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IMPACT of QCD on PRECISION TESTS

I TWO LOOP ELECTROWEAK RESULTS — Weiglein

II PURE QCD CORRECTIONS, QUARK MASS EFFECTS and UNCERTAINTIES — Chetyrkin, K, Kwiatkowski

III QCD, $\Delta\rho$, Δr and m_t — Chetyrkin, K, Steinhauser

IV NONFACTORIZABLE VERTEX CORRECTIONS $Z \to q\bar{q}$, $Z \to b\bar{b}$ — Czarnecki, K, Harlander, Seidensticker, Steinhauser

II PURE QCD CORRECTIONS, QUARK MASS EFFECTS and UNCERTAINTIES

dominant (nonsinglet, $m_Q = 0$) term

$$\boxed{\Gamma(Z \to \text{had}) \propto \left(1 + \frac{\alpha_s}{\pi} + 1.409 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.767 \left(\frac{\alpha_s}{\pi}\right)^3 + \ldots\right)}$$

$\Rightarrow$ optimal way to measure α_s

- cancels in ratios like $\boxed{\Gamma_b/\Gamma_{\text{had}}}$
- remaining terms $\Rightarrow$ impact on precision tests

A) $m_b \neq 0$

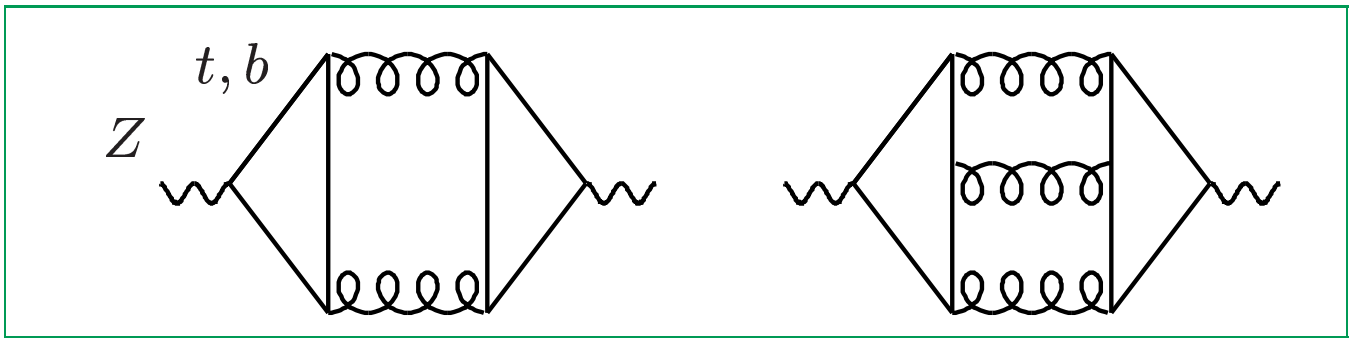
$\Rightarrow$ reduction of $\Gamma(Z \to b\bar{b})$:

$\Delta\Gamma_{m_b} = -1.53$ MeV (m_b(pole)=4.7 $\pm$0.2 GeV)

dominant uncertainty from δm_b:

$$\delta\Delta\Gamma_{m_b} \approx 0.15 \text{ MeV}$$

B) singlet terms



$$\Delta\Gamma_{\text{singl}} = -1.82 \text{ MeV}$$

Uncertainty

from $\delta M_t = \pm 5$ GeV: $\quad \delta\Delta\Gamma_{\text{singl}} = 0.02$ MeV

from $\delta\alpha_s$: $\quad \delta\Delta\Gamma_{\text{singl}} = 0.1$ MeV

$\Rightarrow$ Quark mass and QCD uncertainty:

$$\delta\frac{\Gamma_b}{\Gamma_{\text{had}}} \approx 1\text{–}2 * 10^{-4}$$

III QCD. $\Delta\rho$, Δr, $\Delta\kappa$ and m_t

leading term:

$$\Delta\rho = 3\frac{\sqrt{2}G_F M_t^2}{16\pi^2}$$

large difference between $\overline{\text{MS}}$ and OS mass:

$$M_t(\text{OS}) - M_t(\overline{\text{MS}}) \approx 10 \text{ GeV !}$$

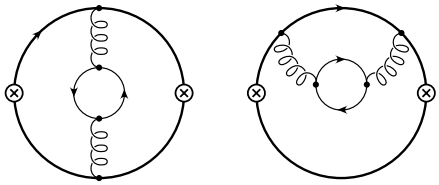
α_s($\widehat{=}$ two loop) and
$\Rightarrow$ α_s^2($\widehat{=}$three loop) required.

Δr, $\Delta\kappa$ are complicated functions of M_t, M_Z and M_W

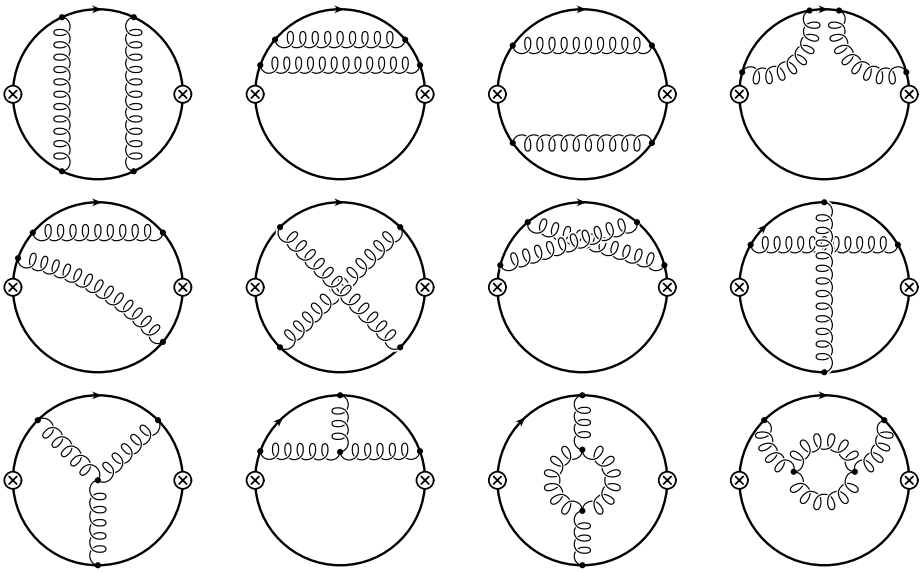
two loop: analytical result
three loop:

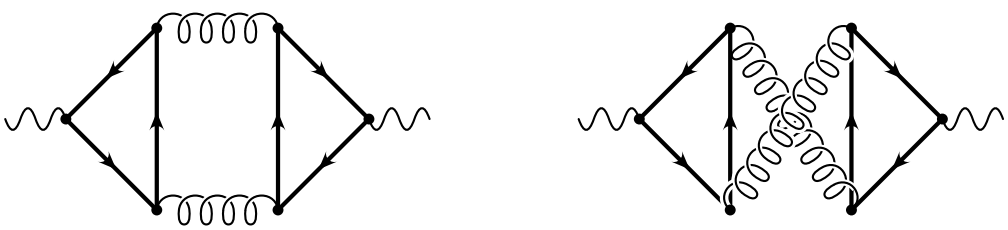
- expansion in $(M_{W,Z}/M_t)^2$ to arbitrary order
- analytic fct of expansion parameter!
- tadpole integrals $\rightsquigarrow$ Fig
- Computer algebra: FORM

Three–Loop Diagrams



Purely gluonic contribution to $\mathcal{O}(\alpha_s^2)$





Result: δM_W in MeV

	α_s^0	α_s^1	α_s^2
M_t^2	611.9	-61.3	-10.9
log+const	136.6	-6.0	-2.6
$1/M_t^2$	-9.0	-1.0	-0.2
Σ	739.5	-68.3	-13.7

α_s^2–term:

13.7 MeV $\quad \widehat{=} \quad \delta m_t = 2$ GeV

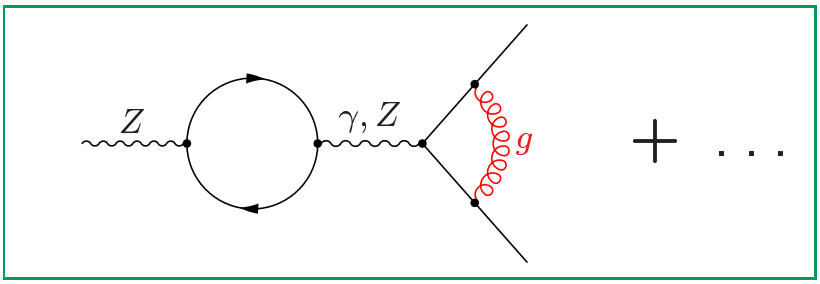
IV NONFACTORIZABLE VERTEX CORRECTIONS

previously

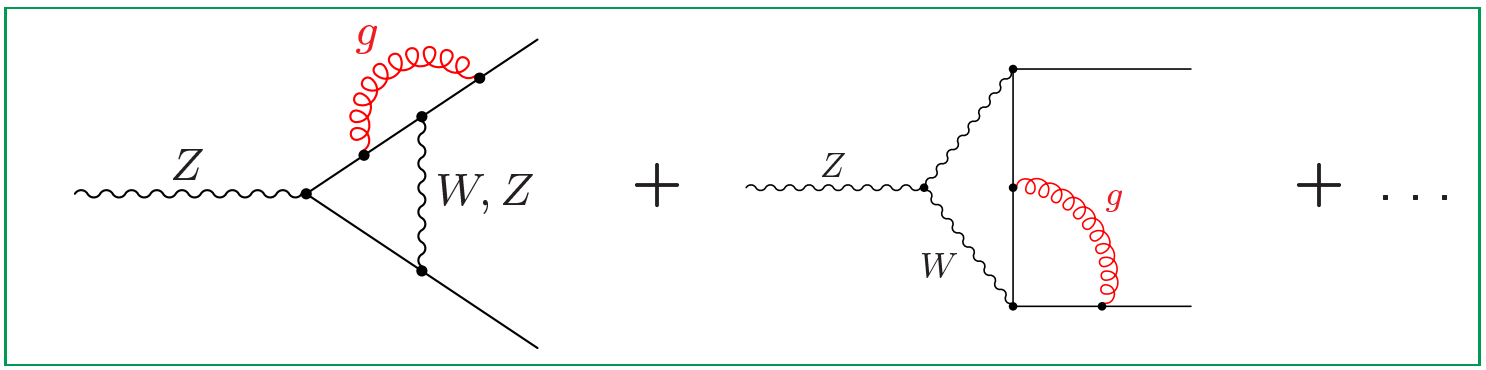
$$\Gamma = \Gamma_{\mathrm{BORN}}(1+\delta_{\mathrm{weak}}+\delta_{\mathrm{strong}})$$
$$\stackrel{?}{=} \Gamma_{\mathrm{BORN}}(1+\delta_{\mathrm{weak}})(1+\delta_{\mathrm{strong}})$$

with $\delta_{\mathrm{strong}} = \frac{\alpha_S}{\pi} + \ldots$

valid for subclass
("oblique" one loop corrections)



not valid for vertex corrections



instructive example: QCD⊗QED
(Kataev)

$$1 + \frac{\alpha_s}{\pi} + Q_q^2 \frac{3}{4}\frac{\alpha}{\pi} - \frac{\alpha_s}{\pi} Q_q^2 \frac{1}{3}\frac{\alpha}{\pi} \qquad \text{correct result}$$

$$\neq \quad \left(1 + \frac{\alpha_s}{\pi}\right)\left(1 + Q_q^2 \frac{3}{4}\frac{\alpha}{\pi}\right) \qquad \text{factorization}$$

QCD⊗EW strategy:

evaluate difference between full result and factorized corrections

⇒ vertex corrections only!

light (u, d, c, s) quarks: (Czarnecki, K.)

(expansion in M_W^2/M_Z^2 and $M_Z^2/4M_W^2$)

$$\delta\Gamma(\text{QCD} \otimes \text{EW}) - \underbrace{\frac{\alpha_s}{\pi}\delta\Gamma(1\text{Loop EW})}_{\text{factorization}} = -0.55 \text{ MeV}$$

$$\widehat{=} \quad \delta\alpha_s \approx 1.10^{-3}$$

$b\bar{b}$

$\alpha_s G_F m_t^2$ (Fleischer et al.)

$\alpha_s \alpha_{weak} \ln m_t^2$ (Kwiatkowski, Steinhauser)

$\alpha_s \alpha_{weak}(\text{const} + 1/m_t^2)$ (Harlander, Seidensticker, Steinhauser, hep-ph/9712228)

expansion in

$$M_Z^2/m_t^2 \quad \text{and} \quad (M_Z/2M_W)^2$$

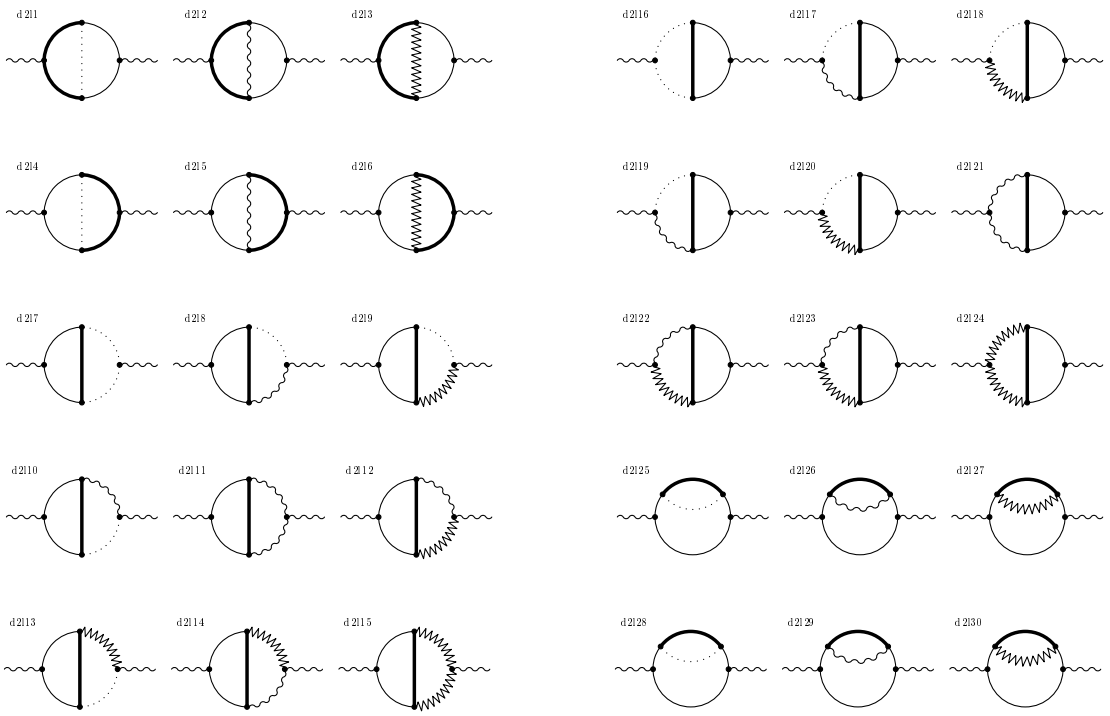
diagrams ($\rightsquigarrow$ Fig)

result:

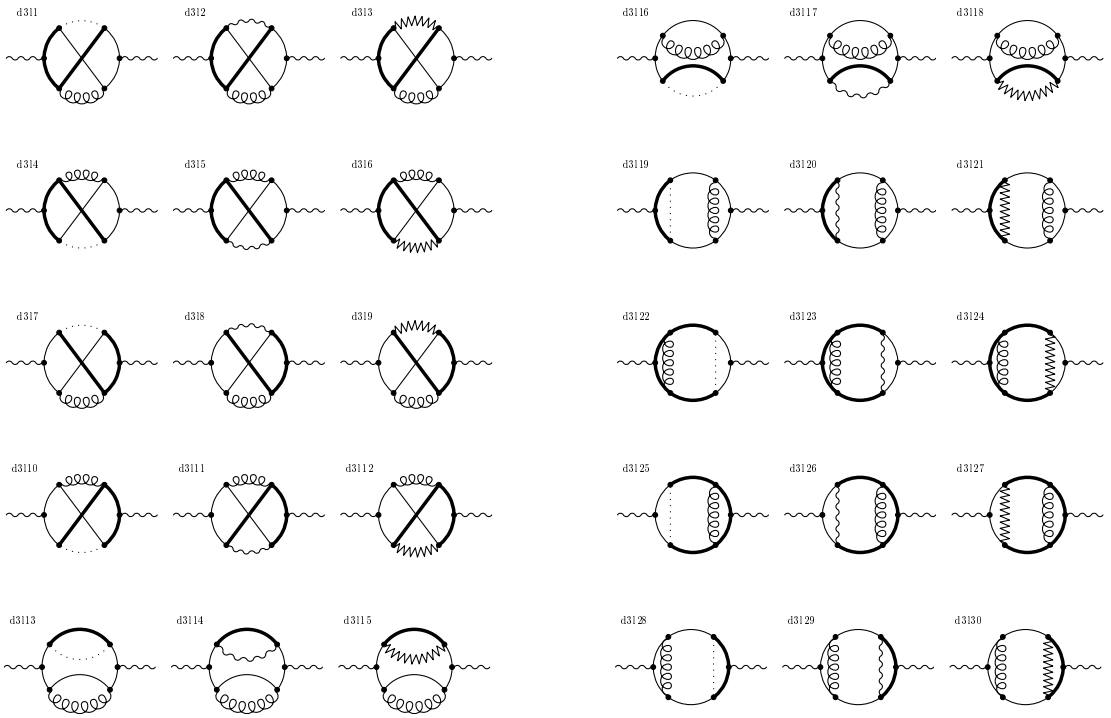
$$\delta\Gamma(Z \to b\bar{b}) - \delta\Gamma(Z \to d\bar{d}) =$$

$$\underbrace{(-5.6}_{m_t^2} \quad \underbrace{-0.79}_{\text{subleading}}\underbrace{)}_{\text{no QCD}} \quad \underbrace{+0.50}_{m_t^2} \quad \underbrace{+0.06)}_{\text{subleading}}\underbrace{\text{ MeV}}_{\mathcal{O}(\alpha_s)}$$

Two–Loop

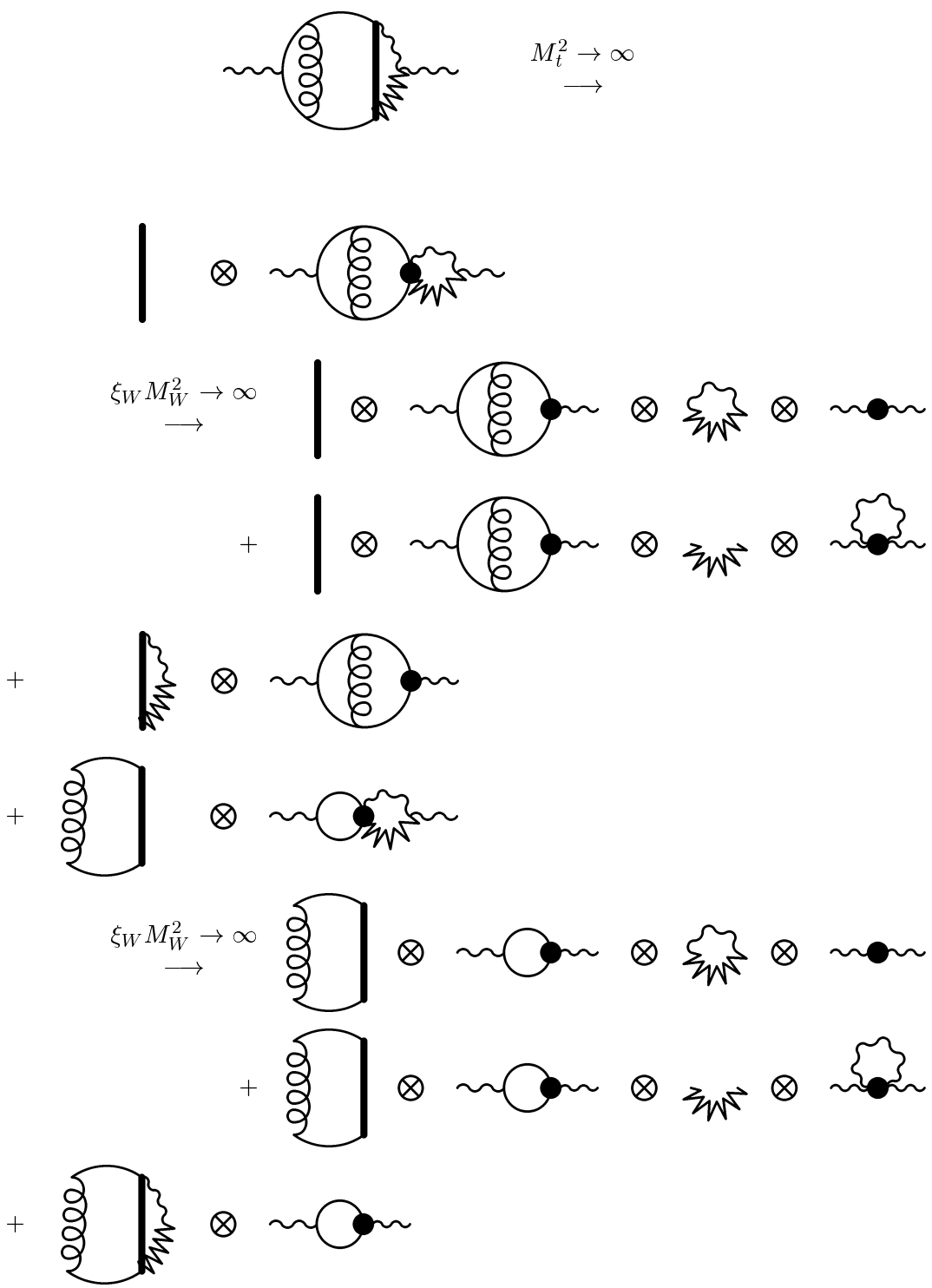


Three–Loop (306 diagrams)



(Seidensticker)

Expansion



(Seidensticker)

large cancellations among subleading terms (const, log $1/m_t^2$) for one– and two–loop contributions!

arbitrary units:

$$
\begin{array}{ll}
\frac{\alpha_s}{\pi} & [1.16 \qquad\qquad\quad\; m_t^2 \\
& +(1.21 - 0.49) \qquad m_t^0(\ln + \text{const}) \\
& +(0.30 - 0.65) \qquad m_t^{-2}(\ln + \text{const}) \\
& +(0.02 - 0.21 + 0.01) \quad m_t^{-4}(\ln^2 + \ln + \text{const}) \\
& +\text{small} \qquad\qquad\quad\;] \\
\\
= & \frac{\alpha_s}{\pi}[1.16 + 0.13]
\end{array}
$$

$m_t^2 + \ln m_t^2$ misleading!

SUMMARY

- mixed two– and three–loop results
  $\sim$ exp. precision ($\delta M_W \sim 20$MeV)

- remaining uncertainty from QCD
  significantly smaller