Precision calculations for the LHC physics

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Outline

1) Basic framework
2) NLO QCD calculations
3) The NNLO QCD frontier: technology
4) The NNLO QCD frontier: examples
Basic framework

QCD -- the theory of strong interactions -- is non-perturbative. Protons -- the primary players at the LHC -- are bound states that are kept together by the strong force whose understanding is, at best, poor.

Yet, in the context of the LHC physics we often talk about precision theoretical predictions and the possibility to use perturbation theory of QCD to achieve this goal. We can get away with that since we are interested in a tiny fraction of all events at the LHC, namely those where large amount of energy is packed into a tiny volume. When this happens, the energy density is so large that heavy (new?) particles can be produced and physics beyond the Standard Model discovered. Such events necessarily require large momentum transfer from one proton to the other. In fact, the momentum transfer must be so large that both protons disintegrate and we get ourselves into a situation of ``double'' deep inelastic scattering, the realm of pQCD and factorization theorems.

\[
d\sigma = \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{\text{part}}(x_1 x_2 s_{\text{hadr}})
\]
The toolkit of perturbative QCD

This approach leads to a systematically improvable, highly predictive framework -- perturbative QCD for collider physics -- that requires four basic ingredients:

1) parton distributions;

2) matrix elements calculated to a required order in perturbation theory (note that calculations in certain kinematic limits require all-order treatment);

3) understanding of how virtual and real emission processes should be combined and the development of parton level event generators capable of describing kinematics of parton scattering processes;

4) parton shower event generators, to describe multiple emissions and detector responses.

\[
d\sigma = \int dx_1 dx_2 f_i(x_1)f_j(x_2)d\sigma_{\text{part}}(x_1x_2s_{\text{hadr}})
\]
Current developments

Thanks to results of HERA, Tevatron, early LHC and countless theoretical work, we have good understanding of all the ingredients required. Nevertheless, the LHC program provides serious motivation for further developments. **The current focus is on**

1) realistic estimates of PDF uncertainties, including theoretical; consistent inclusion of various data into global PDF fits;

2) **calculation of matrix elements to a required order in perturbation theory**: automation of one-loop computations, techniques for two-loop computations for generic multi-particle final states; three-loop computations for low-multiplicity final states;

3) understanding how to combine virtual and real emission processes in a process-independent fashion at **NNLO QCD**; development of NNLO parton level event generators;

4) resummations for complex final states;

5) development of more robust parton shower event generators that include higher order perturbative corrections; merging parton showers and matrix elements.

In what follows I will mostly talk about one- and two-loop computations for the LHC and will leave parton distribution functions, resummations and parton showers out of this discussion.
The NLO story
The reason NLO QCD computations for the LHC attracted so much attention is that this is the first order in pQCD where normalization of production cross-sections can be predicted with reasonable confidence. Complicated final states with large number of leptons, missing energies and jets, that are traditionally employed to search for physics beyond the Standard Model, made it important to compute SM contributions to such processes (backgrounds) through NLO accuracy. The story of one-loop (NLO) calculations in QCD is a success story of community involved with LHC precision physics.

Automation of generic one-loop computations (aMC@NLO, GoSam, OpenLoops, Sherpa, MadGraph, HelacNLO) significantly reduced "labor costs" associated with them and lead to new theoretical ideas about how to use these results for physics.

I will describe a few physics cases where our ability to perform NLO computations for complicated processes is important.
One-loop calculations in QCD

A significant number of impressive NLO computations performed in recent years:

1) jet production (up to five); Blackhat + Sherpa; Njet

2) gauge boson and up to five jets; Blackhat + Sherpa;

3) two gauge bosons and up to two jets; Melia, Rontsch et al., VBFNLO, GoSam + Madevent

4) Top quarks with up to two jets; Denner, Dittmaier, Kallweit, Pozzorini; HelacNLO

5) Top quarks with a gauge boson Schulze, Scharf, Rontsch, HelacNLO, Kardos, Trocsanyi, MCFM

6) Higgs with up to three jets GoSam+ Sherpa + Madevent

Emergent trends in NLO results:

1) strongly reduced scale dependence;
2) usefulness of dynamical kinematic-dependent scales at LO and NLO;
3) complex multi-particle states with spin correlations, corrections to decays etc;
One-loop calculations and unitarity

While it was understood conceptually how one-loop calculations can be performed since at least 1960s, one-loop computations for high-multiplicity processes proved to be prohibitively difficult. Overcoming these technical difficulties lead to a new field-theoretic insights into the technology for one-loop computations whose novelty was driving progress in the field of precision calculations for a significant period of time.

A new conceptual element that appeared in the one-loop technology in recent years is the understanding that unitarity is useful for computing one-loop scattering amplitudes from tree scattering amplitudes; when properly used, it allows us to forgo the use of Feynman diagrams.

\[ T_{if} - T_{if}^+ = \sum T_{in}T_{nf}^{\ast} \]
\[ \mathcal{A} = \sum c_i I_i \]

\[ \text{Im}\mathcal{A} = \sum |A_{\text{tree}}|^2 = \sum c_i \text{Im}I_i \]

In non-trivial applications, a unitarity-based reconstruction of one-loop amplitudes requires that tree-level S-matrix is known in integer-dimensional spaces other than four and for complex external on-shell momenta.
**W+jets production**

Production of $W +$ jets at NLO was thoroughly studied by the Blackhat + Sherpa collaboration using the unitarity methods. Note impressive improvements in stability with respect to changes in renormalization and factorization scales. Note also a choice of a dynamical renormalization scale in the middle plot.

![Diagram of W+jets production](image)

With such a large ``theoretical data set” for $W+n$ jets, one can explore the possibility that high-quality predictions for $W+(n+1)$ jet cross-sections can be obtained by extrapolating cross-sections for lower multiplicities.
Top quark spin correlations and limits on stealthy stops

Recent result by ATLAS excludes stop squarks with the masses 191-200 GeV (stealthy region, stops decaying to tops and missing energy) using spin correlations in top decays. The technique relies on NLO computations for top quark pair production and decay, with all the spin correlations, and on the extension of the likelihood methods to NLO.

An observable discussed originally in the context of discovering the existence of top quark spin correlations at the Tevatron.

Melnikov, Schulze

$R = \frac{|M_{\text{coll}}|^2}{|M_{\text{coll}}|^2 + |M_{\text{uncoll}}|^2}$
On- and off-shell production of top quarks at the LHC

We are used to thinking about top-related processes as a combination of top pair, single top and non-resonance WbWb production. This is motivated by physics and is usually thought to be of importance for computational feasibility. However, automated NLO programs are so powerful, that they can easily treat the production of WbWb final state at NLO independent of its ``resonant'' origin. Apart from admiring exceptional prowess of these programs, we can also use these results as an opportunity to understand the validity of the narrow resonance approximation in top quark physics at the LHC.

In general, the narrow-width approximation is perfectly valid except close to edges of kinematic distributions; such edges might be important in some physics case (the top quark mass).
Summary of NLO computations

Recent developments in the field of one-loop computations for multi-particle processes at the LHC (the "NLO revolution") provided us with powerful tools for phenomenological studies.

Applications of such computations are widely-spread and include

1) studies of specific (complex) observables for multi-particle final states through NLO QCD;

2) testing the consistency of some (simplifying) physics assumptions (e.g. narrow width approximation in top quark pair production, dynamical renormalization scales);

3) providing "computational engines" for upgrading parton shower programs to NLO and merging event samples with different multiplicities;

4) extending advanced theoretical methods (matrix element, likelihood functions) to higher order in perturbation theory;

5) providing essential input for NNLO computations.
NNLO calculations for the LHC

Since application of NLO QCD calculations to the LHC physics is very successful, it is natural to extend the description of hard scattering processes to NNLO.

In many ways, NNLO is replacing NLO as a theoretical frontier for applying perturbative QFT to hadron collider phenomenology. Indeed, there is a NNLO wish-list created as part of the Snowmass community planning exercise that happened in UA last year (2013). Below is a summary of the wish-list and some comments of why certain processes appear there and the technical advances required to reach the NNLO accuracy for some of them:

1) $H + j$  
   Higgs transverse momentum distribution; Higgs decays to observable final states; and loops with massive particles;

2) $H + V$  
   Couplings; $H \rightarrow bb$ @NNLO;

3) $HH$  
   Higgs self-coupling; NLO with exact top mass dependence, virtual corrections;

4) $tt+(jets)$  
   top pair production, including decays; jet bins for forward-backward top asymmetry;

5) single top  
   $tWb$ couplings; top decay through NNO;

6) di-jets  
   PDF fits, contact four-quark operators; quark contributions;

7) tri-jets  
   strong coupling constant; reductions, master integrals;

8) $V + j$  
   PDFs (gluon), backgrounds;

9) $V1V2$  
   anomalous couplings, backgrounds (Higgs); fiducial volume cross-sections;

10) $gg \rightarrow V1 V2$  
   background to Higgs, signal-background interference; loops with massive particles;

11) jets in DIS  
   strong coupling; PDFs etc.

Massive particles in loops; production and decays; 2 -> 3 processes; interface with parton showers.

Many interesting phenomenological applications should be expected in the coming years.
Two-loop calculations in QCD

Calculation of two-loop virtual corrections is an integral part of any NNLO QCD computation for the LHC. Technology for such computations is being continuously developed since early 2000’s. It can be summarized as a sequence of three steps.

1) Parametrization of scattering amplitudes in terms of Lorentz-invariant form-factors; determination of operators that project the amplitude on the individual form-factors;

2) Integration-by-parts identities and reduction of scalar Feynman integrals to master integrals;

3) Calculation of master integrals;

Each of these steps is well-established, but for each of them there are complications;

1) parametrization of scattering amplitudes becomes complicated in cases when we have to deal with the Dirac matrix $\gamma_5$ in closed fermion loops;

2) integration-by-parts can be dealt with using existing programs (AIR, FIRE, REDUZE) but it becomes very difficult for processes with large number of kinematic invariants;

3) calculation of master integrals was always an “art” rather than “science” and it continues to remain this way.
Two-loop calculations in QCD

Calculation of two-loop integrals relies on a large number of various methods (direct integration, Mellin-Barns, differential equations). The method of differential equations has been used to find master integrals for a long time, starting from papers by Kotikov and Remiddi in the early 1990s, however it was never considered to be ``the” method.

An interesting recent development in this field is the suggestion by J. Henn to streamline the application of differential equations in external kinematic variables to compute master integrals

\[ \partial_x \tilde{f} = \epsilon \hat{A}_x (x, y, z, \ldots) \tilde{f} \quad \tilde{f} = \sum_{n=0}^{\infty} \epsilon^n \tilde{f}^{(n)} \]

The important point is that on the right-hand side, the dimensional regularization parameter appears explicitly, and only as a multiplicative pre-factor. It is then possible to solve these equations iteratively order-by-order in \((d-4)\) since in each order of this expansion the above equation contains no homogeneous terms (so that in each order in epsilon, the right-hand side is the source for the left-hand side).

The idea by Henn streamlines and simplifies such computations significantly. This already lead to very impressive advances (e.g. master integrals for Bhabha, V1 V2 production) that will have interesting consequences for phenomenology.
Two-loop virtual corrections: qq->V₁V₂

For the case of double vector boson production, we can identify six different two-loop topologies; the differential equations can be "rationalized" with the following (typical) change of variables

\[
\frac{s}{m_3^2} = (1 + x)(1 + xy),
\]
\[
\frac{t}{m_3^2} = -xz, \quad \frac{m_4^2}{m_3^2} = x^2y
\]
\[
\sqrt{(s - m_3^2 - m_4^2)^2 - 4m_3^2m_4^2} = m_3x(1 - y)
\]

\[
G(a_n, a_{n-1}, \ldots, a_1, t) = \int_0^t \frac{dt_n}{t_n - a_n} G(a_{n-1}, \ldots a_1, t_n)
\]

Important issues: finding a suitable basis; choice of "rational variables"; boundary conditions for solutions of differential equations, analytic continuation. Numerical evaluation of Goncharov’s polylogarithms (GINAC) and their mapping on conventional polylogarithms.

\[
d\vec{f} = \epsilon(dA) \times f, \quad A = \sum A_i \log \alpha_i
\]
\[\alpha = \{x, y, z, 1 + x, 1 - y, 1 + xy, z - y, 1 + y(1 + x) - z, xy + z, 1 + x(1 + y - z), 1 + xz, 1 + y - z, z + x(z - y) + xyz, z - y + yz + xyz\}\]

Caola, Henn, Melnikov, Smirnov, Smirnov
Two-loop virtual corrections: the recap

1) Analytic calculations of two-loop four-point functions are gaining momentum; working with four kinematic invariants appears to be feasible.

2) Even larger number of kinematic invariants (multi-leg, masses etc.) makes such computations increasingly complicated. At the moment, we do not know if two-loop computations for 2->2 amplitudes with larger number of kinematic invariants or 2->3 processes are feasible within this framework;

3) There are interesting attempts to understand if two-loop computations can be done using unitarity techniques, that turned out to be so powerful at one-loop. While there was an impressive progress in this field related to classification of integrand residuals based on techniques from algebraic geometry, there are still many outstanding issues. Currently, the main problem there seems to be the lack of understanding of how to avoid the use of integration-by-parts.

4) Prospects for numerical methods for loop computations are not clear to me. Some impressive work on this is being done (including here in Mainz), but how this will translate into actual phenomenology, is hard to say at the moment.
NNLO calculations: putting everything together

An important achievement of the past few years was the development of technology that allows NNLO QCD computations to be performed for hadron collider processes of a sufficiently general nature (e.g. jets in hadron collisions).

Consider NNLO QCD corrections to a tree process $pp \to X$. There are three sources of infra-red divergencies that must be considered:

1) two-loop virtual corrections to $pp \to X$, where all (d-4)-poles are explicit;

2) one-loop virtual corrections to $pp \to X+g$, where some (d-4)-poles are explicit and some appear only after the integration of the final state gluon;

3) process $pp \to X+g+g$ where all (d-4)-poles appear only after integration over final state gluons is carried out.

The key problem here is that we would like to achieve the cancellation of infra-red singularities at second order in perturbation theory without integrating over kinematic variables of those final state particles that are accessible in experiment, which seems to be impossible given that in real emission processes poles are produced only after phase-space integration...
NNLO calculations: methods

Several theoretical methods were developed to address this issue:

1) antenna subtraction (Gehrmann-de Ridder, Gehrmann, Glover et al.);

2) qt-subtraction (Catani, Grazzini) (so far restricted to color-less final states);

3) sector-improved subtractions (Czakon; Bougezhal, Petriello, K.M.).

All these methods have their merits and all of them have been tested in a variety of realistic applications.

In what follows, I will describe ``sector-improved” subtraction algorithm that is is based on the application of sector decomposition and phase-space partitioning; interestingly, it can be thought of as a direct continuation of the FKS procedure known from NLO.
Phase-space partitioning and sector decomposition at NNLO

The algorithm is easy to explain:

1) start with the phase-space partitioning such that at every sector potential infra-red and collinear divergencies are clearly exposed: in each sector, IR and collinear singularities should be produced when two well-defined final state particles become soft or collinear to a well-defined direction.

2) in each sector, choose parametrization of four-momenta for relevant final state particles in such a way that singularities of Feynman propagators factorize. This step is universal; corresponding changes of variables have been written down explicitly.

3) Perform expansion in plus-distributions to extract coefficients of 1/(d-4) poles.

\[
\int \frac{dx_1}{x_1^{1+\epsilon}} \frac{dx_2}{x_2^{1+a_2\epsilon}} \frac{dx_3}{x_3^{1+a_3\epsilon}} \cdots F(x_1, x_2, x_3, \ldots), \quad F(x_1, x_2, x_3, \ldots) = x_1^{b_1} x_2^{b_2} \cdots |M|^2.
\]

\[
\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon} \delta(x) + \left[ \frac{1}{x} \right]_+ + \ldots
\]

\[
\frac{F(x_1, x_2, x_3, \ldots) - F(0, x_2, x_3, \ldots)}{x_1}
\]

Application of this procedure automatically generates subtraction terms that are required to make phase-space integrals integrable. These subtraction terms correspond to well-defined approximation to scattering amplitudes in soft and collinear regimes. Differential cross-sections are written as the Laurent expansion in dimensional regularization parameter; the coefficients of the Laurent expansion are computed numerically.
Universality of subtraction terms at NNLO

It is possible to show that all the required subtraction terms \( F(x_1=0,x_2=0,...) \) etc. can be obtained without a reference to the underlying process (of course, up to LO and NLO amplitudes) thanks to the universality of soft and collinear limits.

Indeed, vanishing of any the variables that parametrize singular limits (or their combinations), defines kinematic configurations where two unresolved partons are either soft or collinear or both. All relevant singular limits are known since circa 2000 and can be directly borrowed from the relevant papers. Although we only discussed the tree-level case, the same applies to one-loop real-virtual NNLO contributions.

\[
|M(\{n\} + i + j)|^2 = g_s^2 F_{\text{sing}}(i; j; \{n\}) \otimes M(\{n\})M^*(\{n\})
\]

Related work on singular limits by Campbell, Glover, Berends, Giele, Bern, Del Duca, Kilgore, Schmidt
NNLO QCD for t-channel single-top production at the LHC

To show you a concrete example, I will consider the t-channel contribution to single-top production at NNLO QCD.

This process occurs due to an exchange of a W-boson in the t-channel. As the result, there is no color transfer from the light-quark line to the heavy-quark line at LO and NLO. It appears for the first time at NNLO where it is color-suppressed. **We will neglect these contributions in our NNLO computation.**

The relevant two-loop amplitudes are shown below; they involve one-loop corrections applied to heavy- and light-quark lines separately and the two-loop corrections to either heavy- or light-quark lines. The last diagram is the color-suppressed interference effect and we do not consider it. Real emission diagrams of the type similar to what is shown below need to be considered as well.
Ingredients for single-top NNLO computation

1) Two-loop form factors for heavy- (tWb) and light-quark ( qWq’) weak transitions are needed and they are known.
   Bonciani, Ferroglia; Bell; Astarian, Greub and Pecjak; Beneke, Huber and Li; Huber

2) Amplitudes for 0-> tbW(ll’)gg and 0->tbW(ll’)qq and 0->qq’W(ll)gg etc. Such amplitudes are either available or can be computed in a straightforward way;
   R.K. Ellis and J. Campbell

3) Collinear limits of all amplitudes (known in a general, universal form);

4) Soft limits for tree-level amplitudes (known; eikonal factors are slightly more complex for massive particles).

5) Soft limits for one-loop scattering amplitude that include top quarks are less well-known; they require the soft-currents at one loop for the massive fermion.
   Bierenbaum, Czakon and Mitov

6) One-loop amplitudes for bW -> t g are known in a compact form and can be borrowed from e.g. MCFM;
   J. Campbell and F.Tramontano

With these ingredients at place, one needs to perform phase-space partitioning (simple for heavy-quark line since no final state singularities), calculate the relevant limits, remove remaining singularities by performing renormalization (PDFs including). All of this has to be done for a multitude of partonic channels (quark-quark, quark-gluon etc.) -- a bit of a logistic nightmare.
Numerical cancellation of IR singularities

Since calculations are done numerically, cancellation of infra-red and collinear divergencies in the final result are also not exact. In fact, the degree of cancellation provides a useful check on the correctness of the implementation of various contributions.
Main features of the method

The main features of the sector-improved subtraction method for NNLO calculations

1) **It is robust:**
   - it applies equally well to processes with massless and massive particles, colored and not colored;
   - it can be used both for production and decay processes;

2) It uses universal features of scattering amplitudes in soft and collinear regions;

3) It does not require computation of Feynman diagrams -- knowledge of scattering amplitudes is sufficient (this means enormous improvement in efficiency);

4) The process-dependent input are two-loop virtual corrections to a Born process $X$, one-loop virtual corrections to a process $X+g$ and tree amplitudes for $X+gg$ (of course, quark-anti-quark pairs are also assumed).

Therefore, the process-dependent and process-independent input that is required for NNLO computations is similar to what is needed for NLO computations.

The main conceptual difference between NLO and NNLO computations at the moment seems to be the absence of established algorithmic procedures for computing arbitrary two-loop amplitudes.
Examples of NNLO applications
Top production cross-section at the LHC

A very impressive calculation of NNLO QCD corrections to one of the basic process at the LHC. The theory error on the NNLO cross-section prediction is $O(4)$ percent; equal contributions from various sources of uncertainties (scale, top mass, parton distribution functions etc.).

Apart from providing a benchmark for comparing theoretical predictions and experimental results for top pair production cross-sections, this result has been used to provide improved constraints on gluon PDFs and to set limits on stealthy stops;

Further developments will include kinematic distributions (hopefully very soon) and decays of top quarks in the narrow width approximation.

Czakon, Mitov, Fiedler

<table>
<thead>
<tr>
<th>Collider</th>
<th>$\sigma_{\text{tot}}$ [pb]</th>
<th>scales [pb]</th>
<th>pdf [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tevatron</td>
<td>7.009</td>
<td>$+0.259(4.7%)$</td>
<td>$+0.169(2.4%)$</td>
</tr>
<tr>
<td>LHC 7 TeV</td>
<td>167.0</td>
<td>$+6.7(4.0%)$</td>
<td>$-10.7(6.4%)$</td>
</tr>
<tr>
<td>LHC 8 TeV</td>
<td>239.1</td>
<td>$-9.9(3.9%)$</td>
<td>$+6.1(2.5%)$</td>
</tr>
<tr>
<td>LHC 14 TeV</td>
<td>933.0</td>
<td>$+31.8(4.4%)$</td>
<td>$-51.0(5.5%)$</td>
</tr>
</tbody>
</table>

Czakon, Mitov, Fiedler, Rojo, Mangano
t-channel single top production at NNLO

Cross-sections for t-channel single top production at the LHC in dependence of the top quark transverse momentum cut at 8 TeV LHC.

<table>
<thead>
<tr>
<th>$p_\perp$ (GeV)</th>
<th>$\sigma_{\text{LO}, \text{pb}}$</th>
<th>$\sigma_{\text{NLO}, \text{pb}}$</th>
<th>$\delta_{\text{NLO}}$</th>
<th>$\sigma_{\text{NNLO}, \text{pb}}$</th>
<th>$\delta_{\text{NNLO}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>53.8±3.0</td>
<td>55.1±1.6</td>
<td>+2.4%</td>
<td>54.2±0.5</td>
<td>−1.6%</td>
</tr>
<tr>
<td>20</td>
<td>46.6±2.5</td>
<td>48.9±1.2</td>
<td>+4.9%</td>
<td>48.3±0.3</td>
<td>−1.2%</td>
</tr>
<tr>
<td>40</td>
<td>33.4±1.7</td>
<td>36.5±0.6</td>
<td>+9.3%</td>
<td>36.5±0.1</td>
<td>−0.1%</td>
</tr>
<tr>
<td>60</td>
<td>22.0±1.0</td>
<td>25.0±0.2</td>
<td>+13.6%</td>
<td>25.4±0.1</td>
<td>+1.6%</td>
</tr>
</tbody>
</table>

• for the total cross-section next-to-leading order QCD corrections at the central scale are very small, much smaller than their natural O(10%) size; this is a consequence of significant cancellations between different channels -> important to consistently compute corrections to all of them;

• The NNLO result is very close to the NLO result (-1.6%), reduced $\mu$ dependence -> good theoretical control. Also with the cut on the top quark transverse momentum, the size of NLO QCD corrections increases; the NNLO QCD corrections remain small for all values of the transverse momentum.

The ratio of single top and single anti-top production cross-sections appears to be very stable against higher-order QCD corrections and can be used to constrain the ratio of up-quark and down-quark distributions in a proton, for relatively large values of the Bjorken $x$

$$\frac{\sigma_t}{\sigma_{\bar{t}}} = 1.85, \ 1.83, \ 1.83$$

Note strong PDF dependence
H+jet production at NNLO

Process with relatively large cross-section that can be used as an independent source of information on Higgs couplings. Transverse momentum distribution of the Higgs boson is very interesting -- its measurement may reveal if light degrees of freedom contribute to Higgs-gluon form factor. Efficiencies of jet vetoes (resummations, fixed orders etc.)

H+jet cross-sections in dependence of Higgs boson transverse momentum

\[ \sigma_{\text{LO}} = 2.70^{+1.2}_{-0.7} \text{ pb}, \]
\[ \sigma_{\text{NLO}} = 4.38^{+0.76}_{-0.7} \text{ pb}, \]
\[ \sigma_{\text{NNLO}} = 6.20^{+0.20}_{-0.24} \text{ pb} \]

Higgs boson transverse momentum distribution

Jet \( p_T \)-cut 30 GeV, MSTW2008 parton distribution functions

So far results for gg channel only; currently moving towards results for all channels H+jet production cross-sections

Boughezal, Caola, K.M. Petriello, Schulze
See also, Chen, Gehrmann, Matthieu, Glover
Di-jet production at NNLO

Dijet production at NNLO was also computed. So far only gluon contributions are accounted for; quark contributions are in progress. A lot of interesting physics -- strong coupling constant, PDFs, constraints on four-quark operators etc.

Currie, Gehrmann - de Ridder, Gehrmann, Glover, Glover, Pires
Production of VV bosons at NNLO

Production of W+W- bosons at the LHC has attracted a lot of attention recently. The reason is a tantalizing discrepancy between the measured and the expected cross-section that persists in spite of the 7 -> 8 TeV energy increase, etc. The production of ZZ-channel is less discrepant.

Corrections (somewhat) improve the description of data; results call for the calculation of the fiducial-volume cross-sections.

Grazzini, Kallweit, Maierhofer, von Manteufel, Pozzorini, Rathlev, Tancredi, Gehrmann
Conclusions

Perturbative QCD provides a theoretical framework for describing hard collisions at the LHC.

Perturbative computations of scattering cross-sections play important role in all aspects of the LHC physics (input parameters, SM tests, properties of the Higgs, BSM searches).

A great success at NLO where technical difficulties are overcome and results are being used to gain deeper insights into the LHC physics.

An interesting developments with NNLO computations where, finally, we have several methods that allow us to perform fully differential computations at NNLO.

The phenomenology of hard scattering processes at NNLO is starting to appear (top pairs, single top, H+j, dijets) but their applications in physics analyses are still to come.

Another direction that is actively pursued is the combination of various elements required to describe hard scattering at the LHC (fixed orders, multi-jet merging, parton showers etc.). It is to be expected that progress in this direction will further improve the reliability of the LHC results.