HARD SCATTERING AND ELECTROWEAK CORRECTIONS IN THE TeV REGION

J.H. Kühn

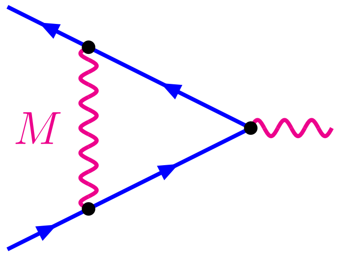
I. Introduction

"Typical" size of electroweak corrections: $\frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2}$

new aspects at LHC: $\sqrt{\hat{s}} \approx$ 1-2TeV $\gg M_{W,Z}$

strong enhancement of negative corrections

one-loop example: massive U(1)



$$\Rightarrow \quad \text{Born} * \left[1 + \frac{\alpha}{4\pi}\left(-\ln^2\frac{s}{M^2} + 3\ln\frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3}\right)\right]$$

$\frac{s}{M^2}$	$-\ln^2\frac{s}{M^2}$	$+3\ln\frac{s}{M^2}$	$-\frac{7}{2}+\frac{\pi^2}{3}$	Σ	$* 4\,\frac{\alpha_{\text{weak}}}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	$+15.15$	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	$+19.31$	-0.21	-22.3	-27%

(four-fermion cross section $\Rightarrow$ factor 4)

- leading $\log^2$ multiplied by (charge)$^2 = I(I+1) = \begin{cases} 3/4 & I = 1/2 \\ 2 & I = 1 \end{cases}$

- important subleading logarithms (NLL+...)

- two-loop terms may be relevant

- interplay between electroweak and QCD corrections

- important differences between fermions and electroweak gauge bosons

- important differences between transversal ($I = 1$) and longitudinal ($I = 1/2$) W-bosons

II. Form Factors & Four-Fermion Scattering at Two Loop

LL: Fadin et al. (2000)

NLL: J.H.K., Penin, Smirnov (2000)
Large (!) subleading corrections
important angular dependent terms

NNLL: J.H.K., Moch, Penin, Smirnov (2001)
Large (!) NNLL terms,
oscillating signs of LL, NLL, NNLL
$\Rightarrow$ compensations

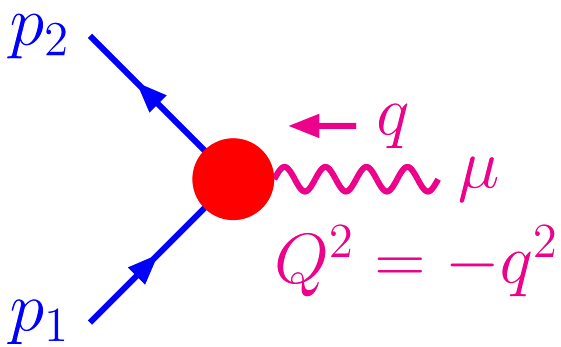
$N^3LL + N^4LL$ Jantzen, J.H.K., Penin, Smirnov (2003-2005)

Massive SU(2) $\Rightarrow$ basic aspects

Additional complication in SM: massless photon

$$|Q^2| \gg M_{W,Z}^2 \gg m_\gamma^2$$

A) Form Factor and Evolution Equations



Born:

$\mathcal{F}_{\mathsf{Born}} = \bar{\psi}(p_2)\gamma_\mu \psi(p_1)$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow\ \mathcal{F} = \mathcal{F}_{\mathsf{Born}}\, F_0(\alpha(M^2))\ \exp\left\{\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x} \frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: N^{4}LL $\Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \Big[\underset{\text{LL}}{\mathcal{L}^{2n}} + \underset{\text{NLL}}{\mathcal{L}^{2n-1}} + \underset{\text{NNLL}}{\mathcal{L}^{2n-2}} + \underset{\text{N}^3\text{LL}}{\mathcal{L}^{2n-3}} + \underset{\text{N}^4\text{LL}}{\mathcal{L}^{2n-4}} \Big]$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

NNLL requires running of α (i.e. β_0 and β_1) and:

$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$	up to $\mathcal{O}(\alpha)$	(one-loop)
$\gamma(\alpha)$	up to $\mathcal{O}(\alpha^2)$	(massless two loop)

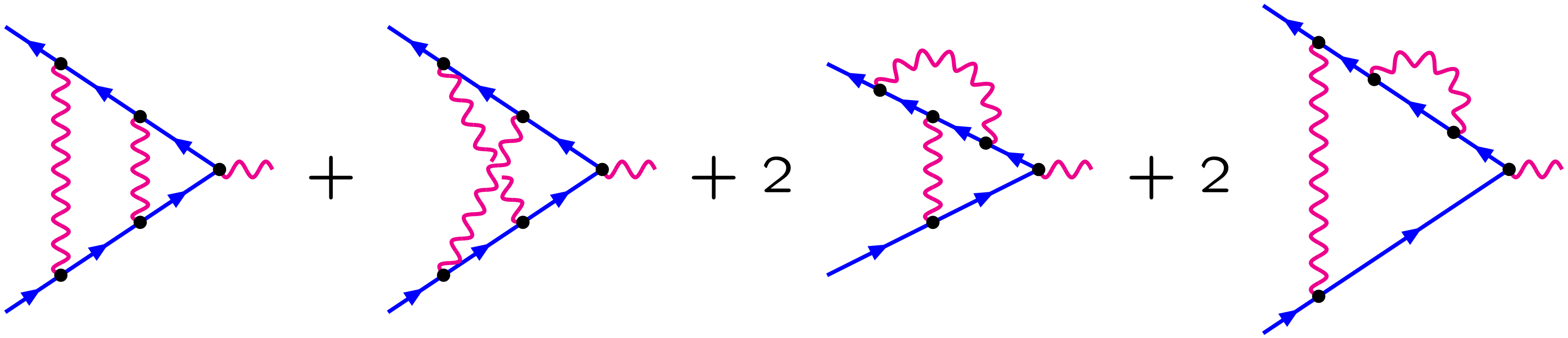
N^{3}LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

Important and highly non-trivial calculation (expansion by regions!)

N^{4}LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

B) Two-Loop Results: Massive U(1) Model

$$\mathcal{F}_\alpha(M,Q) = \mathcal{F}_{\text{Born}}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \dots\right]$$



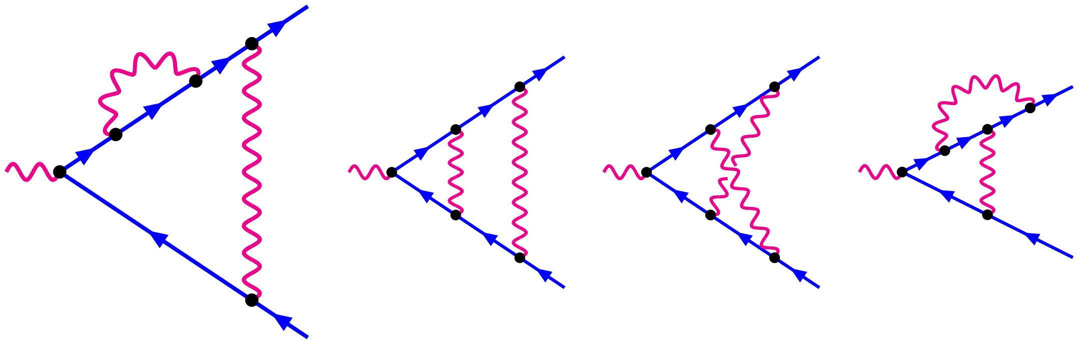
$$f^{(2)} = \frac{1}{2}\mathcal{L}^4 - 3\,\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right)\mathcal{L}$$
$$+\frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3}\ln^4 2 + 256\,\text{Li}_4\left(\frac{1}{2}\right)$$

$\mathcal{L} \equiv \ln(Q^2/M^2)$

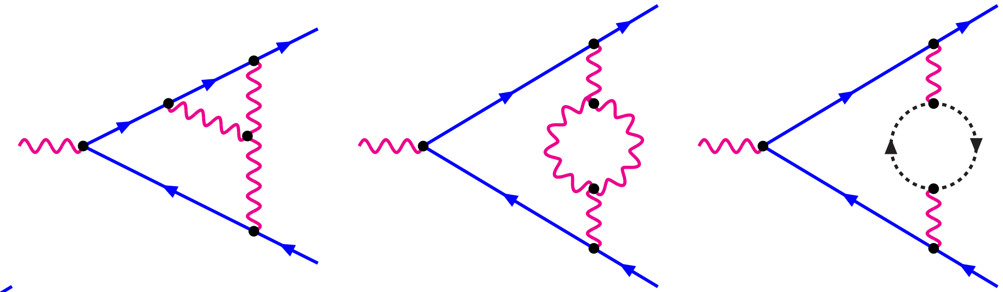
C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

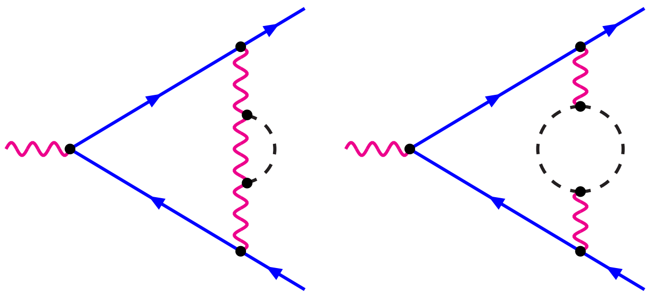
Abelian (C_F^2):



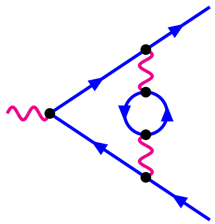
non-Abelian ($C_F C_A$): last 2 +



Higgs:



fermion ($C_F T_F n_f$):



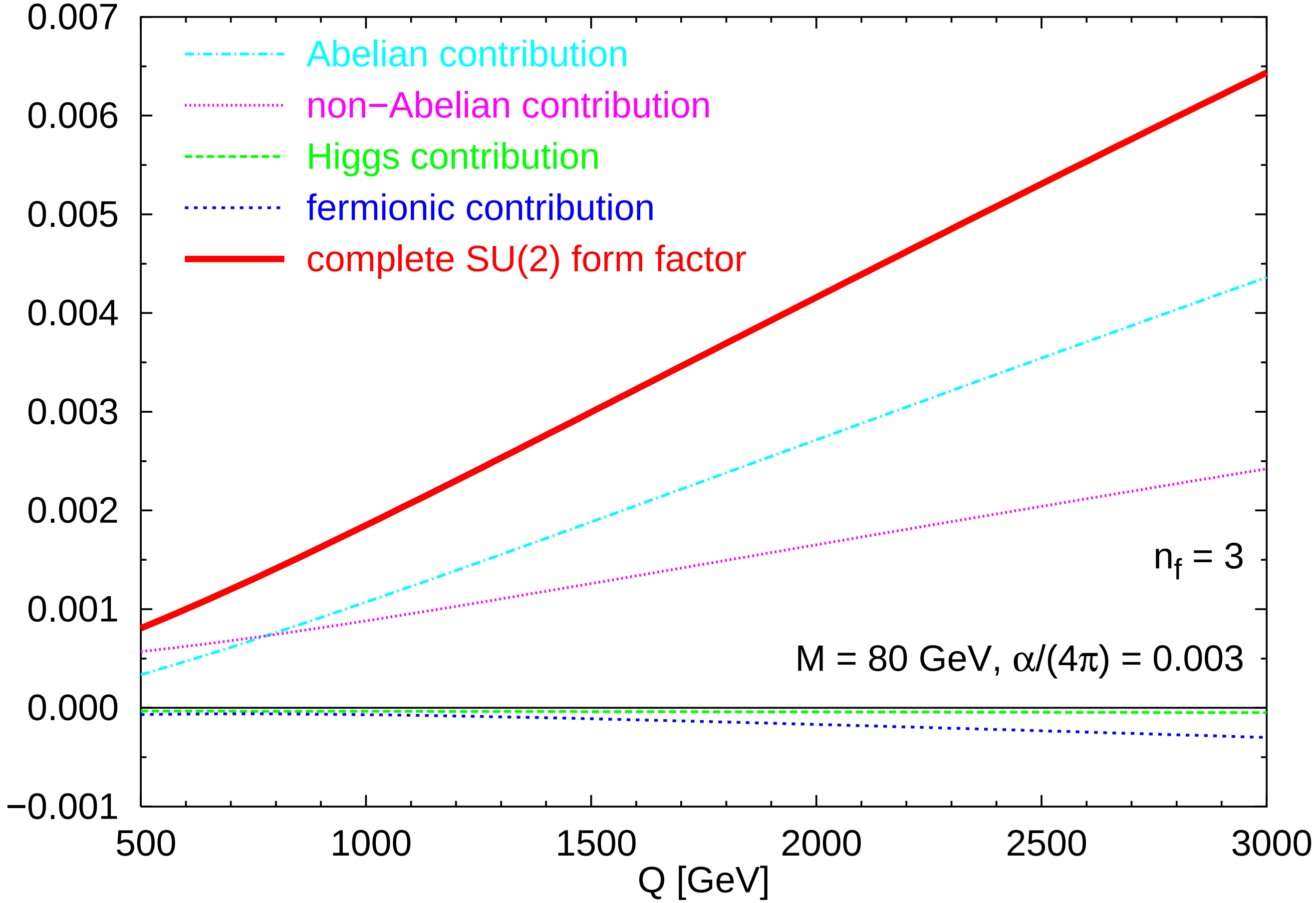
+ 1-loop×1-loop corrections + renormalization

$$f_2 = +\frac{9}{32}\mathcal{L}^4 - \frac{19}{48}\mathcal{L}^3 - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\mathcal{L}^2 + \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\mathcal{L}$$

(Expansion by regions! Result also needed in other evaluations, e.g. Manohar + . . .)

individual contributions

(N^{3}LL approximation, $M_{\text{Higgs}} = M$, $n_f = 3$, Feynman-'t Hooft gauge)



4-fermion cross section $\Rightarrow$ factor 4!

D) Four fermion scattering

Evaluation in the high energy limit

define

$$\begin{aligned}\mathcal{A}^{\lambda} &= \bar{\psi}_2 t^a \gamma_\mu \psi_1 \bar{\psi}_4 t^a \gamma_\mu \psi_3 \\ \mathcal{A}^{\lambda}_{LL} &= \bar{\psi}_{2L} t^a \gamma_\mu \psi_{1L} \bar{\psi}_{4L} t^a \gamma_\mu \psi_{3L} \\ \mathcal{A}^{d}_{LR} &= \bar{\psi}_{2L} \gamma_\mu \psi_{1L} \bar{\psi}_{4R} \gamma_\mu \psi_{3R}\end{aligned}$$

define "reduced" amplitude $\tilde{\mathcal{A}}$

$$\mathcal{A} = \frac{ig^2}{s} \mathcal{F}^2 \tilde{\mathcal{A}}$$

evolution equation

$$\frac{\partial}{\partial \ln Q^2} \tilde{\mathcal{A}} = \chi(\alpha(Q^2)) \tilde{\mathcal{A}}$$

$\tilde{\mathcal{A}}$: vector in isospin/chiral basis

χ: matrix

N^{3}LL **requires:**

- form factor up to N^{3}LL
- χ up to two loop, as obtained from hard contribution to single pole part of 4-fermion scattering amplitude

e.g. pure massive SU(2) theory with SSB:

$$\sigma^{(2)} = \Bigg[\frac{9}{2}\mathcal{L}^4 - \frac{449}{6}\mathcal{L}^3 + \left(\frac{4855}{18} + \frac{37}{3}\pi^2\right)\mathcal{L}^2 + \left(\frac{34441}{216} - \frac{1247}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\mathcal{L}\Bigg]\sigma_B$$

for identical isospin in initial and final state

Electroweak theory

- infrared logs must be separated

- NNLL
  - result insensitive to form of gauge-boson mass generation
  - term of order $1 - M_W^2/M_Z^2 = \sin^2\theta$ included

- N^3LL
  - sensitive to details of mass generation, gauge boson mixing
  - Approximation: terms of $\mathcal{O}(\sin^2\theta)$ neglected

Result for the correction factor

$$R(e^+e^- \to Q\bar{Q}) = 1 - 1.66\,L(s) + 5.60\,l(s) - 8.39\,a + 1.93\,L^2(s) - 11.28\,L(s)\,l(s) + 33.79\,l^2(s) - 150.95\,l(s)\,a$$

$$R(e^+e^- \to q\bar{q}) = 1 - 2.18\,L(s) + 20.94\,l(s) - 35.07\,a + 2.79\,L^2(s) - 51.98\,L(s)\,l(s) + 321.34\,l^2(s) - 603.43\,l(s)\,a$$

$$R(e^+e^- \to \mu^+\mu^-) = 1 - 1.39\,L(s) + 10.12\,l(s) - 21.26\,a + 1.42\,L^2(s) - 20.33\,L(s)\,l(s) + 112.57\,l^2(s) - 260.15\,l(s)\,a$$

with

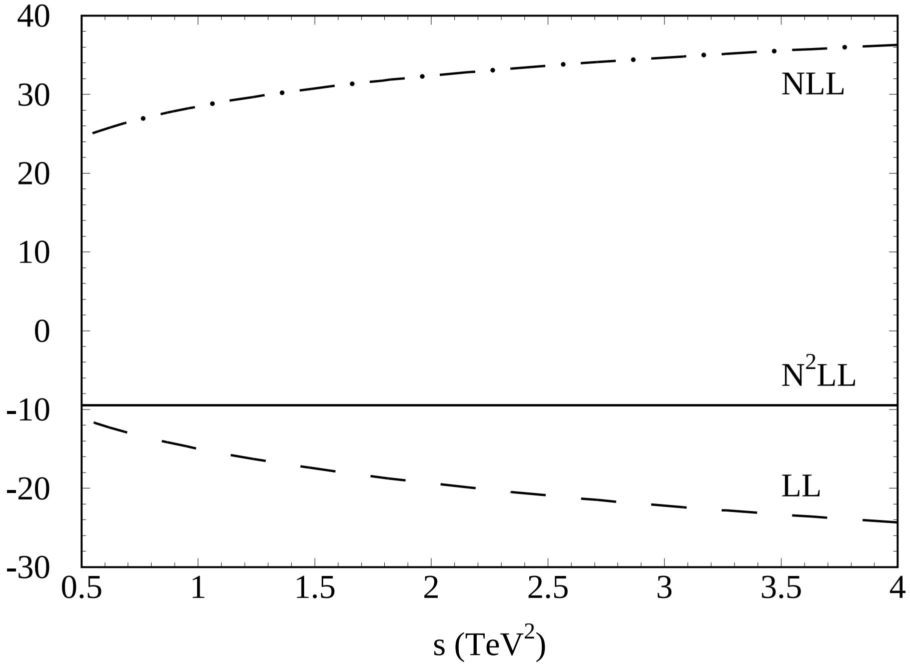
$$L(s) = \frac{g^2}{16\pi^2}\ln^2\left(\frac{s}{M^2}\right) = 0.07 \quad (0.11)$$

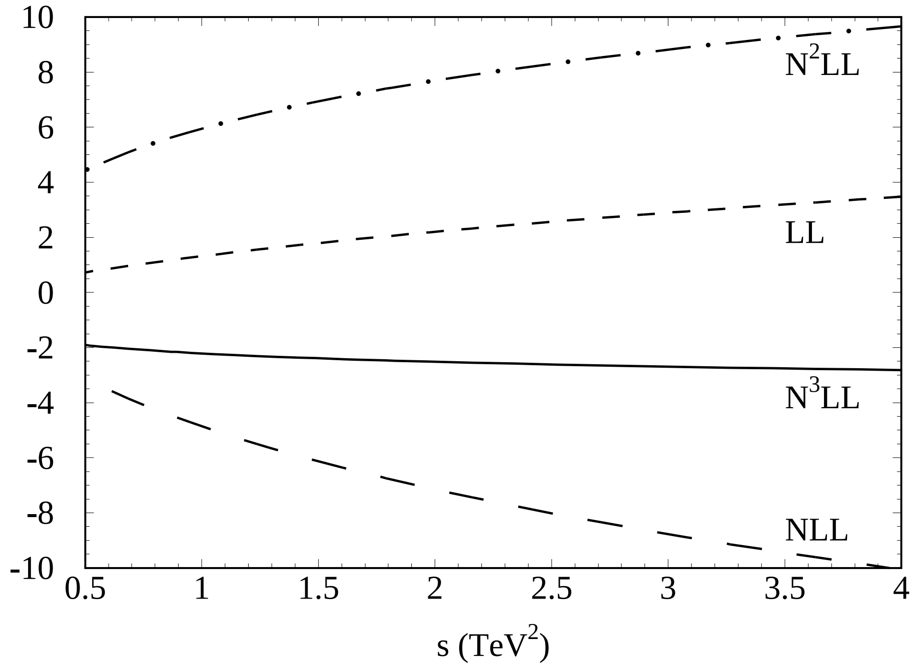
$$l(s) = \frac{g^2}{16\pi^2}\ln\left(\frac{s}{M^2}\right) = 0.014\ (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

for $\sqrt{s} = 1$ TeV (2 TeV)

Separate logarithmic contributions to $R(e^+e^- \to q\bar{q})$ in % to the Born approximation





one-loop LL $\left(\ln^2(s/M^2)\right)$, NLL $\left(\ln^1(s/M^2)\right)$ and N²LL $\left(\ln^0(s/M^2)\right)$

two-loop LL $\left(\ln^4(s/M^2)\right)$, NLL $\left(\ln^3(s/M^2)\right)$, NNLL $\left(\ln^2(s/M^2)\right)$ and N³LL $\left(\ln^1(s/M^2)\right)$

Large cancellations!

III. W-Pair Production at a Linear Collider

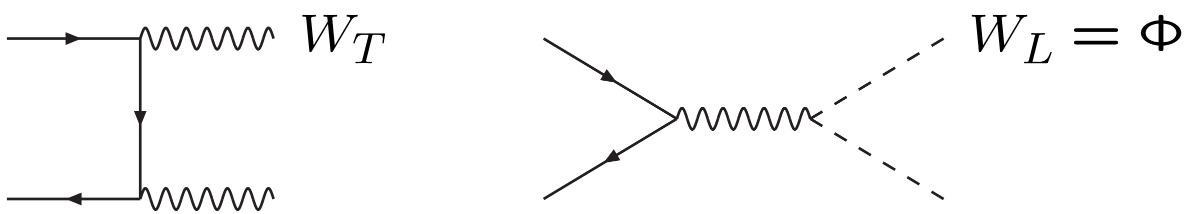
J.H.K., Metzler, Penin

NPB 795,277 (2008); + in preparation

Born:
dominance of transverse vs longitudinal Ws.
dominance of lefthanded initial states.

Sudakov Logarithms:
techniques similar to fermion scattering: evolution equation & separation of QED
new aspects:

- longitudinal vs transverse Ws (I=1 vs I=1/2; equivalence theorem)



- NNLL: $\log^2$, log, const. required in 1 loop

transverse amplitude

$$\mathcal{A}_T = \mathcal{Z}_\psi \mathcal{Z}_A \tilde{\mathcal{A}}_T \,,$$

with

$$\mathcal{Z}_i = \exp\left\{\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x} \frac{\mathrm{d}x'}{x'}\gamma_i(\alpha(x')) + \zeta_i(\alpha(x)) + \xi_i(\alpha(M^2))\right]\right\}$$

and

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}}_T = \boldsymbol{\chi}_T(\alpha(Q^2))\tilde{\mathcal{A}}_T \,,$$

$$\tilde{\mathcal{A}}_T = \mathrm{Pexp}\left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\boldsymbol{\chi}_T(\alpha(x))\right]\mathcal{A}_{0T}(\alpha(M^2))\,.$$

anomalous dimensions γ, ζ and $\boldsymbol{\chi}_T$ are mass independent

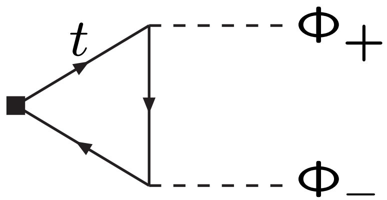
$$\gamma_\psi^{(1)} = -3/2\,, \quad \gamma_\psi^{(2)} = -\frac{65}{3} + \pi^2 + \frac{5}{6} n_f\,, \qquad \zeta_\psi^{(1)} = \frac{9}{4}, \quad \zeta_A^{(1)} = 0\,.$$

$$\boldsymbol{\chi}_T^{(1)} = \begin{pmatrix} -2(\ln(x_-) + i\pi) & 0 & \ln(\frac{x_+}{x_-})) \\ 0 & -2(\ln(x_+) + i\pi) & \ln(\frac{x_-}{x_+}) \\ (\ln(x_+) + i\pi) & (\ln(x_-) + i\pi) & 0 \end{pmatrix}.$$

$\mathcal{A}_{0T}$ from initial conditions
($\widehat{=}$ one loop result in high energy limit: $\ln^2$, ln, const. from Beenakker, Denner, Dittmaier, Mertig, Sack; transformed to $\overline{MS}$, QED singularities subtracted)

longitudinal amplitude: $W^{\pm} \hat{=} \Phi^{\pm}$

as above, additional contribution to anomalous dimension matrix $\zeta^{(1)}$ from Yukawa coupling in NLL:

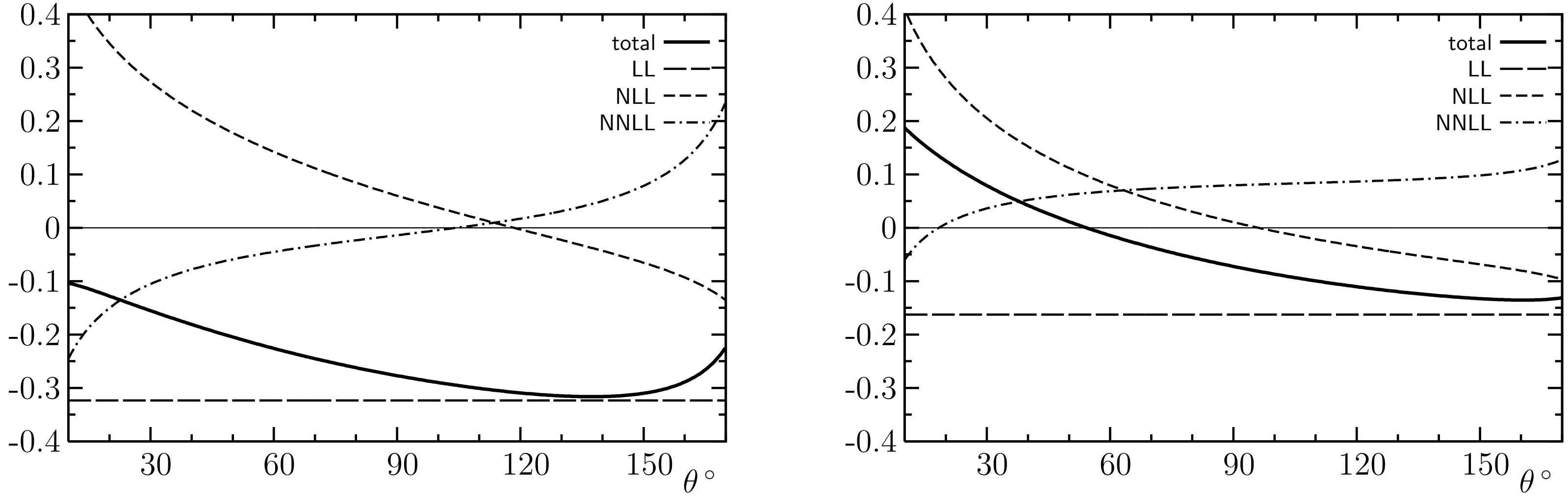


($\Rightarrow \sim$ log in 1 loop)

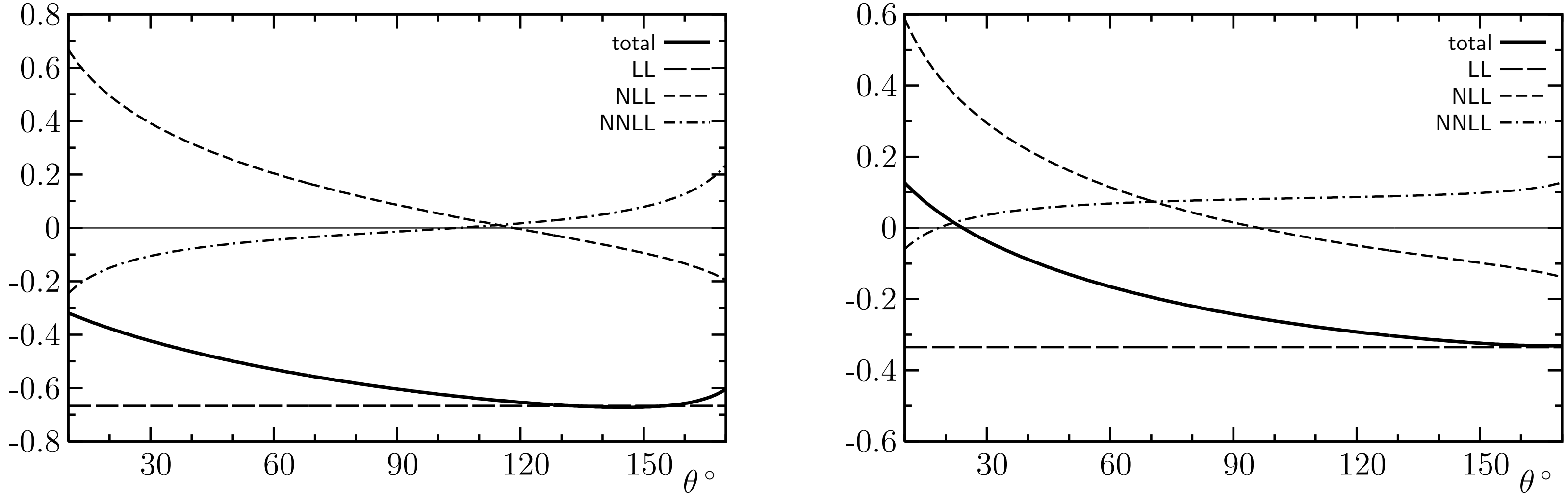
contribution to longitudinal Ws.

$$\zeta^{(1)} = \frac{1}{4}\begin{pmatrix} 12 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix} + \frac{m_t^2}{4M_W^2}\begin{pmatrix} 0 & 0 & 6 & 0 & -6 \\ 0 & 0 & 0 & 6 & -6 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & -1 & -1 & -1 & 0 \end{pmatrix}$$

1-loop

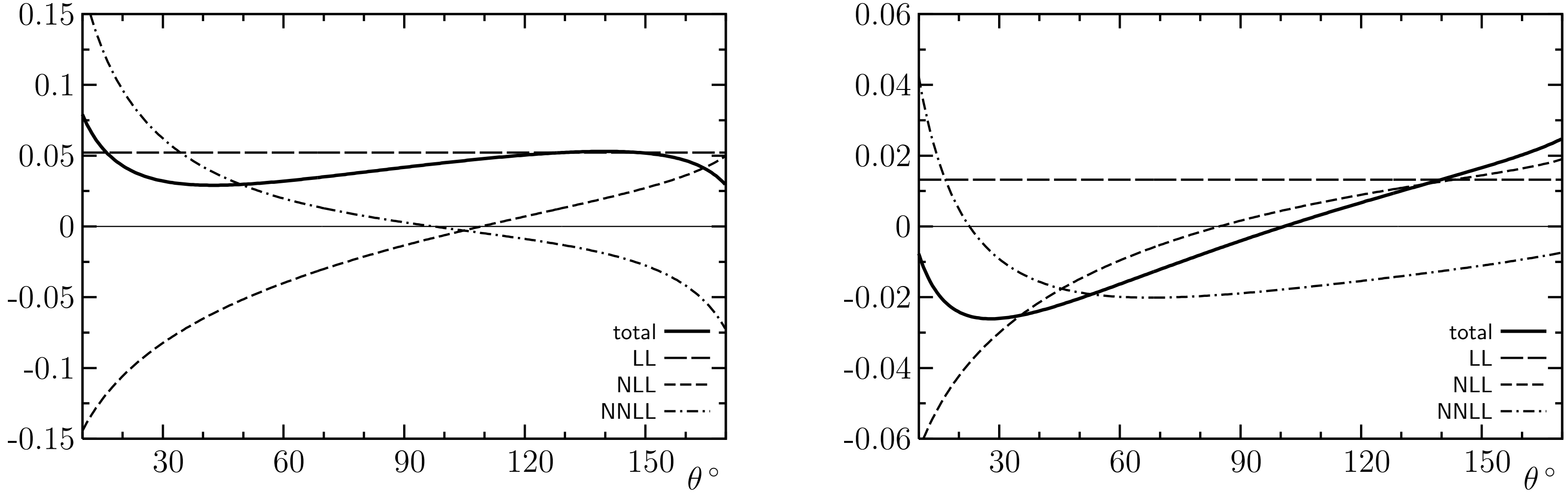


The one-loop logarithmic corrections to the differential cross section relative to the Born approximation at $\sqrt{s} = 1$ TeV as functions of the production angle for transverse (left) and longitudinal (right) polarization of the gauge bosons.

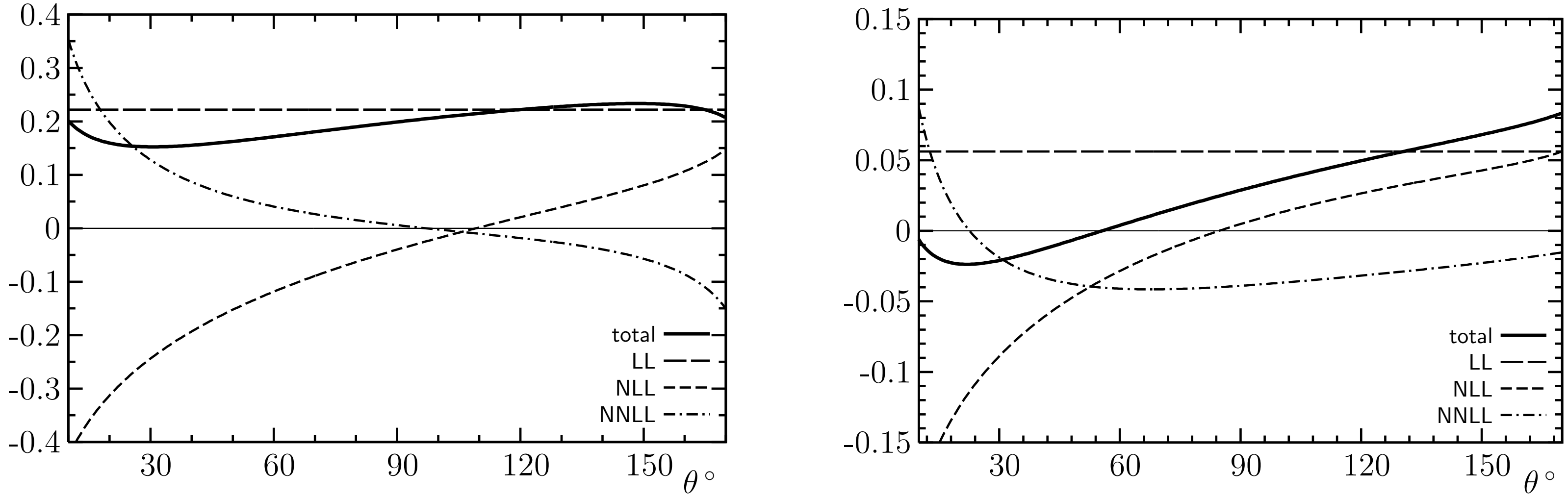


The same as above but for $\sqrt{s} = 3$ TeV.

2-loop



The two-loop logarithmic corrections to the differential cross section relative to the Born approximation at $\sqrt{s} = 1$ TeV as functions of the production angle fortransverse (left) and longitudinal (right) polarization of the gauge bosons.



The same as above but for $\sqrt{s} = 3$ TeV.

IV. W-Pair Production at the LHC

J.H.K., Metzler, Penin, Uccirati

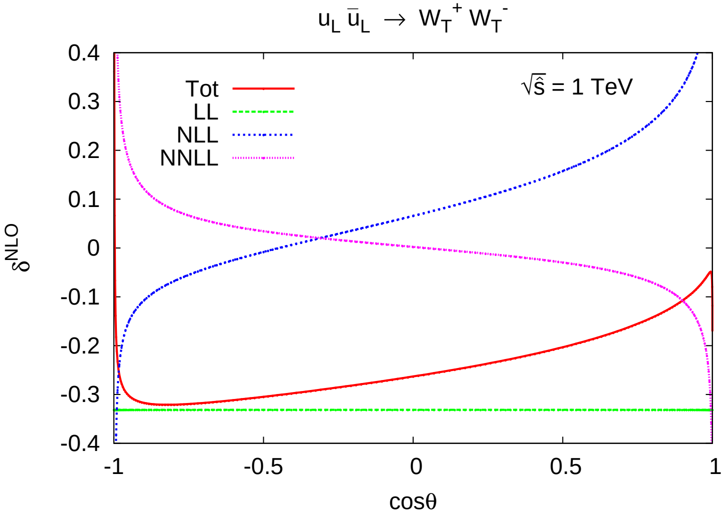
Hadronic initial states:

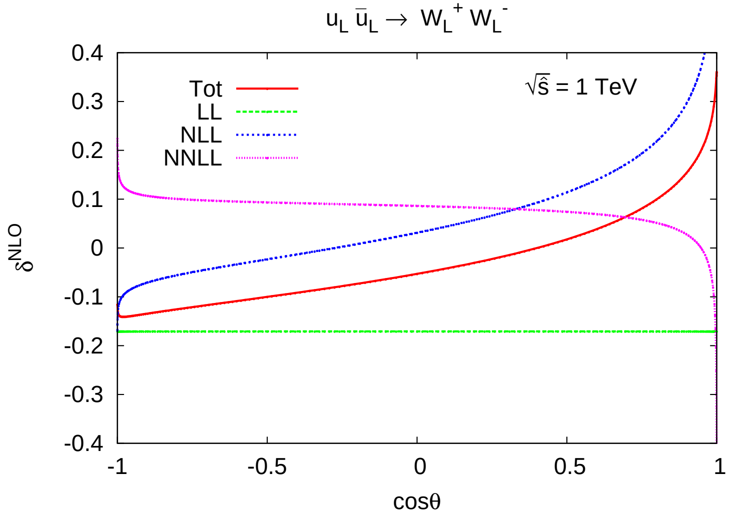
One-loop results not available ($\ln^2$, ln, const.)

$\longrightarrow$ independent one-loop calculation.
separation of IR-logs.
results qualitatively similar.

Logarithmic correction to the differential cross section (relative to the Born) at $\sqrt{s} = 1$ TeV

1-loop:

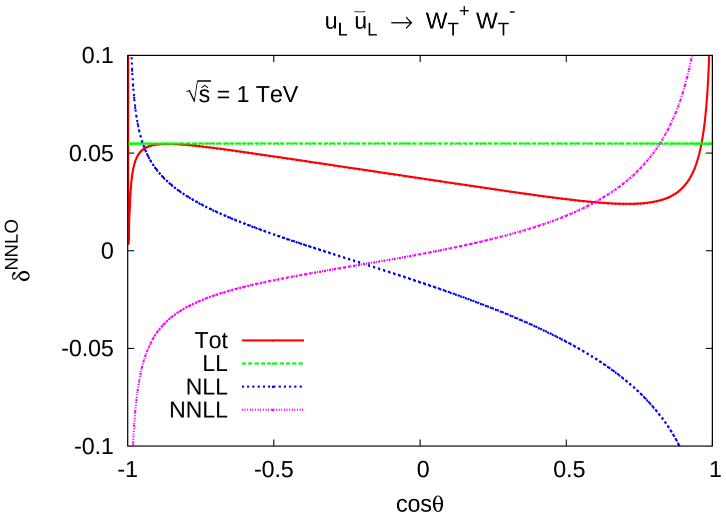


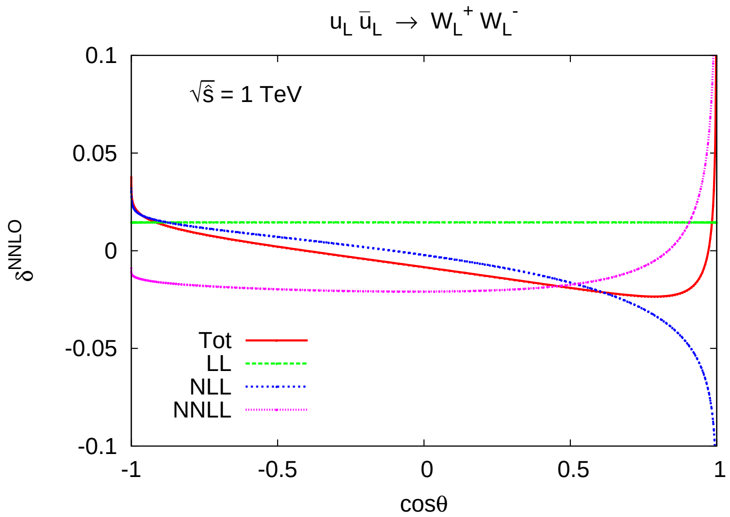


transverse

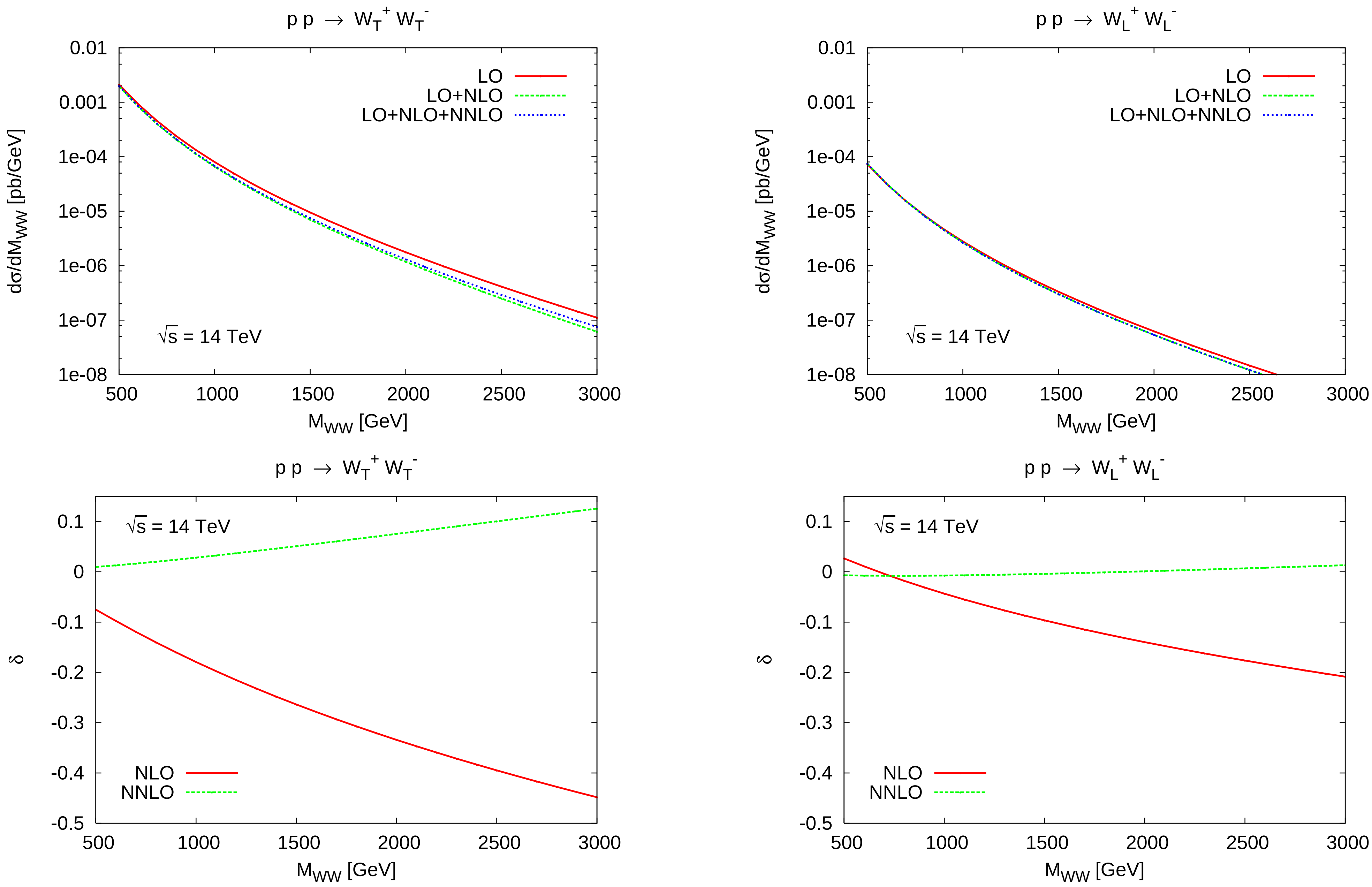
longitudinal

2-loop:





Invariant Mass Distribution ($\theta_{cut} = 30^\circ$)



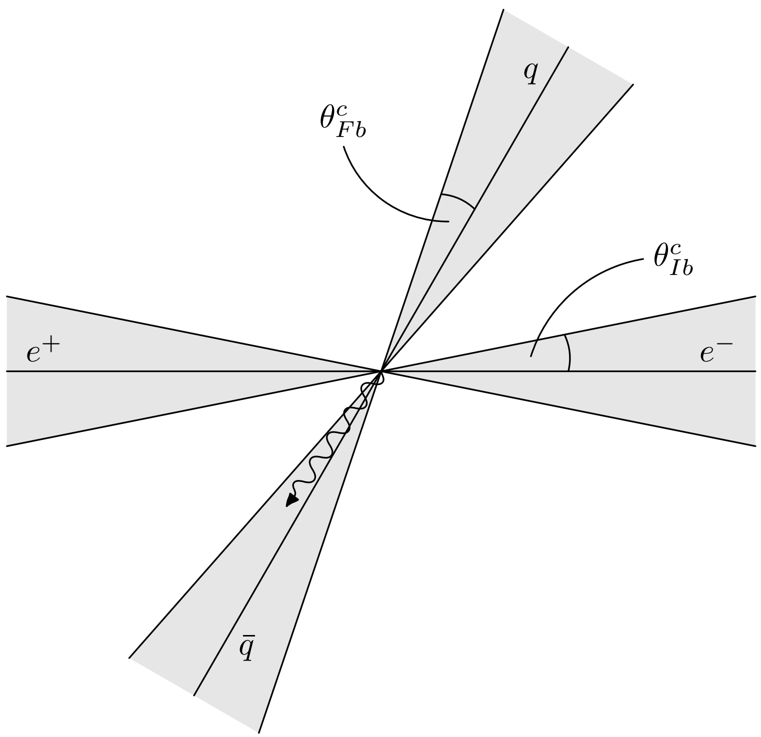
(larger effects for W_T! I=1)

V. Impact of real radiation

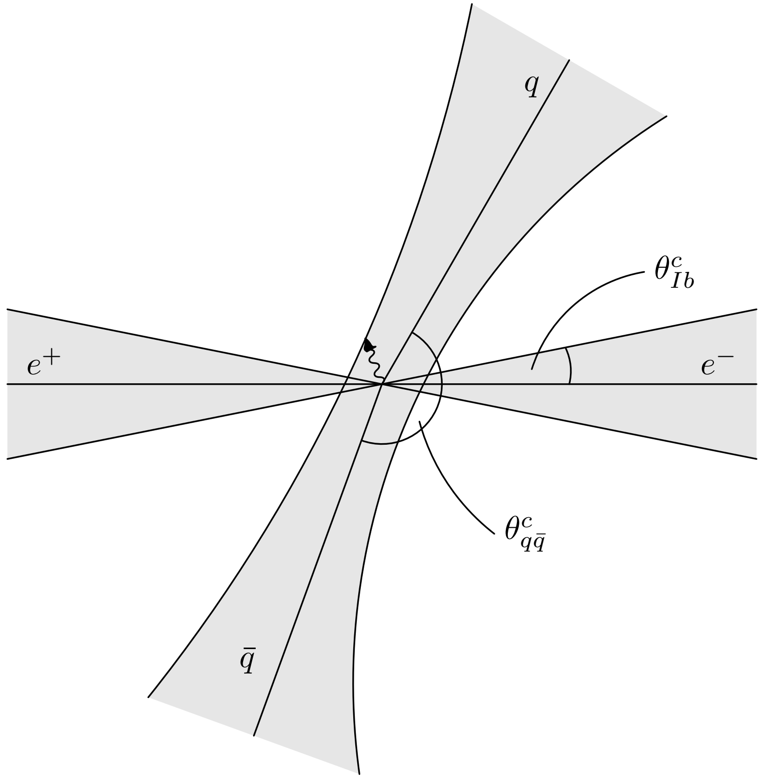
Bell, J.H.K., Rittinger

arXiv:1004.4117

soft and/or collinear radiation might (partially) compensate the reduction



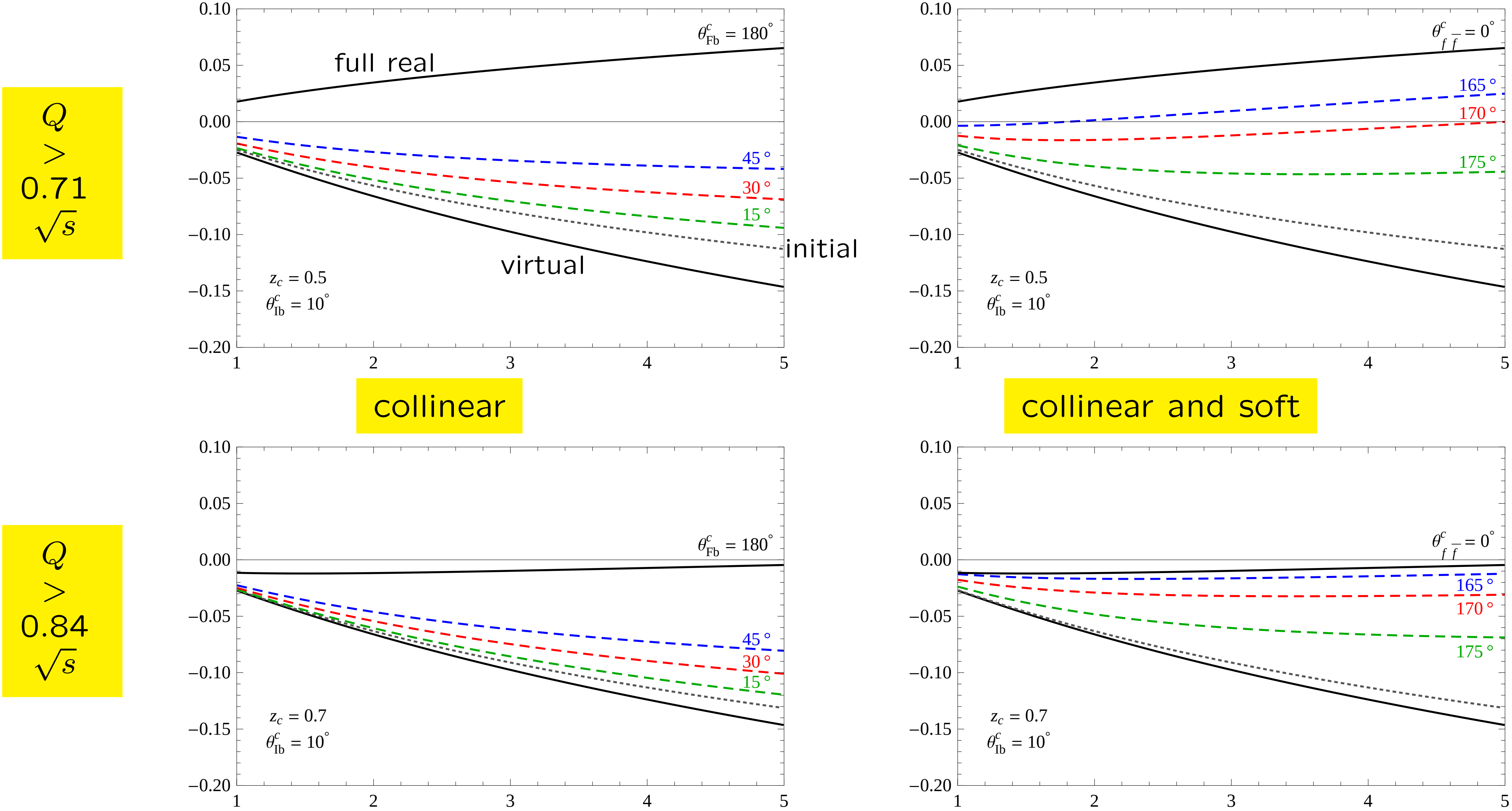
only collinear radiation



collinear and soft radiation

Summation over isospin in the final state (Z and W radiation allowed)

$e^+e^- \to q\bar{q} + (V)$:



$(\sigma_R + \sigma_V)/\sigma_0$ with 1 TeV $< \sqrt{s} <$ 5 TeV. Virtual only (lower solid line), only initial state radiation (black dotted line), initial and final state radiation (green, red and blue line) and real radiation with full angular phase space (upper solid line)

VI. Conclusions

- ILC and LHC will explore the TeV-region: $\hat{s} \big/ M_W^2 \gg 1$
- electroweak corrections amount to $\mathcal{O}(10\% - 20\%)$ in the interesting kinematic region
- form factors and four-fermion scattering in two loop
- two-loop terms might become relevant
- large effects for W pair production
- different behaviour of transverse and longitudinal Ws
- interplay between gauge and Yukawa terms
- how well can we separate real radiation?