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PERTURBATIVE QCD POTENTIAL and $t\bar{t}$ THRESHOLD

Peter	NPB501(1997)471
Ježabek, Peter, Sumino	PLB428(1998)352
Ježabek, K, Peter, Sumino, Teubner	Abstract #346, PRD58(1998)0140006

I WHY

II STATUS of PERTURBATIVE CALCULATIONS

III MOMENTUM versus COORDINATE SPACE: PREDICTIONS and AMBIGUITIES

I WHY

$\boxed{V_{pQCD}}$

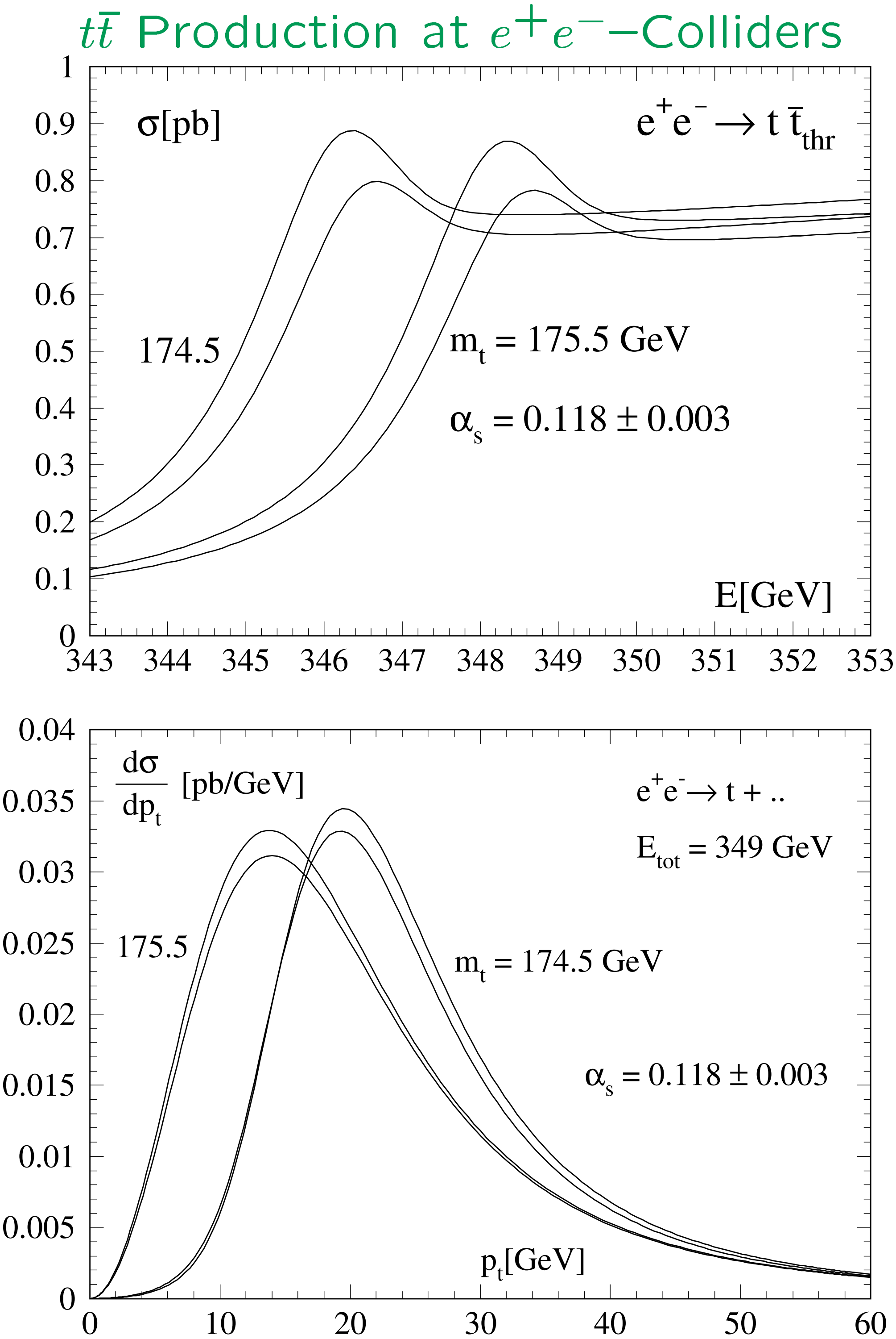
important ingredient for

- $\boxed{\text{QUARKONIUM spectroscopy}}$

- $\boxed{\text{sum rules}}$

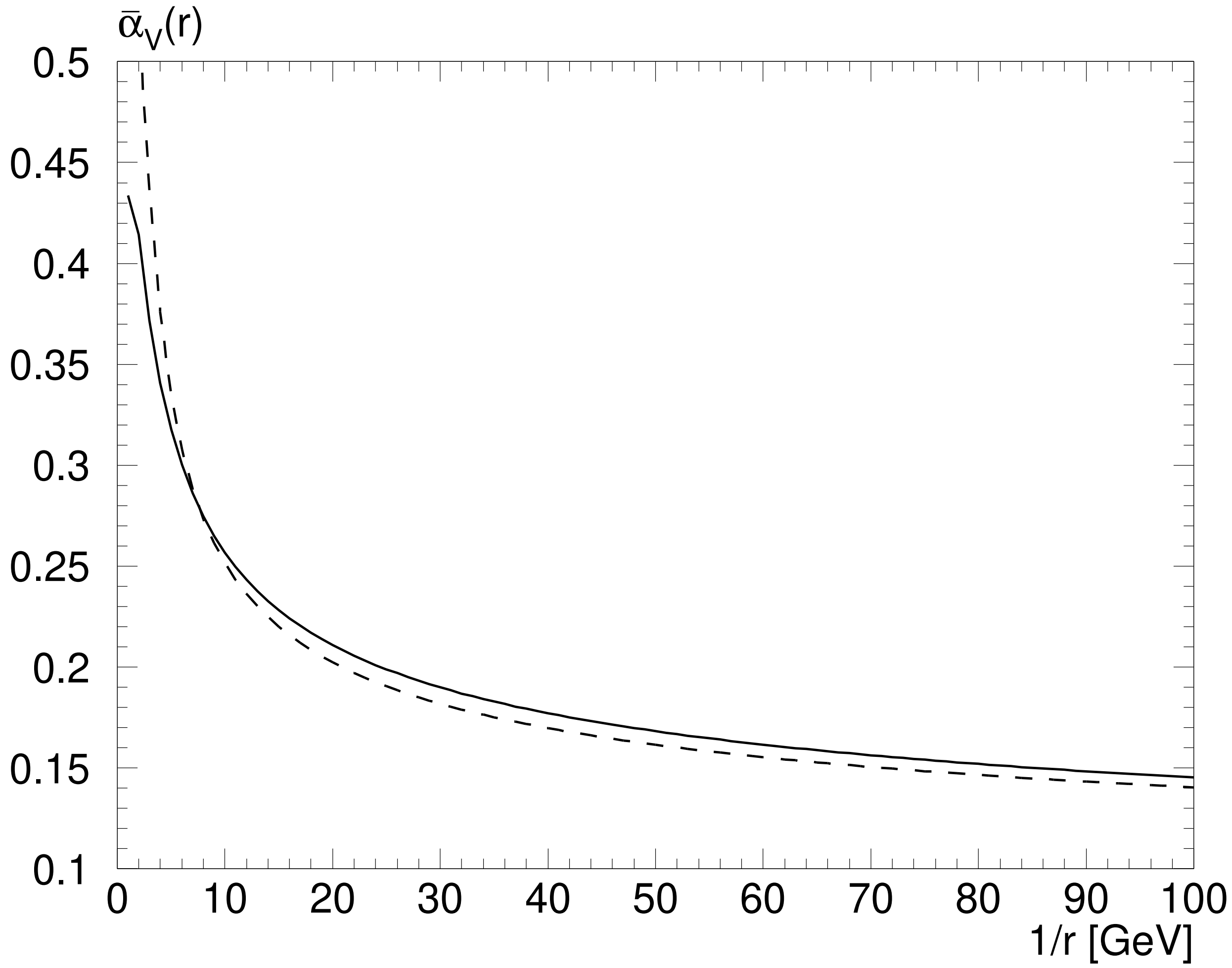
- $t\bar{t}$ threshold behaviour at e^+e^- collider
  $\rightarrow m_t$, Γ_t, weak couplings
  $\rightsquigarrow$ Fig

- $\boxed{\text{test of pQCD}}$



$t\bar{t}$ Production at e^+e^-–Colliders
σ[pb]
$e^+e^- \to t\,\bar{t}_{thr}$
174.5
m_t = 175.5 GeV
α_s = 0.118 ± 0.003
E[GeV]
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
343 344 345 346 347 348 349 350 351 352 353
$\frac{d\sigma}{dp_t}$ [pb/GeV]
$e^+e^- \to t + ..$
E_{tot} = 349 GeV
175.5
m_t = 174.5 GeV
α_s = 0.118 ± 0.003
p_t[GeV]
0 0.005 0.01 0.015 0.02 0.025 0.03 0.035 0.04
0 10 20 30 40 50 60

Effective Charge



solid line: numerical Fourier–Transformation from momentum space momentum space:

$$V_{\mathrm{PQCD}} \equiv -C_F \frac{4\pi\alpha_{\mathrm{V}}(q^2)}{q^2}$$

$$\alpha_{\mathrm{V}} = \alpha_{\overline{\mathrm{MS}}} \Big(\underset{\substack{\uparrow \\ \text{LO}}}{1} \quad + \ldots \underset{\substack{\uparrow \\ \text{NLO} \\ \text{Fischler(1977)} \\ \text{Billoire(1980)}}}{\alpha_{\overline{\mathrm{MS}}}} \quad + \ldots \underset{\substack{\uparrow \\ \text{NNLO} \\ \text{Peter(1997)}}}{\alpha^2_{\overline{\mathrm{MS}}}} \Big)$$

few NNLO results for rigorous QCD tests available (R_{had}, DIS moments, V_{QCD})

II STATUS OF CALCULATIONS

$$\alpha_{\mathrm{V}}(q^2) = \alpha_{\overline{\mathrm{MS}}}(\mu^2) \sum \tilde{a}_n \left(\frac{\mu^2}{q^2}\right) \left(\frac{\alpha_{\overline{\mathrm{MS}}}\left(\mu^2\right)}{4\pi}\right)^n$$

$$\boxed{\begin{aligned} \tilde{a}_0 &= 1 \\ \tilde{a}_1 &= a_1 + \beta_0 \ln\frac{\mu^2}{q^2} \\ \tilde{a}_2 &= a_2 + \beta_0^2 \ln^2\frac{\mu^2}{q^2} + (\beta_1 + 2\beta_0 a_1)\ln\frac{\mu^2}{q^2} \end{aligned}}$$

$$a_1 = \frac{31}{9}C_A - \frac{20}{9}T_F n_f$$

Fischler, Billoire

$$\begin{aligned} a_2 = & \left(\frac{4343}{162} + 6\pi^2 - \frac{\pi^2}{4} + \frac{22}{3}\zeta_3\right) C_A^2 \\ & - \left(\frac{1798}{81} + \frac{56}{3}\zeta_3\right) C_A T_F n_f \\ & - \left(\frac{55}{3} - 16\zeta_3\right) C_F T_F n_f + \left(\frac{20}{9}T_F n_f\right)^2 \end{aligned}$$

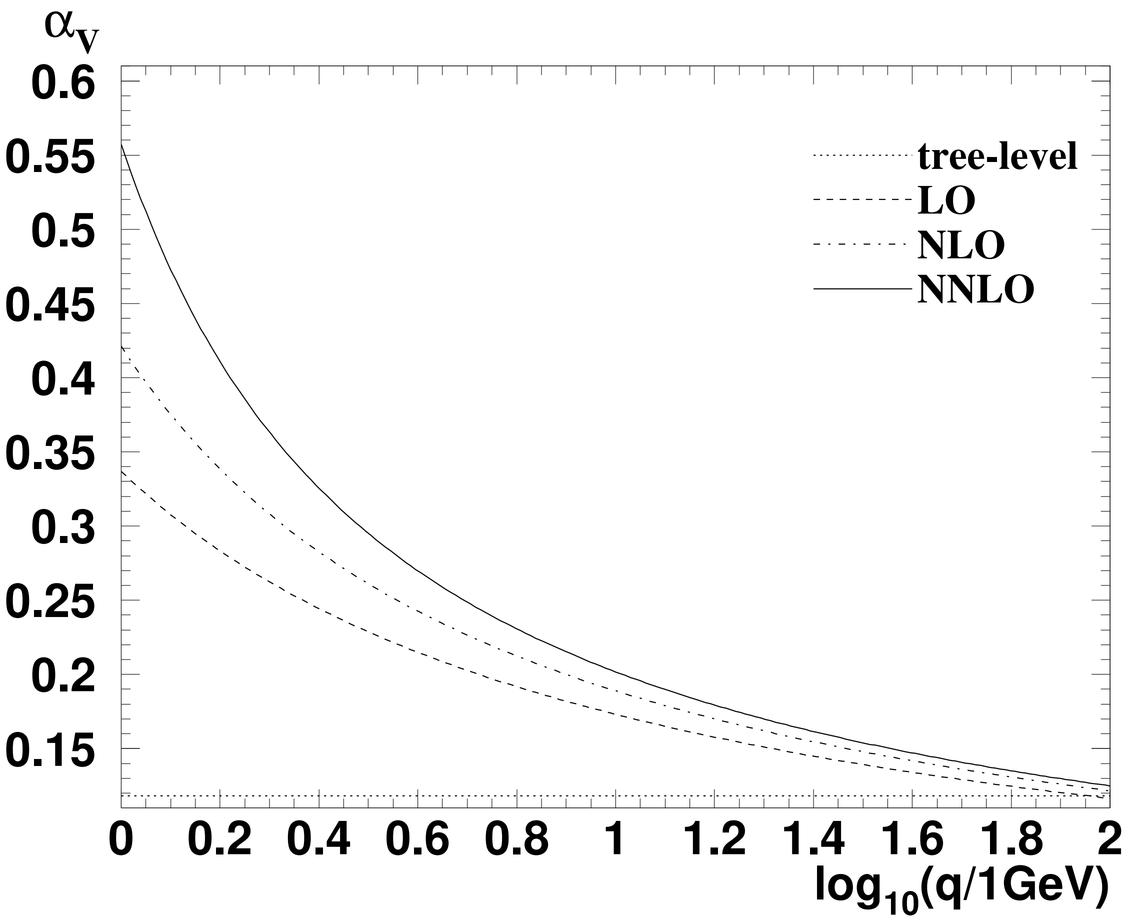
Peter

a_2: numerically large ($n_f = 5$)

$$\begin{aligned} a_1 &= 10.33 - 1.11 \cdot n_f &&= 4.78 \\ a_2 &= 630. - 65. \cdot n_f + 1.2 \cdot n_f^2 &&= 333.5 \end{aligned}$$

momentum space potential:

- RG improvement: $\mu^2 = q^2$
- $\alpha_{\overline{\rm MS}}(q^2)$ through RG (employing β_0, β_1, β_2)



large deviations from coulombic behaviour, enhanced by higher orders

III TRANSITION TO COORDINATE SPACE

Fixed order result (momentum space):

$$\boxed{\begin{aligned}V(q^2) = -C_F\frac{4\pi\alpha_{\overline{\rm MS}}(\mu^2)}{q^2}\Bigg[1 + \frac{\alpha_{\overline{\rm MS}}(\mu^2)}{4\pi}\left(a_1 + \beta_0 \ln\frac{\mu^2}{q^2}\right) \\ + \frac{\alpha_{\overline{\rm MS}}(\mu^2)}{4\pi}\left(a_2 + \beta_0 \ln^2\frac{\mu^2}{q^2} + (\beta_1 + 2\beta_0 a_1)\ln\frac{\mu^2}{q^2}\right)\Bigg]\end{aligned}}$$

Fourier Transformation $\Rightarrow$
Fixed order result (coordinate space):

$$\boxed{\begin{aligned}V(r) &= -C_F\frac{\alpha_{\overline{\rm MS}}(\mu^2)}{r}\Big[1 + \frac{\alpha_{\overline{\rm MS}}(\mu^2)}{4\pi}\left(2\beta_0\ln(\mu r') + a_1\right) \\ &\quad + \left(\frac{\alpha_{\overline{\rm MS}}(\mu^2)}{4\pi}\right)^2\Big(\beta_0^2(4\ln^2(\mu r') + \frac{\pi^2}{3}) \\ &\quad + 2(\beta_1 + 2\beta_0 a_1)\ln(\mu r') + a_2\Big) + \ldots\Big]\end{aligned}}$$

with $r' = e^{\gamma_E} r$

RG: $\mu = r'$

$$\boxed{V(r) \equiv -C_F \frac{\bar{\alpha}_{\mathrm{V}}(\frac{1}{r})}{r}}$$

with

$$\bar{\alpha}_{\mathrm{V}}(\frac{1}{r}) = \alpha_{\mathrm{V}}(q = \frac{1}{r'}) \cdot$$

$$\Big[\underbrace{(1 + 1.225\alpha_{\mathrm{V}}^2)}_{\text{fixed order}} + \underbrace{(5.60\alpha_{\mathrm{V}}^3 + 32.2\alpha_{\mathrm{V}}^4)}_{\text{additional RG and Fourier transform}} + \ldots \Big]$$

asymptotic series!

(in explicit form checked for Richardson potential) two choices (equivalent in NNLO)

$$
\begin{tikzcd}
\underset{\text{fixed order}}{\mathsf{V}(q)} \arrow[r, "\text{FOURIER}"'] \arrow[d, "\mathrm{RG}(\overline{\mathrm{MS}})"] & \underset{\text{fixed order}}{\overline{\mathsf{V}}(1/r)} \arrow[d, "\mathrm{RG}(\overline{\mathrm{MS}})"'] \\
\underset{\mathrm{RG}}{\mathsf{V}(q)} \arrow[r, leftrightarrow, "?"] & \underset{\mathrm{RG}}{\overline{\mathsf{V}}(1/r)}
\end{tikzcd}
$$

V and $\overline{\mathsf{V}}$ are not exactly the mutual Fourier transforms, difference formally higher order

numerically important: $\boxed{t\bar{t}: \quad \alpha_{\mathsf{V}} = 0.138}$

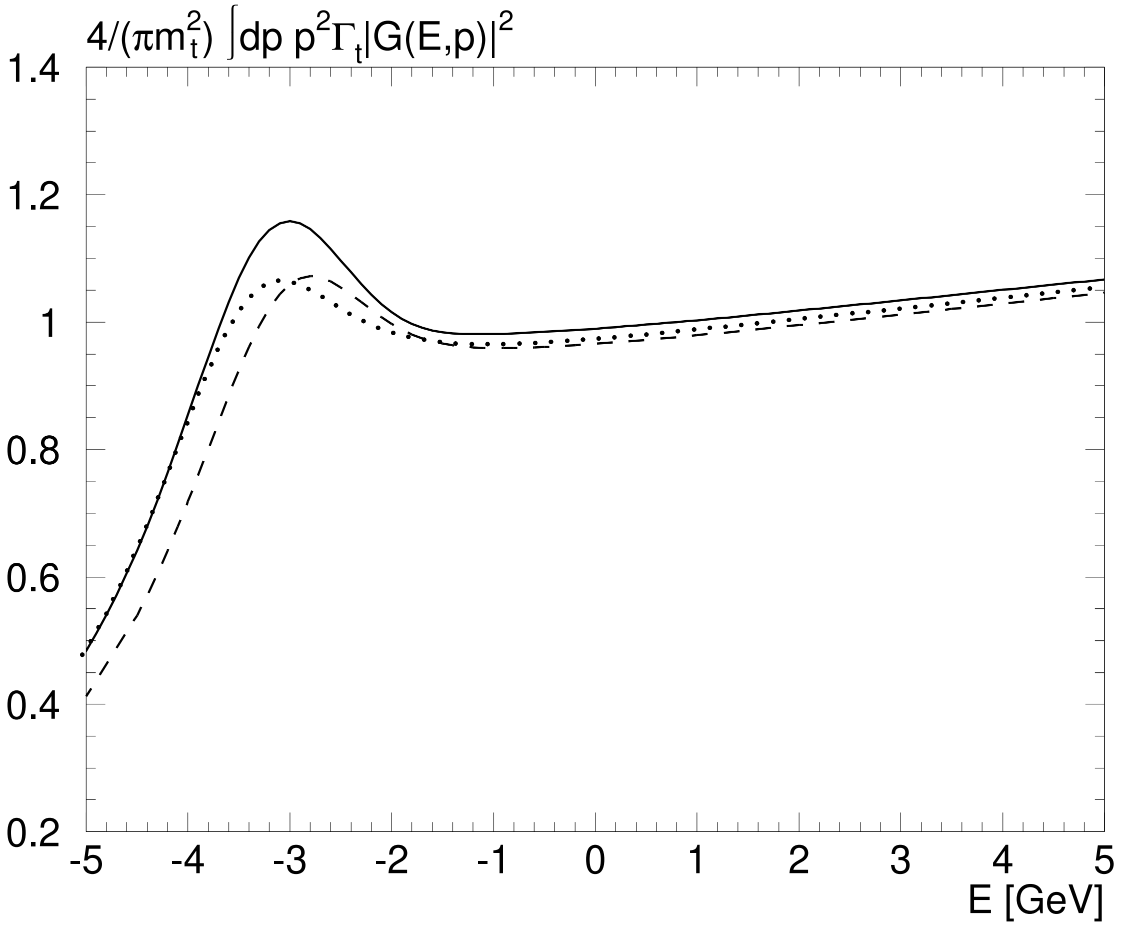
relative difference between momentum and coordinate space:

$$\boxed{\Delta\alpha_s/\alpha_s \approx 5.6\alpha_{\mathsf{V}}^3 + 32.2\alpha_{\mathsf{V}}^4 \approx 0.025}$$

- "peak" cross section $\sim \alpha_S^3$
- continuum close to threshold $\sim \alpha_S$

$\Rightarrow$ difference between momentum and coordinate space approach $\sim$ 7% on peak
2.5% continuum

consistent with numerical study



difference remains stable under variation of scale, . . .

SUMMARY

- $\star$ $\boxed{\text{NNLO result available for } \alpha_{\mathrm{V}} \leftrightarrow \alpha_{\overline{\mathrm{MS}}}}$

- $\star$ $\boxed{\text{numerically large corrections}}$

- $\star$ significant differences between coordinate and momentum space approach

- $\star$ uncertainty for $t\bar{t}$ cross section $\delta\sigma/\sigma \sim 2.5\% - 7.8\%$ (peak)

- $\star$ slight preferrence for momentum space (smaller corrections)