HARD SCATTERING AND ELECTROWEAK CORRECTIONS AT THE LHC

J.H. Kühn

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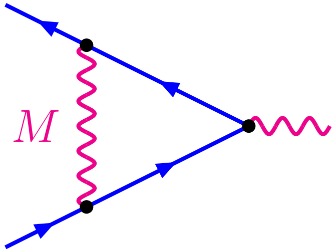
I. Introduction

"Typical" size of electroweak corrections: $\frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2}$

new aspects at LHC: $\sqrt{\hat{s}} \approx$ 1-2TeV $\gg M_{W,Z}$

strong enhancement of negative corrections

one-loop example: massive U(1)



$$\Rightarrow \quad \text{Born} * \left[1 + \frac{\alpha}{4\pi}\left(-\ln^2\frac{s}{M^2} + 3\ln\frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3}\right)\right]$$

$\frac{s}{M^2}$	$-\ln^2\frac{s}{M^2}$	$+3\ln\frac{s}{M^2}$	$-\frac{7}{2}+\frac{\pi^2}{3}$	Σ	$* \, 4\,\frac{\alpha_{\text{weak}}}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	$+15.15$	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	$+19.31$	-0.21	-22.3	-27%

(four-fermion cross section ⇒ factor 4)

- leading $\log^2$ multiplied by $(\text{charge})^2 = I(I+1) = \begin{cases} 3/4 & I = 1/2 \\ 2 & I = 1 \end{cases}$

- important subleading logarithms (NLL+...)

- two-loop terms may be relevant

- interplay between electroweak and QCD corrections

- important differences between fermions and electroweak gauge bosons

II. Form Factors & Four-Fermion Scattering at Two Loop

LL: Fadin et al. (2000)

NLL: J.H.K., Penin, Smirnov (2000)
Large (!) subleading corrections
important angular dependent terms

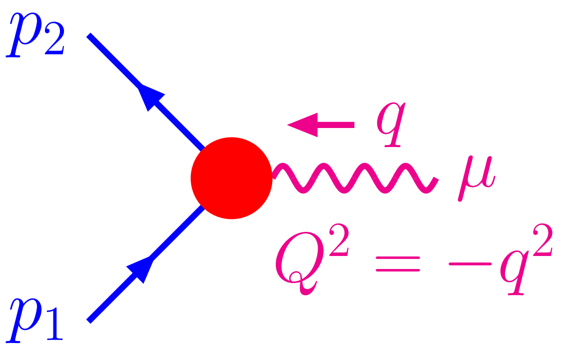
NNLL: J.H.K., Moch, Penin, Smirnov (2001)
Large (!) NNLL terms,
oscillating signs of LL, NLL, NNLL
⇒ compensations

N^3LL+N^4LL Jantzen, J.H.K., Penin, Smirnov (2003-2005)

Additional complication in SM: massless photon

$$|Q^2| \gg M^2_{W,Z} \gg m^2_\gamma$$

A) Form Factor and Evolution Equations



Born:

$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow \quad \mathcal{F} = \mathcal{F}_{\text{Born}}\, F_0(\alpha(M^2)) \exp\left\{\int\limits_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int\limits_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: N^{4}LL ⇒ corresponds to all terms of the form:

$$\alpha^n \left[\underset{\text{LL}}{\mathcal{L}^{2n}} + \underset{\text{NLL}}{\mathcal{L}^{2n-1}} + \underset{\text{NNLL}}{\mathcal{L}^{2n-2}} + \underset{\text{N}^3\text{LL}}{\mathcal{L}^{2n-3}} + \underset{\text{N}^4\text{LL}}{\mathcal{L}^{2n-4}} \right]$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

NNLL requires running of α (i.e. β_0 and β_1) and:

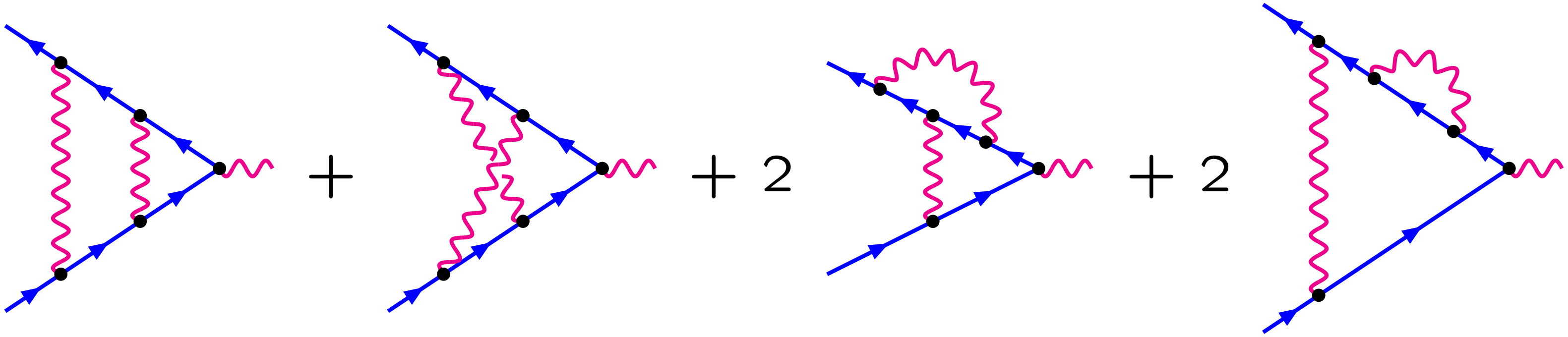
$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$ up to $\mathcal{O}(\alpha)$ (one-loop)
$\gamma(\alpha)$ up to $\mathcal{O}(\alpha^2)$ (massless two loop)

N^{3}LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

N^{4}LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

B) Two-Loop Results: Massive U(1) Model

$$\mathcal{F}_\alpha(M,Q) = \mathcal{F}_{\text{Born}}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \ldots\right]$$



$$f^{(2)} = \frac{1}{2}\mathcal{L}^4 - 3\,\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right)\mathcal{L}$$

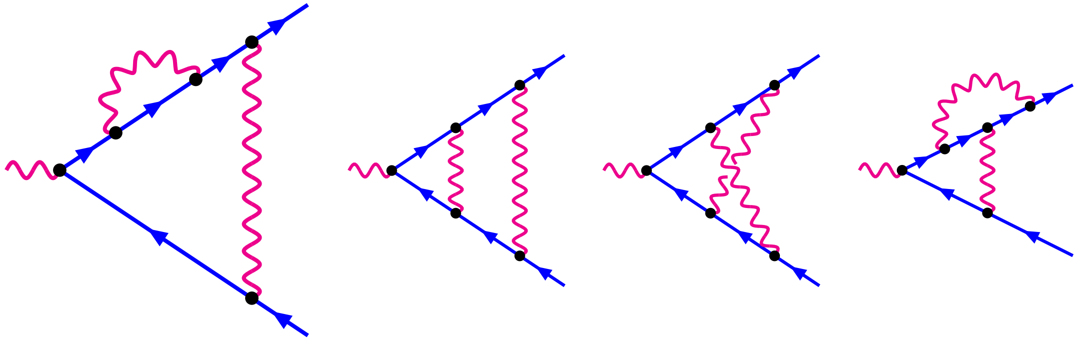
$$+\frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2\ln^2 2 + \frac{32}{3}\ln^4 2 + 256\,\mathrm{Li}_4\left(\frac{1}{2}\right)$$

$\mathcal{L} \equiv \ln(Q^2/M^2)$

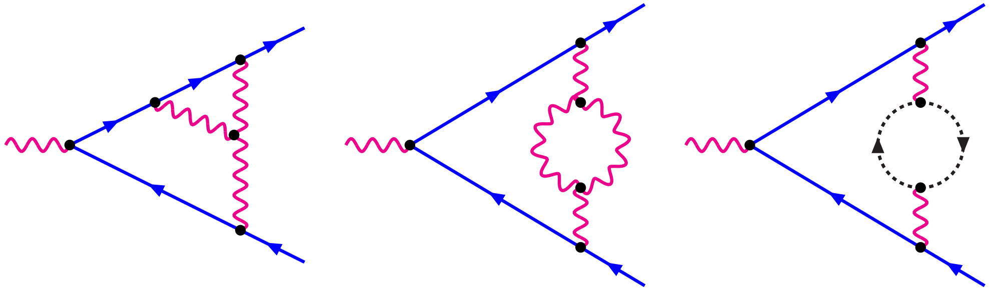
C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

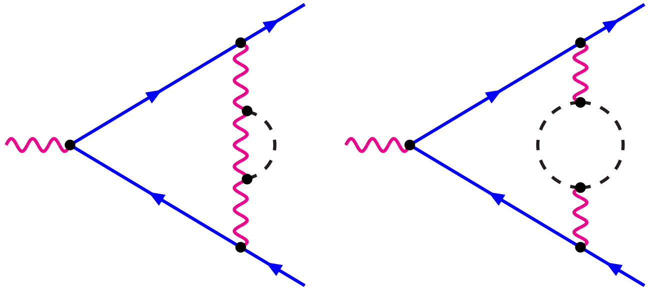
Abelian (C_F^2):



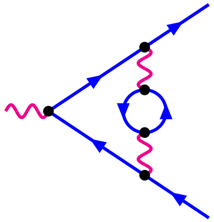
non-Abelian ($C_F C_A$): last 2 +



Higgs:



fermion ($C_F T_F n_f$):



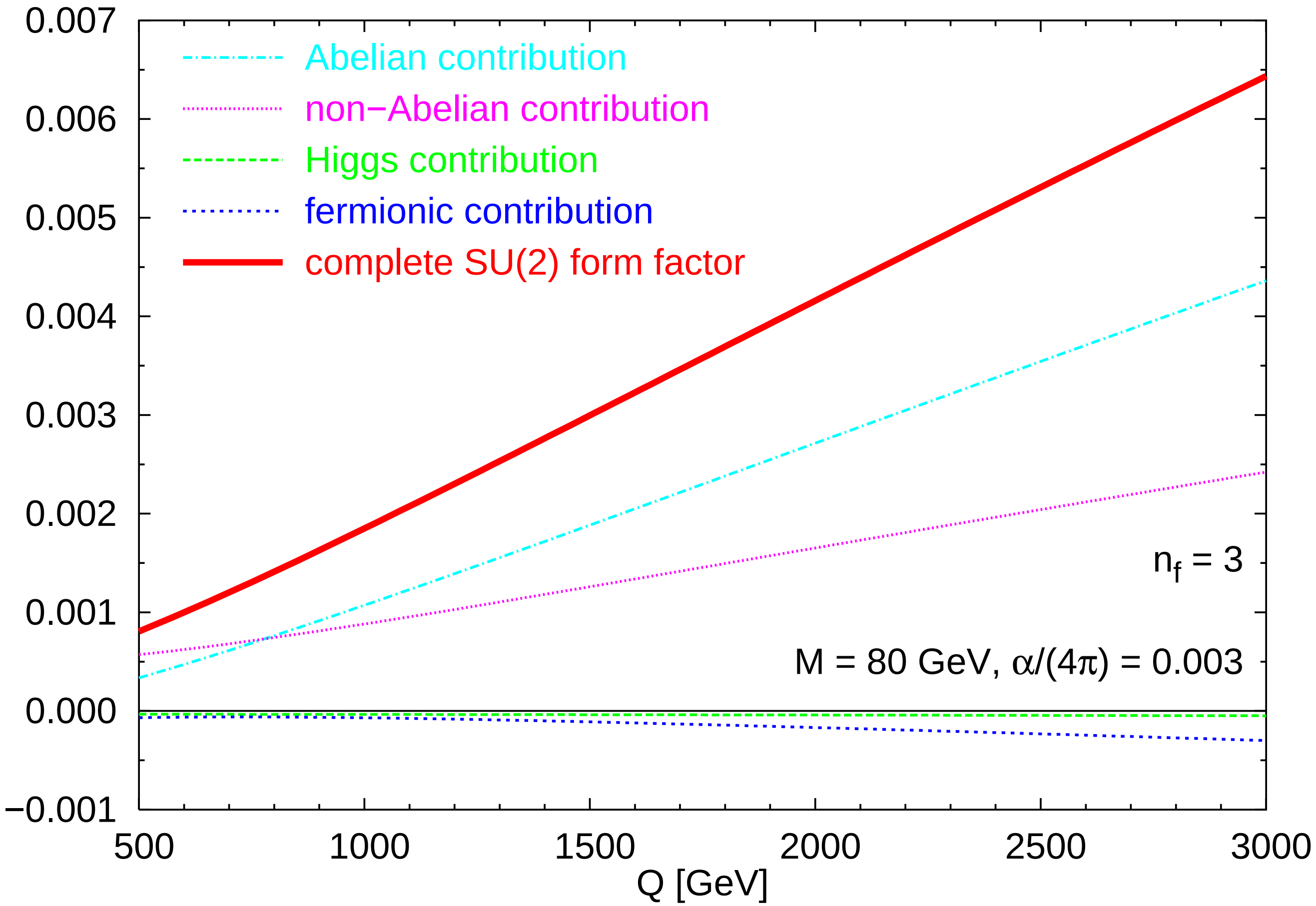
+ 1-loop×1-loop corrections + renormalization

$$f_2 = +\frac{9}{32}\mathcal{L}^4 - \frac{19}{48}\mathcal{L}^3 - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\mathcal{L}^2$$

$$+ \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\mathcal{L}$$

individual contributions

(N^{3}LL approximation, $M_{\text{Higgs}} = M$, $n_f = 3$, Feynman-'t Hooft gauge)



4-fermion cross section ⇒ factor 4!

D) Four fermion scattering

Evaluation in the high energy limit

define

$$\begin{aligned} \mathcal{A}^\lambda &= \bar{\psi}_2 t^a \gamma_\mu \psi_1 \bar{\psi}_4 t^a \gamma_\mu \psi_3 \\ \mathcal{A}^\lambda_{LL} &= \bar{\psi}_{2L} t^a \gamma_\mu \psi_{1L} \bar{\psi}_{4L} t^a \gamma_\mu \psi_{3L} \\ \mathcal{A}^d_{LR} &= \bar{\psi}_{2L} \gamma_\mu \psi_{1L} \bar{\psi}_{4R} \gamma_\mu \psi_{3R} \end{aligned}$$

define "reduced" amplitude $\tilde{\mathcal{A}}$

$$\mathcal{A} = \frac{ig^2}{s} \mathcal{F}^2 \tilde{\mathcal{A}}$$

evolution equation

$$\frac{\partial}{\partial \ln Q^2} \tilde{\mathcal{A}} = \chi(\alpha(Q^2)) \tilde{\mathcal{A}}$$

$\tilde{\mathcal{A}}$: vector in isospin/chiral basis

χ: matrix

N^{3}LL **requires:**

- form factor up to N^{3}LL
- χ up to two loop, as obtained from hard contribution to single pole part of 4-fermion scattering amplitude

e.g. pure massive SU(2) theory with SSB:

$$\begin{aligned}\sigma^{(2)} = &\left[\frac{9}{2}\mathcal{L}^4 - \frac{449}{6}\mathcal{L}^3 + \left(\frac{4855}{18} + \frac{37}{3}\pi^2\right)\mathcal{L}^2\right.\\ &\left.+\left(\frac{34441}{216} - \frac{1247}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\mathcal{L}\right]\sigma_B\end{aligned}$$

for identical isospin in initial and final state

Electroweak theory

- infrared logs must be separated

- NNLL
  - result insensitive to form of gauge-boson mass generation
  - term of order $1 - M_W^2/M_Z^2 = \sin^2\theta$ included

- N^3LL
  - sensitive to details of mass generation, gauge boson mixing
  - Approximation: terms of $\mathcal{O}(\sin^2\theta)$ neglected

Result for the correction factor

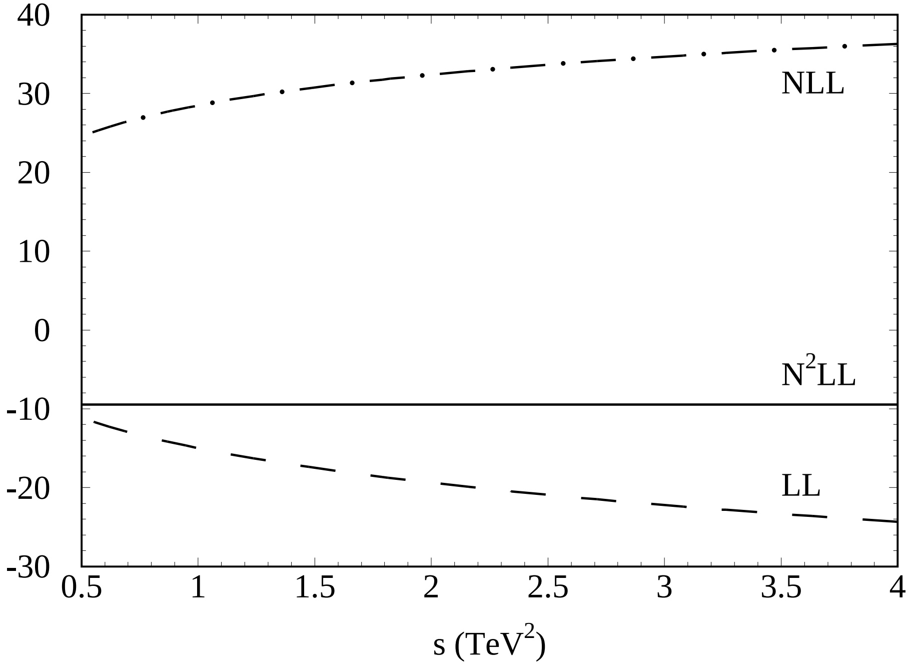
$$\begin{aligned}
R(e^+e^- \to Q\bar{Q}) &= 1 - 1.66\,L(s) + 5.60\,l(s) - 8.39\,a + 1.93\,L^2(s) \\
&\quad -11.28\,L(s)\,l(s) + 33.79\,l^2(s) - 150.95\,l(s)\,a \\
R(e^+e^- \to q\bar{q}) &= 1 - 2.18\,L(s) + 20.94\,l(s) - 35.07\,a + 2.79\,L^2(s) \\
&\quad -51.98\,L(s)\,l(s) + 321.34\,l^2(s) - 603.43\,l(s)\,a \\
R(e^+e^- \to \mu^+\mu^-) &= 1 - 1.39\,L(s) + 10.12\,l(s) - 21.26\,a + 1.42\,L^2(s) \\
&\quad -20.33\,L(s)\,l(s) + 112.57\,l^2(s) - 260.15\,l(s)\,a
\end{aligned}$$

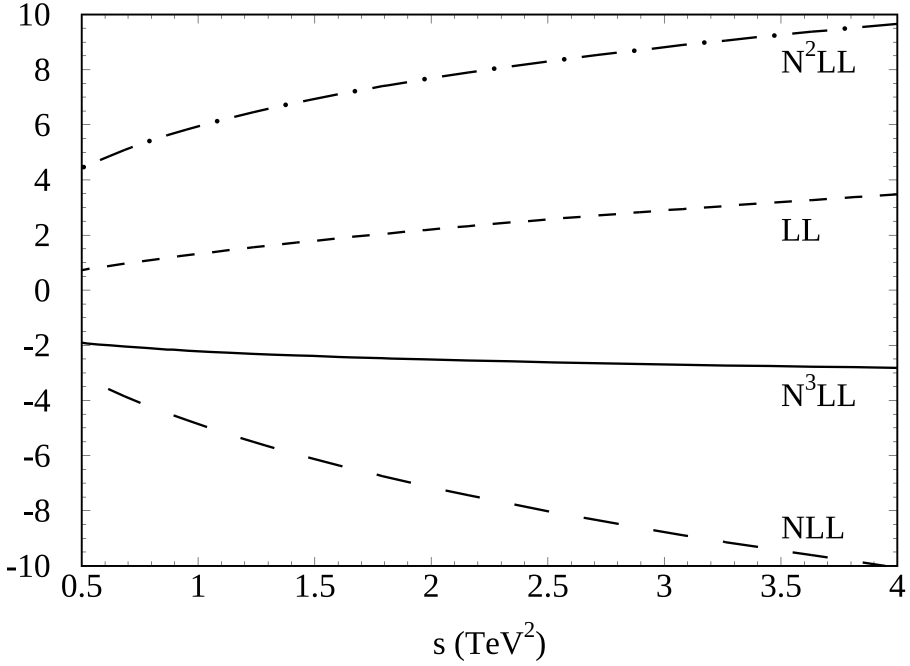
with

$$\begin{aligned}
L(s) &= \frac{g^2}{16\pi^2}\ln^2\left(\frac{s}{M^2}\right) = 0.07 \quad (0.11) \\
l(s) &= \frac{g^2}{16\pi^2}\ln\left(\frac{s}{M^2}\right) = 0.014\ (0.017) \\
a &= \frac{g^2}{16\pi^2} = 0.003
\end{aligned}$$

for $\sqrt{s} = 1$ TeV (2 TeV)

Separate logarithmic contributions to $R(e^+e^- \to q\bar{q})$ in % to the Born approximation





one-loop LL $\left(\ln^2(s/M^2)\right)$, NLL $\left(\ln^1(s/M^2)\right)$ and N^{2}LL $\left(\ln^0(s/M^2)\right)$

two-loop LL $\left(\ln^4(s/M^2)\right)$, NLL $\left(\ln^3(s/M^2)\right)$, NNLL $\left(\ln^2(s/M^2)\right)$ and N^{3}LL $\left(\ln^1(s/M^2)\right)$

Large cancellations!

III. Z, W and Photon Production at LHC

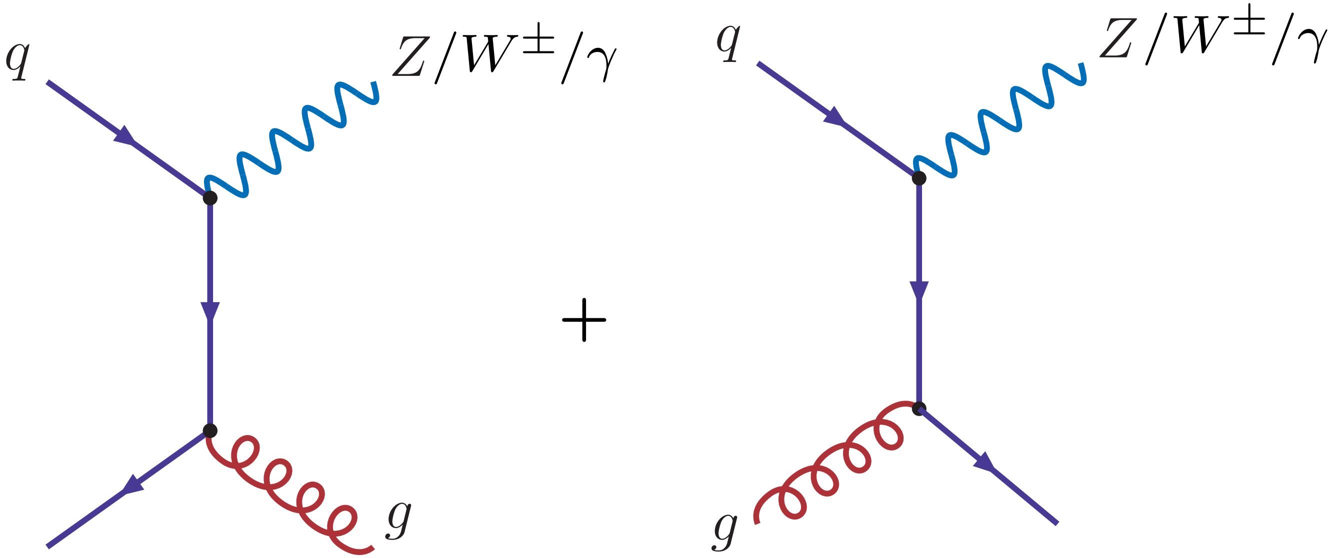
J.H.K., Kulesza, Pozzorini, Schulze

Phys. Lett. B609(2005) 277, Nucl. Phys. B727(2005) 368,

JHEP 0603:059,2006, Phys.Lett. B651(2007), NPB797 (2008) 27

sizable rate at large p_{T} (1-2 TeV)

Large electroweak corrections ($\hat{s} \gg M_{W,Z}^2$)



High energy limit

consider $q\bar{q} \to Zg$

NLL $\widehat{=}$ double + single logarithmic terms

$$H_1^{\mathrm{A}} \overset{\mathrm{NLL}}{=} -\left[\log^2\left(\frac{|\hat{s}|}{M_W^2}\right) - 3\log\left(\frac{|\hat{s}|}{M_W^2}\right)\right] H_0,$$

$$H_1^{\mathrm{N}} \overset{\mathrm{NLL}}{=} -\left[\log^2\left(\frac{|\hat{t}|}{M_W^2}\right) + \log^2\left(\frac{|\hat{u}|}{M_W^2}\right) - \log^2\left(\frac{|\hat{s}|}{M_W^2}\right)\right] H_0$$

- remaining subleading terms $\leq$ 2.5%
- NNLL includes non-enhanced terms (angular dependent)
- compact formulae

size of the correction:

$$\sqrt{\hat{s}} \;=\; 200\ \mathrm{GeV}: \qquad \frac{\delta\sigma}{\sigma} \leq 0.3\%$$

$$\sqrt{\hat{s}} \;=\; 4000\mathrm{GeV}: \qquad \frac{\delta\sigma}{\sigma} \approx 20-30\%$$

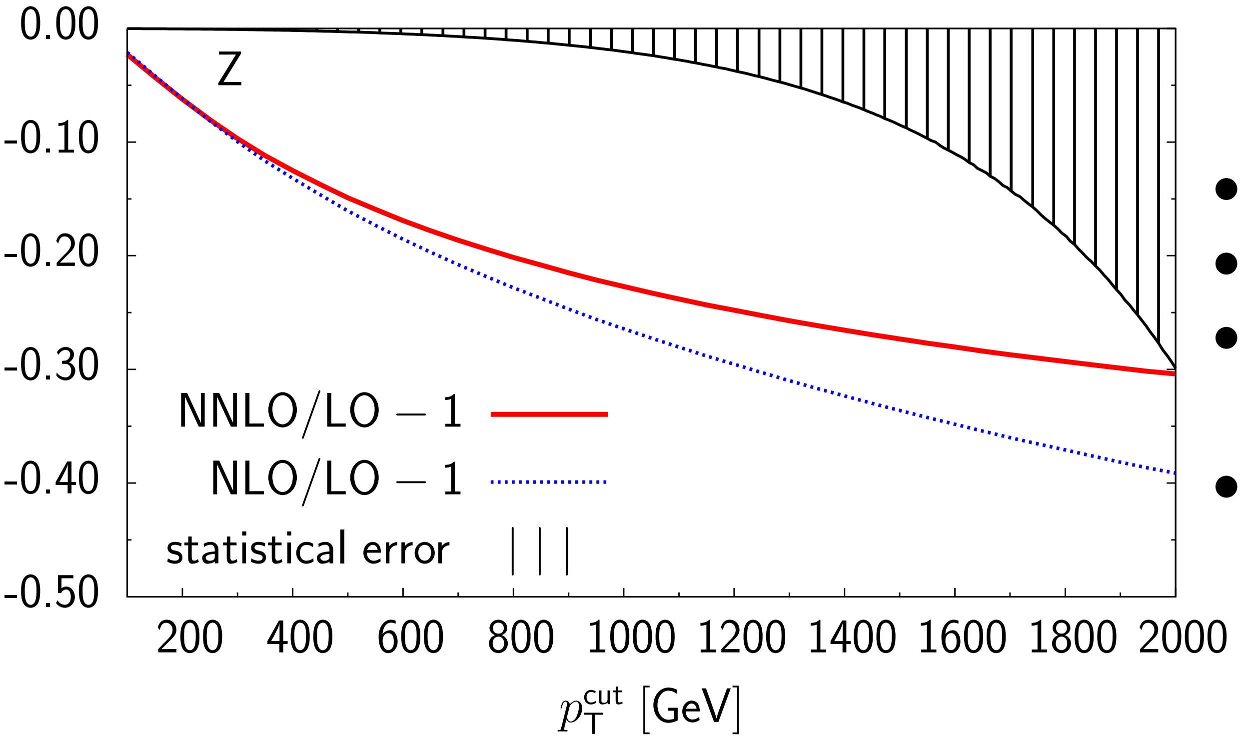
Result consistent with general considerations

two-loop (NLL):

$$
\begin{aligned}
A^{(2)} = \sum_{\lambda=\mathrm{L,R}} \Bigg\{ & \frac{1}{2}\left(I^Z_{q_\lambda} C^{\mathrm{ew}}_{q_\lambda} + \frac{c_\mathrm{W}}{s_\mathrm{W}^3} T^3_{q_\lambda}\right)\left[I^Z_{q_\lambda} C^{\mathrm{ew}}_{q_\lambda}\left(\mathsf{L}^4_{\hat{s}} - 6\mathsf{L}^3_{\hat{s}}\right)\right.\\
& \left. + \frac{c_\mathrm{W}}{s_\mathrm{W}^3} T^3_{q_\lambda}\left(\mathsf{L}^4_{\hat{t}} + \mathsf{L}^4_{\hat{u}} - \mathsf{L}^4_{\hat{s}}\right)\right] - \frac{T^3_{q_\lambda} Y_{q_\lambda}}{8 s_\mathrm{W}^4}\left(\mathsf{L}^4_{\hat{t}} + \mathsf{L}^4_{\hat{u}} - \mathsf{L}^4_{\hat{s}}\right)\\
& + \frac{1}{6} I^Z_{q_\lambda}\left[I^Z_{q_\lambda}\left(\frac{b_1}{c_\mathrm{W}^2}\left(\frac{Y_{q_\lambda}}{2}\right)^2 + \frac{b_2}{s_\mathrm{W}^2} C_{q_\lambda}\right) + \frac{c_\mathrm{W}}{s_\mathrm{W}^3} T^3_{q_\lambda} b_2\right]\mathsf{L}^3_{\hat{s}} \Bigg\}
\end{aligned}
$$

with $\mathsf{L}^n_{\hat{r}} = \log^n\left(\frac{|\hat{r}|}{M_W^2}\right)$, $b_1 = -41/(6c_\mathrm{W}^2)$ and $b_2 = 19/(6s_\mathrm{W}^2)$

Complete one loop calculation
NLL approximation at two loops



- one-loop $\sim 30\%$ at $p_T \sim 1$TeV
- two-loop relevant above 1 TeV
- important angular-dependent logarithmic terms
- experiment: p_T up to 2 TeV

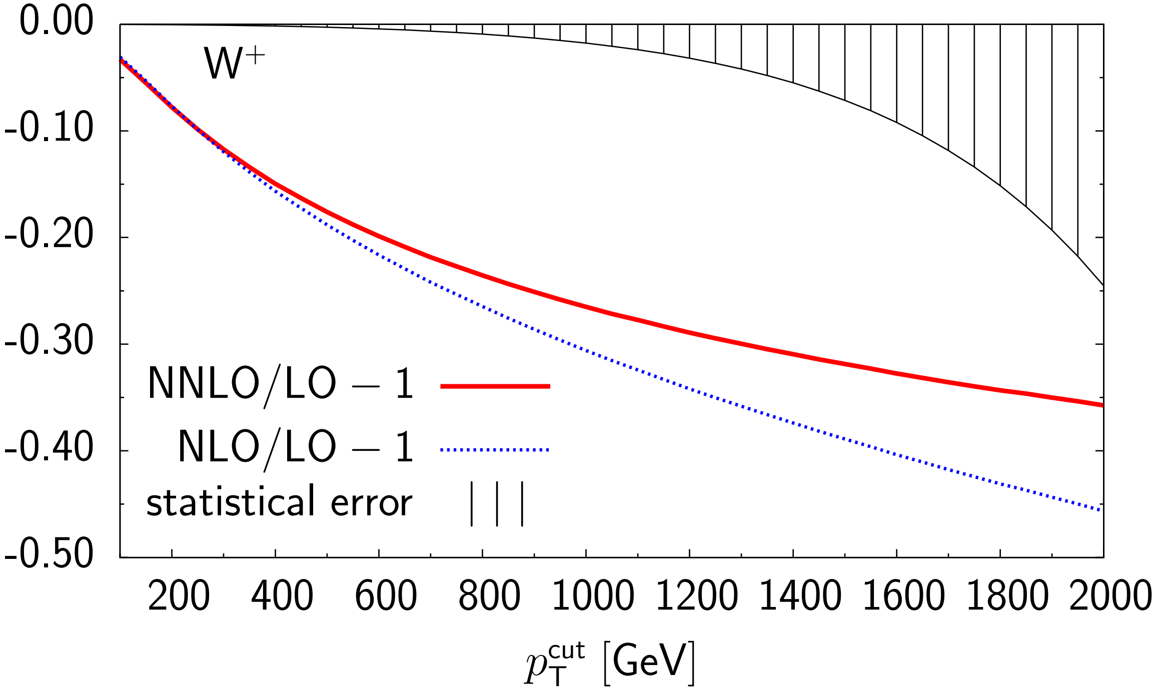
Relative **NLO** and **NNLO** corrections w.r.t. the **LO** and statistical error for the unpolarized integrated cross section for $pp \to Zj$ at $\sqrt{s} = 14$ TeV.

(Similarly, but smaller by a factor 2 for jet+γ)

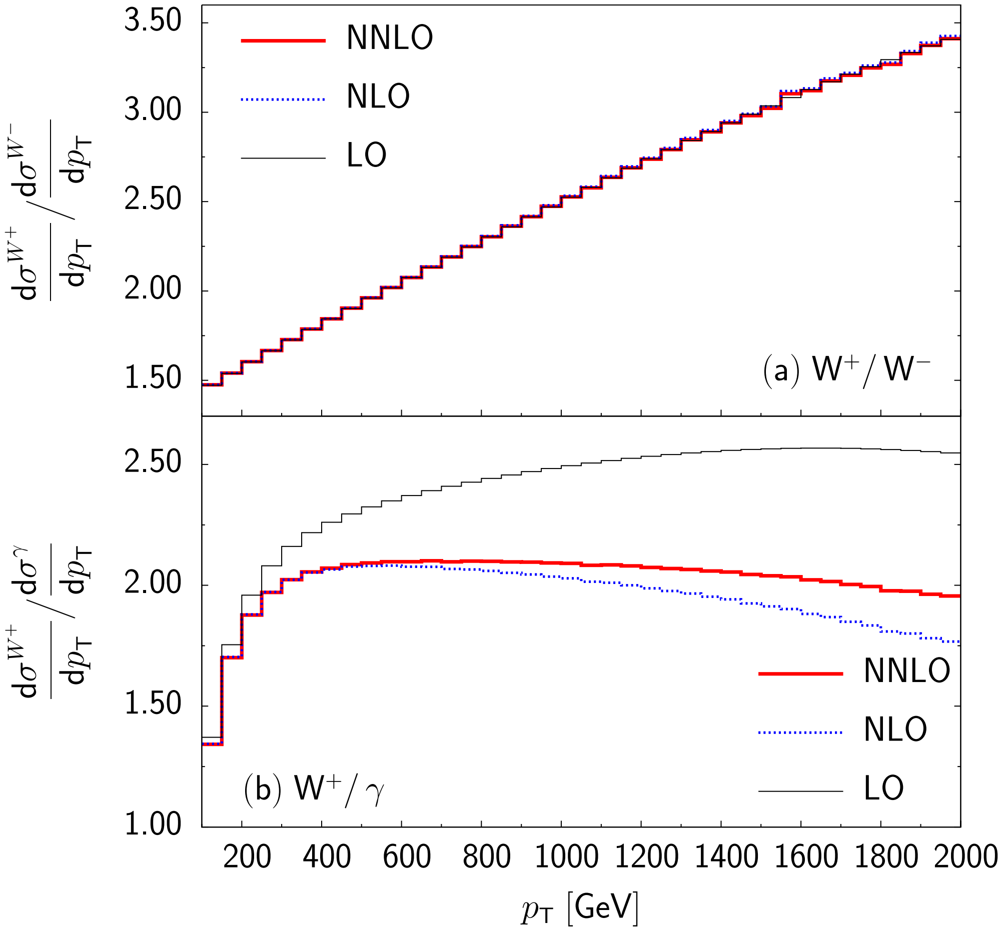
W production

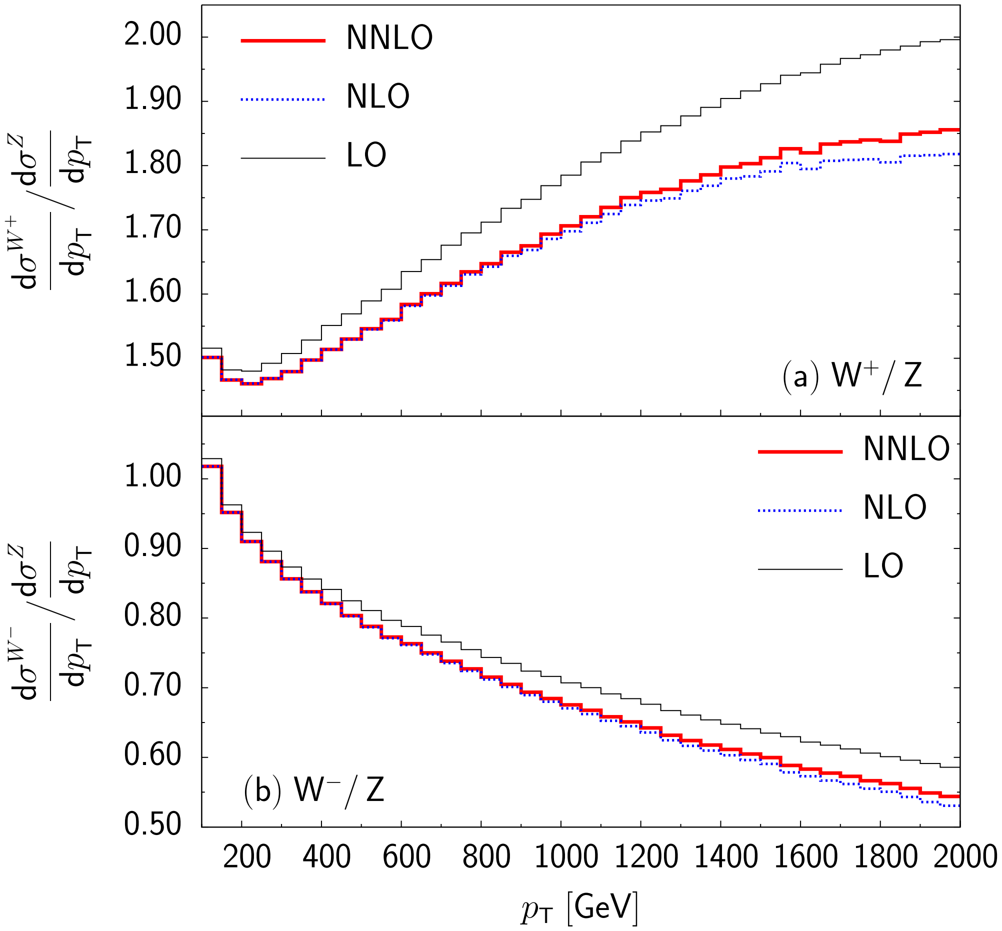
additional complications:

- photon radiation as necessary part of virtual corrections (gauge invariance)
- IR singularities must be compensated by real radiation
- $p_{\mathrm{T}}(W) = p_{\mathrm{T}}(\text{jet}) + p_{\mathrm{T}}(\gamma)$



(related results: Dittmaier, Kasprzik, ...)





ratios are less sensitive to QCD corrections

IV. Top and Bottom Production:

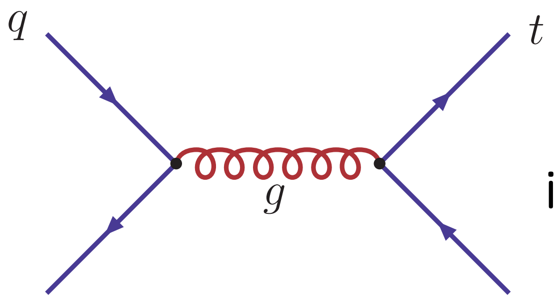
J.H.K., Scharf, Uwer

Eur. Phys. J. C45(2006) 139

Eur. Phys. J. C51(2007) 37

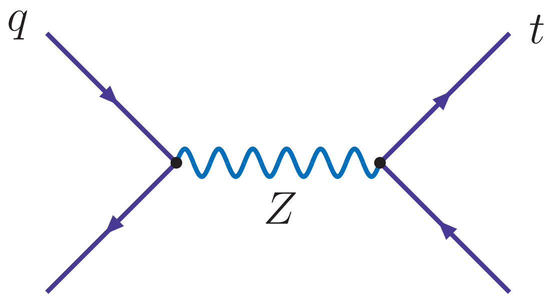
arXiv: 0909.0059

$q\,\bar{q} \longrightarrow t\,\bar{t}$:



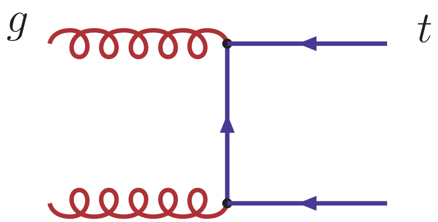
$\sim \mathcal{O}(\alpha_{\mathsf{s}})$

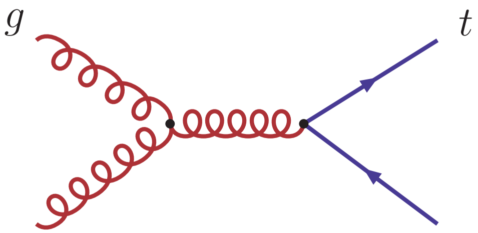
no
interference
with



$\sim \mathcal{O}(\alpha_{\mathsf{weak}})$

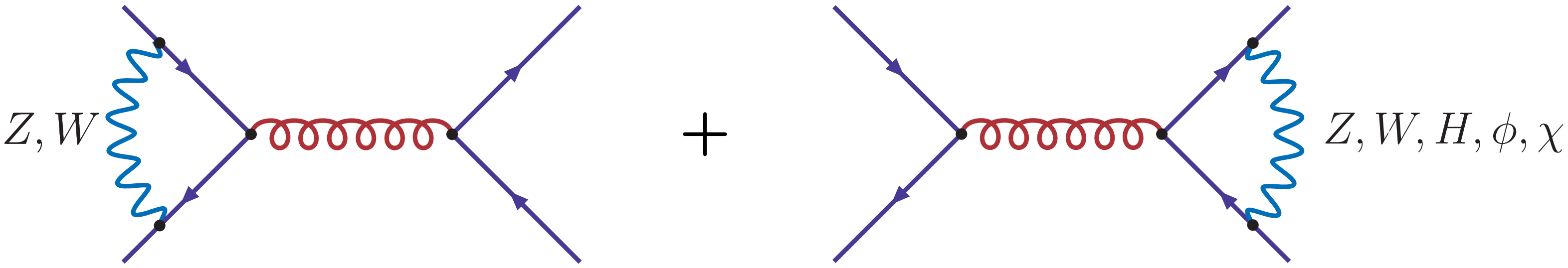
$g\,g \longrightarrow t\,\bar{t}$:

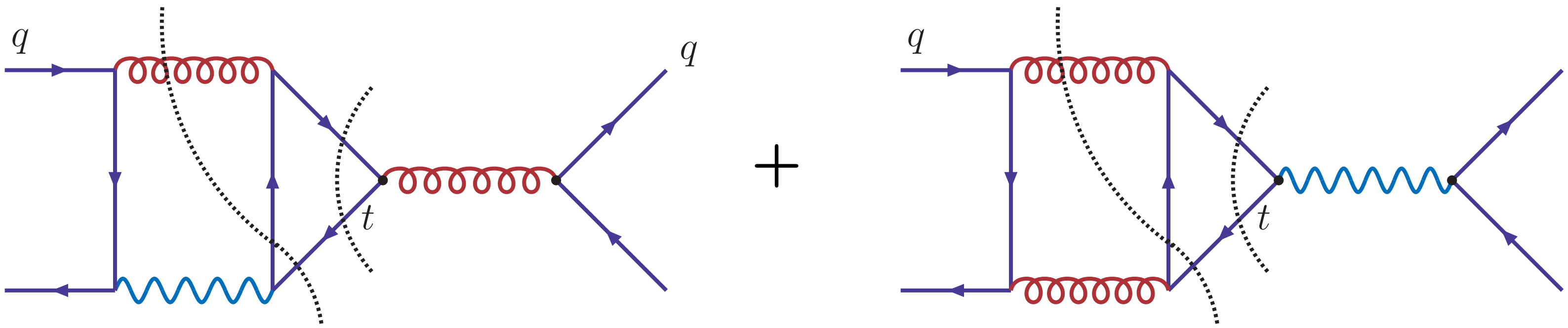




$\sim \mathcal{O}(\alpha_{\mathsf{s}})$

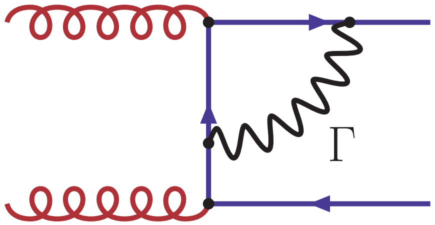
$\mathcal{O}(\alpha_s^2\alpha_{\text{weak}})$ weak corrections ($q\,\bar{q} \to t\,\bar{t}$)

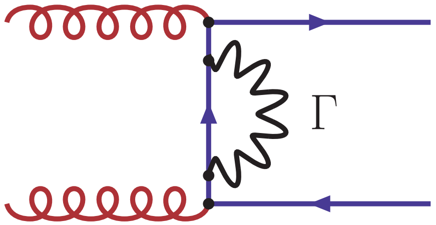


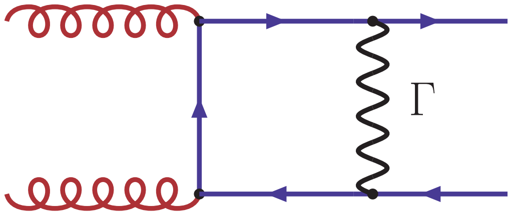


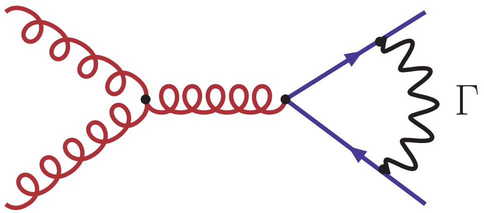
cuts of second group individually IR-divergent

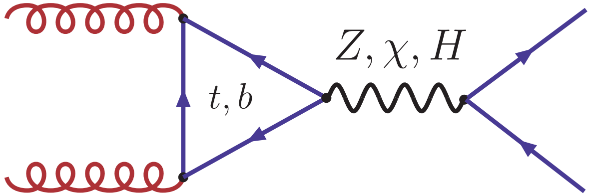
$\mathcal{O}(\alpha_{\mathsf{s}}^2\alpha_{\mathsf{weak}})$ weak corrections $(g\,g \to t\,\bar{t}\,)$











- analytical & numerical results available
  (earlier partial results by Beenakker *et al.*, some disagreements)
  independent evaluation by Bernreuther & Fücker

- $(\text{box contribution})_{\text{up}-\text{quark}} = -(\text{box contribution})_{\text{down}-\text{quark}}$
  $\Rightarrow$ suppression

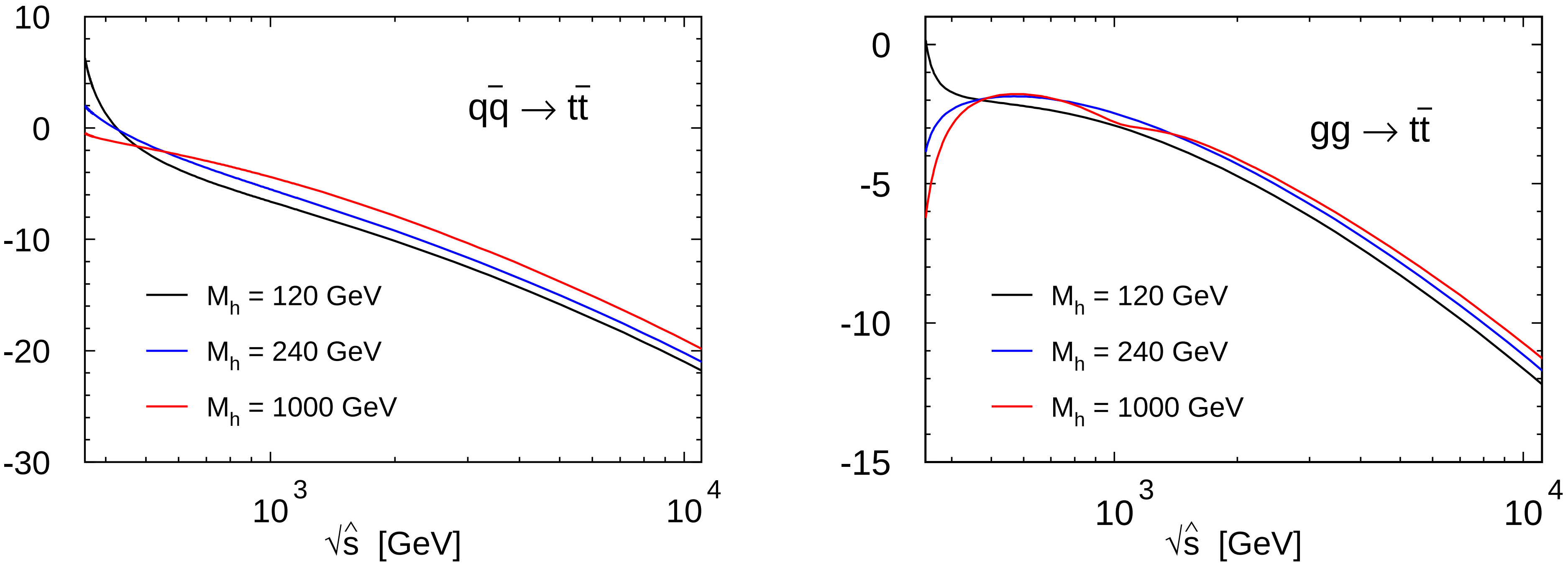
- box contribution moderately $\widehat{s}$-dependent

- strong increase with $\widehat{s}$

- sizable M_{h}-dependence, large effect close to threshold

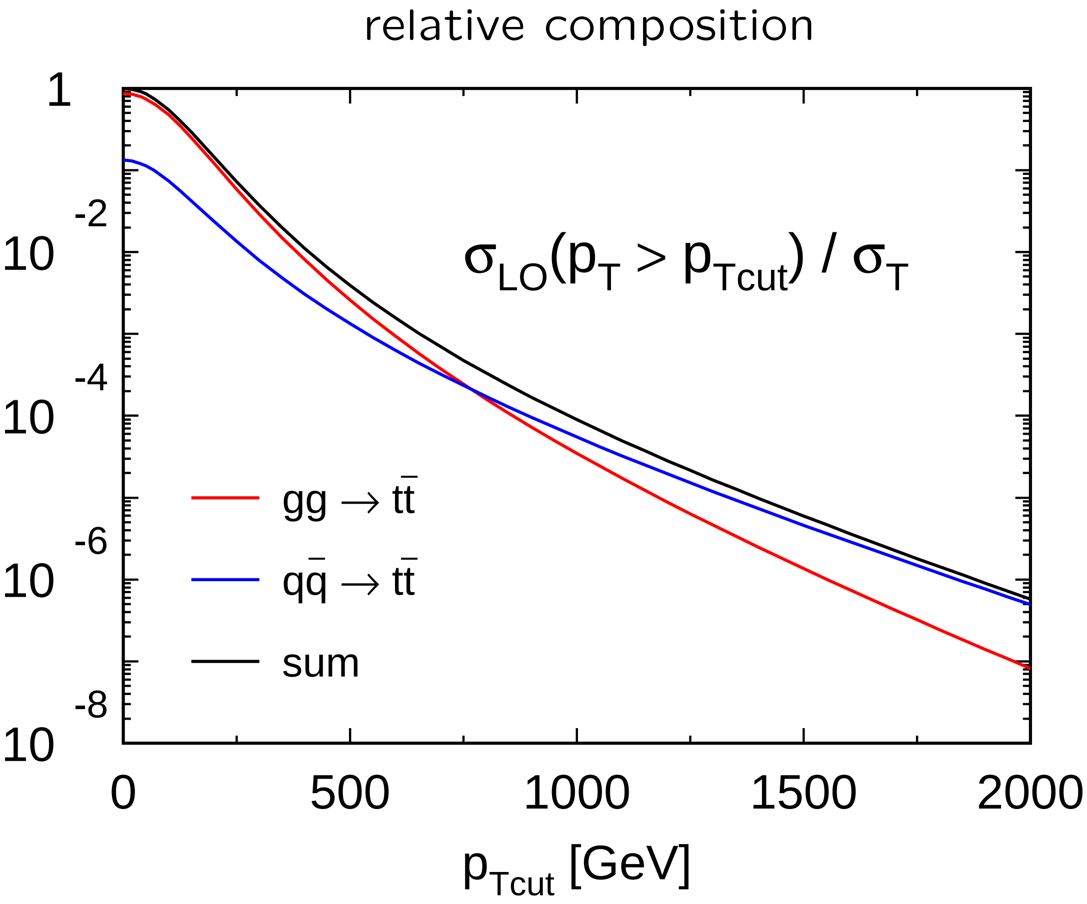
large corrections for large $\sqrt{\hat{s}}$

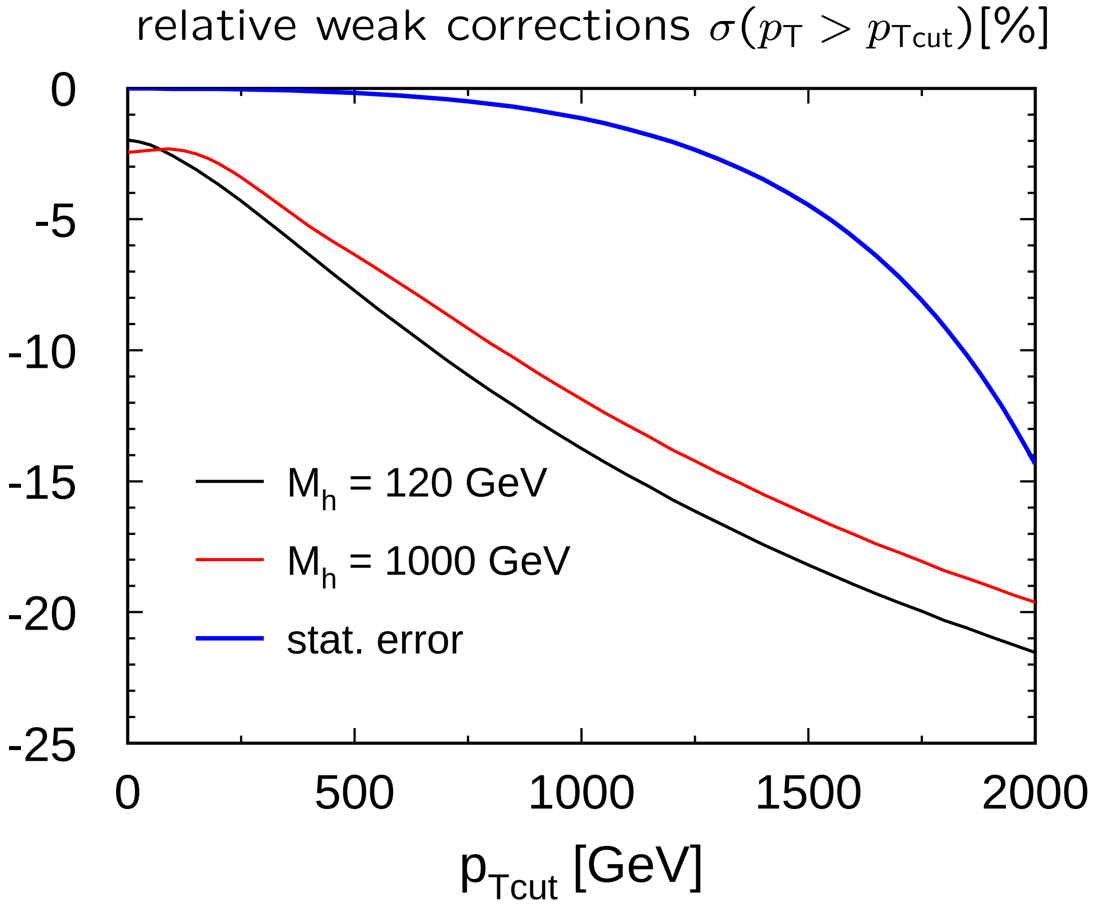
sizable M_h-dependence



(relative weak corrections [%])

Transverse momentum dependence


relative composition


relative weak corrections $\sigma(p_T > p_{Tcut})$[%]

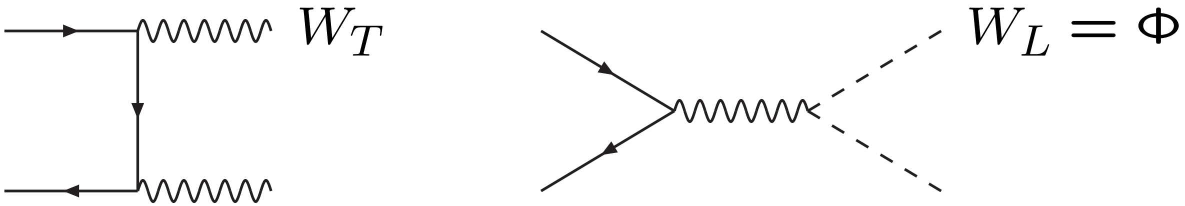
V. W-Pair Production ($e^+e^- \to W^+W^-$; $q\bar{q} \to W^+W^-$)

J.H.K., Metzler, Penin, Uccirati

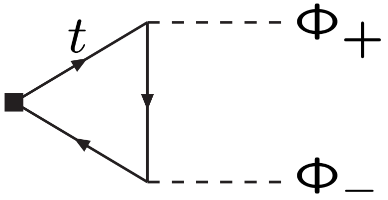
NPB 795,277 (2008); + in preparation

similar techniques: evolution equation & spearation of QED
new aspects:

- longitudinal vs transverse Ws (I=1 vs I=1/2; equivalence theorem)



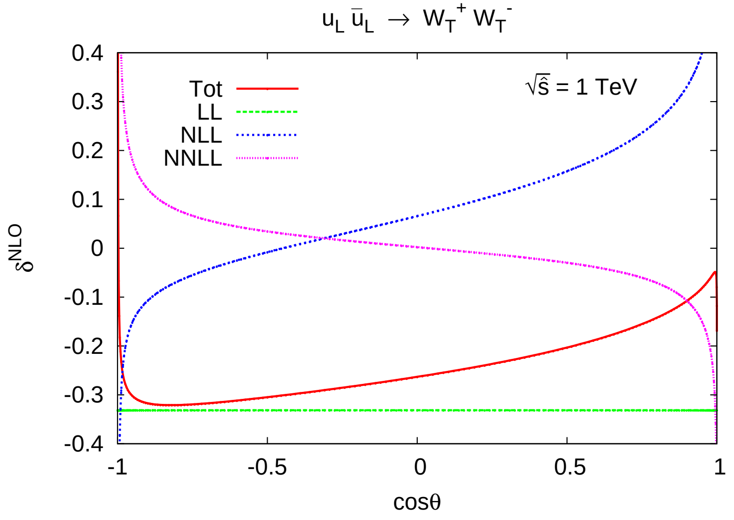
- Yukawa coupling in NLL:

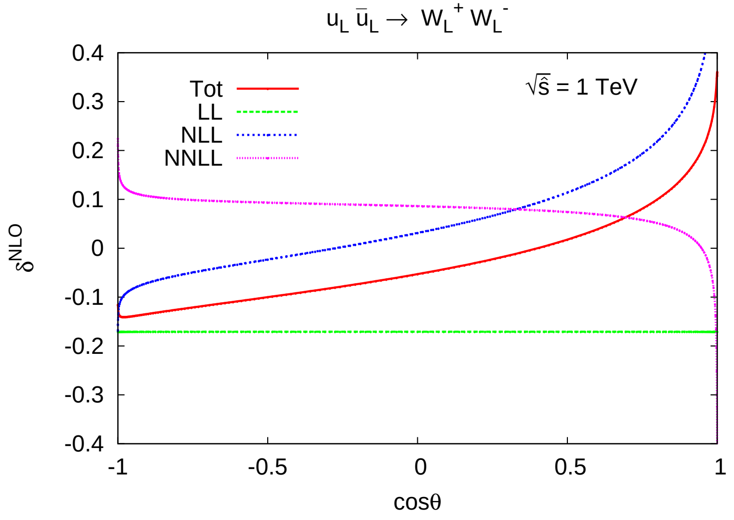


- NNLL: $\log^2$, log, const. required in 1 loop

Logarithmic correction to the differential cross section (relative to the Born) at $\sqrt{s} = 1$ TeV

1-loop:

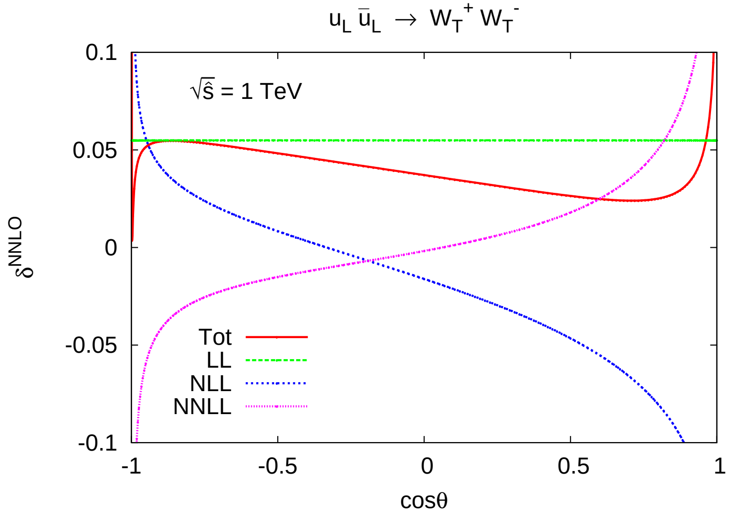


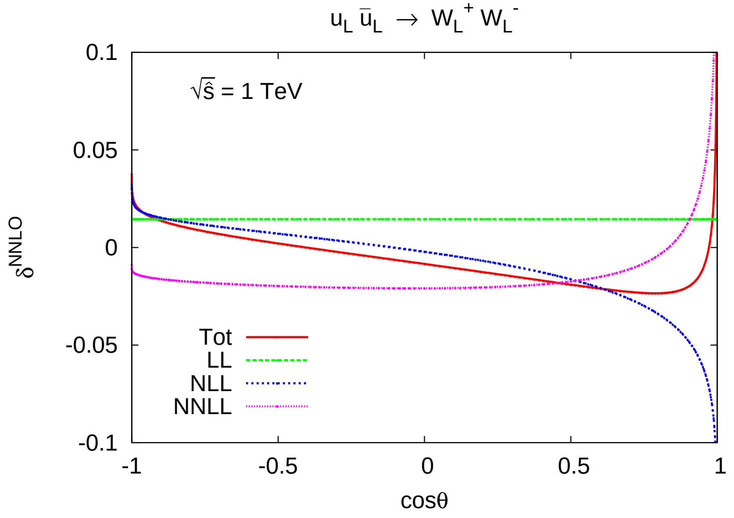


transverse

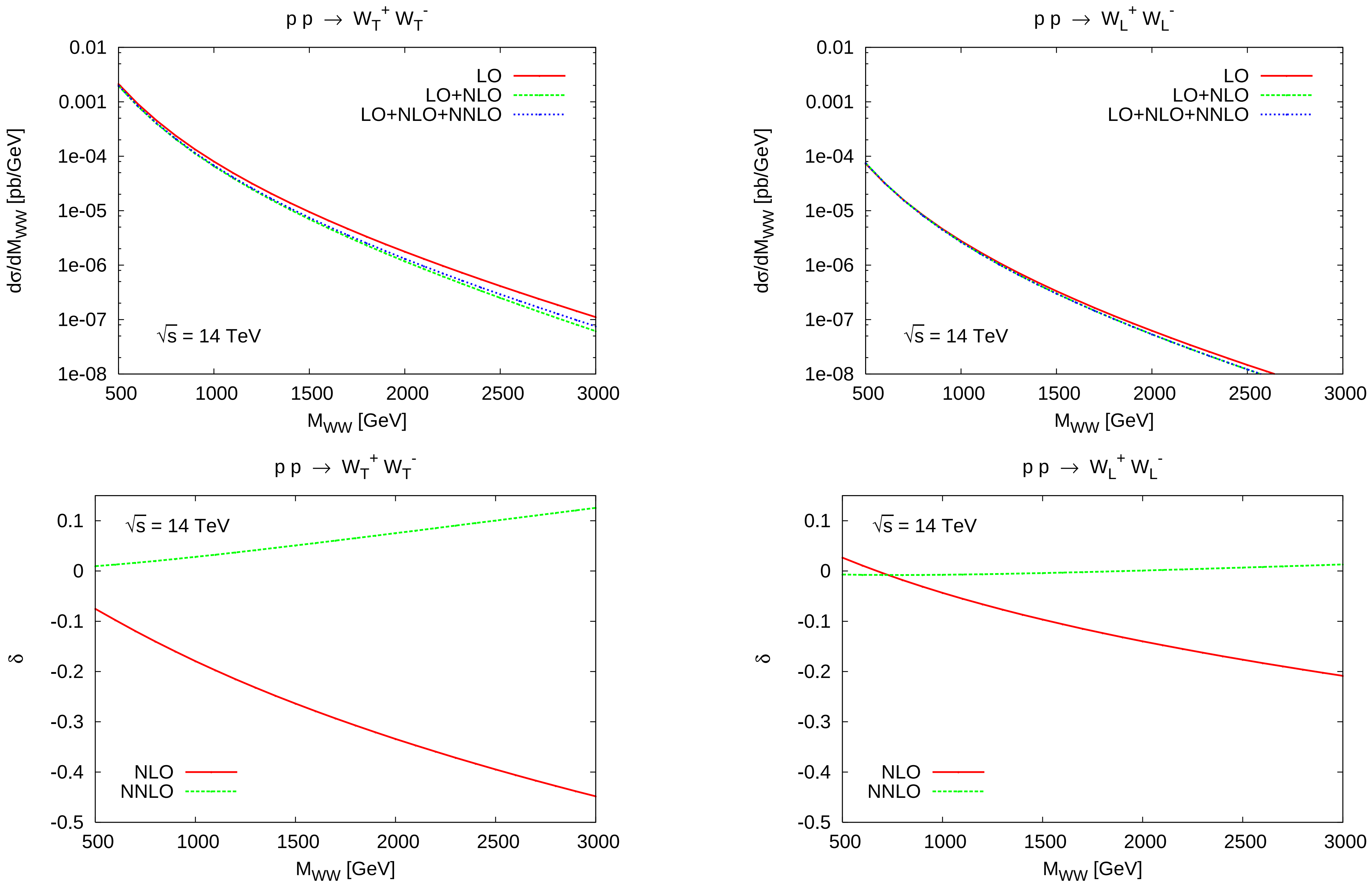
longitudinal

2-loop:





Invariant Mass Distribution ($\theta_{cut} = 30^\circ$)



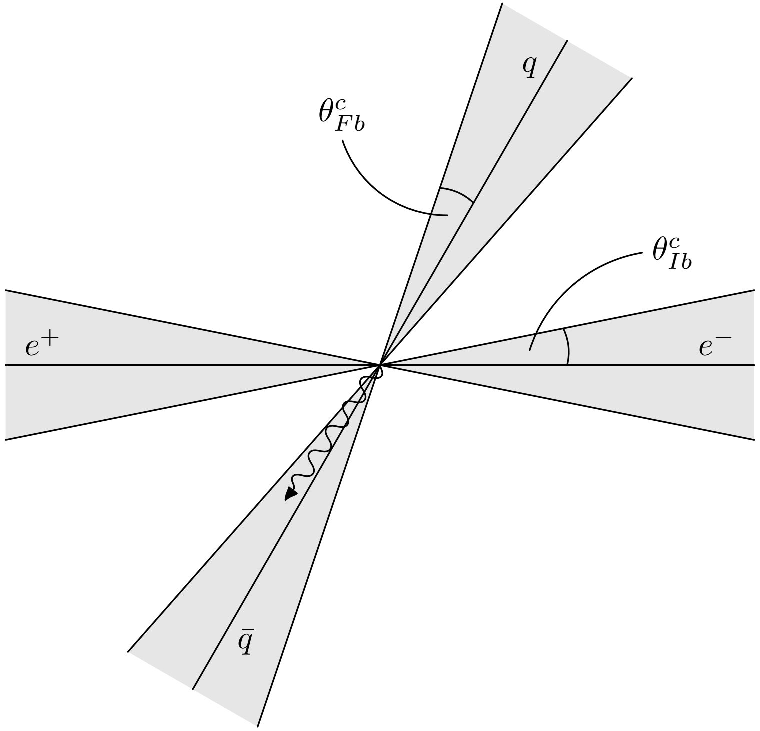
(larger effects for W_T! I=1)

VI. Impact of real radiation

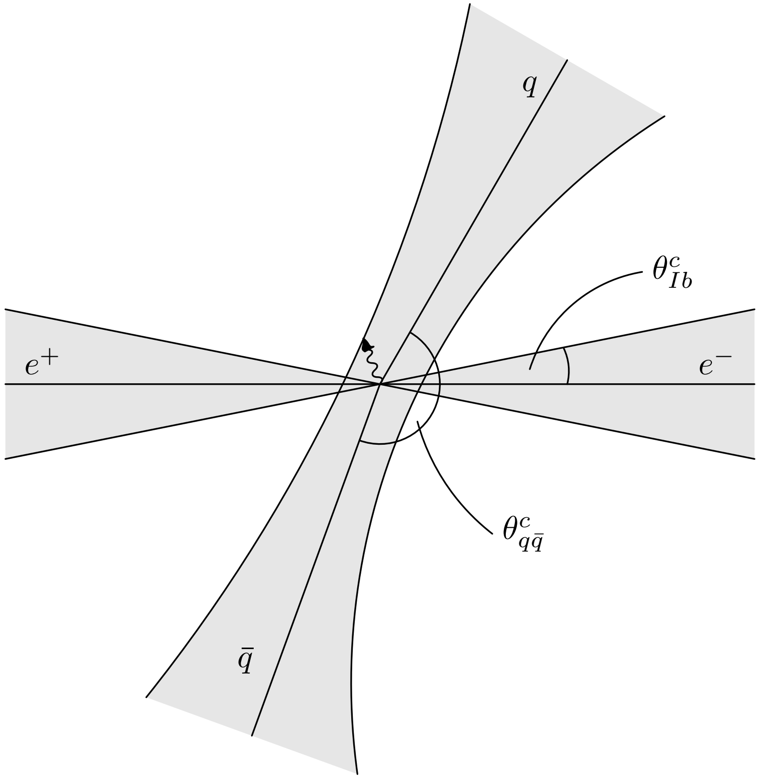
Bell, J.H.K., Rittinger

arXiv:1004.4117

soft and/or collinear radiation might (partially) compensate the reduction



only collinear radiation



collinear and soft radiation

Summation over isospin in the final state (Z and W radiation allowed)

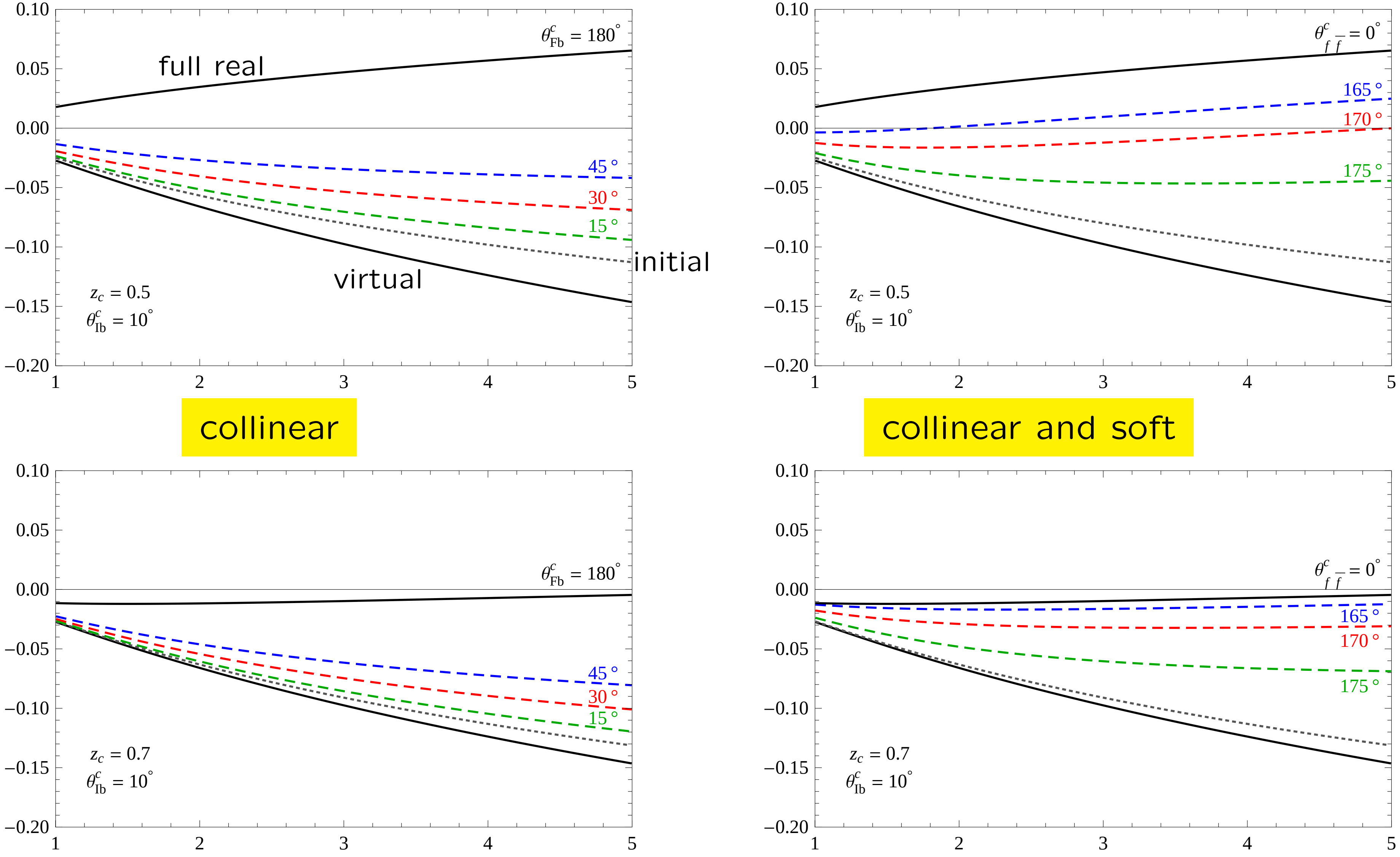
$e^+e^- \to q\bar{q} + (V)$:

$Q > 0.71\sqrt{s}$

$Q > 0.84\sqrt{s}$

collinear

collinear and soft



$(\sigma_R + \sigma_V)/\sigma_0$ with 1 TeV $< \sqrt{s} <$ 5 TeV. Virtual only (lower solid line), only initial state radiation (black dotted line), initial and final state radiation (green, red and blue line) and real radiation with full angular phase space (upper solid line)

IV. Conclusions

- LHC will explore the TeV-region: $\hat{s}\big/M_W^2 \gg 1$
- electroweak corrections amount to $\mathcal{O}(10\% - 20\%)$ in the interesting kinematic region
- form factors and four-fermion scattering in two loop
- p_{T}-distributions of Z, W, γ and their ratios will be strongly affected
- two-loop terms might become relevant
- top-quark distributions at large $\hat{s}$ are strongly modified
- sizable M_{h}-dependence
- large effects for W pair production
- how well can we separate real radiation?