neglect the scale dependence of α (i.e., use $\alpha = 1/137$).

a) Knowing that the parity and charge conjugation of a $f\bar{f}$ bound state are given as $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$ with L and S the orbital angular momentum and total spin, can you understand why the $\eta_{c,b}$ are pseudoscalar states and decay into an even number of photons, while the J/ψ and Υ are vectors and decay into an odd number of photons? Generalize to the gluonic decays, and explain why $J/\psi \to g^* \to$ *hadrons* is forbidden but $J/\psi \to \gamma^* \to$ *hadrons* is allowed. (Hint: remember that photons and gluons are vector particles, with $J^{PC} = 1^{--}$).

b) Extract $\alpha_S(m_c)$ by comparing $\eta_c \to \gamma\gamma$ and $\eta_c \to hadrons$ (i.e., $\eta_c \to gg$).

c) Extract $\alpha_S(m_b)$ by comparing $\Upsilon(1S) \to hadrons$, $\Upsilon(1S) \to \gamma + hadrons$, and $\Upsilon(1S) \to \gamma^* \to f\bar{f}$ (Hint: write the total $\Upsilon(1S)$ width as the sum over these decay channels, with $f = e, \mu, \tau, u, d, s, c$). The Zweig rule states that meson decays are suppressed if they proceed through the annihilation of the constituent quarks. Compare the width of $\Upsilon(1S)$ and $\Upsilon(4S)$, which can decay into $B\bar{B}$. Using the results for $\alpha_S(m_b)$ and $\Gamma(\Upsilon \to ggg)$, estimate what would have been the $\Upsilon(1S)$ width if the mode $\Upsilon(1S) \to g^* \to f\bar{f}$ was allowed, and compare again with the $\Upsilon(4S)$ width.

d) Using

$$\Lambda = \mu e^{-2\pi/\beta_0\alpha(\mu)} \left(\frac{4\pi + 2\beta_1\alpha(\mu)/\beta_0}{\beta_0\alpha(\mu)} \right)^{\beta_1/\beta_0^2} , \tag{3}$$

with

$$\beta_0 = 11 - \frac{2}{3} n_f , \tag{4}$$

$$\beta_1 = 51 - \frac{19}{3} n_f , \tag{5}$$

get $\Lambda^{(3)}$ for $n_f = 3$ from $\alpha_S(m_c)$, $\Lambda^{(4)}$ for $n_f = 4$ from $\alpha_S(m_b)$, and $\Lambda^{(5)}$ for $n_f = 5$ from the world-average value

$$\alpha_S(M_Z) = 0.1176 \pm 0.0020. \tag{6}$$

How do you interpret these values? What do they tell you about the perturbativity of QCD?

e) Compare $\alpha_S(m_c)$ and $\alpha_S(m_b)$ by running them up to $\alpha_S(M_Z)$, and compare them with the world-average (6). Be careful crossing thresholds: use $n_f = 4$ between m_c and m_b since only the u, d, s, c are dynamical in that energy range, and $n_f = 5$ between m_b and M_Z since all flavors but the top are dynamical there.

f) Compute the decay rate for $\eta_c \to \gamma\gamma$ using the formula (1). First compute the cross-section $\sigma(q\bar{q} \to \gamma\gamma)$ as a function of the center-of-mass energy E, and then take the limit $v_{rel} \to 0$ (with $v_{rel} = 2\beta\sqrt{\beta^2 - 1}$, $\beta = E/m_q$).