To discuss quantization of non-abelian gauge fields, it is useful to introduce the so-called path integral formalism to describe quantum fields. We will first discuss it in ordinary quantum mechanics.

Consider a quantum mechanical system described by a Hamiltonian
\[ H = \frac{\hat{P}^2}{2m} + V(\hat{Q}), \]
where \( \hat{P} \) & \( \hat{Q} \) are momentum and position operators.

Let's imagine that at a time \( t = t_i \) our system is in a state with definite coordinate \( x = x_i \); we would like to find the probability amplitude that at a time \( t = t_f \) our system is in a state with definite coordinate \( x = x_f \). These states are defined as \( \hat{Q} \ket{x_i} = x_i \ket{x_i} \), \( \hat{Q} \ket{x_f} = x_f \ket{x_f} \).

The amplitude is computed as:
\[ i \frac{\partial \ket{\Psi}}{\partial t} = \hat{H} \ket{\Psi} \Rightarrow \ket{\Psi(t)} = e^{-i \hat{H}(t-t_i)} \ket{\Psi(t_i)}, \]
where \( \ket{\Psi(t_i)} \) is the state at \( t = t_i \). The amplitude is
\[ U(x_f, x_i; t_f, t_i) = \bra{x_f} e^{-i \hat{H}(t_f-t_i)} \ket{x_i}. \]

Our goal is to write this matrix element in a particular way.
To this end, let us split the time interval into \( n \) "small" intervals:

\[
\begin{align*}
\cdots & t_1 \quad t_2 \quad \cdots \quad t_{n-1} \quad t_f \\
\end{align*}
\]

The length of each interval is \( \delta t = (t_f - t_i)/n \to 0 \), i.e. \( n \to \infty \).

\[
U(x_f, x_i; t_f, t_i) = \langle x_f | \mathcal{E}^{-\mathfrak{i} \mathcal{H} \delta t} \mathcal{E}^{\mathfrak{i} \mathcal{H} \delta t} | x_i \rangle^n \text{ times}
\]

A trick is to insert complete sets of states into "strategic places". We use

\[
\hat{\Pi} = \int dx_k \quad |x_k\rangle \langle x_k|,
\]

and write

\[
U(x_f, x_i; t_f, t_i) = \int \prod_{k=1}^{n-1} dx_k \quad \langle x_f | e^{-\mathfrak{i} \mathcal{H} \delta t} | x_{n-1} \rangle \otimes \langle x_{n-1} | e^{\mathfrak{i} \mathcal{H} \delta t} | x_{n-2} \rangle \cdots \otimes \langle x_1 | e^{-\mathfrak{i} \mathcal{H} \delta t} | x_i \rangle.
\]

We see that the primary object to investigate is \( \langle q_a | e^{-\mathfrak{i} \mathcal{H} \delta t} | q_b \rangle \), in the limit \( \delta t \to 0 \). To this end, we expand the exponential to first order in \( \delta t \):

\[
e^{-\mathfrak{i} \mathcal{H} \delta t} \approx 1 - \mathfrak{i} \mathcal{H} \delta t = 1 - \mathfrak{i} \left( \frac{\hat{p}^2}{2m} + V(q) \right)
\]

Next \( \langle q_a | q_b \rangle = \delta(q_a - q_b) \)

\( \langle q_a | V(q) | q_b \rangle = \delta(q_a - q_b) V\left( \frac{q_a + q_b}{2} \right) \)

and

\[
\langle q_a \left| \frac{\hat{p}^2}{2m} \right| q_b \rangle = \int \frac{dp_a}{2\pi} \int \frac{dp_b}{2\pi} \quad \langle q_a | p_a \rangle \langle p_a \left| \frac{\hat{p}^2}{2m} \right| p_b \rangle \langle p_b | q_b \rangle.
\]
Using \( \langle q_a p_a \rangle = e^{i p_a q_a} \) and \( \langle p_b q_b \rangle = \langle q_b p_b \rangle \)
and \( \langle p_a | \frac{\hat{p}^2}{2m} | p_b \rangle = (2\pi)^3 \delta(p_a-p_b) \frac{p_a^2}{2m} \)
we find
\[
\langle q_a | \frac{\hat{p}^2}{2m} | q_b \rangle = \int \frac{d p_a}{2\pi} \frac{d p_b}{2\pi} e^{i p_a q_a - i p_b q_b} (2\pi)^3 \delta(p_a-p_b) \frac{p_a^2}{2m} = \int \frac{d p_a}{2\pi} e^{i p_a (q_a - q_b)} \frac{p_a^2}{2m}.
\]
Now, using \( \delta(q_a - q_b) = \int \frac{d p_a}{2\pi} e^{i p_a (q_a - q_b)} \)
we write
\[
\langle q_a | e^{i H_0 t} | q_b \rangle = \int \frac{d p_a}{2\pi} e^{i p_a (q_a - q_b) - i \frac{p_a^2}{2m} + \frac{1}{2} V(q_a^2 + q_b^2)} \]

Where \( \psi \) is the transition amplitude and find
\[
\psi(x_f, x_i; t_f, t_i) = \int \prod_{k=1}^{n-1} dx_k \prod_{k=1}^{n} \frac{d p_k}{2\pi} e^{i \sum_{k=1}^{n} p_k (x_k - x_{k-1})} \otimes e^{-i \frac{p_k^2}{2m} + \frac{1}{2} V(x_k^2 + x_{k-1}^2)}
\]
where \( x_n = x_f \) and \( x_0 = x_i \)

We notice that the integral over \( p \) is **Gaussian**, so it is straightforward to perform it.
We have \( x_k - x_{k-1} = \xi_k \) 

\[
\int \frac{dk}{2\pi} e^{i \mathbf{p}_k \cdot \mathbf{x}_k} - i \delta t \frac{p_k^2}{2m} = \int \frac{dk}{2\pi} e^{-\frac{i}{2m} \left( \mathbf{p}_k - \mathbf{m} \xi_k \right)^2} \xi_k^{\frac{1}{2}} e^{i \xi_k^{\frac{1}{2}} / (2m \delta t)} = \sqrt{\frac{mi}{2\pi \delta t}} e^{i \left( \frac{x_k - x_{k-1}}{2m \delta t} \right)^2} \cdot e^{i \frac{\xi_k^{\frac{1}{2}}}{(2m \delta t)}}.
\]

Next, we use this result in the expression for \( U(x_f, x_i; t_f, t_i) \) and obtain

\[
U(x_f, x_i; t_f, t_i) = \int d^n x_k \left( \frac{mi}{2\pi \delta t} \right)^n \exp \left[ - \frac{i}{2m \delta t} \frac{\left( x_k - x_{k-1} \right)^2}{2} - i \frac{\sqrt{\xi_k^{\frac{1}{2}}}}{\delta t} \right].
\]

We can now realize that this integral can be written as

\[
\left| U(x_f, x_i; t_f, t_i) \right| = \int \mathcal{D} \mathbf{x}(t) \left| e^{i \int_{t_i}^{t_f} \mathcal{L}(x, \dot{x}) \, dt} \right| \left. \begin{array}{l}
x_j = x(t_f) \\
x_i = x(t_i)
\end{array}\right| \]

with \( \mathcal{L}(x, \dot{x}) = \frac{\dot{x}^2}{2m} - V(x) \), being the Lagrangian of the system and \( \mathcal{D} \mathbf{x}(t) \) a symbolic notation for the measure of the integral over trajectories that take a particle from \( x_i \) at \( t = t_i \) to \( x_f \) at \( t = t_f \). Note that we have to integrate over all trajectories and not only those that minimize the action

\[
S = i \int_{t_i}^{t_f} \mathcal{L}(x, \dot{x}) \, dt.
\]
To a large extent, quantum field theory is quantum mechanics with \( \infty \) degrees of freedom. So the formula that we just derived should apply directly. We then have

\[
\langle \phi_0 (\vec{x}) | e^{-iHt} | \phi_a (\vec{x}) \rangle = \int \mathcal{D}\phi \, e^{i \int dt \, L},
\]

where \( L = \frac{1}{2} (\partial\phi)^2 - V(\phi) \) and the fields \( \phi(x,t) \) are supposed to satisfy \( \phi(t=0, x) = \phi_0 (x) \) and \( \phi(t=T, x) = \phi_b (x) \).

The main quantities that we need to construct QFT are the Green's functions, so we need to understand how to compute them. We will focus on a 2-point correlation function; the other ones are computed by simple generalization of the arguments that we now discuss.

Let us consider an integral

\[
I = \int \mathcal{D}\phi(x) \, \phi(x_1) \phi(x_2) \exp \left[ i \int dx \, L(\phi) \right]
\]

where \( \phi(T, x) = \phi_a (x) \) and \( \phi(T, x) = \phi_b (x) \). Let us now split the integration measure \( \mathcal{D}\phi(x) \) by separately integrating over fields at \( t = x_1^0 \) and \( t = x_2^0 \):

\[
\int \mathcal{D}\phi(x) = \int \mathcal{D}\phi_1(x_1) \, \mathcal{D}\phi_2(x_2) \int \mathcal{D}\phi(x)
\]

\[
\phi(x_1^0, x) = \phi_1(x)
\]

\[
\phi(x_2^0, x) = \phi_2(x)
\]
Now consider $x_1^0 < x_2^0$. Then, the path integral splits into 3 parts:

$$I = \int D\varphi_1(x_1) D\varphi_2(x_2) \langle \varphi_0 | e^{-iH(T-x_2^0)} | \varphi_2 \rangle$$

$$\otimes \varphi_2(x_2) \langle \varphi_2 | e^{-iH(x_2^0-x_1^0)} | \varphi_1 \rangle \varphi_1(x_1)$$

$$\otimes \langle \varphi(x_t) | e^{-iH(x_1^0+T)} | \varphi_a \rangle$$

To simplify this formula, write

$$\hat{\varphi}(x_t) | \varphi_1 \rangle = \varphi_1(x_1) | \varphi_1 \rangle$$

and

$$I = \int D\varphi_1 D\varphi_2 \langle \varphi_0 | e^{-iH(T-x_2^0)} \hat{\varphi}(x_2) | \varphi_2 \rangle$$

$$\langle \varphi_1 | e^{-iH(x_2^0-x_1^0)} \hat{\varphi}(x_1) e^{-iH(x_1^0+T)} | \varphi_a \rangle =$$

$$= (\text{completeness}) = \int \langle \varphi_0 | e^{-iH(T-x_2^0)} \hat{\varphi}(x_2) e^{-iH(x_2^0-x_1^0)} \hat{\varphi}(x_1) e^{-iH(x_1^0+T)} | \varphi_a \rangle =$$

$$I = \langle \varphi_0 | e^{-iHT} \varphi_H(x_2) \varphi_H(x_1) e^{iHT} | \varphi_a \rangle$$

where $\varphi_H(x) = e^{iHx_0^0} \hat{\varphi}(x) e^{-iHx_0}$ is the field operator in the Heisenberg representation.

Of course, if we have chosen $x_2^0 < x_1^0$, we'll have $\varphi_H(x_2)$ and $\varphi_H(x_1)$ to appear in the opposite order. Hence, we conclude that

$$I = \langle \varphi_0 | e^{-iHT} \varphi_H(x_2) \varphi_H(x_1) e^{iHT} | \varphi_a \rangle$$
This is almost that we want kept apart from external states, \( \Psi_a \) & \( \Psi_b \). However, we can use the trick that we used already when discussing the \( T \to \infty \) limit of a perturbative expansion of Green's functions: take \( T \to T(1-i\epsilon) \) & take the limit \( T \to \infty \).

As we do,

\[
\begin{align*}
\sum_n e^{-i E_n T} |n\rangle \langle n| \Psi_a & \rightarrow \sum_n e^{-i E_n T} |n2\rangle \langle n2| \Psi_a \\
& \rightarrow e^{-i E_0 T} |12\rangle \langle 12| \Psi_a,
\end{align*}
\]

where \( |12\rangle \) is the vacuum state. With this,

\[
I = e^{-2i E_0 T} \langle \Psi_b | 12\rangle \langle 12| \Psi_a \rangle \langle \Psi_a | T^2 \phi_H(x_2) \phi_H(x_1) | 12\rangle
\]

\[
I = \frac{1}{e^{-2i E_0 T} \langle \Psi_b | 12\rangle \langle 12| \Psi_a \rangle} \langle \Psi_a | T^2 \phi_H(x_2) \phi_H(x_1) | 12\rangle
\]

Now, use

\[
\langle \Psi_b | 12\rangle \langle 12| \Psi_a \rangle e^{-2i E_0 T} = \langle \Psi_b | e^{-i HT} | \Psi_a \rangle = \int \mathcal{D} \phi \exp \left[ -i \int d^4x \mathcal{L} \right] \frac{1}{e^{-2i E_0 T} \langle \Psi_b | 12\rangle \langle 12| \Psi_a \rangle}
\]

\[
\langle \Psi_a | T^2 \phi_H(x_2) \phi_H(x_1) | 12\rangle = \lim_{T \to \infty} \frac{1}{1-i\epsilon} \int \mathcal{D} \phi e^{i S_T}
\]

where \( S_T = \int d^4x \mathcal{L} \), with

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi).
\]
It is clear that higher correlation functions require just taking more insertions of the $\phi$-fields under the integration sign in the numerator of the above expression. We can make this very clear and compact by defining the generating functional for the Green's function $Z[J]$:

$$Z[J] = \int \mathcal{D}\phi \, e^{iS[\phi, J]}$$

where

$$S[\phi, J] = \int dx \left( \mathcal{L}(\phi) + \phi(x) \cdot J(x) \right)$$

Then

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \cdots \mathcal{O}_n \rangle = \frac{1}{Z[0]} \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \cdots \frac{\delta}{\delta J(x_n)} Z[J].$$

It is possible to continue along these lines and study the case of the free field and then develop perturbation theory for quantum fields, but we will not do that. Instead, we will focus on another aspect of the functional integral which becomes very helpful for quantization of gauge fields.
We will start with QED. The key object is an integral \( \int \mathcal{D}A \, e^{iS[A]} \), with

\[
\mathcal{D}A = \mathcal{D}A_0 \mathcal{D}A_1 \mathcal{D}A_2 \mathcal{D}A_3 \quad \text{and} \quad S[A] = \int dx \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)
\]

Suppose we want to compute this integral.

To this end, we write

\[
-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{2} \nabla_{\nu} A_{\mu} F^{\mu\nu} = \frac{1}{2} A_{\nu} \left( \partial_{\mu} - \partial_{\nu} \right) A_{\mu}.
\]

If we write \( A_{\mu}(x) = \int \frac{dk}{(2\pi)^4} e^{ikx} A_{\mu}(k) \), we find \( \mathcal{D}A(x) \rightarrow \mathcal{D}A_{\mu}(k) \) and

\[
S[A] = \frac{1}{2} \int \frac{dk}{(2\pi)^4} A_{\mu}(k) \left[ -k^2 g_{\mu\nu} + \partial_{\mu} \partial_{\nu} \right] A_{\nu}(k).
\]

This is a typical Gaussian integral, of the type

\[
\int d^N \mathbf{x} \, e^{\mathbf{A} \cdot \mathbf{x}} = \text{const} \left[ \det(A) \right]^{-1/2}, \quad \text{(we do not need to know what the constant is)}
\]

The matrix \(-k^2 g_{\mu\nu} + \partial_{\mu} \partial_{\nu}\) has an eigenvalue \(0\), since \( \overline{A_{\nu}(k)} = \partial_{\nu} f(k^2) \) gives

\[
(-k^2 g_{\mu\nu} + \partial_{\mu} \partial_{\nu}) \overline{A_{\nu}(k)} = \Phi \Rightarrow \left[ \det(A) \right] = 0
\]

\(\Rightarrow\) the integral over \( A_{\mu}(k) \) of \( e^{iS(A)} \)

will diverge (i.e., does not exist).

The problem, as was recognized by L. Faddeev and V. Popov, is that when we try to perform the integral over \( A_{\mu}(x) \), we...
Integrate over infinitely many physically-equivalent field configurations that differ from each other by a gauge transform:

\[ A_\mu(x) \rightarrow A_\mu(x) + \frac{1}{e} \partial_\mu \alpha(x) \]

They suggested that the integration over \( DA_\mu \) should be re-arranged in such a way that the integral over non-equivalent physical field configurations is identified & extracted.

To this end, let us introduce the following integral:

\[
1 = \int D\alpha(x) \delta(G(A_\alpha)) \times \text{det} \left[ \delta G(A_\alpha) \right]_{\alpha}
\]

Here, \( G(A_\alpha) = \partial_\mu A_\mu + \frac{1}{e} \partial^2 - \omega(x) \).

Since \( \text{det} \left[ \delta G(A_\alpha) \right] = \text{det} \left[ \frac{1}{e} \partial^2 \right] \) is independent of \( \alpha(x) \), for the purpose of the following discussion it is irrelevant. So let us write

\[
\int DA_\mu e^{iS[A_\mu]} = \int DA_\mu D\alpha \delta(G[A_\alpha]) \times \text{det} \left[ \frac{1}{e} \partial^2 \right] \times e^{iS[A_\alpha]}
\]

Here, we can now perform a gauge transformation \( \equiv \) charge integration variables \( A_\mu \rightarrow A_\mu^0 = A_\mu \pm \frac{1}{e} \partial_\mu \alpha \).
Then \( \mathcal{D}A_\mu = \mathcal{D}\bar{A}_\mu \) and \( S[A] = S[\bar{A}] \) gives

\[
\int \mathcal{D}A_\mu \, e^{iS[A]} = \det \left[ \frac{1}{e^2} \right] \int d\alpha \int \mathcal{D}\bar{A} e^{iS[\bar{A}]} \times \delta \left( \partial_\mu \bar{A}^\mu - \omega(x) \right).
\]

Now, we achieved what we wanted; the integral over \( \bar{A} \) restricted to configurations that satisfy the gauge-fixing condition \( \delta \)

the infinite integral \( \int d\alpha \) that describes gauge redundancies. We can make the final result somewhat nicer by integrating over all functions \( \omega \), centered around \( \omega(x) = 0 \) since the result of functional integration can not depend on \( \omega(x) \) [one more gauge transformation will remove \( \omega(x) \) explicitely].

We write

\[
1 = N(\bar{\varepsilon}) \int \mathcal{D}\omega \exp \left[ -i \int dx \frac{\omega^2(x)}{2\bar{\varepsilon}} \right]
\]

\[
\int \mathcal{D}A_\mu \, e^{iS[A]} = \det \left[ \frac{1}{e^2} \right] N(\varepsilon) \int d\alpha \int \mathcal{D}\bar{A} e^{iS[\bar{A}]}
\]

\[
\times \int \mathcal{D}\omega \exp \left[ -i \int dx \frac{\omega^2(x)}{2\varepsilon} \right] \times \delta \left( \partial_\mu \bar{A}^\mu - \omega(x) \right) = \det \left[ \frac{1}{e^2} \right] N(\varepsilon) \int d\alpha \int \mathcal{D}\bar{A} e^{iS_\varepsilon[\bar{A}]},
\]

where

\[
S_\varepsilon[\bar{A}] = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\varepsilon} (\partial_\mu \bar{A}^\mu)^2.
\]

Now, if we imagine computation of Green's functions of gauge-invariant operators, all the computations...
that we just discussed go through and we find
\[ \langle \Omega \left| \mathcal{T} \mathcal{O}(A) \right| \Omega \rangle = \frac{\int DA \, \mathcal{O}(A) \, e^{iS_\xi[A]}}{\int DA \, e^{iS_\xi[A]}}. \]

For the action \( S_\xi[A] \), we can easily find the photon propagator
\[ D^{\mu\nu}(k) = \frac{-i}{k^2 + i\varepsilon} \left( g^{\mu\nu} - (1 - \varepsilon) \frac{k^\mu k^\nu}{k^2} \right) \]
and more general Feynman rules if we couple the theory to e.g. \( \gamma \) Dirac fermion fields. We'll not go into this direction.

Our next step concerns non-abelian gauge fields.

In that case, we'll repeat the same procedure as in QED, except that the integration over gauge configurations becomes tricky. We write \( \Omega = e^{i\alpha^a T^a} \)
\[ G[A] = \mathcal{F}_\mu \left[ \mathcal{D}^\alpha \mathcal{A}^\alpha + i \frac{g}{2} \mathcal{F}_{\mu\nu} \mathcal{A}^{\nu} \right]. \]

To compute \( \frac{\delta G[A]}{\delta A} \), we consider
\[ \frac{\delta A}{\delta A} \]
infinitesimal transformation:
\[ (A^a)_{\mu}' = A^a_{\mu} + \frac{1}{g} \partial _\mu \alpha^a + f^{abc} A^b_{\mu} \alpha^c = A^a_{\mu} + \frac{1}{g} (D^a)_{\mu} \]
where \( D^a_{\mu} \) is the covariant derivative in the adjoint representation.
Then \[ \frac{\delta G[A^a]}{\delta A^b} = \frac{1}{g} (\partial^a D_\mu)^a b = \frac{1}{g} \partial^a [\partial_\mu \delta^{ab} + f^{abc} \delta_{\mu \nu}] \]

The difference between abelian & non-abelian cases is that \[ \text{det} \left[ \frac{\delta G[A^a]}{\delta A^b} \right] \] depends on the field \( A^a_\mu \) in the non-abelian case an, therefore, remains "inside" the functional integral, so that we have

\[
\int D A^a e^{i S[A]} = \left[ \int D x \right] \int D A^a e^{i S[A]} \delta(G[A])
\]

\[ \times \text{det} \left[ \frac{\delta G[A^a]}{\delta A^b} \right] \]

Since \( \delta(G[A^a]) = \delta(\partial^a A^a_A) \), it can be treated in the same way as in the PED case.

If gives \( e^{i S[A]} \delta(G[A]) \)

\[ 
\rightarrow \quad e^{i S_\xi[A]} , \text{ where} \\
S_\xi[A] = - \frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu} - \frac{1}{2} (\partial^a A^a_\mu)^2 .
\]

However, \[ \text{det} \left[ \frac{\delta G[A^a]}{\delta A^b} \right] \] remains.

Faddeev & Popov suggested to represent this determinant as the functional integral over new set of \text{anti-commuting} fields --- the Faddeev-Popov ghosts:

\[ \text{det} \left[ \frac{1}{g} \partial^a D_\mu \right] = \int D c \ D \bar{c} \ \exp \left[ i \int d x \bar{c} \ (\partial^a D_\mu) c \right] \]

where

\[ \bar{c} \ (\partial^a D_\mu) c = \bar{c}^a (- \partial^2 g^{ac} + g \partial^a f^{abc} A^b_\mu) c \]
With this trick, the path integral for the non-abelian gauge fields becomes:

\[
\int DA^a \; Dc \; D\bar{c} \; e^{iS_{\xi}[A] + iS_{\text{ghost}}},
\]

where

\[
i[S_{\xi} + S_{\text{ghost}}] = i \int d^4x \left[ -\frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu} - \frac{1}{2\xi} (\partial^\mu A^a_\mu)^2 
+ \bar{c}^a \left(- \partial^2 \delta^{ac} - g \partial^\mu f^{abc} A^b_\mu \right) c^c \right].
\]

With this, quantization procedure of the Yang-Mills theory becomes straightforward.

**Feynman rules (including fermions)**

\[
a_{\alpha \beta} = \frac{-i \delta_{\alpha \beta}}{k^2 + i\epsilon} \quad \left( \delta_{\mu \nu} - (1 - \xi) \frac{k_\mu k_\nu}{k^2} \right)
\]

gluon propagator

\[
\quad \frac{-i \delta_{\alpha \beta}}{k^2} \quad i \rightarrow i = \frac{i \delta_{ij}}{k^2} \quad \text{quark prop.}
\]

**Vertices:**

\[
\begin{array}{ll}
a_{\alpha \beta} & = ig \gamma^\mu(t^a)_{ij} \\
& = gf^{abc} \left[ g \mu \nu (k-p)^\rho + g^{\nu \sigma}(p-q)^\rho \\
& \quad + g^{\mu \rho}(q-k)^\nu \right];
\end{array}
\]

\[
\begin{array}{ll}
a_{\alpha \beta} & = -ig^2 \left[ f^{abc} \right. \\
& \quad + f^{ace} f^{bde} \left( g^{\mu \rho} g_{\nu \sigma} - g^{\mu \sigma} g_{\nu \rho} \right) \\
& \quad + f^{ade} f^{bce} \left( g^{\mu \rho} g_{\nu \sigma} - g^{\mu \sigma} g_{\nu \rho} \right) \\
& \left. \right] g^{\rho \nu} g^{\mu \sigma} g^{\nu \rho} \\
& \equiv -gf^{abc} p^\mu .
\end{array}
\]