Phenomenology

of

Multi-Loops

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Encyclopaedia Britannica: **phenomenology**

a philosophical movement originating in the 20th century, the primary objective of which is the direct investigation and description of phenomena as consciously experienced, without theories about their causal explanation and **as free as possible from unexamined preconceptions and presuppositions**. The word itself is much older, however, going back at least to the 18th century, when the Swiss-German mathematician and philosopher Johann Heinrich Lambert applied it to that part of **his theory of knowledge that distinguishes truth from illusion and error**.

Multi-Loops

how to attack them

what are they good for

Lots of technicalities:

have to have a goal

need some inspiration

don't stumble across your feet

Selected Results

(no technicalities)

α_s

ρ-parameter: $m_t \Longleftrightarrow M_W \Longleftrightarrow M_H$

m_c und m_b

α_s from $\sigma(e^+e^- \Rightarrow \text{had})$

and τ-decays

$\mathcal{R}(s) = \left(\sum Q_f^2\right)\Big($		1	parton model	
1955	$+$	$\frac{\alpha_s}{\pi}$	QED	Källen, Sabry; Schwinger
1979	$+$	$\#\left(\frac{\alpha_s}{\pi}\right)^2$	QCD, gives meaning to α_s	Chetyrkin, Kataev, Tkachov; Dine, Sapirstein; Celmaster, Gonsalves
1988/1991	$+$	$\#\left(\frac{\alpha_s}{\pi}\right)^3$	required for precision	Gorishny, Kataev, Larin; Surguladze, Samuel; general gauge: Chetyrkin (1996)
2008	$+$	$\#\left(\frac{\alpha_s}{\pi}\right)^4$	remove theory error slight shift in α_s by 0.0005	Baikov, Chetyrkin, JK

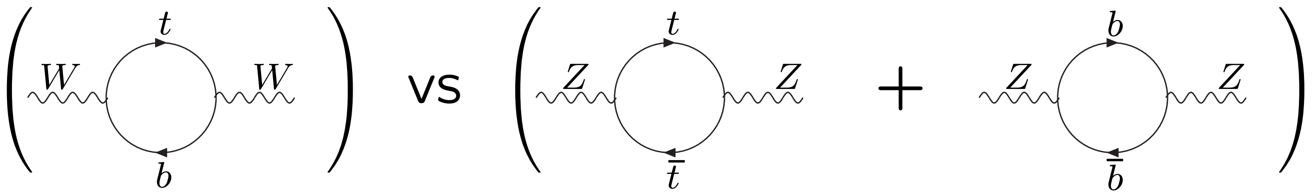
$$\left.\begin{array}{r} \alpha_s(\text{from Z}) = 0.1190 \pm 0.0026 \\ \alpha_s(\text{from } \tau) = 0.1202 \pm 0.0019 \end{array}\right\} \xrightarrow{\text{N}^3\text{LO}} \alpha_s = 0.1198(15)$$

ρ - parameter

$$m_t \Longleftrightarrow M_W \Longleftrightarrow M_H$$

relative shift of M_Z and M_W

from $(t\bar{t})$, $(b\bar{b})$ and $(t\bar{b})$ fluctuations



leading term: $\Delta\rho = 3\frac{\sqrt{2}G_F M_t^2}{16\pi^2}$ (Veltman)

$\Longrightarrow$ early limit on M_t ($\lesssim$ 200 GeV)

Large difference between $\overline{\text{MS}}$ and OS mass

$$M_t - \overline{m}_t(\overline{m}_t) \approx 10\,\text{GeV}$$

$\Longrightarrow$ importance of higher orders for $\Delta\rho$

1 Loop 1977	$G_F\, m_t^2$	Veltman
2 Loop 1987	$\alpha_s G_F\, m_t^2$	Djouadi, Verzegnassi; Kniehl, JK, Stuart
3 Loop 1995	$\alpha_s^2 G_F\, m_t^2$	Chetyrkin, JK, Steinhauser; Fleischer, Tarasov, Jegerlehner
3 Loop 2001-2003	$\alpha_s \left(G_F\, m_t^2\right)^2$ $\left(G_F\, m_t^2\right)^3$	... Chetyrkin, ...
4 Loop	$\alpha_s^3 G_F\, m_t^2$	Chetyrkin + Karlsruhe; Czakon + ...

Result: δM_W in MeV

	α_s^0	α_s^1	α_s^2	α_s^3	$\alpha_s \alpha_{weak}$
m_t^2	611.9	– 61.3	– 10.9	– 2.1	2.5
log + const	136.6	– 6.0	– 2.6	– –	– –
$\frac{1}{m_t^2}$	– 9.0	– 1.0	– 0.2	– –	– –
Σ	739.5	– 68.3	– 13.7	– 2.1	2.5

α_s^2-term: 13.7 MeV $\widehat{=}\ \delta m_t = 2$ GeV (TEVATRON)

α_s^3-term: 2.1 MeV $\widehat{=}\ \delta m_t = 0.3$ GeV (ILC)

Conversely: M_{Pole} fixed

$$\delta \alpha_s = 2 \cdot 10^{-3} \quad \Longrightarrow \quad \delta M_W = 1.7\ \text{MeV}$$

m_c und m_b

from ITEP sum rules to precise quark masses

The concept

$$\mathcal{M}_n^{\text{exp}} \equiv \int \frac{\mathrm{d}s}{s^{n+1}} \mathcal{R}_Q(s)$$

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!} \left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\Bigg|_{q^2=0} = \frac{9}{4} Q_Q^2 \left(\frac{1}{4m_Q^2}\right)^n \overline{C}_n$$

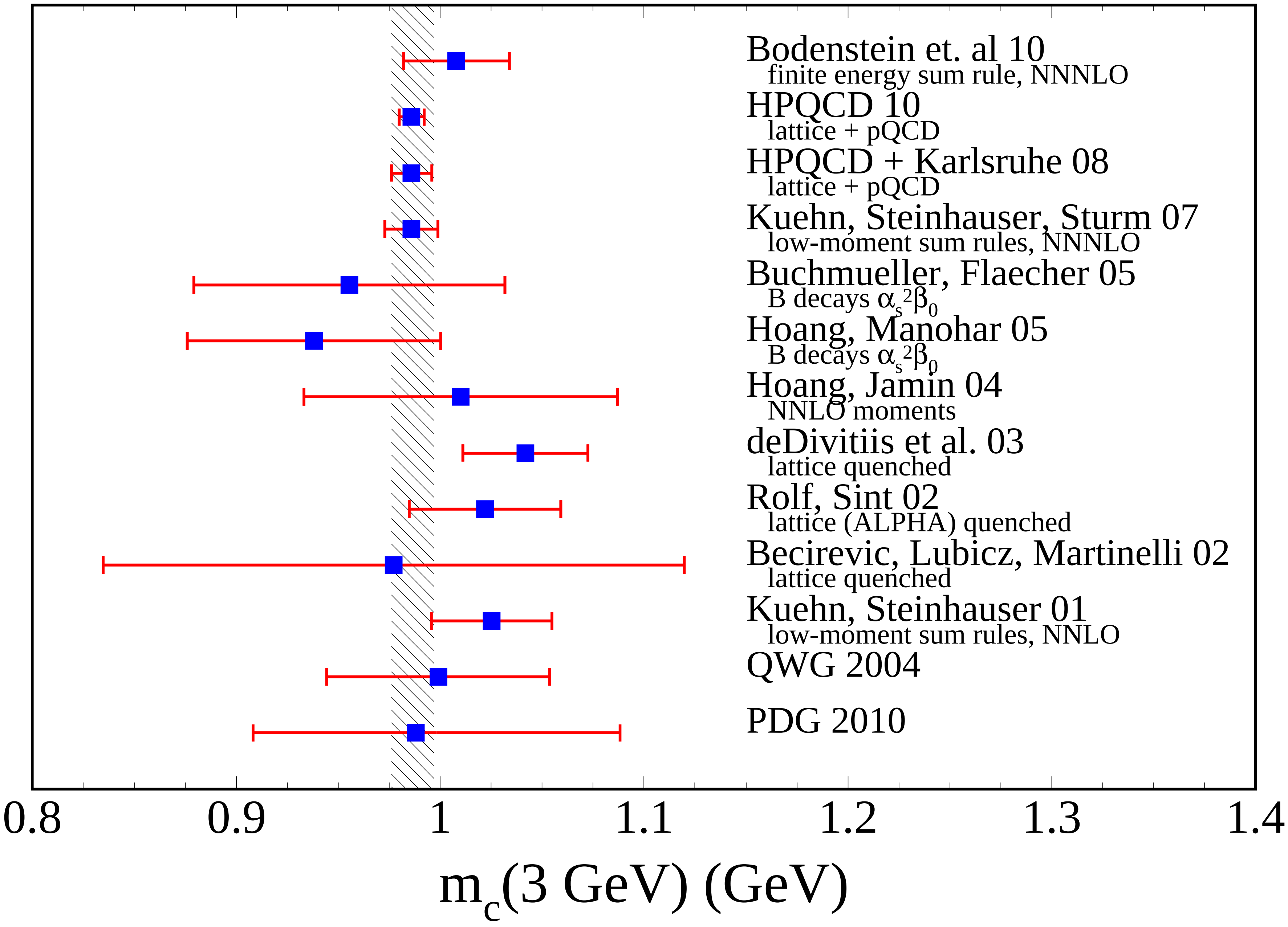
$$\mathcal{M}_n^{exp} = \mathcal{M}_n^{th} \qquad \Longrightarrow \qquad m_Q = \frac{1}{2}\left(\frac{9}{4} Q_Q^2 \frac{\overline{C}_n}{\mathcal{M}_n^{exp}}\right)^{\frac{1}{2n}}$$

$\overline{C}_n$ can be evaluated pertubatively

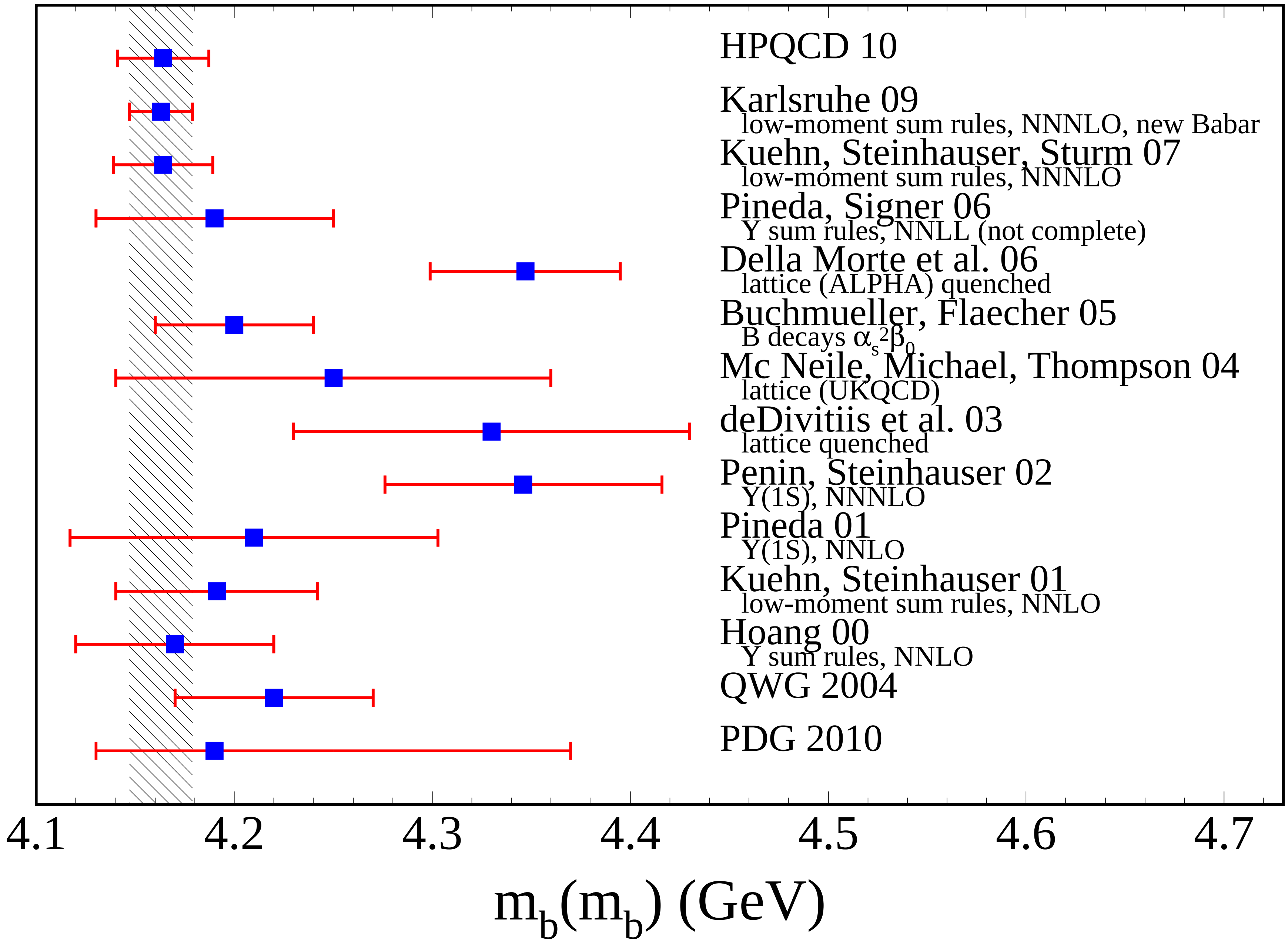
$q^2 = 0 \Rightarrow$ tadpoles

$\overline{C}_n(s) = \quad \overline{C}_n^{(0)}$

2 Loop 1977/1978	$+ \quad \frac{\alpha_s}{\pi} \overline{C}_n^{(1)}$	short distance mass	ITEP
3 Loop 1996/2001	$+ \quad \left(\frac{\alpha_s}{\pi}\right)^2 \overline{C}_n^{(2)}$	precise m_Q	Chetyrkin, JK, Steinhauser
4 Loop 2006/2008	$+ \quad \left(\frac{\alpha_s}{\pi}\right)^3 \overline{C}_n^{(2)}$	reduction of theor. error, application to lattice	Chetyrkin + KA , Czakon +



Bodenstein et. al 10
finite energy sum rule, NNNLO
HPQCD 10
lattice + pQCD
HPQCD + Karlsruhe 08
lattice + pQCD
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Hoang, Manohar 05
B decays $\alpha_s^2\beta_0$
Hoang, Jamin 04
NNLO moments
deDivitiis et al. 03
lattice quenched
Rolf, Sint 02
lattice (ALPHA) quenched
Becirevic, Lubicz, Martinelli 02
lattice quenched
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
QWG 2004
PDG 2010
0.8
0.9
1
1.1
1.2
1.3
1.4
m_c(3 GeV) (GeV)


HPQCD 10
Karlsruhe 09
low-moment sum rules, NNNLO, new Babar
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Pineda, Signer 06
Y sum rules, NNLL (not complete)
Della Morte et al. 06
lattice (ALPHA) quenched
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Mc Neile, Michael, Thompson 04
lattice (UKQCD)
deDivitiis et al. 03
lattice quenched
Penin, Steinhauser 02
Y(1S), NNNLO
Pineda 01
Y(1S), NNLO
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
Hoang 00
Y sum rules, NNLO
QWG 2004
PDG 2010
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$ (GeV)

Moving on: the artistic aspect