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Übungsblatt 3
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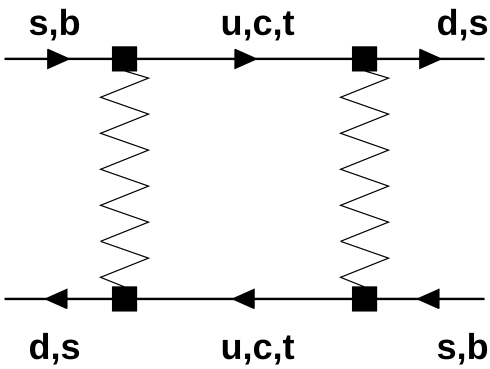
Problem 1: Meson-antimeson mixing

The M^0-$\bar{M}^0$ mixing amplitude M_{12}, where $M = K, B_d$ or B_s, can be written as

$$M_{12} = A_M \sum_{i,j=u,c,t} \lambda_i^M \lambda_j^M \tilde{S}(x_i, x_j) \tag{1}$$

if one neglects QCD corrections. Here $x_i = m_i^2/M_W^2$ and $\tilde{S}(x_i, x_j) = \tilde{S}(x_j, x_i)$ is obtained from the box diagram:



For $M = K, B_d, B_s$ one has:

	$M = K$	$M = B_d$	$M = B_s$
A_M [GeV]	$(6.2 \pm 0.8) \cdot 10^{-11}$	$(1.3 \pm 0.4) \cdot 10^{-9}$	$(2.0 \pm 0.6) \cdot 10^{-9}$
λ_i^M	$V_{is} V_{id}^*$	$V_{ib} V_{id}^*$	$V_{ib} V_{is}^*$

The uncertainties stem from poorly known hadronic quantities.

a) Use CKM unitarity to eliminate λ_u^M in favour of λ_c^M and λ_t^M to write

$$M_{12} = A_M [(\lambda_t^M)^2 S(x_t) + 2\lambda_c^M \lambda_t^M S(x_c, x_t) + (\lambda_c^M)^2 S(x_c)] \tag{2}$$

and find the relation between S in (2) and $\tilde{S}$ in (1), setting $x_u = 0$. Verify the GIM mechanism: Which function vanishes for (i) $x_c = x_t$ and (ii) $x_c = 0$?

b) With $S(x_c) = 2.6 \cdot 10^{-4}$, $S(x_c, x_t) = 2.3 \cdot 10^{-3}$ and $S(x_t) = 2.3$ compute the three terms in (2) and assess the relevance of the second and third term for the three meson systems. Mixing-induced CP asymmetries determine $\sin(2 \arg M_{12})$ (assuming the standard CKM phase convention). How big is the relative deviation of $\arg M_{12}(B_d)$ from $2\beta \simeq 46^\circ$ and of $\arg M_{12}(B_s)$ from $-2\beta_s \simeq -2.2^\circ$?

c) The mass difference between the mass eigenstates is given by $\Delta m = 2|M_{12}|$. Compute Δm from the formula above and assess the relevance of neglected QCD effects by comparing your results with the experimental numbers

$$\begin{aligned} \Delta m_K &= (3.483 \pm 0.006) \cdot 10^{-15}\,\text{GeV} \quad , \\ \Delta m_{B_d} &= (3.34 \pm 0.03) \cdot 10^{-13}\,\text{GeV} \quad , \\ \Delta m_{B_s} &= (1.170 \pm 0.008) \cdot 10^{-11}\,\text{GeV} \quad . \end{aligned}$$

d) The CP-violating quantity $|\varepsilon_K|$ in K^0-$\bar{K}^0$ mixing,

$$|\varepsilon_K| \approx \frac{1}{2\sqrt{2}} \arg M_{12},$$

has been measured as $|\varepsilon_K| = (2.28 \pm 0.02) \cdot 10^{-3}$. Express the λ_c^K and λ_t^K in terms of Wolfenstein parameters (to leading non-vanishing order in λ) and determine the constraint on $(\bar{\rho}, \bar{\eta})$ found from $|\varepsilon_K|$.