electron and proton through
by introducing physical fields.
Electric charge and bare mass
is written in terms of bare fields, there
\[ e = \gamma \alpha_0, \quad F = \frac{\gamma}{\sqrt{r^2 + \beta^2}} \]
\[ \chi = -\frac{1}{\gamma} \int \rho \left( \frac{r^2}{r^2 + \beta^2} \right) \]
problem for QED. The Lagrangian

\[ \mathcal{L} = \text{work and the details of this}\]

\[ \text{other quantities by the common terms to satisfy}\]
\[ \text{by invariance condition on Lagrangian function}\]
\[ \text{are not known, a priori, but can be determined}\]
\[ \text{on a quantity, a term - the counter-terms -}\]
\[ \text{can be introduced that at the price of}\]
\[ \text{other the original Lagrangian. The latter}\]
\[ \text{physically observable quantities. The former}\]
\[ \text{between force (unphysical) quantities and}\]
\[ \text{recall that the basic idea is to distinguish}\]
\[ \text{no generalization that only the case of QED}\]

\[ \text{renormalization of the theory. We will}\]
\[ \text{of previous lectures we discussed}\]

\[ \text{Charge renormalization}\]
\[ \text{in QED: wave function, masses, and}\]
\[ \text{Lecture 8: One-loop renormalization}\]
\[ \chi = -\frac{1}{2} \sum_{+} \sum_{-} \sum_{\mu} F_{\mu +} - \sum_{\mu} F_{\mu -} \]
To NH, we need to write

Get us explore the first one. Further, the conditions are important.

$$\phi \equiv (\phi) \cap \{}$$

$$\phi = \left\{ \begin{array}{l}
\frac{d}{dx} \phi = 0 \\
\phi = w \text{ and } \phi = (\phi) \cap \end{array} \right\}$$

This condition has that we interest are

The function $W$ is defined by

Contribution. The space in QED is similar.

in terms of $F$-monence integrable self-energy.

By theory, this condition can be expressed

three-particle poles. At the zero in case of

do not receive corrections close to their

Chern's functions. To this end, we

follows from the conditions of an approximate

Definition of physical parameters
The photon field is canonically normalized. The photon field is massless and that from the photon polarization vectors implies that it is possible to choose the assumed normalized field.

\[ \phi = (p) \]

The propagator \( S(p) \)

\[ \frac{\delta - m}{E} = (p) \]

Therefore, if

\[ (d) \left( \frac{p - m}{E} \right) \]

\[ \Rightarrow \]

\[ \Rightarrow \left[ (d) \right] \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]

\[ \Rightarrow \]
\[
\int_0^1 \frac{\left[ x^m - (x-\lambda)^m \right]^2}{(x + \sqrt{d})^2} \, dx = \frac{\left[ (\lambda + \sqrt{d})^2 - \lambda^2 \right]^2}{4}.
\]

**Integral Calculus:**

And familiar rules for Feynman diagrams. We can use standard chain rule. Specifically:

\[
\int_0^1 \frac{\left[ x^m - (x-\lambda)^m \right]^2}{(x + \sqrt{d})^2} \, dx = \frac{\left[ (\lambda + \sqrt{d})^2 - \lambda^2 \right]^2}{4}.
\]

The loop integral \( Z(p) \) reads

\[ Z(p) = \frac{1}{4} \int \frac{d\lambda}{(\lambda + \sqrt{d})^2} \]

to find \( \pi \) and \( \sigma \).

As usual, counterterms are not known; we need to use physical conditions for \( Z(p) \).

\[ Z(p) = \frac{1}{4} \int \frac{d\lambda}{(\lambda + \sqrt{d})^2} \]

We begin with the Feynman self-energy.

We will now derive one-loop results for the counterterms, these are called one-loop counterterms.

We will now describe the loop corrections.

The theory requires the following primary condition: To this end, we fix the physical condition.

Our final condition describes this loop.
Finally, \( q(n) \equiv 0 \), we find

\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin^2 x} \, dx = \frac{\pi}{2}
\]

The value of \( I(z) \), for \( p = m \), becomes

\[
I(z) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin^2 x} \, dx
\]

\[
= \left\{ \begin{array}{ll}
\frac{\pi}{2} & \text{if } 1 - 2z \geq 0 \\
\frac{1 - 2z}{3 - 2z} & \text{if } 1 - 2z < 0
\end{array} \right.
\]

Thus, for \( p = m \), we have a simple result.

In fact, there is a general expression that is

\[
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin^2 x} \, dx = \frac{\pi}{2}
\]

\[
\int_{0}^{\infty} e^{-\frac{\pi}{2}} \, dx = \frac{\pi}{2}
\]

\[
\int_{0}^{\infty} e^{-\frac{\pi}{2}} \, dx = \frac{\pi}{2}
\]

over all values of the parameter.
The second condition requires calculation of \( \frac{\partial \Sigma(p)}{\partial p} \) \( \mid_{p=m} \). To compute this derivative, remember that \( \hat{p} \cdot \hat{p} = p^2 \). Therefore, \( \hat{p} = \frac{2 \hat{p}}{p} \). Therefore, \( \frac{\partial \hat{p}}{\partial p} = \frac{2 \hat{p}}{p} \). Then, the expression turns to

\[
\frac{\partial}{\partial \hat{p}} (2 \hat{p} - \hat{p} \cdot \hat{p}) = \frac{2}{p} - \frac{2 \hat{p} \cdot \hat{p}}{p^2}
\]

We set \( \hat{p} \) to \( m \), to obtain the derivative of the squared term:

\[
\frac{\partial}{\partial \hat{p}} (2 \hat{p} - \hat{p} \cdot \hat{p}) = \frac{2}{m} - \frac{2 \hat{p} \cdot \hat{p}}{m^2}
\]

Therefore, the condition of the squared term is

\[
\frac{\partial}{\partial \hat{p}} (2 \hat{p} - \hat{p} \cdot \hat{p}) = 0 \Rightarrow \frac{2}{m} - \frac{2 \hat{p} \cdot \hat{p}}{m^2} = 0
\]

We obtain, after expanding \( \hat{p} \) and \( m \),

\[
\begin{align*}
0 &= \delta_2 + e^2 \Gamma (1+\epsilon) m^{-2\epsilon} \frac{e^{-2\epsilon}}{4\pi} \ln \frac{(1 - 2E)}{(1 - 2E) \ln (\hat{p} - 5\epsilon)} \\
&+ \frac{(2 \hat{p} - \hat{p} \cdot \hat{p})}{(4\pi) \ln (\hat{p} - 5\epsilon)}
\end{align*}
\]
Photon field renormalization.

Next, we discuss calculation of the counterterms. In the renormalized higher-order calculation, this equation holds as an accident and we see that \[ \mathcal{Z}^m = \mathcal{Z}^m \]

\[
\left( 1 + \frac{\mathcal{Z}}{\mathcal{Z}} \right)^{\frac{1}{2}} \left( 1 + \frac{\mathcal{Z}}{\mathcal{Z}} \right) = 1 - \mathcal{Z}^m
\]

Recall that \[ m = m^0 - m^2 \mathcal{Z}^m \]

and then we define \[ m = m^0 \] and that for $Z^2$, we get \[ Z^2 = Z + \frac{1}{2} \mathcal{Z}^m \]

For $Z^2$, we tune \[ m = m^0 \] in the expressions for the renormalization constants $Z^2$ and $Z$. We will now turn to the mass terms on page 6.
The contribution is

\[
\sum_{\text{coulomb}} = -\varepsilon_e (n a - n b) \varepsilon^z \frac{(x-y)(x-y)}{(x-y)^2}
\]

We arrive at

\[
\frac{\left[ (x-y)^2 \right]}{(x-y)^2} \int_0^1 e^{-|x-y|} (x-y) \sum_{\text{coulomb}} \frac{\varepsilon^z (n a - n b)}{(x-y)^2} dx
\]

We obtain, for the photon vacuum polarization

\[
\sum_{\text{coulomb}} = -\varepsilon_e (n a - n b) \varepsilon^z \frac{(x-y)(x-y)}{(x-y)^2}
\]
\[ P^2 = p_1^2 = m^2 \text{ and } P^2 = p_2^2 = m^2 \]

From the approximation, we need to compute the one-loop

The first renormalization constant that

\[ \frac{1}{2} \left( \frac{m}{3 + 1} \right)^2 \frac{e^3}{h} = \frac{1 + 3}{2} = 3 \]

This gives the expression, to the lowest order of the

The one-loop electron-photino interaction

we need to compute the \( Z \) factor.

The

\[ \mathcal{L} = \frac{1}{2} \left( \frac{m}{3 + 1} \right)^2 \frac{e^3}{h} = -8 \approx 3 \]

Finally, \( U(0) \) we find

\[ \int_{\mathcal{E}} \left[ \frac{1}{2} \left( \frac{m}{3 + 1} \right)^2 \frac{e^3}{h} \right] \left( \frac{m}{3 + 1} \right)^2 \frac{e^3}{h} - \frac{g_1}{\mu} \left( \frac{m}{3 + 1} \right)^2 \frac{e^3}{h} \right] = \left( \frac{g_1}{\mu} \left( \frac{m}{3 + 1} \right)^2 \frac{e^3}{h} \right) \]

contract for the photino.
There are two ways to satisfy
\[ \langle \phi, \psi \rangle = 0 \text{ and } (d, \rho) \text{ are Dirac equations.} \]

Hence, \( \phi \) and \( \psi \) are linear functions that

\[ \int x \rho \left( \int \hat{\rho}(x) \right) \]
Since it is zero, we can drop it.

To check what is non-nil at $x=0$,

function.

Next we that we make the $O(h)$ term in the numerator.

As we mentioned, this is the reason we need

we find, not necessarily a diagonal at $x=0$.

We consider

\[ \int_{0}^{b} x \frac{\partial}{\partial x} \phi \left( x \right) \, dx = \frac{3}{2} \left( x \phi \right) \left( 3 + \frac{1}{2} \right) \int_{0}^{b} \phi \left( x \right) \, dx \]

where $o=b^{2}$.

Thus, $g = o$, we find

Do integration over $f$ and taking

check if $f$ is true in a second.

meaningful. The equation

The is done because unknown are removed.

We kept $(d)(N)m(x)$ in the expansion. So, there

Note that

If $Z$, then so that case we require $P \neq 0$.

The above expressions are general. We
and Fig. 2 (a) depicts these

formalized and the same way as

electron wave function \( \psi (x) \) is
given that \( \psi (x) = \psi _{1} - \epsilon _{2} \psi _{2} \). To

We therefore find that \( s = \frac{1}{2} \)

Then the find \( \psi _{1} = - \epsilon _{2} \psi _{2} \). This

The correction (one-loop) to electron-phonon

\[
\frac{4\pi \rho^2}{2 (1+\epsilon)} \left( \frac{e}{\hbar} \right) \frac{1}{3} \frac{4\pi \rho^2}{2 (1+\epsilon)} \left( \frac{e}{\hbar} \right) \frac{1}{3}
\]

and expand in \( \epsilon \). We find

and extensively in \( \epsilon \). We find

exact agreement by numerical simulation

need it. The calculation of \( \lambda _{n} \) can be

to \( \text{Num}_{1} (s) = C \). Here, the analysis do not

turns out that \( \epsilon > 0 \). The treatment for

Nam (s) is defined on page 11 and is
We will discuss in the next lecture.

Reason is the gauge invariance of \( \Phi \).

The order in the interaction theory. The more general, it is natural to act at one-loop. However, this equality is

\[ e = \frac{g}{\sqrt{2}} \]

We showed the equality of \( \mathbb{Z}_2 \) and \( \mathbb{Z}_2 \).

In other words:

\[ e \mathbb{Z}_2 = \frac{g}{\sqrt{2}} \mathbb{Z}_2 \]