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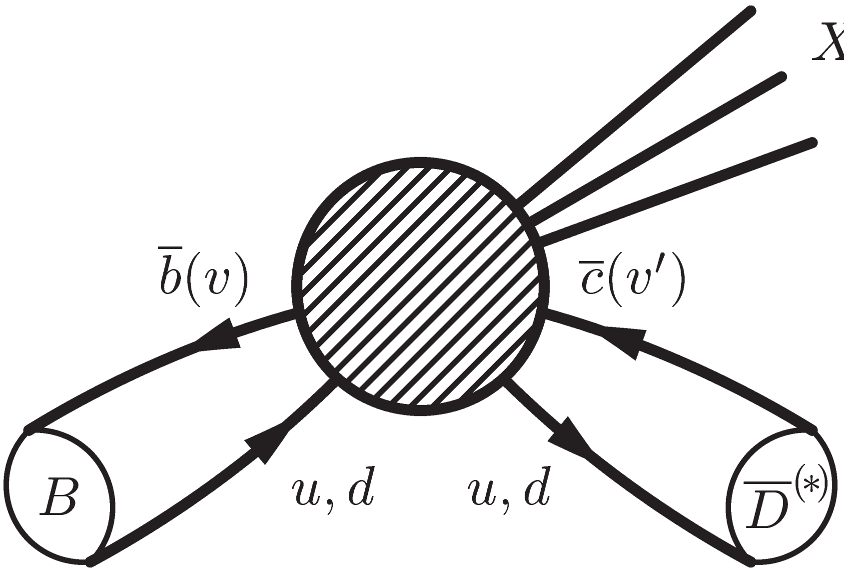
QCD-Based Description of One-Particle Inclusive B-Decays

Work in collaboration with Thomas Mannel and Xavier Calmet
vgl. hep-ph/9907378, TTP99–31, submitted to Phys. Rev. D



8. Schwarzwaldworkshop des Graduiertenkollegs
"Elementarteilchenphysik an Beschleunigern"
der Fakultät für Physik der Universität Karlsruhe
im internationalen Forum Burg Liebenzell
Bad Liebenzell im Schwarzwald
30. September bis 2. Oktober 1999

1. Introduction and Motivation

Status of the description of B decays

- ⋆ $1/m_b$ expansion in exclusive decays $B \to D^{(*)}\ell\nu$:
  Heavy Quark Effective Theory
- ⋆ $1/m_b$ expansion in inclusive decays $B \to X\ell\nu$:
  HQET and Operator Product Expansion
- ⋆ no good method in semi-inclusive decays $B \to DX$:
  let's try to find it...

One particle inclusive decays are interesting

- ⋆ large branching ratios: $\mathrm{Br}(\mathrm{B} \to \overset{(-)}{D}\mathrm{X}) \approx 85\%$
- ⋆ lots of admixture data from CLEO and ARGUS
  with interesting features (spin & charge, spectra)
- ⋆ individual flavour & charge channels
  accessible to LEP I analysis, e.g. in DELPHI
- ⋆ CP-violating effects might show up at CDF II

2. Towards a QCD based method for one-particle inclusive heavy meson decays

2.1. The effective weak Hamiltonian

$$H_{eff} = H_{eff}^{(sl)} + H_{eff}^{(nl)}$$

$$H_{eff}^{(sl)} = \frac{G_F}{\sqrt{2}} V_{cb} \, (\bar{b}c)_{V-A} \, (\bar{\ell}\nu)_{V-A} + h.c.$$

$$H_{eff}^{(nl)} = \frac{G_F}{\sqrt{2}} \sum_{k=1}^{2} C_k \, V_{cb} \, (V_{ud}^* O_k + V_{cs}^* P_k) + h.c.$$

$$\left.\begin{array}{rcl} O_1 & = & (\bar{b}c)_{V-A} \, (\bar{u}d)_{V-A} \\ P_1 & = & (\bar{b}c)_{V-A} \, (\bar{c}s)_{V-A} \end{array}\right\} \text{singlets}$$

$$\left.\begin{array}{rcl} O_2 & = & (\bar{b}T^a c)_{V-A} \, (\bar{u}T^a d)_{V-A} \\ P_2 & = & (\bar{b}T^a c)_{V-A} \, (\bar{c}T^a s)_{V-A} \end{array}\right\} \text{octets}$$

Neglect

- ⊗ penguin operators
- ⊗ octet operators
- ⊗ Cabibbo-suppressed operators
- ⊗ radiative corrections

2.2. Definition of the matrix elements

In the following:

Use the heavy mass limit for both the b and c quarks

$\Longrightarrow$ regard D and D^* as degenerate ground states,

postpone the discussion of D^* decay

$$G(M^2) = \sum_X \left| \langle B | H_{eff} | \overline{D}^{(*)} X \rangle \right|^2 (2\pi)^4 \delta^4 (p_B - p_{\overline{D}} - p_X)$$

$$d\Gamma(B \to \overline{D}^{(*)} X) = \frac{1}{2m_B} \, d\Phi_{\overline{D}} \, G(M^2)$$

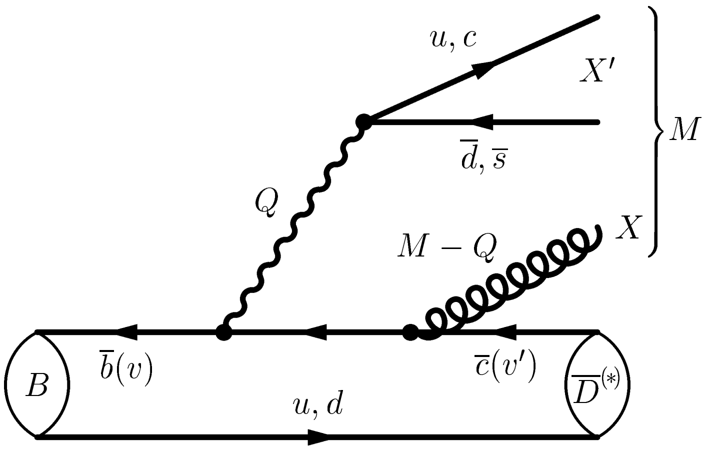
An example to study in detail:

The contribution of the channel $b \to c\bar{u}d$

$$G_1(M^2) = \frac{G_F^2}{2} \left| V_{cb} V_{ud}^* \right|^2 \left| C_1 \right|^2 \sum_X (2\pi)^4 \delta^4 (M - p_X)$$
$$\left\langle B(p_B) \middle| \left(\bar{b}\gamma_\mu (1-\gamma_5) c\right) \left(\bar{u}\gamma^\mu (1-\gamma_5) d\right) \middle| \overline{D}^{(*)}(p_{\overline{D}}) X \right\rangle$$
$$\left\langle \overline{D}^{(*)}(p_{\overline{D}}) X \middle| \left(\bar{d}\gamma^\nu (1-\gamma_5) u\right) \left(\bar{c}\gamma_\nu (1-\gamma_5) b\right) \middle| B(p_B) \right\rangle$$

2.3. Factorization in the large N_C limit

$$\langle B|O_1|\overline{D}^{(*)}X\rangle \to \langle B|(\bar{b}c)_{V-A}|\overline{D}^{(*)}X\rangle\langle 0|(\overline{u}d)_{V-A}|X'\rangle$$



$$G_1(M^2) = \frac{G_F^2}{2}|V_{cb}V_{ud}^*|^2|C_1|^2 \int \frac{d^4Q}{(2\pi)^4} K_{\mu\nu}P^{\mu\nu}$$

$$K_{\mu\nu}(p_B, M, Q) = \sum_X (2\pi)^4\delta^4(M - Q - p_X)$$

$$\langle B|\,\bar{b}\,\gamma_\mu(1-\gamma_5)\,c\,|\overline{D}^{(*)}X\rangle\langle\overline{D}^{(*)}X|\,\bar{c}\,\gamma_\nu(1-\gamma_5)\,b\,|B\rangle$$

$$P_{\mu\nu}(Q) = \sum_{X'} (2\pi)^4\delta^4(Q - p_{X'})$$

$$\langle 0|\,\overline{u}\,\gamma_\mu(1-\gamma_5)\,d\,|X'\rangle\langle X'|\,\overline{d}\,\gamma_\nu(1-\gamma_5)\,u\,|0\rangle$$

2.4. Short distance expansion (SDE)

$$P_{\mu\nu}(Q) = \sum_{X'} (2\pi)^4 \delta^4(Q - p_{X'})$$
$$\left\langle 0 \,\middle|\, \left(\overline{u}\gamma_\mu(1-\gamma_5)d\right) \,\middle|\, X' \right\rangle \left\langle X' \,\middle|\, \left(\overline{d}\gamma_\nu(1-\gamma_5)u\right) \,\middle|\, 0 \right\rangle$$

Apply the optical theorem:

$$P_{\mu\nu}(Q) = \int d^4x\, e^{-iQx}$$
$$\left\langle 0 \,\middle|\, \left(\overline{u}(x)\gamma_\mu(1-\gamma_5)d(x)\right) \left(\overline{d}(0)\gamma_\nu(1-\gamma_5)u(0)\right) \,\middle|\, 0 \right\rangle .$$

Q is large: $\mathcal{O}(m_b - m_c)$ in most of the phase space

$\Longrightarrow$ use the leading term of the SDE:

$$P_{\mu\nu}(Q) = \frac{N_C}{3\pi} \left(Q_\mu Q_\nu - g_{\mu\nu} Q^2\right) \Theta(Q^2)$$

This is the partonic result for massless quarks.

2.5. Heavy quark symmetry

Rephasing of the heavy quark fields $q = q_v e^{-imvx}$

$$K_{\mu\nu}(m_b v, M, Q) = \int d^4 z \sum_X \exp[-i(M-Q)z]$$
$$\langle B(v)|\bar{b}_v(z)\gamma_\mu(1-\gamma_5)c_{v'}(z)|\overline{D}^{(*)}(v')X\rangle$$
$$\langle X\overline{D}^{(*)}(v')|\bar{c}_{v'}(0)\gamma_\nu(1-\gamma_5)b_v(0)|B(v)\rangle$$

Spin projection matrices for the heavy mesons:

$$\begin{aligned} H_B(v) &= \frac{1}{2}\sqrt{m_B}(1+\not{v})\gamma_5 \\ H_{\overline{D}^{(*)}}(v') &= \frac{1}{2}\sqrt{m_D}(1+\not{v}')\begin{cases} \not{\epsilon} & \text{(vector meson)} \\ \gamma_5 & \text{(pseudo-scalar m.)} \end{cases} \end{aligned}$$

Due to heavy quark symmetry we have:

$$K_{\mu\nu} \propto \overline{H}_B\gamma_\mu(1-\gamma_5)H_{\overline{D}^{(*)}} \otimes \overline{H}_{\overline{D}^{(*)}}\gamma_\nu(1-\gamma_5)H_B$$

2.6. Right charm contribution from $b \to c\bar{c}s$

$$G_2(M^2) = \frac{G_F^2}{2} |V_{cb}V_{cs}^*|^2 |C_1|^2 \int \frac{d^4Q}{(2\pi)^4} K_{\mu\nu} P'^{\mu\nu}$$

$$P'_{\mu\nu}(Q) = \sum_{X'} (2\pi)^4 \delta^4(Q - p_{X'})$$
$$\langle 0|\bar{c}\gamma_\mu(1-\gamma_5)s|X'\rangle\langle X'|\bar{s}\gamma_\nu(1-\gamma_5)c|0\rangle$$

Leading term of the short distance expansion:

$$P'_{\mu\nu}(Q) = \left(A(Q^2)\, Q_\mu Q_\nu - B(Q^2)\, Q^2 g_{\mu\nu}\right) \Theta(Q^2 - m_c^2)$$

$$\begin{aligned} A(Q^2) &= \frac{N_C}{3\pi}\left(1 - \frac{m_c^2}{Q^2}\right)^2 \left(1 + 2\frac{m_c^2}{Q^2}\right) \\ B(Q^2) &= \frac{N_C}{3\pi}\left(1 - \frac{m_c^2}{Q^2}\right)^2 \left(1 + \frac{1}{2}\frac{m_c^2}{Q^2}\right) \end{aligned}$$

This is the partonic result for one massive quark.

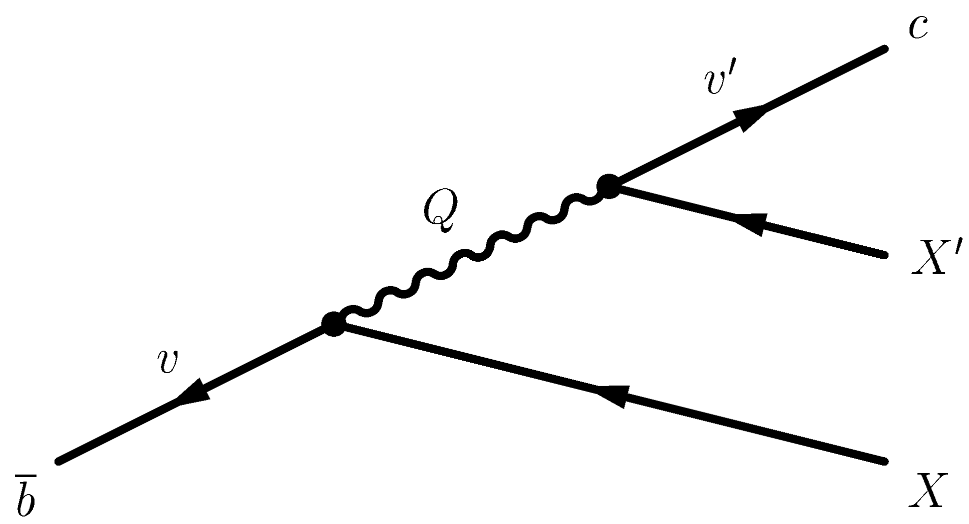
Experimental rates are reproduced using an effective charm quark mass of $m_c^{\text{eff}} = 1.0\,\text{GeV}$.

2.7. Wrong charm contribution

$$G_3 = \frac{G_F^2}{2} \left|V_{cb} V_{cs}^*\right|^2 \left|C_1\right|^2 \sum_X (2\pi)^4 \delta^4(p_B - p_D - p_X)$$

$$\left\langle B(p_B) \,\middle|\, \left(\bar{b}\gamma_\mu(1-\gamma_5)c\right) \left(\bar{c}\gamma^\mu(1-\gamma_5)s\right) \middle|\, D^{(*)} X \right\rangle$$

$$\left\langle D^{(*)}(p_D) X \,\middle|\, \left(\bar{s}\gamma^\nu(1-\gamma_5)c\right) \left(\bar{c}\gamma_\nu(1-\gamma_5)b\right) \middle|\, B(p_B) \right\rangle$$



2.8. Factorization in the wrong charm case

$$G_3(p_B, p_D) = \frac{G_F^2}{2} \left|V_{cb}V_{cs}^*\right|^2 \left|C_1\right|^2 \int \frac{d^4Q}{(2\pi)^4} K'_{\mu\nu} R^{\mu\nu}$$

$$K'_{\mu\nu}(p_B, Q) = \sum_X (2\pi)^4 \delta^4(p_B - p_X - Q)$$
$$\langle B|\bar{b}\gamma_\mu(1-\gamma_5)c|X\rangle\langle X|\bar{c}\gamma_\nu(1-\gamma_5)b|B\rangle$$

Use a short distance expansion:

$$K'_{\mu\nu}(m_b v, Q) = 2\pi\delta\left((m_b v - Q)^2 - m_c^2\right)$$
$$\mathrm{Tr}[\overline{H}_B(v)\gamma_\mu(1-\gamma_5)(m_b \not{v} - \not{Q})\gamma_\nu(1-\gamma_5)H_B(v)]$$

$$R_{\mu\nu}(p_D, Q) = \sum_{X'} (2\pi)^4 \delta^4(Q - p_D - p_{X'})$$
$$\langle 0|\bar{c}\gamma_\mu(1-\gamma_5)s|X'D^{(*)}\rangle\langle D^{(*)}X'|\bar{s}\gamma_\nu(1-\gamma_5)c|0\rangle$$

Constraint by heavy quark symmetry:

$$R_{\mu\nu}(m_c v', Q) \propto \overline{H}_{D^{(*)}}(v')\,\gamma_\mu(1-\gamma_5) \otimes \gamma_\nu(1-\gamma_5)\, H_{D^{(*)}}(v')$$

3. Modelling nonperturbative functions

3.1. The right charm contribution

$$K_{\mu\nu} \propto \overline{H}_B \gamma_\mu (1-\gamma_5) H_{\overline{D}^{(*)}} \otimes \overline{H}_{\overline{D}^{(*)}} \gamma_\nu (1-\gamma_5) H_B$$

Many ways to contract the four light quark indices...
Same current matrix element as in the semileptonic case
$\Longrightarrow$ use the same contraction

$$\begin{aligned} K_{\mu\nu}(p_B, M, Q) = (2\pi)^4 \delta^4(M-Q)\, \eta(vv') \\ \mathrm{Tr}[\overline{H}_B(v)\gamma_\mu(1-\gamma_5)H_{\overline{D}^{(*)}}(v')] \\ \mathrm{Tr}[\overline{H}_{\overline{D}^{(*)}}(v')\gamma_\nu(1-\gamma_5)H_B(v)] \end{aligned}$$

Simplest ansatz for the single non-perturbative function:
Saturation assumption $\Gamma(B \to DX\ell\nu) = \Gamma(B \to D^*\ell\nu)$
near the non-recoil point $v = v'$

$$\eta(vv') = |\xi(vv')|^2$$

$$\xi(vv') = 1 - a(vv' - 1), \qquad a = 0.84$$

3.2. The wrong charm contribution

$$R_{\mu\nu}(m_c v', Q) \propto \overline{H}_{D^{(*)}}(v')\, \gamma_\mu (1-\gamma_5) \otimes \gamma_\nu (1-\gamma_5)\, H_{D^{(*)}}(v')$$

Try two of the many possible contractions:

Model 1 — inspired by factorization:

$$R_{\mu\nu}(m_c v', Q) = 2\, F(v'Q, Q^2)$$
$$\mathrm{Tr}[\overline{H}_{D^{(*)}}(v')\, \gamma_\mu (1-\gamma_5)]\ \mathrm{Tr}[\gamma_\nu (1-\gamma_5)\, H_{D^{(*)}}(v')]$$

Model 2 — inspired by the partonic result:

$$R_{\mu\nu}(m_c v', Q) = 2\, \tilde{F}(v'Q, Q^2)$$
$$\mathrm{Tr}[\overline{H}_{D^{(*)}}(v')\gamma_\mu (1-\gamma_5)(\not{Q} - m_c \not{v}')\gamma_\nu (1-\gamma_5) H_{D^{(*)}}(v')]$$

Massless $s \Longrightarrow$ assume on-shell hadronic recoil:

$$\begin{aligned} F(v'Q, Q^2) &= f\ 2\pi\delta\left((Q - m_c v')^2\right) \\ \tilde{F}(v'Q, Q^2) &= \tilde{f}\ 2\pi\delta\left((Q - m_c v')^2\right) \end{aligned}$$

Fit to CLEO measurement of the wrong charm rate:

$$\begin{aligned} f &= 0.147\, GeV \\ \tilde{f} &= 0.121 \end{aligned}$$

3.3. Effects of $D^* \to D$ decays

Off the heavy mass limit, no degeneracy of D and D^*
$\Longrightarrow$ D^* suffers decay into $D\pi$ or $D\gamma$

$$\begin{aligned} \mathrm{Br}(D^{*-} \to D^- Y) &= 0.32 \\ \mathrm{Br}(D^{*-} \to \overline{D}^0 Y) &= 0.68 \\ \mathrm{Br}(\overline{D}^{*0} \to \overline{D}^0 Y) &= 1 \end{aligned}$$

$\Longrightarrow$ Feeddown:

$$\begin{aligned} \mathrm{Br}(B \to D^- X) &= \mathrm{Br}(B \to D^-_{\mathrm{dir}} X) + \\ &\quad \mathrm{Br}(B \to D^{*-} X)\mathrm{Br}(D^{*-} \to D^- Y) \\ \mathrm{Br}(B \to \overline{D}^0 X) &= \mathrm{Br}(B \to \overline{D}^0_{\mathrm{dir}} X) + \mathrm{Br}(B \to \overline{D}^{*0} X) + \\ &\quad \mathrm{Br}(B \to D^{*-} X)\mathrm{Br}(D^{*-} \to \overline{D}^0 Y) \end{aligned}$$

Use the results of Section 2 for the following channels:

$$\begin{array}{cccc@{\qquad}cccc} B^+ & \to & \overline{D}^0_{\mathrm{dir}} & X & B^0 & \to & D^-_{\mathrm{dir}} & X \\ B^+ & \to & \overline{D}^{*0} & X & B^0 & \to & D^{*-} & X \end{array}$$

All other non-spectator-like channels are neglected.

4. Results and comparison with data

4.1. Admixture branching ratios

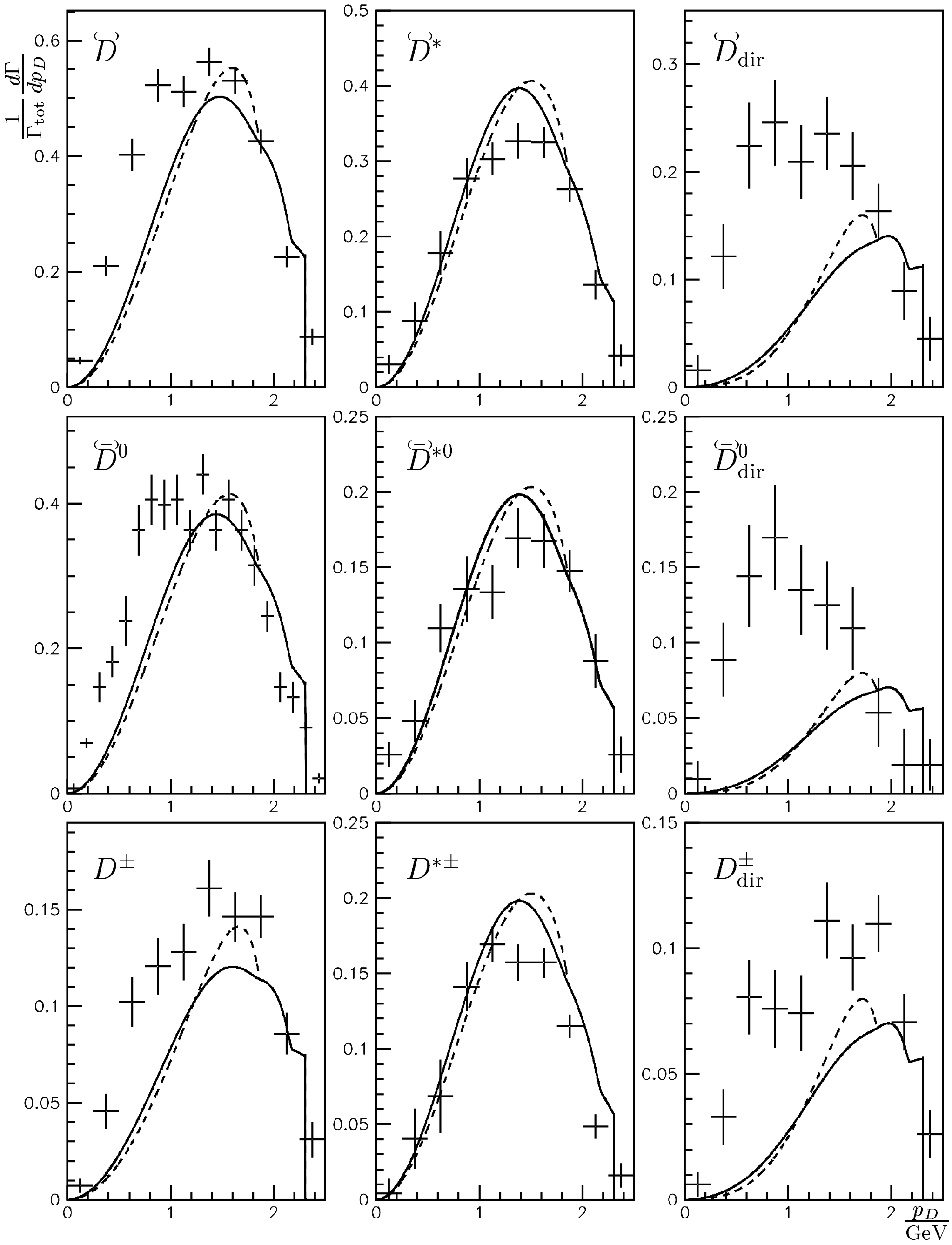
Mode	Theory	Experiment
$B \to \overset{(-)}{D} X$	68.8%	$(87.2 \pm 3.5)\%$
$B \to \overset{(-)}{D}{}^{*} X$	51.8%	$(48.7 \pm 3.1)\%$
$B \to \overset{(-)}{D}_{\mathrm{dir}} X$	17.1%	$(38.5 \pm 4.7)\%$
$B \to \overset{(-)}{D}{}^{0} X$	52.0%	$(63.1 \pm 2.9)\%$
$B \to D^{\pm} X$	16.8%	$(24.1 \pm 1.9)\%$
$B \to \overset{(-)}{D}{}^{*0} X$	25.9%	$(26.0 \pm 2.7)\%$
$B \to D^{*\pm} X$	25.9%	$(22.7 \pm 1.6)\%$
$B \to \overset{(-)}{D}{}^{0}_{\mathrm{dir}} X$	8.5%	$(21.7 \pm 4.1)\%$
$B \to D^{\pm}_{\mathrm{dir}} X$	8.5%	$(16.8 \pm 2.9)\%$
$B \to D^{-} \ell^{+} \nu_{\ell} X$	2.0%	$(2.7 \pm 0.8)\%$
$B \to \overline{D}{}^{0} \ell^{+} \nu_{\ell} X$	6.5%	$(7.0 \pm 1.4)\%$
$B \to D^{*-} \ell^{+} \nu_{\ell} X$	3.3%	$(2.8 \pm 0.4)\%$
$B \to \overline{D}{}^{*0} \ell^{+} \nu_{\ell} X$	3.3%	$(3.2 \pm 0.7)\%$

4.2. Single channel branching ratios

Vector Modes		Pseudoscalar Modes	
$B^0 \to \overline{D}^{*0} X$	0%	$B^0 \to \overline{D}^0 X$	31.8%
$B^0 \to D^{*-} X$	46.8%	$B^0 \to D^- X$	29.1%
$B^+ \to \overline{D}^{*0} X$	46.8%	$B^+ \to \overline{D}^0 X$	60.9%
$B^+ \to D^{*-} X$	0%	$B^+ \to D^- X$	0%
$B^0 \to D^{*0} X$	2.5%	$B^0 \to D^0 X$	5.7%
$B^0 \to D^{*+} X$	2.5%	$B^0 \to D^+ X$	2.2%
$B^+ \to D^{*0} X$	2.5%	$B^+ \to D^0 X$	5.7%
$B^+ \to D^{*+} X$	2.5%	$B^+ \to D^+ X$	2.2%

$\hookrightarrow$ to be measured,
e.g. by DELPHI at LEP I...

4.3. $\overset{(-)}{D}$ momentum spectra



5. Conclusions

5.1. Achievements

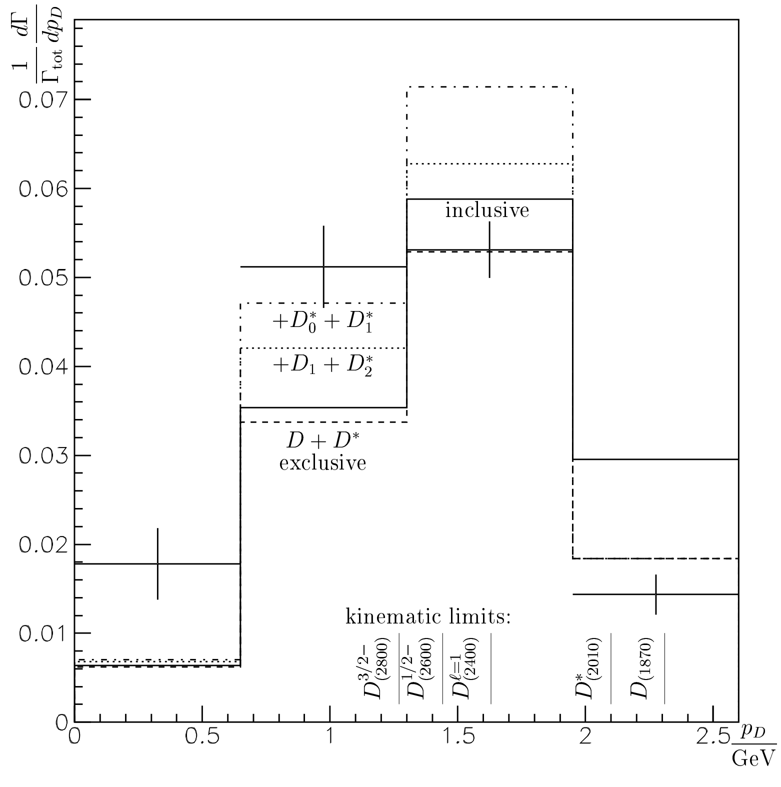
- ⋆ QCD-based descriptions of the decays $B \to \overset{(-)}{D} X$, in principle allowing to calculate corrections in $\Lambda_{QCD}/(m_b - m_c)$, $1/N_C$ and $\alpha_s(m_c)$
- ⋆ Simple heavy quark spin symmetry relations between $\overset{(-)}{D}$ and $\overset{(-)}{D}{}^*$ rates and spectra
- ⋆ First simple suggestions for the non-perturbative functions involved
- ⋆ Predictions for rates and spectra for various individual decay channels
- ⋆ Shapes of the $\overset{(-)}{D}{}^*$ spectra are fine
- ⋆ Charge counting $\dfrac{\mathrm{Br}(B \to \overset{(-)}{D}{}^0 X)}{\mathrm{Br}(B \to D^\pm X)} = \begin{cases} 3.1 & \text{(theo)} \\ 2.6 \pm 0.3 & \text{(exp)} \end{cases}$

5.2. Remaining Problems

- ⊗ Spin counting — known problem, still mysterious

$$\frac{\mathrm{Br}(B \to \overset{(-)}{D}{}^{*} X)}{\mathrm{Br}(B \to \overset{(-)}{D}_{\mathrm{dir}} X)} = \begin{cases} 3.0 & \text{(theo)} \\ 1.3 \pm 0.2 & \text{(exp)} \end{cases}$$

- ⊗ Shapes of the $\overset{(-)}{D}$ spectra are wrong, particularly for low $\overset{(-)}{D}$ momenta, even in the semi-leptonic case:



5.3. Further Theoretical Possibilities

- ◇ Refine the non-perturbative functions
  by considering (non-)resonant $D(n\pi)$ decays

- ◇ Construct a complete set
  of non-perturbative functions
  and perform a global fit to the data

5.4. Next Projects

EXP: Analyse DELPHI Z^0 data
regarding individual channels *(Ingo Schwarze)*

TH: Construct CP-violation observables
for one-particle inclusive decays *(Xavier Calmet)*