QUARK MASSES

Johann H. Kühn

I. GENERALITIES

– The Concept of Quark Masses

– Why

II. CHARM and BOTTOM MASSES

– m_Q from Sum Rules

– Experimental Analysis: m_c

– Experimental Analysis: m_b

III. TOP

IV. SUMMARY

I. GENERALITIES

I. 1. The Concept of Quark Masses

Bavarian

“eine Mass” (⇒ approx. one quart)

“Masskrug” (⇒ beer mug, stein)

1 liter ⇒ 1 kg

French

Platinum-Iridium-Alloy

(Sevres, Paris)

Avogadro's project

12 g $\equiv N_A$ atoms of carbon-12

define Avogadro's constant $N_A \equiv 6.022141\cdots \times 10^{23}$

in practice

count the number of Si-28 in a "perfect" sphere

input:

relative atomic mass of Si-28: 27.9769265325(19)

$(\delta = 6.39 \times 10^{-11})$

Watt balance

mechanical power = electrical power

$$m\ g\ v\ =\ U^2\ /\ R\ =\ \text{constants} \times (\text{frequency})^2\ \hbar$$

$$U \sim \nu\ \frac{\hbar}{e} \quad ; \qquad R \sim \frac{\hbar}{e^2}$$

g: gravitational acceleration

v: velocity

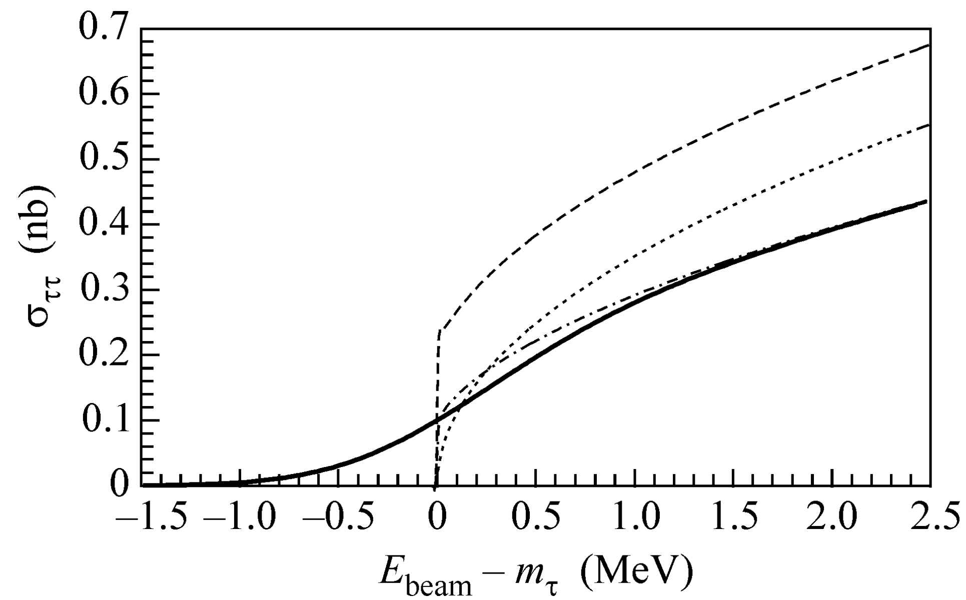
U: Josephson

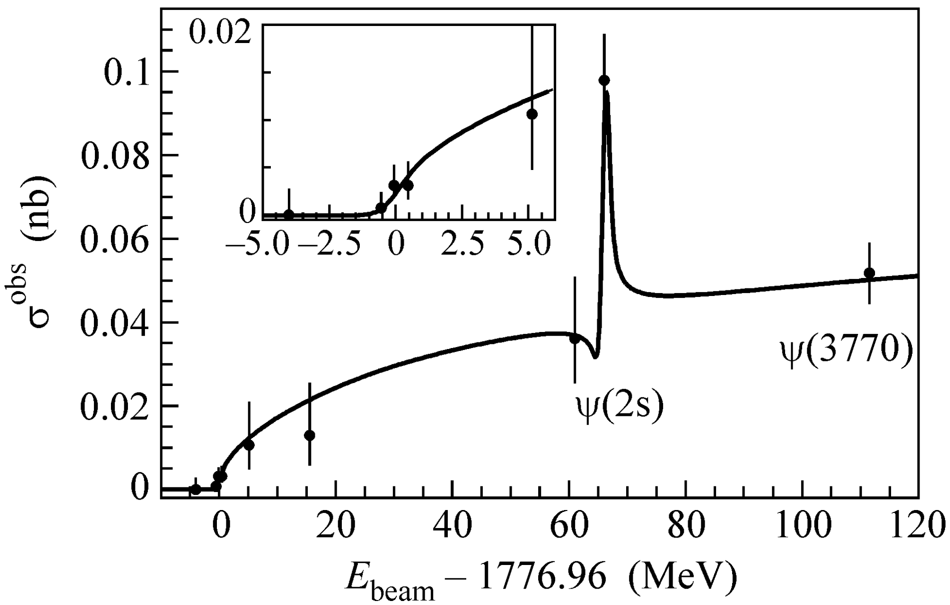
R: quantum Hall resistance

Masses of elementary particles

$m_p = 1.007\,276\,466\,77(10)$ in "atomic units" ($\equiv$ carbon/12)

τ-lepton: $m_\tau = 1776.82 \pm 0.16\ MeV$





kinematics: location of threshold

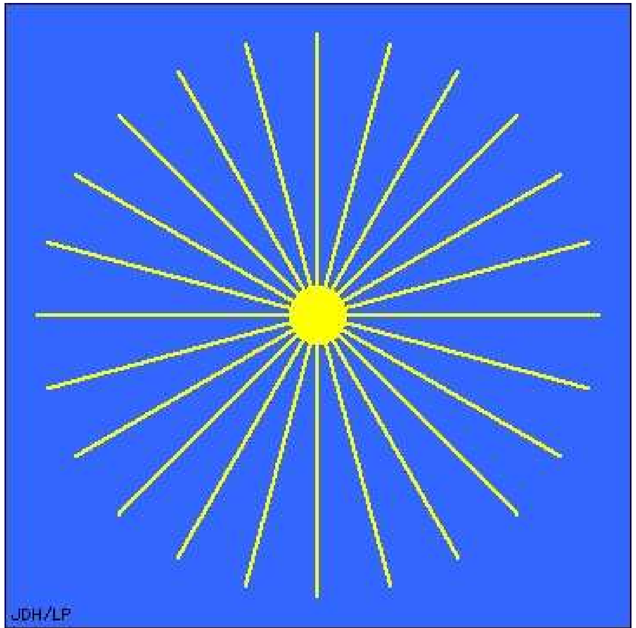
"pole mass" $M^2_{\text{pole}} = E^2 + \vec{p}^{\,2}$

intuitively clear

(Novosibirsk,

Jetp Letts 85, 347)

but: can we separate core and cloud (field energy)?

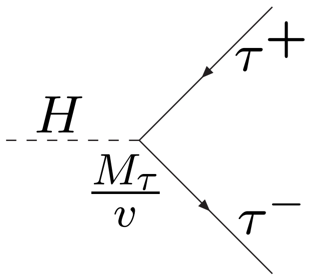


(a) Electric field lines around an electron.

kick the particle hard

$\Rightarrow$ perhaps only short distance part

consider Higgs boson decay: $H \to \tau^+ \tau^-$



$$v^2 = G_F\sqrt{2} = (246\,GeV)^2$$

$$\Gamma(H \to \tau^+\tau^-) \approx \frac{G_F M_\tau^2}{4\pi\sqrt{2}} M_H$$

Which mass M_τ?

Calculate quantum corrections:

QED: correction factor $\left(1 - \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{M_H^2}{M_\tau^2} - \frac{9}{4}\right)\right)$

large logarithm: negative;

can be absorbed in "running mass"

$$M_\tau(M_H) = M_\tau\left(1 - \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{M_H^2}{M_\tau^2} + 1\right)\right)$$

constant term: depends on choice of mass definition

(concept of "running" mass (and coupling) was introcuced in QED before QCD was discovered) convenient choice for QCD calculations

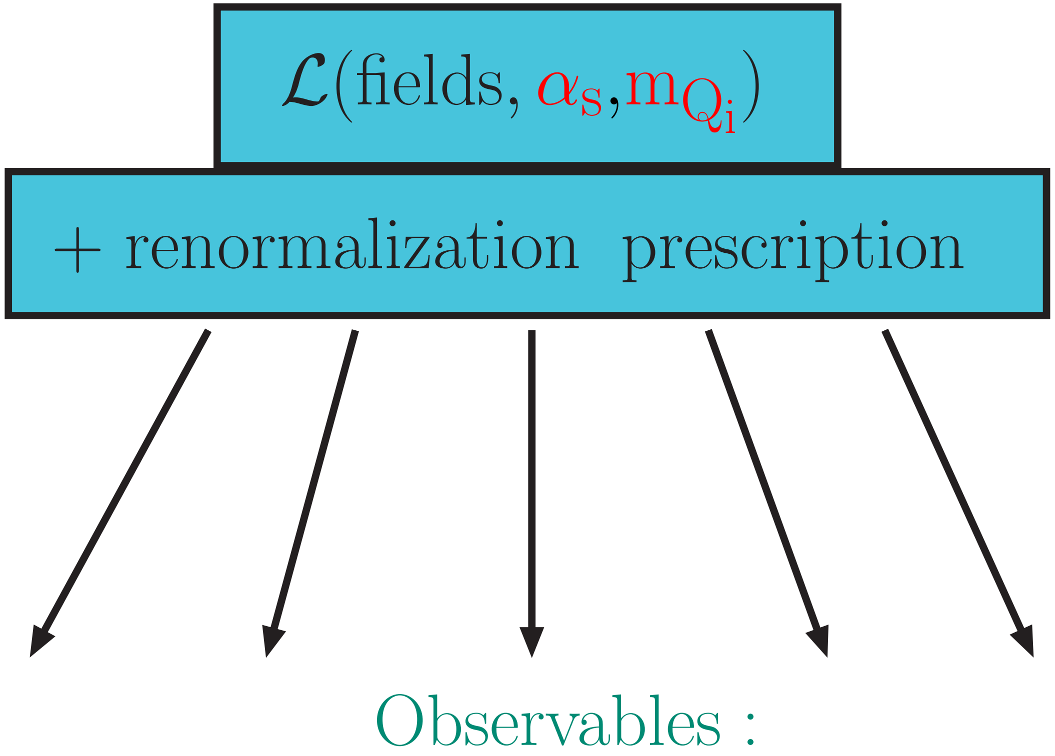
$\overline{MS}$ convention:

perform calculation in dimensional regularisation: $d = 4 - 2\,\varepsilon$

divergencies appear as poles in ε

$\Rightarrow$ subtract poles (and convention dependent constants)

m is a convention-dependent parameter in the Lagrangian and depends on the renormalization scale μ.


$\mathcal{L}$(fields, α_s, m_{Q_i})
+ renormalization prescription
Observables :

conversions: $M \Leftrightarrow \overline{m_b}(\mu)$

$$\overline{m_b}(\mu) = M\left\{1 - a_s\left[\frac{4}{3} + \ln\frac{\mu^2}{M^2}\right] - a_s{}^2\left[\# + \ln + \ln^2\right] + a_s{}^3\left[\# + ...\right]\right\}$$

$a_s{}^3$: Chetyrkin+Steinhauser; Melnikov+Ritbergen

examples: $M_t = 171\text{GeV} \quad \Rightarrow \quad m_t(m_t) = 161\text{GeV}$

$m_b(m_b) = 4165\text{MeV} \quad \Rightarrow \quad M_b = 4796\text{MeV}$

large logarithms for $\mu^2 \gg M^2$ $\rightarrow$ renormalization group

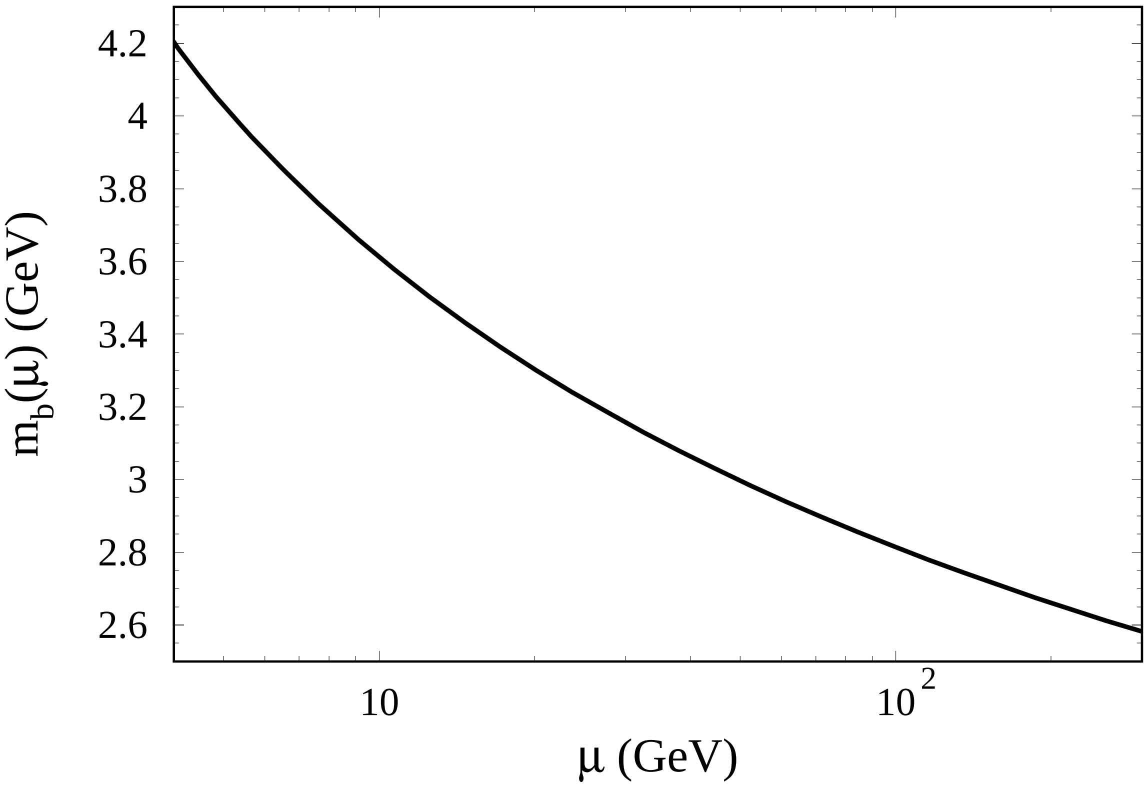
$$\mu^2 \frac{d}{d\mu^2}\bar{m}(\mu) = \bar{m}(\mu)\,\gamma(\alpha_s)$$

$\gamma(\alpha_s) = -\sum_{i\geq 0}\gamma_i a_s{}^{i+1}$, (known up to γ_3, Chetyrkin; Larin+...)

+matching

solve RGE numerically or perturbatively

$$\overline{m}(\mu) = \overline{m}(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma_m^0/\beta_0}\left[1+\left(\frac{\gamma_m^1}{\beta_0}-\frac{\beta_1\gamma_m^0}{\beta_0^2}\right)\left(\frac{\alpha_s(\mu)}{\pi}-\frac{\alpha_s(\mu_0)}{\pi}\right)+\ldots\right]$$



$m_b(m_b) = 4165$ MeV

$m_b(10\text{GeV}) = 3610$ MeV

$m_b(M_Z) = 2836$ MeV

$m_b(161\text{GeV}) = 2706$ MeV

$\overline{\text{MS}}$- vs. Pole-Mass

Pole-Mass (M_{pole}): close to intuition

- t $\longrightarrow$ b W

  $M_{\text{pole}}(\text{b W}) = (171.4 \pm 2.1)$ GeV $\pm \mathcal{O}(\Lambda?)$

- $e^+e^- \longrightarrow \text{t}\,\bar{\text{t}}$

  "peak" at $2M_{\text{pole}} + \mathcal{O}(\alpha_s^2)$

- $M_{\text{B}} \approx M_{\text{pole}} + \mathcal{O}(\Lambda)$

  5280 MeV $\approx (4820 + 460)$MeV

But: large corrections for observables involving large momentum transfers

Implication for Higgs decay: (e.g. $M_H = 120\ GeV$)

$$\Gamma(H \to b\bar{b}) \approx \frac{G_F \overline{m}_b(M_H)^2}{4\pi\sqrt{2}} M_H\, 3\Big[1 + 5.667\Big(\frac{\alpha_s}{\pi}\Big) + \#\Big(\frac{\alpha_s}{\pi}\Big)^2 + \#\Big(\frac{\alpha_s}{\pi}\Big)^3 + \#\Big(\frac{\alpha_s}{\pi}\Big)^4\Big]$$

$\Big(\overline{m}_b(M_H)/M_b\Big)^2 \approx (2.8/4.8)^2 \approx 0.34$

$[1 + \ldots] \approx [1 + 0.207 + 0.039 + 0.002 - 0.001] \approx 1.247$

$\Rightarrow$ dominant corrections from running mass!

Large corrections (often, not always!) absorbed by running mass.

other schemes:

pole mass contains unphysical long distance contributions

$\rightarrow$ subtract unphysical long distance terms

"potential subtracted (PS) mass" (Beneke)

"1 s-mass" (Hoang+Manhohar)

often used for B-meson decays, Y-spectroscopy, closer to pole mass definition

$\rightarrow$ residual uncertainty often larger than uncertainty of m_b in $\overline{MS}$-scheme.

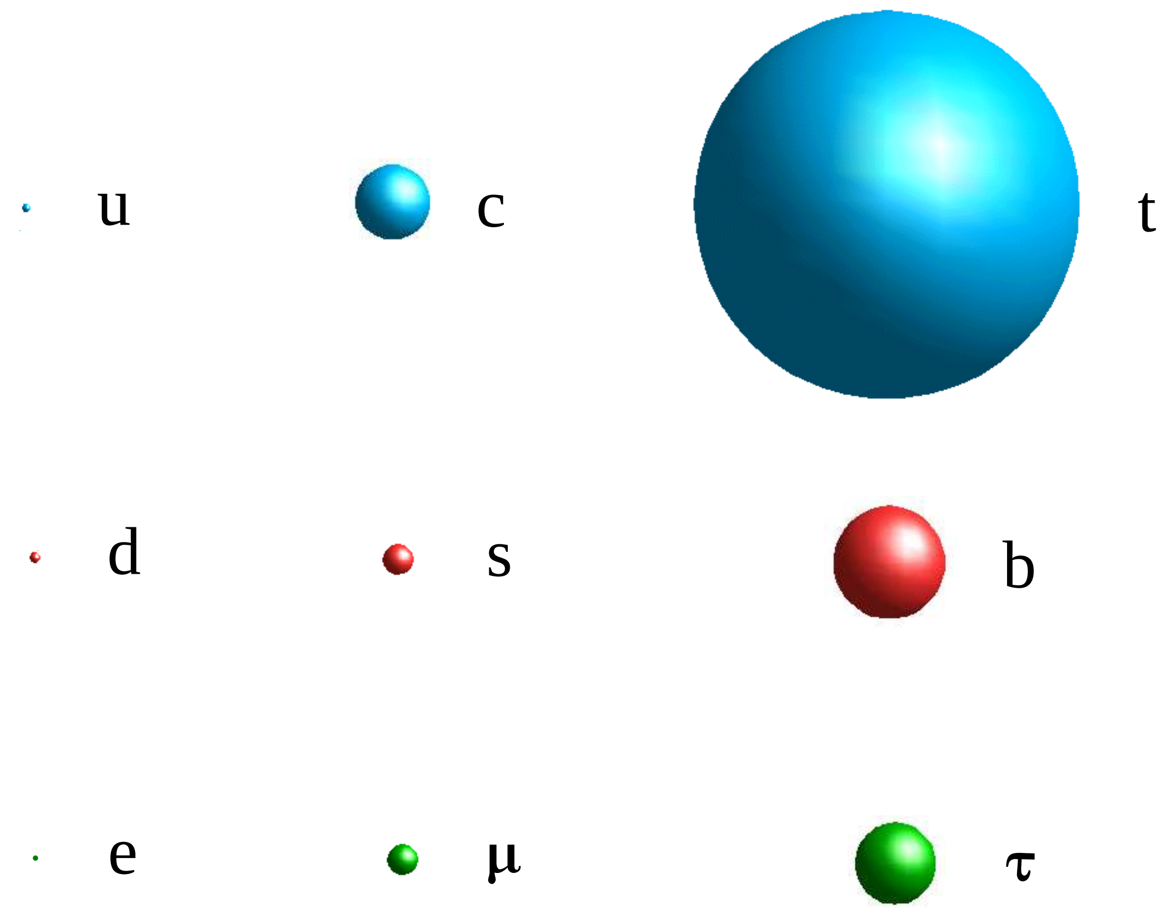
The Puzzle

$m_u = 1.7 - 3.3$ MeV $\quad m_c = 1270^{+70}_{-90}$ MeV $\quad m_t = 172 \pm 0.9 \pm 1.3$ GeV

$m_d = 4.1 - 5.8$ MeV $\quad m_s = 101^{+29}_{-21}$ MeV $\quad m_b = 4.19^{+0.18}_{-0.06}$ GeV

$m_e = 0.511$ MeV $\quad m_\mu = 106$ MeV $\quad m_\tau = 1.7773$ GeV

PDG


u
c
t
d
s
b
e
μ
τ

WHY precise masses?

B-decays:

$$\Gamma(B \to X_{\mathrm{u}} l \bar{\nu}) \sim G_{\mathrm{F}}^2 \, m_{\mathrm{b}}^5 \, |V_{\mathrm{ub}}|^2$$

$$\Gamma(B \to X_{\mathrm{c}} l \bar{\nu}) \sim G_{\mathrm{F}}^2 \, m_{\mathrm{b}}^5 \, f(m_{\mathrm{c}}^2/m_{\mathrm{b}}^2) \, |V_{\mathrm{cb}}|^2$$

moments of $\frac{\mathrm{d}N}{\mathrm{d}E_l}$, $\frac{\mathrm{d}N}{\mathrm{d}m(l\bar{\nu})}$,

$B \to X_{\mathrm{s}}\gamma$: moments of m_{had}^2

Υ-spectroscopy:

$$m\left(\Upsilon(1\mathrm{s})\right) = 2M_{\mathrm{b}} - \left(\tfrac{4}{3}\alpha_{\mathrm{s}}\right)^2 \frac{M_{\mathrm{b}}}{4} + \ldots$$

sum rules:

$$\int \frac{\mathrm{d}s}{s^{n+1}}\, R_{\mathrm{Q}}(s) \sim \frac{1}{m_{\mathrm{Q}}^{2n}}$$

dominant decay mode for light Higgs

perturbative vs. lattice:

recently (HPQC)

$\overline{m}_c(3\text{ GeV}) = 986(6)\text{ MeV}$

$\overline{m}_b(10\text{ GeV}) = 3617(25)\text{ MeV}$

Yukawa Unification

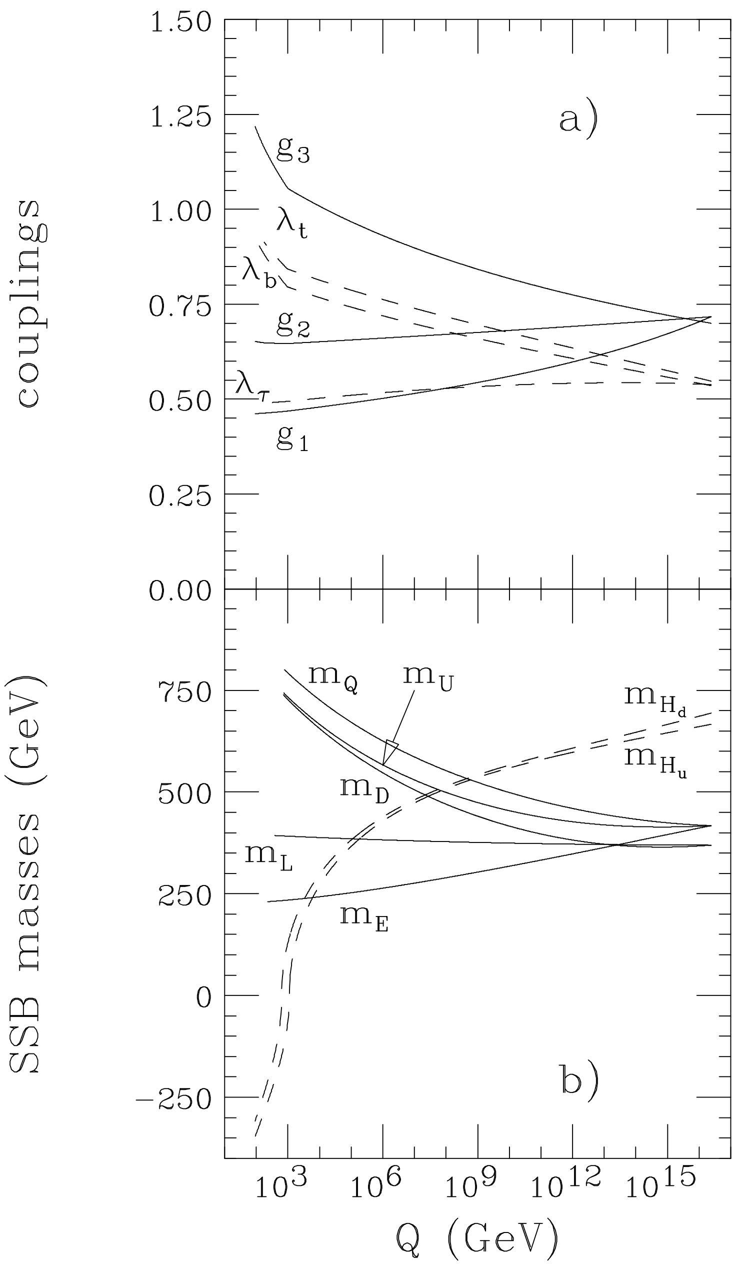
$$\lambda_\tau = \lambda_b \quad \text{or} \quad \lambda_\tau = \lambda_b = \lambda_t$$

identical coupling to Higgs boson(s) at GUT scale

top-bottom $\rightarrow m_t / m_b \sim$ ratio of vacuum expectation values

request $\frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t}$

$\delta m_t \approx 1$ GeV $\Rightarrow \delta m_b \approx 25$ MeV



Baer *et al.*

Phys.Rev.D61,2000

II. CHARM and BOTTOM MASSES

in collaboration with

K. Chetyrkin, Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard, A. Smirnov,

M. Steinhauser, C. Sturm and the HPQCD Collaboration

NPB 619 (2001) 588

EPJ C48 (2006) 107

NPB 778 (2008) 05413

PLB 669 (2008) 88

NPB 823 (2009) 269

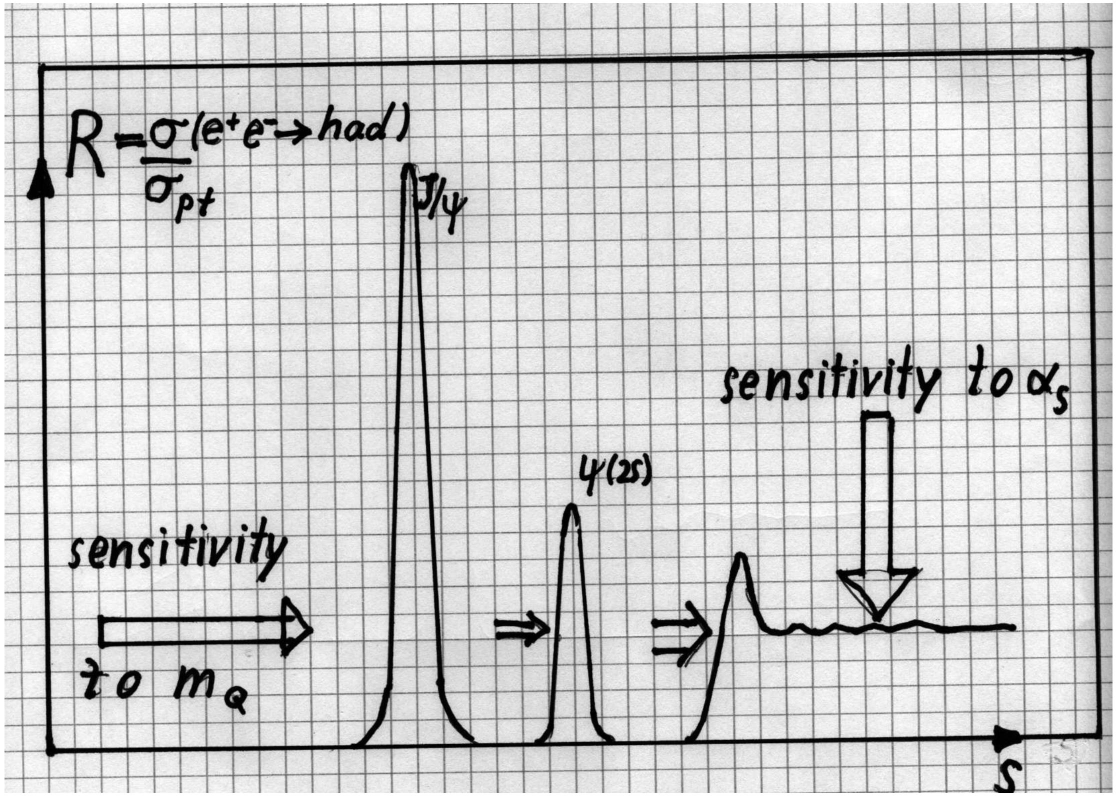
PRD 80 (2009) 074010

NPB 824 (2010) 1

arXiv:1010.6157

II. 1. m_Q from SVZ Sum Rules, Moments and Tadpoles

Main Idea (SVZ)



Some definitions:

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right) \Pi(q^2) \;\equiv\; i \int \mathrm{d}x\, e^{iqx} \langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ.

$$R(s) \;=\; 12\pi\, \mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

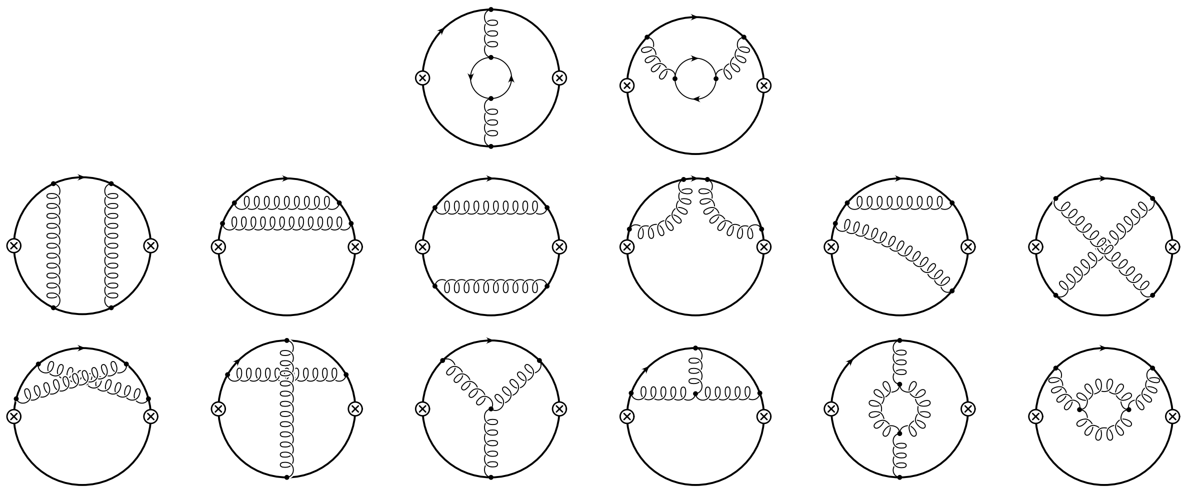
Taylor expansion: $\Pi_Q(q^2) \;=\; Q_Q^2\, \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n\, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s}{\pi}\bar{C}_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}_n^{(3)} + \ldots$$

Analysis in NNLO

Coefficients $\bar{C}_n$ from three-loop one-scale tadpole amplitudes with "arbitrary" power of propagators;

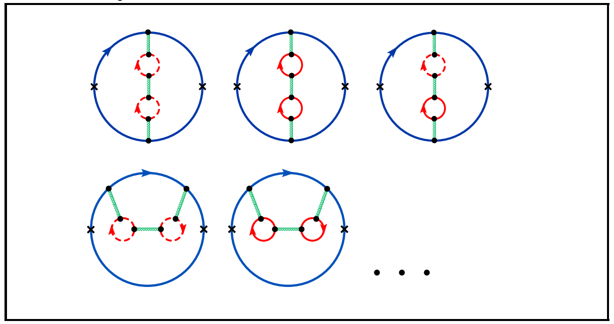


Analysis in N^{3}LO

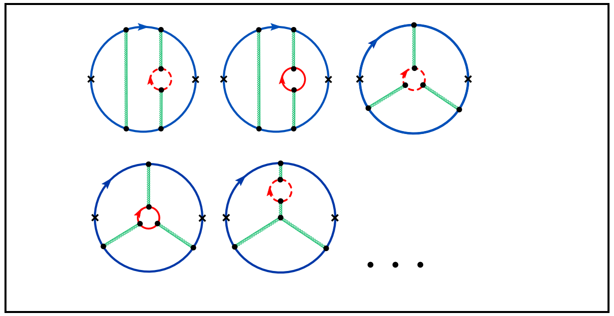
Algebraic reduction to 13 master integrals (Laporta algorithm);

numerical and analytical evaluation of master integrals

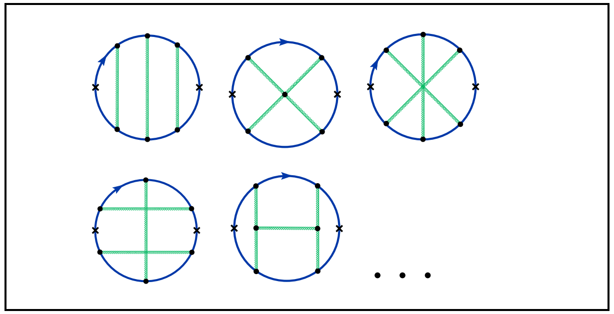
n_f^2-contributions



n_f^1-contributions



n_f^0-contributions



 : heavy quarks,  : light quarks,

n_f: number of active quarks

$\Rightarrow$ About 700 Feynman-diagrams

Relation to measurements

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

Perturbation theory: $\bar{C}_n$ is function of α_s and $\ln \frac{m_c^2}{\mu^2}$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\, \frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

$$\Rightarrow \mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint: $\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$

$$\boxed{\Rightarrow m_c = \frac{1}{2}\left(\frac{9}{4}Q_c^2\,\bar{C}_n/\mathcal{M}_n^{\text{exp}}\right)^{1/(2n)}}$$

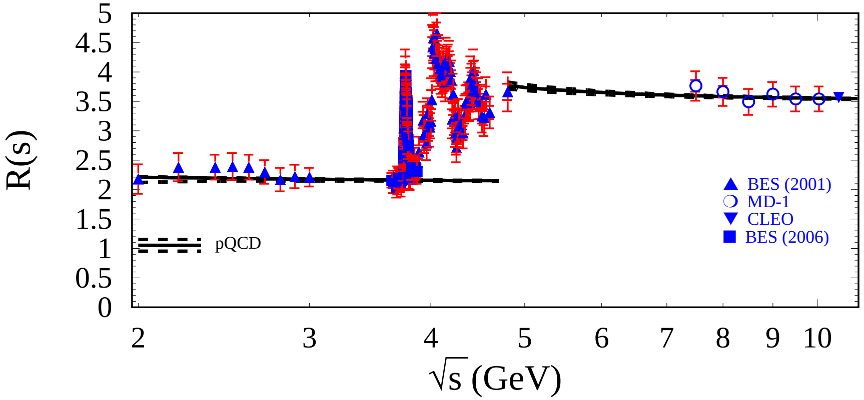
qualitative considerations

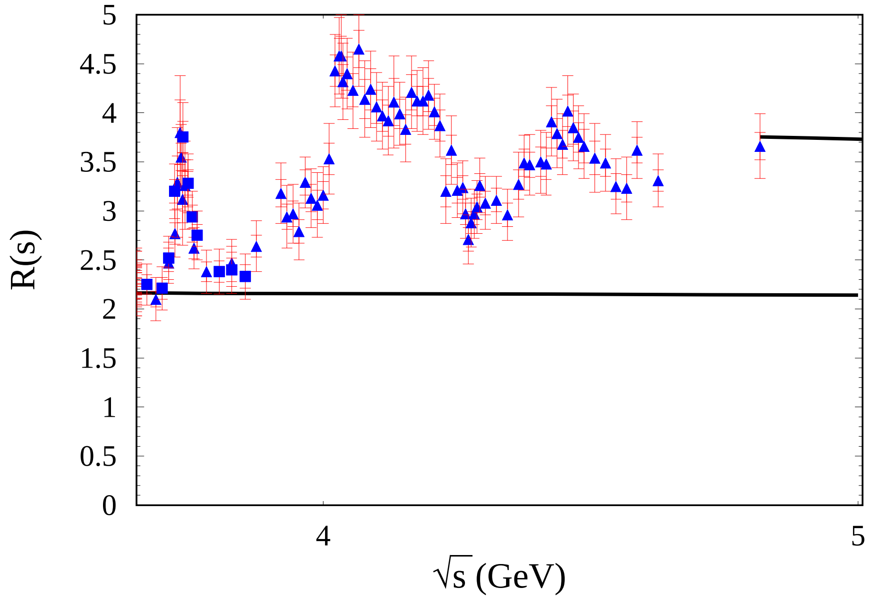
$$\mathcal{M}_n = \int_{threshold} \frac{d\,s}{s^{n+1}} R_c(s) \sim \text{dimension } (m_c)^{-2n}$$

- depends moderately on α_s!
- $\Pi(q^2)$ is an analytic function with branch cut from $(2m_D)^2$ to ∞.
- averaging over resonances reduces influence of long distances (binding effects).
- $\Pi(q^2 = 0)$ and its derivatives at $q^2 = 0$ are short distance quantities.
  $\Rightarrow$ pQCD is applicable.

II. 2. Experimental Analysis: m_c

$$\mathcal{M}_n^{\text{exp}} \equiv \int \frac{d\,s}{s^{n+1}}\,R_{\text{charm}}(s)$$





Ingredients (charm)

experiment:

- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & BABAR (PDG)
- $\psi(3770)$ and $R(s)$ from BES
- $\alpha_s = 0.1187 \pm 0.0020$

Results (m_c)

n	$m_c(3\,\text{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

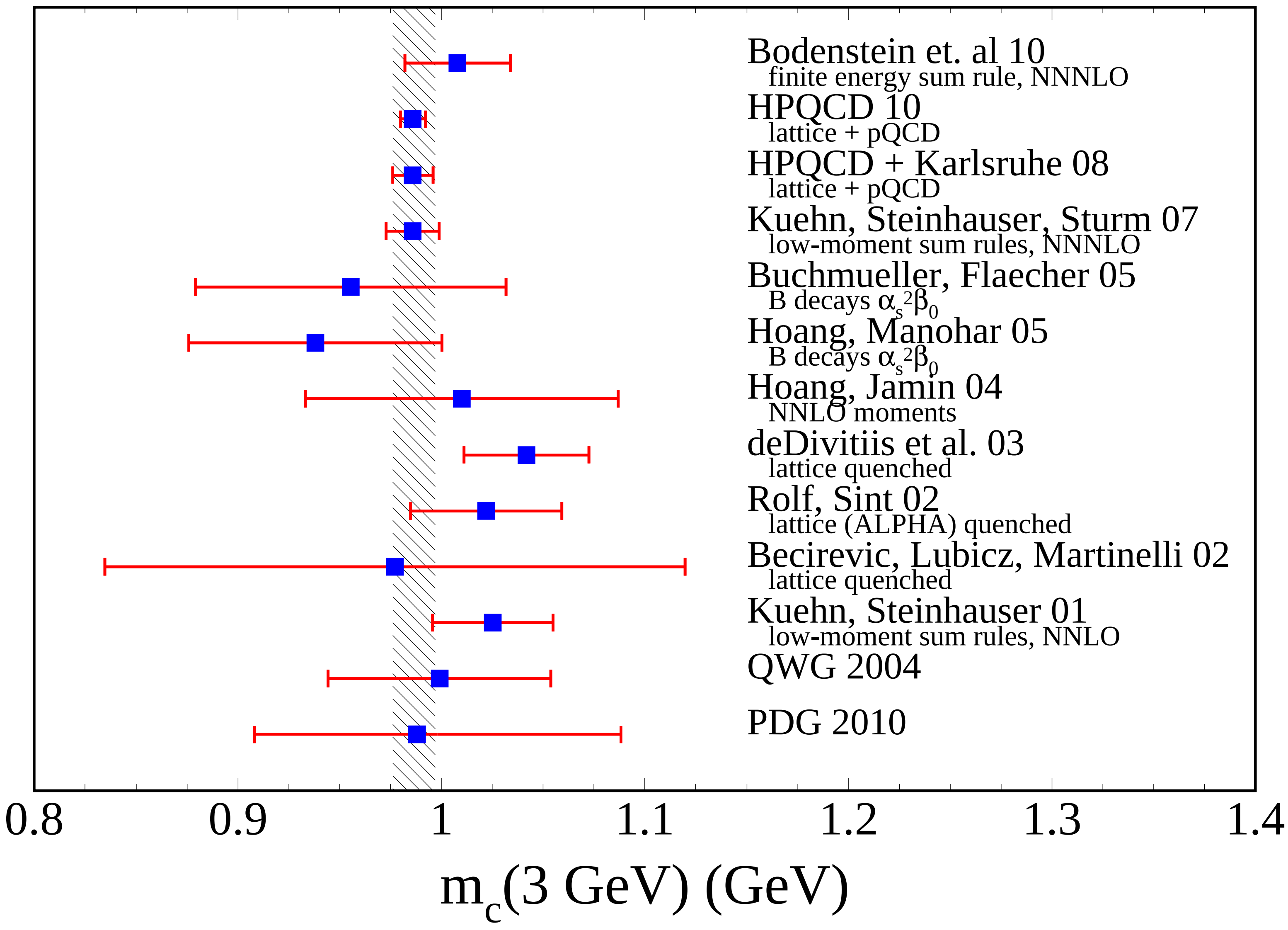
Remarkable consistency between $n = 1, 2, 3, 4$

and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

prefered scale: $\mu = 3\,\text{GeV}$,

$$m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$$

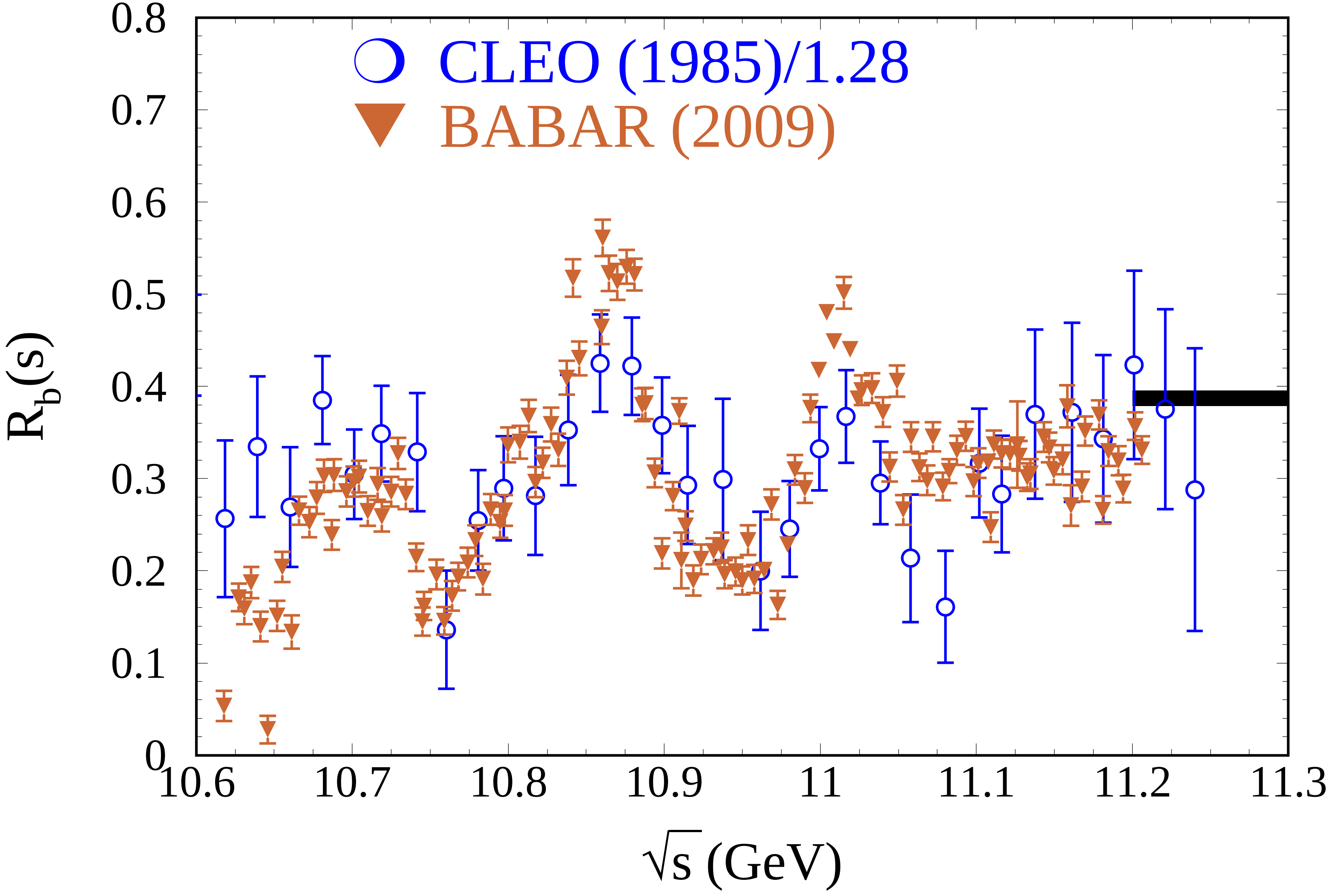
$\Big($ conversion to $m_c(m_c)$: $m_c(m_c) = 1279 \pm 13\,\text{MeV}$ $\Big)$


Bodenstein et. al 10
finite energy sum rule, NNNLO
HPQCD 10
lattice + pQCD
HPQCD + Karlsruhe 08
lattice + pQCD
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Hoang, Manohar 05
B decays $\alpha_s^2\beta_0$
Hoang, Jamin 04
NNLO moments
deDivitiis et al. 03
lattice quenched
Rolf, Sint 02
lattice (ALPHA) quenched
Becirevic, Lubicz, Martinelli 02
lattice quenched
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
QWG 2004
PDG 2010
0.8
0.9
1
1.1
1.2
1.3
1.4
m_c(3 GeV) (GeV)

II. 3. Experimental Ingredients for m_b

Contributions from

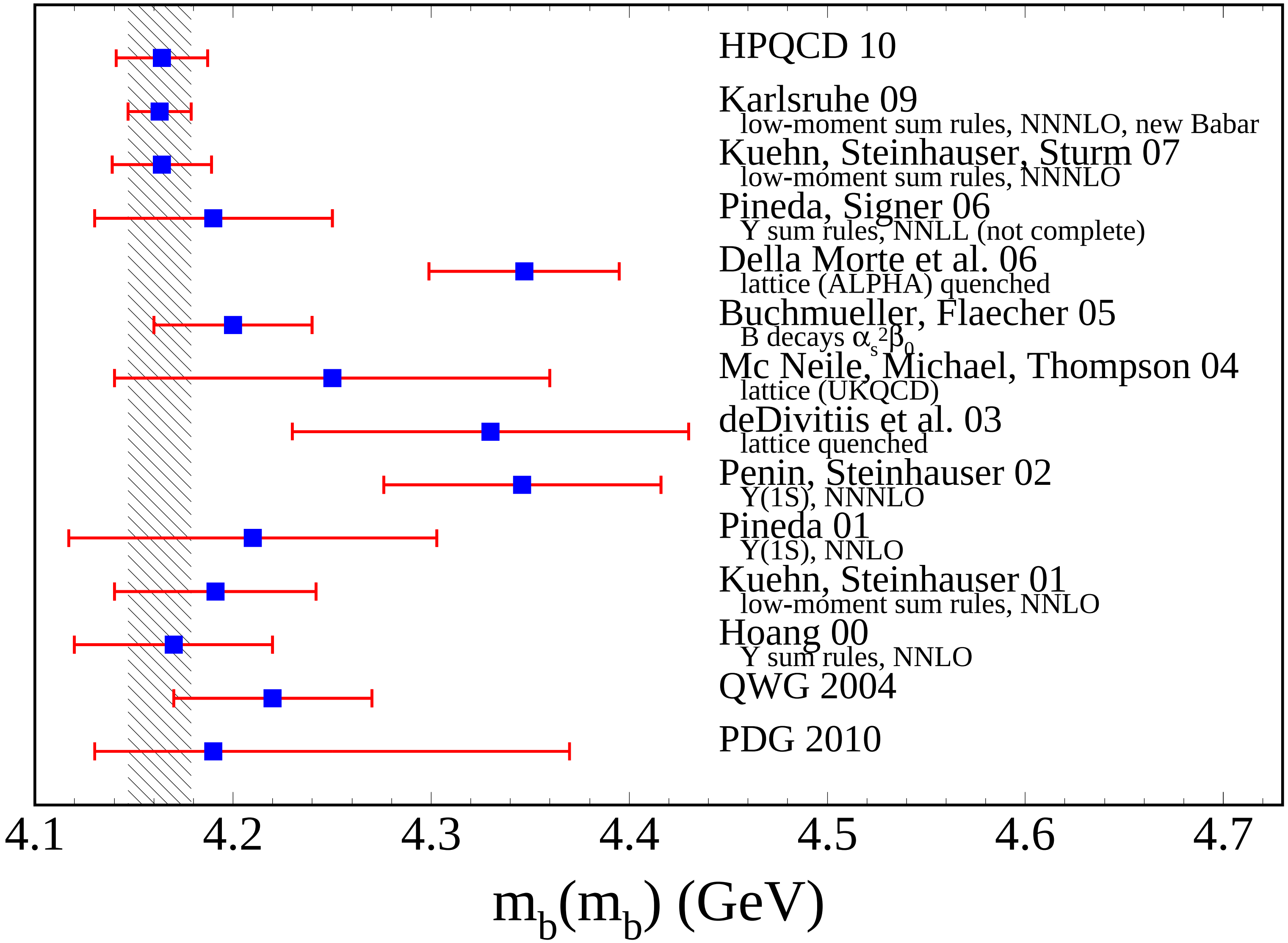
- narrow resonances ($\Upsilon(1S) - \Upsilon(4S)$) (PDG)
- threshold region (10.618 GeV − 11.2 GeV) (BABAR 2009)
- perturbative continuum ($E \geq 11.2\,\text{GeV}$) (Theory)
- different relative importance of resonances vs. continuum for $n = 1, 2, 3, 4$


CLEO (1985)/1.28
BABAR (2009)
$R_b(s)$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3
$\sqrt{s}$ (GeV)

n	$m_b(10\,\text{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

Consistency ($n = 1, 2, 3, 4$) and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

- $m_{\text{b}}(10\,\text{GeV}) = 3610 \pm 16\,\text{MeV}$
- $m_{\text{b}}(m_{\text{b}}) = 4163 \pm 16\,\text{MeV}$


HPQCD 10
Karlsruhe 09
low-moment sum rules, NNNLO, new Babar
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Pineda, Signer 06
Υ sum rules, NNLL (not complete)
Della Morte et al. 06
lattice (ALPHA) quenched
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Mc Neile, Michael, Thompson 04
lattice (UKQCD)
deDivitiis et al. 03
lattice quenched
Penin, Steinhauser 02
Υ(1S), NNNLO
Pineda 01
Υ(1S), NNLO
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
Hoang 00
Υ sum rules, NNLO
QWG 2004
PDG 2010
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$ (GeV)

lattice & pQCD (HPQCD + Karlsruhe, Phys. Rev. D78, 054513)

lattice evaluation of pseudoscalar correlator

$\Rightarrow$ replace experimental moments by lattice simulation

input: $M(\eta_c) \widehat{=} m_c\,, \quad M(\Upsilon(1S)) - M(\Upsilon(2S)) \widehat{=} \alpha_s$

pQCD for pseudoscalar correlator available:

"all" moments in $\mathcal{O}(\alpha_s^2)$, four lowest moments in $\mathcal{O}(\alpha_s^3)$.

update: HPQCD 2010

$\alpha_s(3\text{GeV}) \Rightarrow \alpha_s(M_Z) = 0.1183(7)$

$m_c(3\text{GeV}) = 986(6)$ MeV

$m_b(10\text{GeV}) = 3617(25)$ MeV

III. TOP

$\Gamma(t \to b + W) \approx \frac{G_F m_t^3}{8\pi\sqrt{2}} \sim 1.3$ GeV

$\Rightarrow$ no top mesons!

(formation time would be too long)

kinematic reconstruction of top from decay products $W + b$-jet

$\Rightarrow$ pole mass !?

But: decay products are color singlet!

$\Rightarrow$ no perfect separation of t, $\bar{t}$ and underlying event conceptually possible

$\Rightarrow$ $\delta m \sim$ several 100 MeV (guess!)

result based on MC simulation, templates, ...

$m_t = 173.3 \pm 1.1$ GeV (Shabalina, ICHEP 2010)

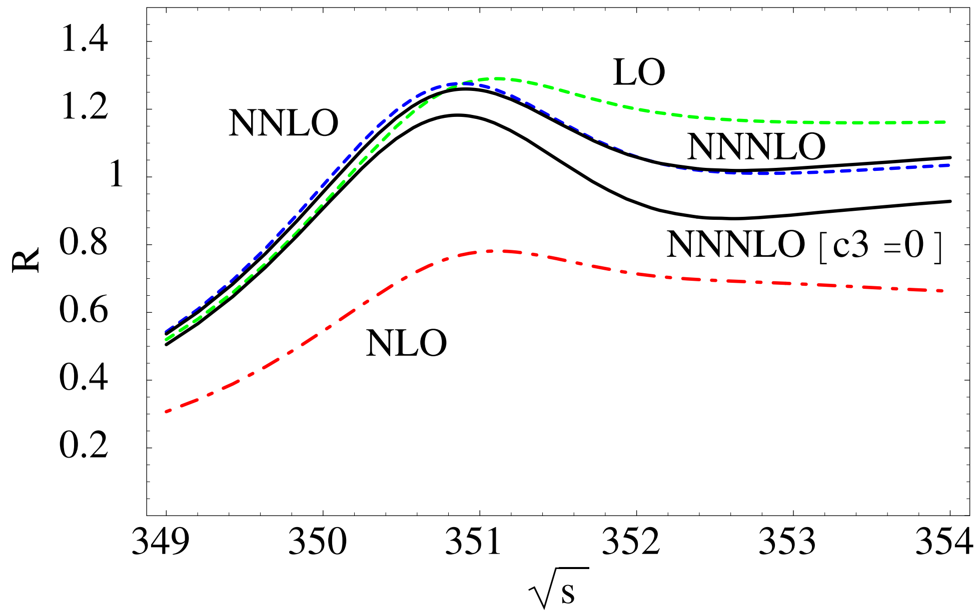
note: $\overline{m}_t(\overline{m}_t) = 173.30 - 7.53 - 0.96 - 0.14$ $\Rightarrow$ dramatic difference

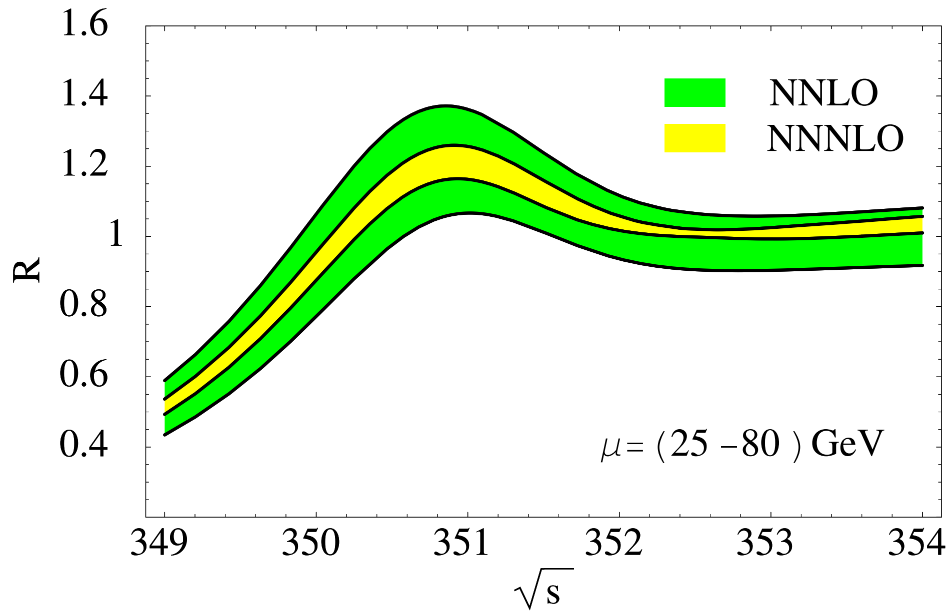
m_t is important ingredient in electroweak precision test:

$\overline{m}_t(\overline{m}_t)$ seems to be the natural parameter for electroweak precision tests (ρ-parameter)

$\delta M_W \sim 10$ MeV $\frac{\delta m_t}{1.5\text{GeV}}$ (cf $M_W = 80.399(23)$ GeV PDG) how can we do better?

ILC: theoretically clean prediction for $e^+e^- \to t\bar{t}$ in threshold region

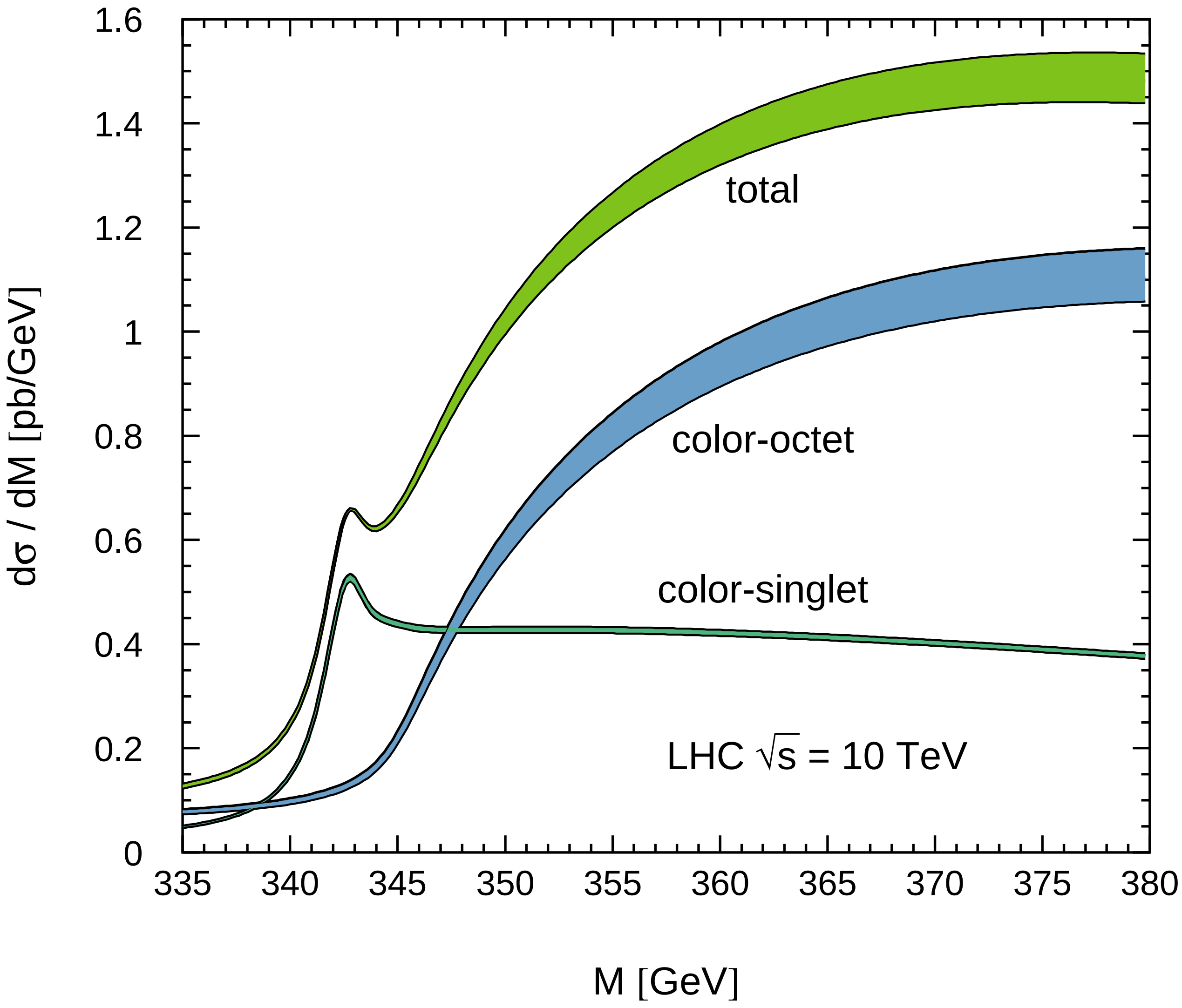




experimental precision: $\delta m_t = \mathcal{O}(20\text{MeV})$!

theoretical interpretation: $\delta m_t =?$

LHC: substitute ?



IV. SUMMARY

multiloop results + precise data

$m_c(3\,\text{GeV}) = 986(13)$ MeV	$e^+e^- +$ pQCD
$m_c(3\,\text{GeV}) = 986(6)$ MeV	lattice + pQCD
$m_b(10\,\text{GeV}) = 3610(16)$ MeV $m_b(m_b) = 4163(16)$ MeV	$e^+e^- +$ pQCD
$m_b(10\,\text{GeV}) = 3617(25)$ MeV	lattice + pQCD

Top at hadron colliders

$$m_t = 173.3 \pm 1.1 \text{ GeV}$$

Essential improvements from ILC + NRQCD for threshold behaviour

$\Rightarrow \delta m_t < 50$ MeV feasible!