The Radiative Return at Φ- and B-Meson Factories

Karlsruhe Institute of Technology

J.H. KÜHN
INSTITUT FÜR THEORETISCHE TEILCHENPHYSIK

KARLSRUHER INSTITUT FÜR TECHNOLOGIE

I Basic Idea

II Monte Carlo Generators: Status & Perspectives

III Charge Asymmetry and Radiative Φ-Decays

IV Nucleon Form Factors at B-Factories

V Pion and Kaon Form Factors at large Q^2 and $\tau \to \nu K^- K^0$

VI Experimental Results

VII Conclusions

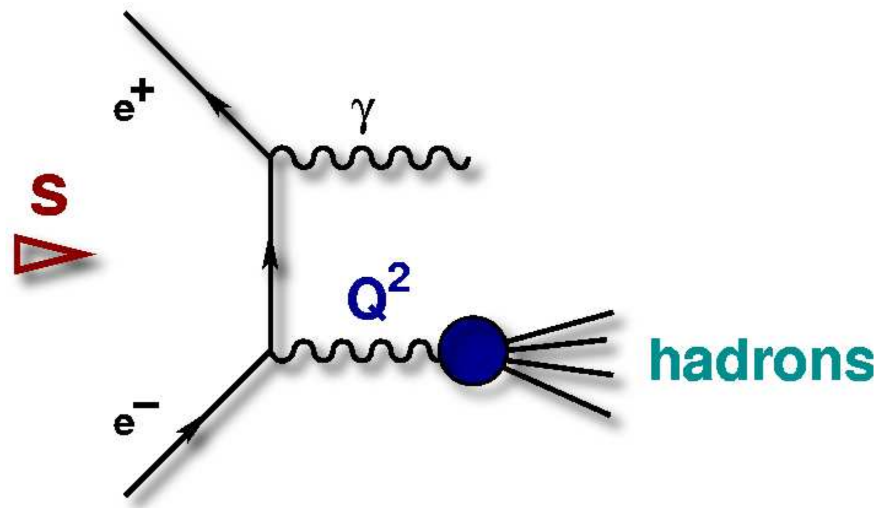
(with H. Czyż, G. Rodrigo, K. Melnikov;

and S. Binner, A. Grzelinska, E. Nowak, G. Rodrigo, A. Wapienik)

I BASIC IDEA

photon radiated off the initial e^+e^- (ISR) reduces the effective energy of the collision

$$d\sigma(e^+e^- \to \text{hadrons} + \gamma) = H(Q^2, \theta_\gamma)\, d\sigma(e^+e^- \to \text{hadrons})$$



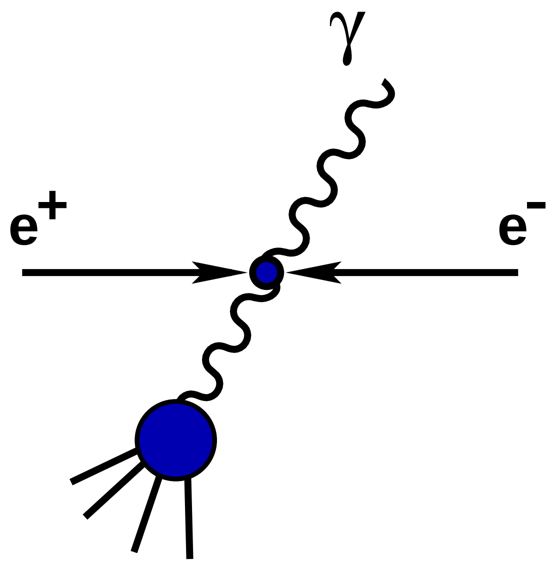
- measurement of $R(s)$ over the full range of energies, from threshold up to $\sqrt{s}$
- large luminosities of factories compensate α/π from photon radiation
- radiative corrections essential (NLO)
- advantage over energy scan (BES, CMD2, SND): systematics (e.g. normalization) only once

High precision measurement of the hadronic cross-section at DAΦNE, CLEO-C, B-factories

DAΦNE versus B-factories:

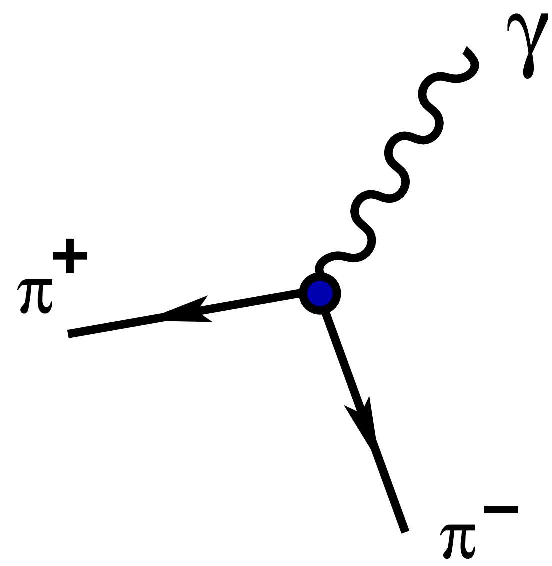
configurations in the cms - frame

10 GeV



very hard photon: clear kinematic separation between photon and hadrons

1 GeV



no natural kinematic separation

➪ cuts to control FSR versus ISR

(two step process: $e^+e^- \to \gamma\ \rho(\to \gamma\pi\pi) \Rightarrow$ see below)

Rough estimates (1999) for rates:

$\pi^+ \pi^- \gamma$: $E_\gamma > 100 MeV$

$\sqrt{s}$ $[GeV]$	$\int \mathfrak{L}$ $[fb^{-1}]$	#events, $\theta_{min} = 7^\circ$
1.02	1.35	16 $\cdot 10^6$
10.6	100	3.5 $\cdot 10^6$

multi-hadron-events (R $\equiv 2$) $\sqrt{s} = 10.6\, GeV$

Q^2-interval $[GeV]$	#events, $\theta_{min} = 7^\circ$
$[\,1.5\,,\ 2.0\,]$	9.9 $\cdot 10^5$
$[\,2.0\,,\ 2.5\,]$	7.9 $\cdot 10^5$
$[\,2.5\,,\ 3.0\,]$	6.6 $\cdot 10^5$
$[\,3.0\,,\ 3.5\,]$	5.8 $\cdot 10^5$

actually (2010): $\int \mathfrak{L} \sim 3 fb^{-1}$ (KLOE)

$\int \mathfrak{L} \sim 500 fb^{-1}$ (BABAR)

Lowest order

$$\frac{d\sigma}{dQ^2}\left(e^+e^- \to \gamma \; + \; \mathrm{had}(Q^2)\right) = \sigma\left(e^+e^- \to \mathrm{had}(Q^2)\right)$$

$$\times \frac{\alpha}{\pi s} \left\{ \begin{array}{l} \frac{s^2+Q^4}{s(s-Q^2)}\left(\log(s/m_e^2) - 1\right), \text{ no angular cut} \\ \frac{s^2+Q^4}{s(s-Q^2)} \log\left(\frac{1+\cos\theta_{min}}{1-\cos\theta_{min}}\right) - \frac{s-Q^2}{s}\cos\theta_{min} \end{array} \right\}$$

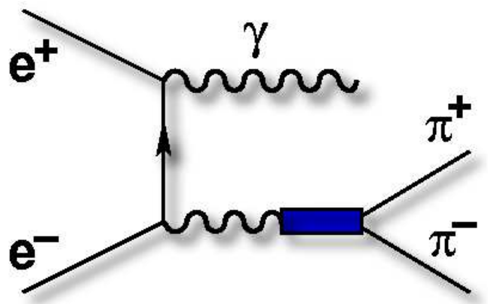
$\Rightarrow$ differential luminosity: $\frac{dL}{dQ^2}\left(Q^2, s\right) = \frac{\alpha}{\pi s}\left\{\cdots\right\} L(\text{at } s)$

all errors dominated by systematics

$\Rightarrow$ Monte Carlo Generator

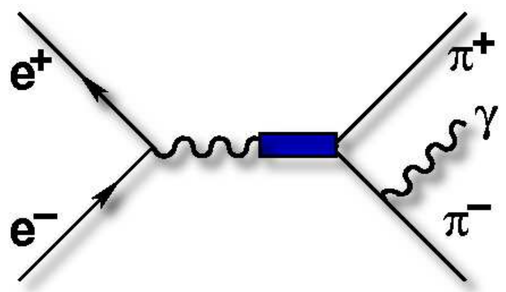
Basic Ingredients for Pion Formfactor

- ISR



pion form factor
⇨ to be tested

- FSR



radiation from point-like pions (probably overestimated)

⇐ virtual Compton scattering

- **additional radiation:** collinear (EVA MC) (Binner, JK, Melnikov)
  or NLO calculation (PHOKHARA MC)

II MONTE CARLO GENERATORS

P
H
O
K
H
A
R
A

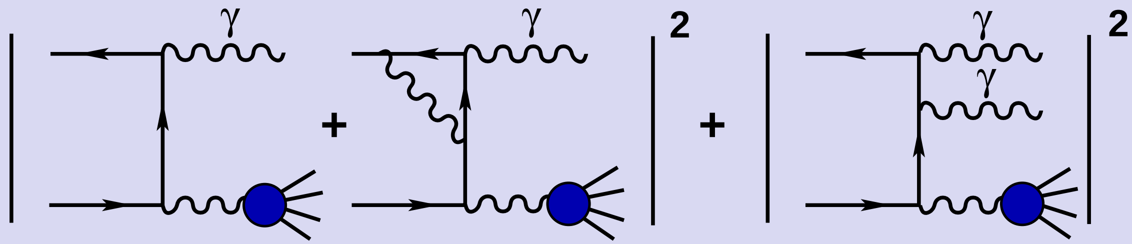
References etc. $\longrightarrow$ http://ific.uv.es/∼rodrigo/phokhara

QED corrections at leptonic side

$\Longrightarrow$ basic building block for all hadronic final states

PHOKHARA 2.0:
$\pi^+\pi^-$, $\mu^+\mu^-$, 4π

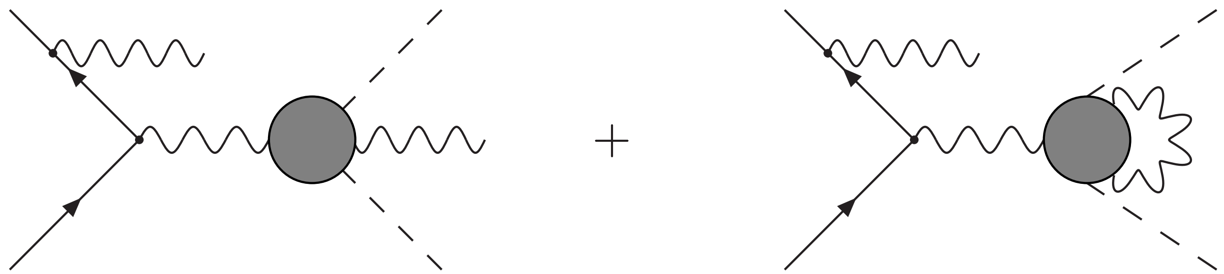
- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level



- FSR at LO: $\pi^+\pi^-$, $\mu^+\mu^-$
- tagged or untagged photons
- modular structure

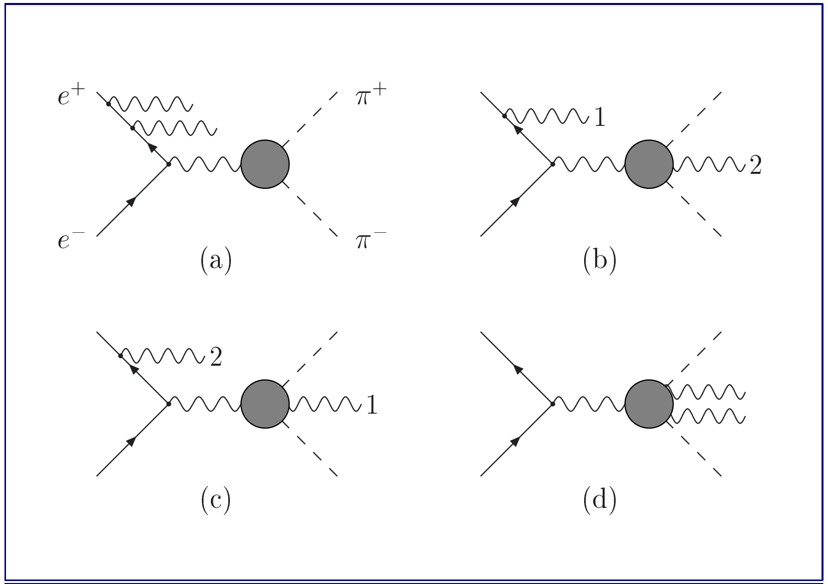
PHOKHARA 3.0

- specifically developed for $\pi^+\pi^-$ (plus photons)
- allows for simultaneous emission of photons
  from initial and final state,
  including virtual corrections (interference neglected).

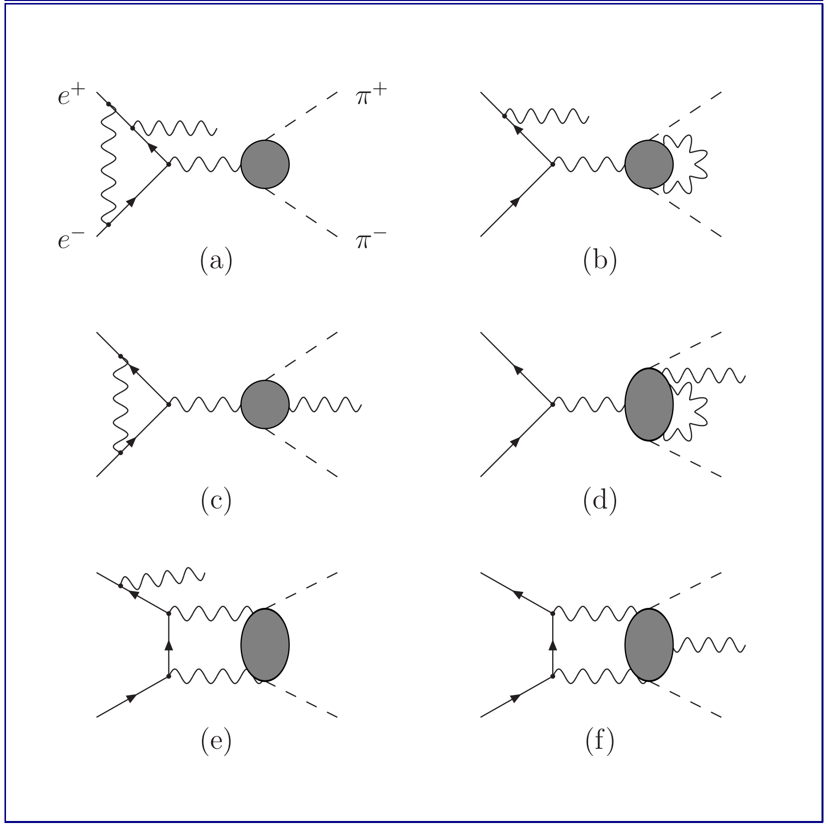


$\Rightarrow$ dominated by "two step process": $e^+e^- \to \gamma\ \rho\ (\to \gamma\ \pi\pi)$

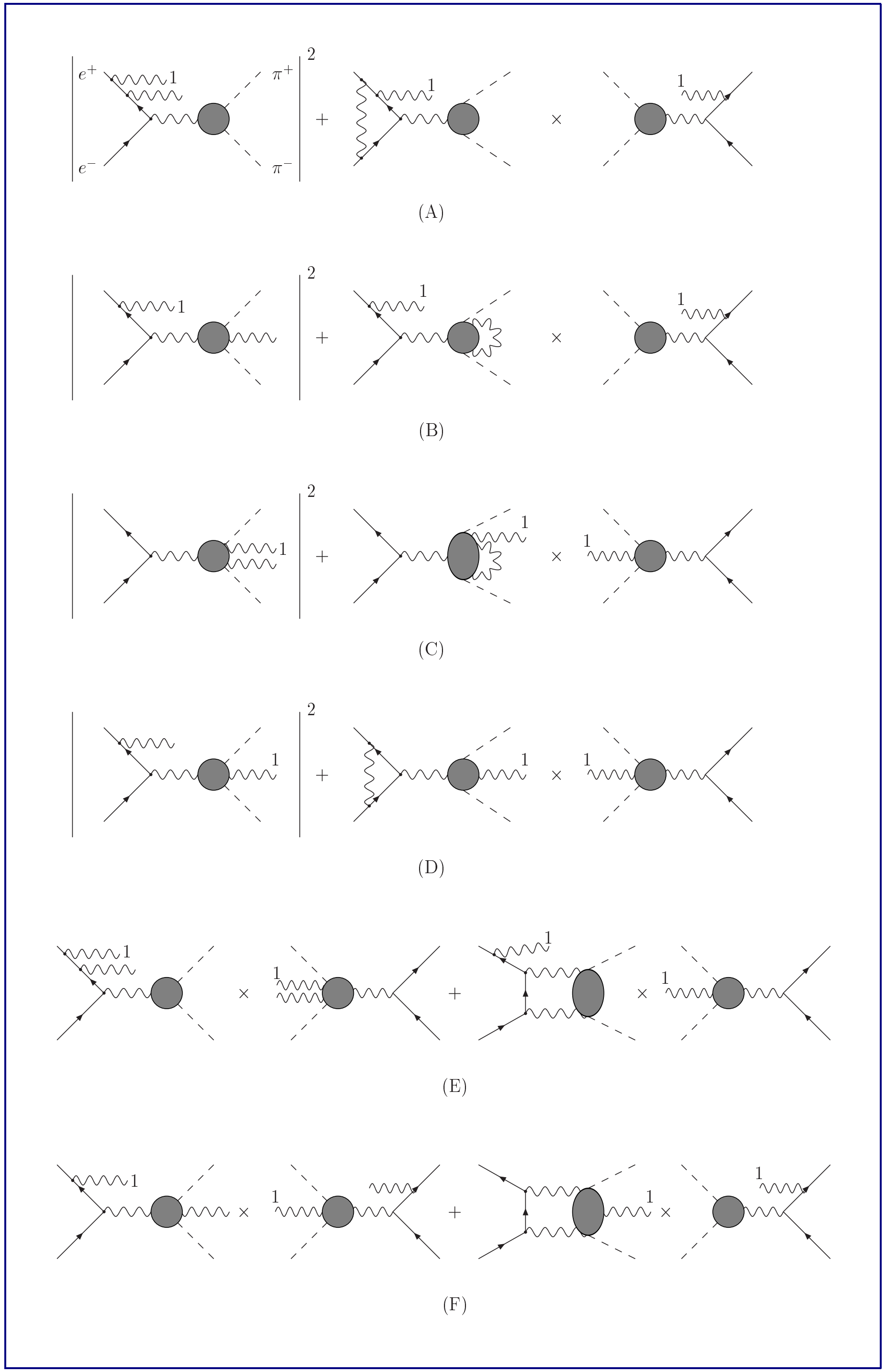
$\Rightarrow$ importance of $\pi\pi\gamma$ as input for a_μ



e^+
e^-
π^+
π^-
(a)
(b)
(c)
(d)

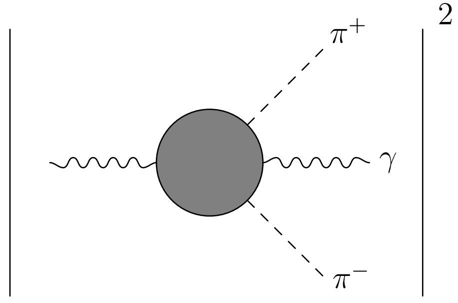


e^+
e^-
π^+
π^-
(a)
(b)
(c)
(d)
(e)
(f)



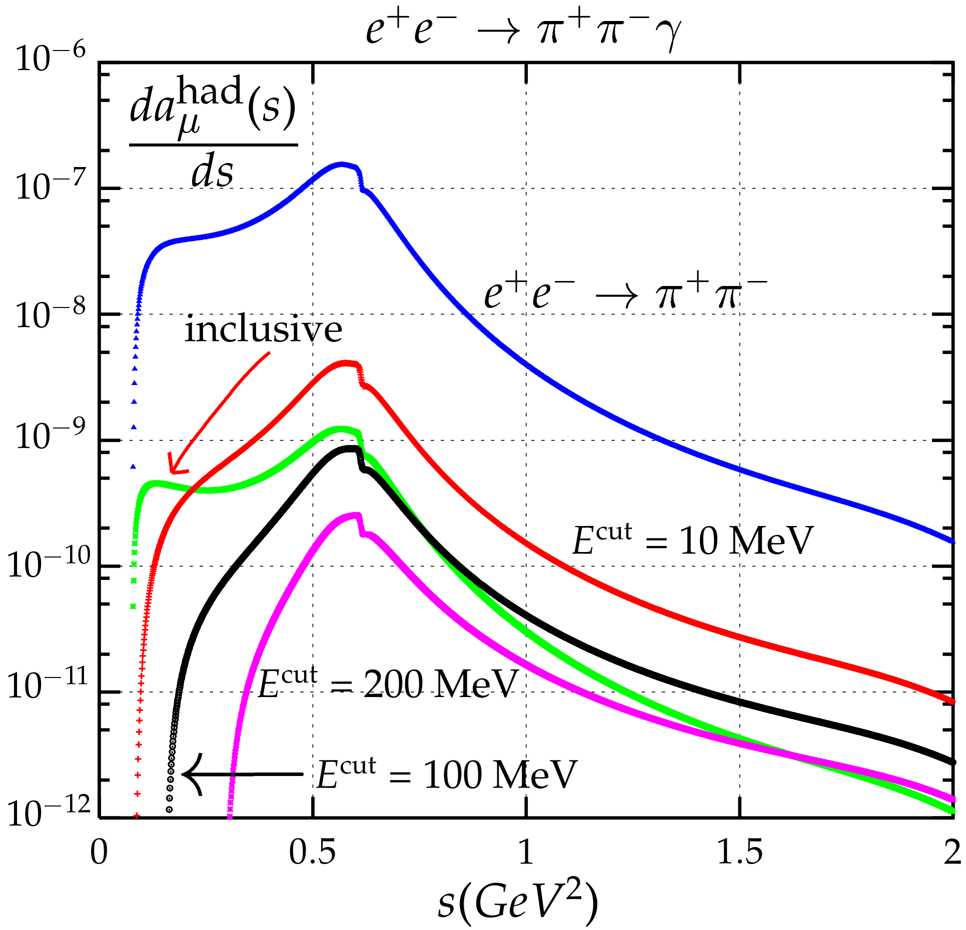
(A)
(B)
(C)
(D)
(E)
(F)

Contributions to a_μ from

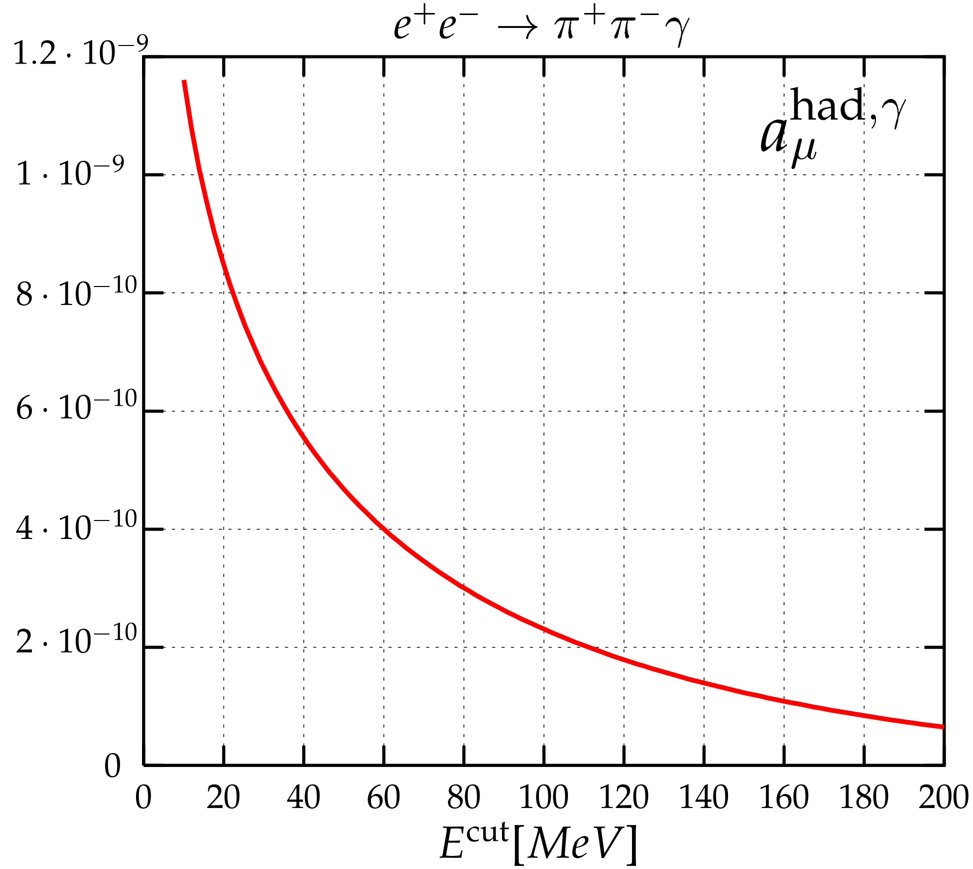


Estimates

$$\delta a_\mu(\text{quark}, \gamma, m_q = 180\ \text{MeV}) = 1.880 \times 10^{-10}$$
$$\delta a_\mu(\text{quark}, \gamma, m_q = 66\ \text{MeV}) = 8.577 \times 10^{-10}$$
$$\delta a_\mu(\pi^+\pi^-, \gamma) = 4.309 \times 10^{-10}$$

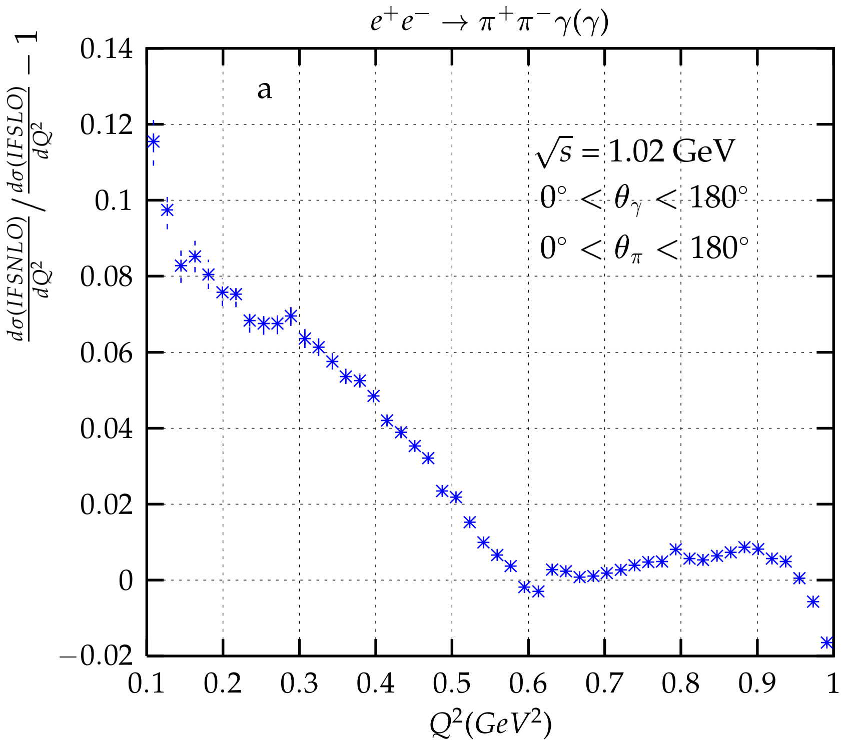


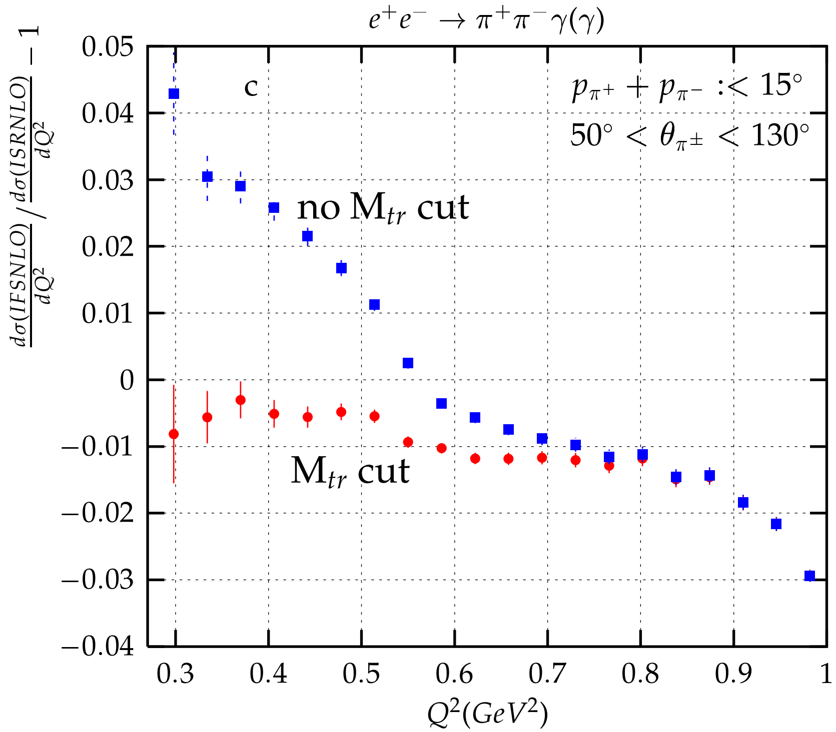
Differential contribution to $a_\mu^{\text{had},\gamma}$ from $\pi^+\pi^-\gamma$ intermediate states for different cutoff values compared with the complete contribution (virtual plus real corrections, labelled 'inclusive') evaluated in sQED (FSR), as well as with the contribution from the $\pi^+\pi^-$ intermediate state.



Integrated contribution to $a_\mu^{\text{had},\gamma}$ as a function of the cutoff E^{cut}.

Large effect for $Q^2 < m_\rho^2$ eliminated by suitable cuts on $\pi^+\pi^-$ configuration (suppress 2γ events)



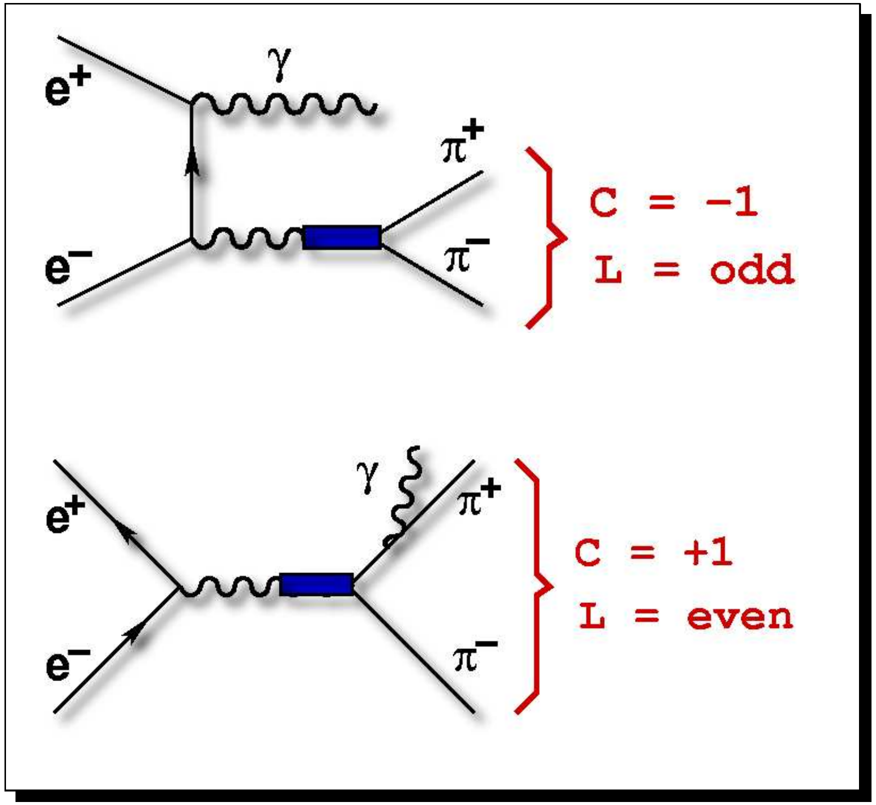


or measure photon

III Charge Asymmetries and Radiative Φ-Decays

(H. Czyż, A. Grzelinska, JK)

interference:



- interference odd under $\pi^+ \leftrightarrow \pi^-$
- asymmetric differential distribution: $\int$ interf. $= 0$

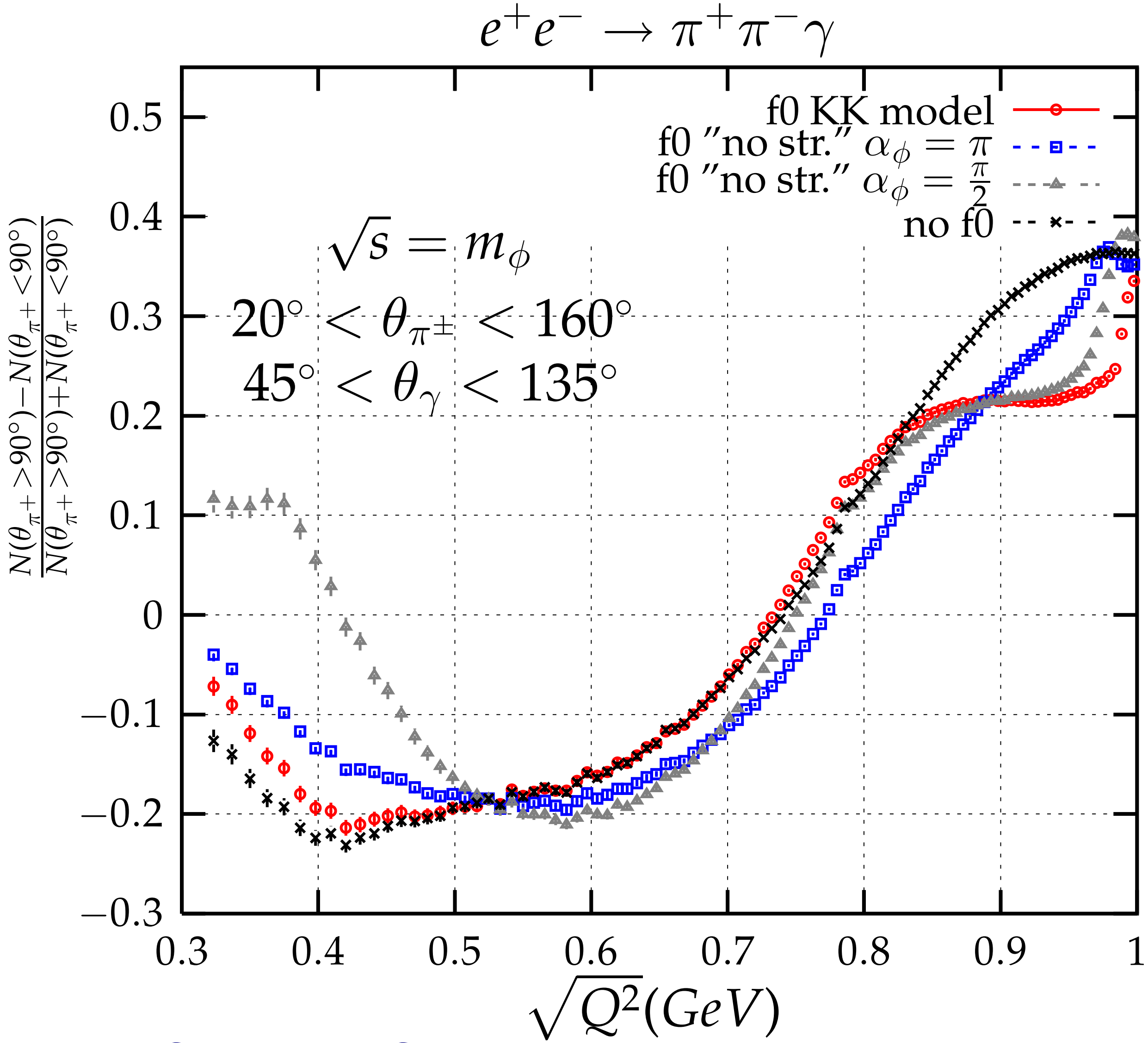
$$A(\theta) = \frac{N^{\pi^+}(\theta) - N^{\pi^-}(\theta)}{N^{\pi^+}(\theta) + N^{\pi^-}(\theta)}$$

Photon coupled to "pointlike" pion

additional contribution on top of Φ-resonance (KLOE !)

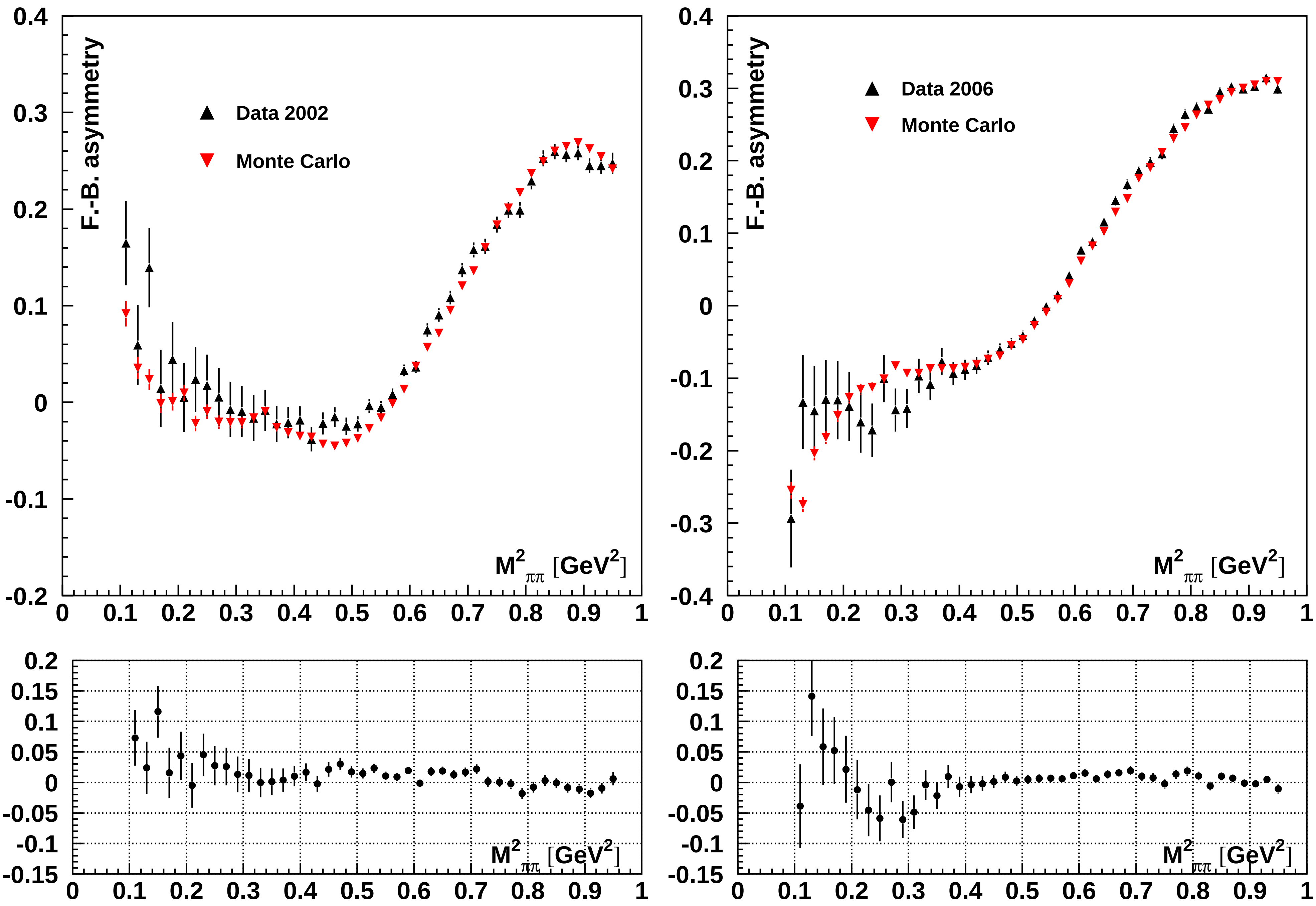
$e^+e^- \to \Phi \to \gamma\ \mathrm{f}_{0,2}\ (\to \pi^+\pi^-)$ interference !

Significant influence of scalar resonances on charge asymmetry



Interference between ISR and FSR

$\Rightarrow$ amplitude for $\Phi \to \gamma\pi\pi$


F.-B. asymmetry
Data 2002
Monte Carlo
$M^2_{\pi\pi}$ [GeV^2]
F.-B. asymmetry
Data 2006
Monte Carlo
$M^2_{\pi\pi}$ [GeV^2]
$M^2_{\pi\pi}$ [GeV^2]
$M^2_{\pi\pi}$ [GeV^2]

IV NUCLEON FORM FACTORS

(PHOKHARA 4.0)

$Q^2 \gtrsim 4m_N^2$ accessible at B-factories

$\Rightarrow$ study $e^+e^- \to \gamma N\bar{N}$ (with $N = p$ or n)

hadronic current:

$$J_\mu = -ie \cdot \bar{u}(q_2) \left(F_1^N(Q^2)\,\gamma_\mu - \frac{F_2^N(Q^2)}{4m_N}\,[\gamma_\mu, \not{Q}] \right) v(q_1)\,,$$

$$Q = q_1 + q_2\,, \quad q = (q_1 - q_2)/2$$

or

$$G_M = F_1 + F_2\,, \quad G_E = F_1 + \frac{Q^2}{4m^2}\,F_2$$

Result:

$$d\sigma = \frac{1}{2s} L_{\mu\nu} H^{\mu\nu} \, d\Phi_2(p_1 + p_2; Q, k) \, d\Phi_2(Q; q_1, q_2) \, \frac{dQ^2}{2\pi} \, ,$$

$$L_{\mu\nu} H^{\mu\nu} = \frac{(4\pi\alpha)^3}{Q^2} \Bigg\{ \left(|G_M^N|^2 - \frac{1}{\tau} |G_E^N|^2 \right) \times \frac{32s}{\beta_N^2 (s - Q^2)} \left(\frac{1}{y_1} + \frac{1}{y_2} \right) \left(\frac{(p_1 \cdot q)^2 + (p_2 \cdot q)^2}{s^2} \right) + 2 \left(|G_M^N|^2 + \frac{1}{\tau} |G_E^N|^2 \right) \left[\left(\frac{1}{y_1} + \frac{1}{y_2} \right) \frac{(s^2 + Q^4)}{s(s - Q^2)} - 2 \right] \Bigg\} \, ,$$

where

$$y_{1,2} = \frac{s - Q^2}{2s} (1 \mp \cos\theta_\gamma) \, , \quad \tau = \frac{Q^2}{4m_N^2} \, , \quad \beta_N^2 = 1 - \frac{4m_N^2}{Q^2}$$

Separation of $|G_M|^2$ and $|G_E|^2$ through angular distribution:

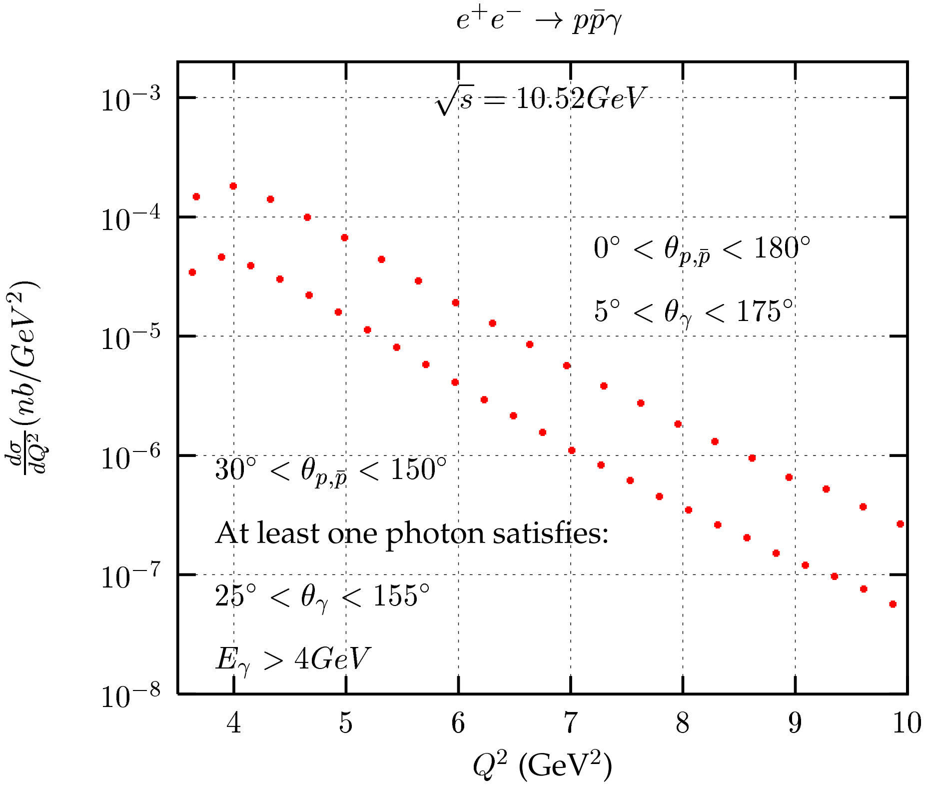
$$L_{\mu\nu}H^{\mu\nu} = \frac{(4\pi\alpha)^3}{Q^2}\frac{(1+\cos^2\theta_\gamma)}{(1-\cos^2\theta_\gamma)} \times 4\left(|G_M^N|^2\,(1+\cos^2\hat{\theta}) + \frac{1}{\tau}|G_E^N|^2\,\sin^2\hat{\theta}\right)$$

$\hat{\theta}$ = angle of nucleon with respect to γ-direction in hadronic rest frame

(valid for $s/Q^2 \ll 1$, corrections and "optimal frame" $\longrightarrow$ EPJ C 35 (2004) 527)

Similarity to $e^+e^- \longrightarrow N\bar{N}$:

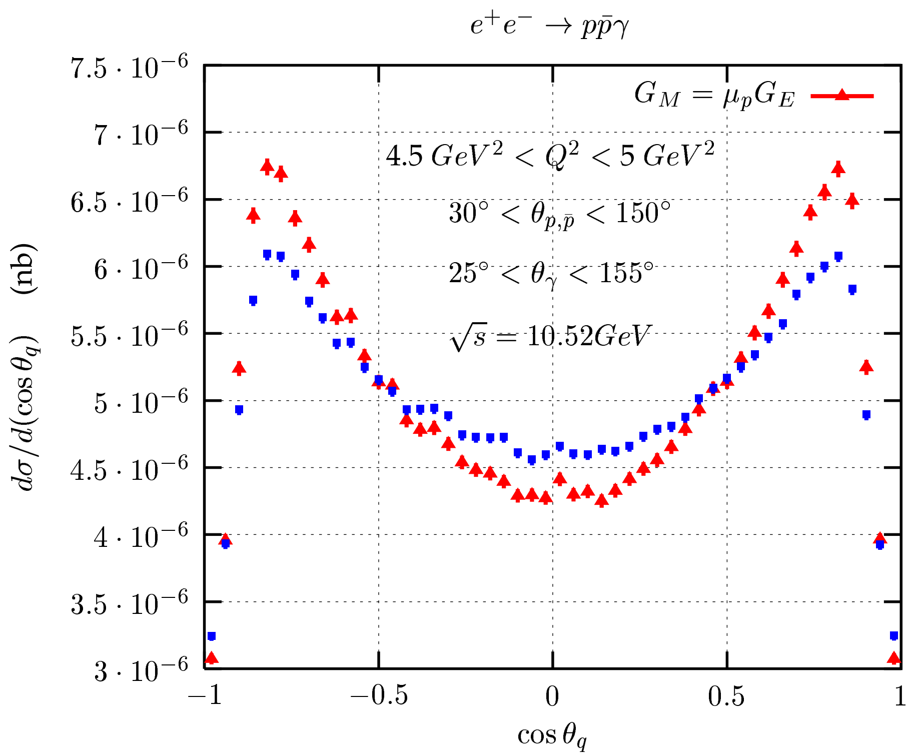
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta_N}{4Q^2}\left(|G_M^N|^2\,(1+\cos^2\theta) + \frac{1}{\tau}|G_E^N|^2\,\sin^2\theta\right)$$



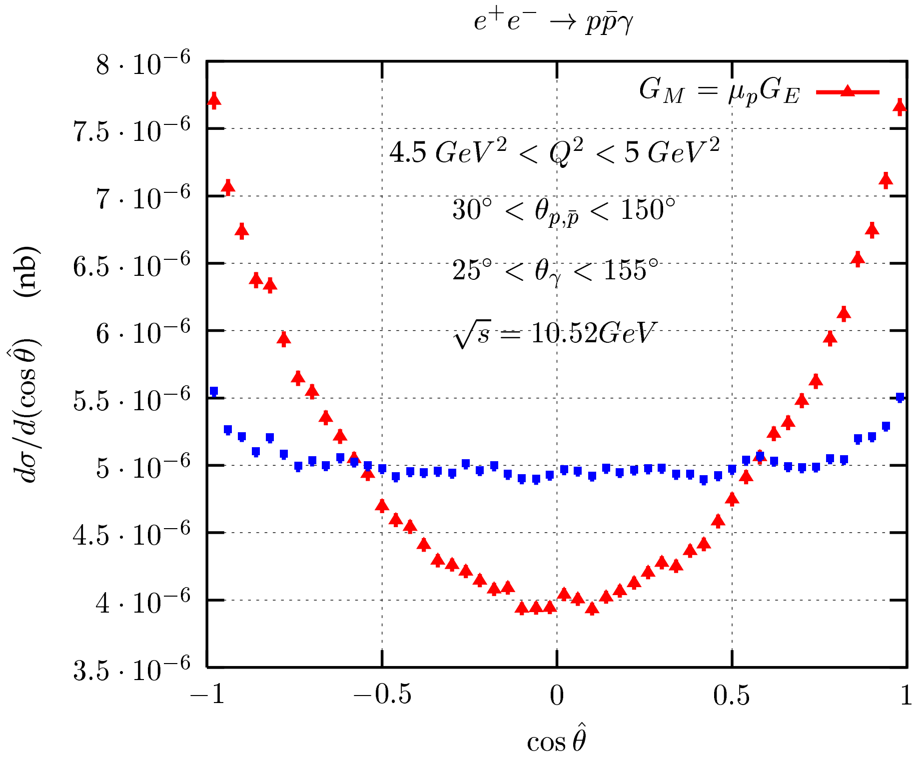
$e^+e^- \longrightarrow p\bar{p}\,\gamma$

implementation in PHOKHARA

Angular distributions of nucleon



lab frame



hadronic rest frame

(two choices for G_M/G_E)

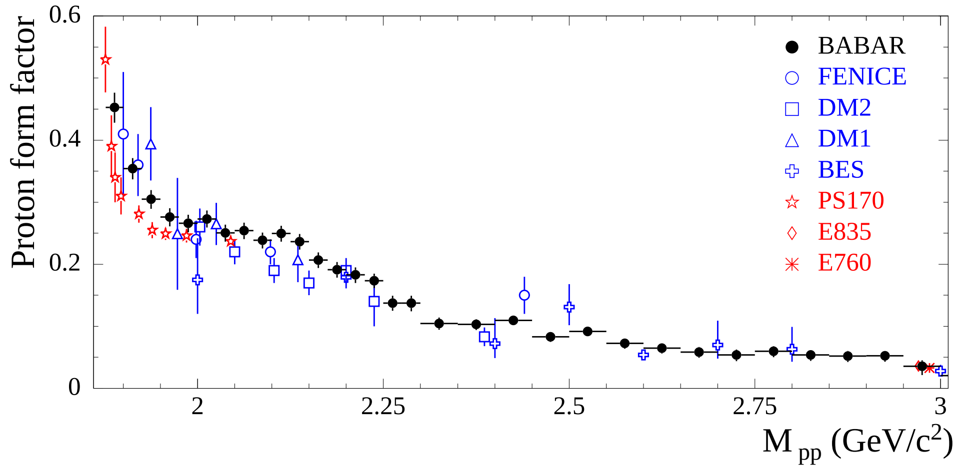
Comments

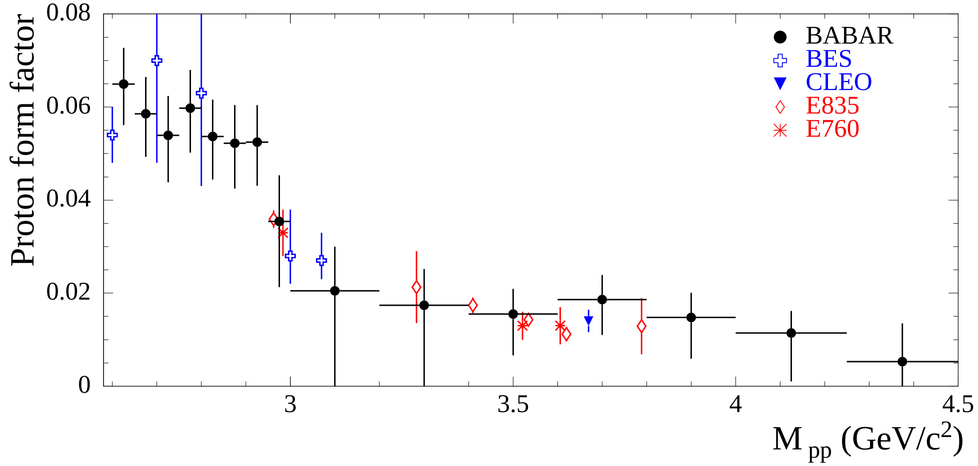
- **similar results for neutron pair or $\Lambda\overline{\Lambda}$ production**
- **NLO corrections from ISR included** (corrections $\sim$ 1–2%)
- **no FSR**
- **$\Lambda\overline{\Lambda}$ can be studied with analysis of Λ-polarization**

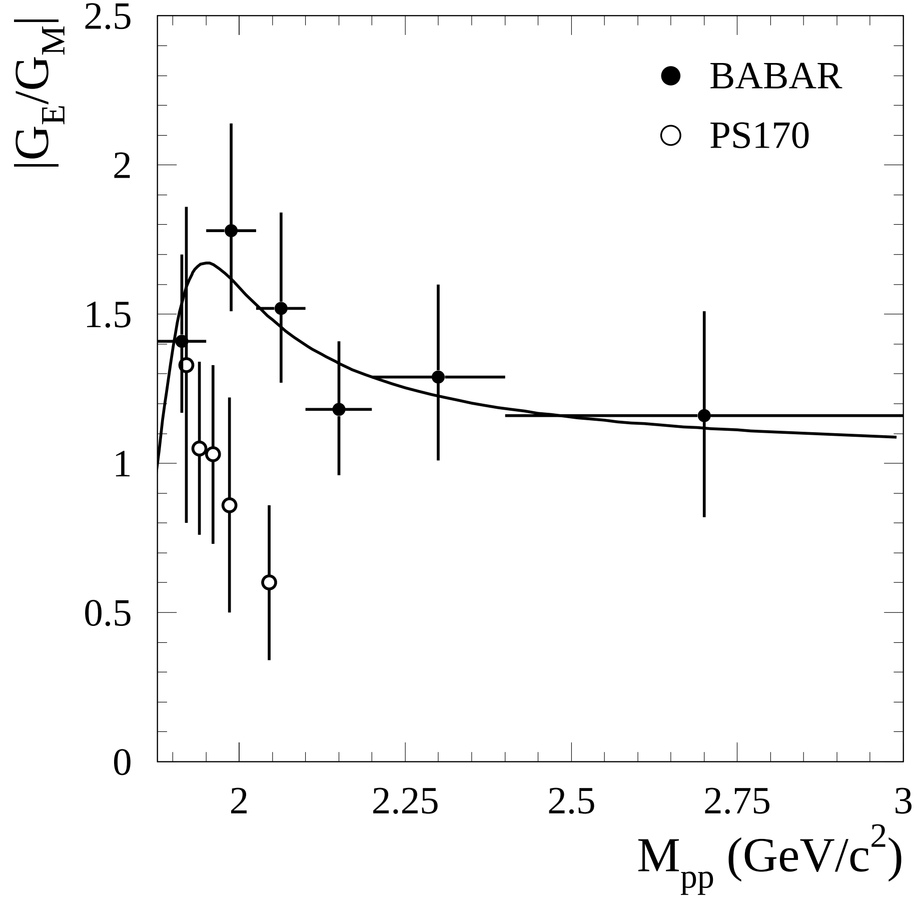
thousands of events around 4–5 GeV^2
several events up to 7–8 GeV^2

Results (from BABAR)

PRD 73, 012005 (2006)







Recent improvements

- model amplitudes
- Improved formfactors for

  $\pi^+\pi^-2\pi^0, \quad 2\pi^+2\pi^-,$

  $\pi^+\pi^-$ (high Q^2 region)

  $K^+K^-, K^0\overline{K}^0$

- narrow resonances (J/ψ, ψ')
  (direct vs. indirect amplitudes)

V MESON FORM FACTORS at LARGE Q^2

Khodjamirian, JK, EPJ C 39 (2004) 41

Czyz, Grzelinska, JK, PRD 81, 094014 (2010)

radiative return will explore large Q^2

convenient representation for F_π :

generalized VDM with ρ, ρ', $\ldots$

combined with Veneziano-type tower of resonances (Dominguez)

$$F_\pi(s) = \sum_{n=0}^{\infty} c_n \frac{m_n^2}{m_n^2 - s},$$

$$c_n = \frac{(-1)^n \Gamma(\beta - 1/2)}{\sqrt{\pi}(\frac{1}{2} + n)\Gamma(n+1)\Gamma(\beta - 1 - n)},$$

$$m_n^2 = m_\rho^2 (1 + 2n),$$

$\beta =$ free parameter

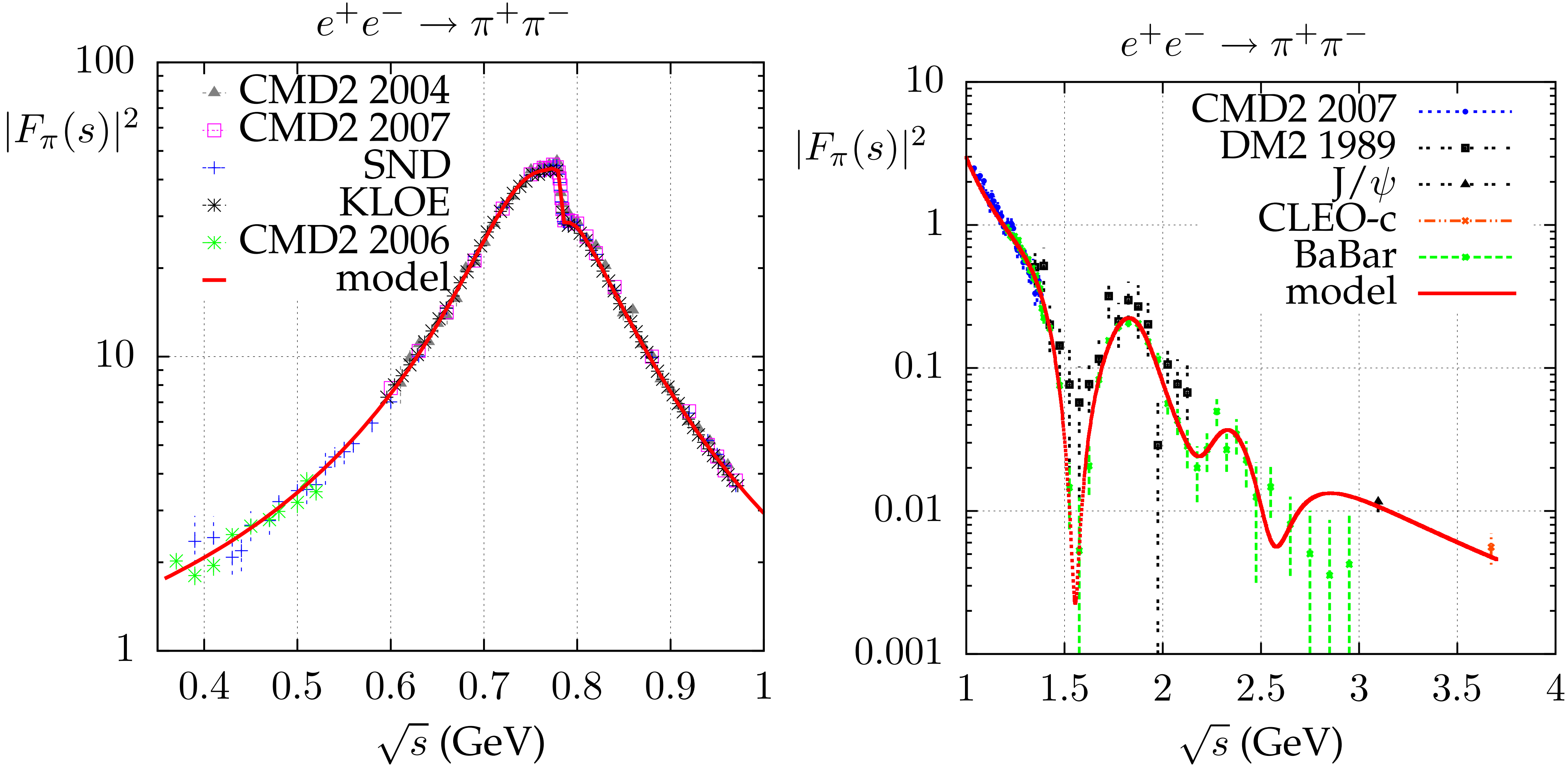
$$\sum c_n = 1$$

Modifications:

- finite widths
- parameters of ρ, ρ', ρ'' fitted to data
- Breit-Wigner for ρ, ρ', ρ'' with Q^2-dependent widths
  $\Rightarrow$ reasonable agreement between model and fit

Parameter	model(fit)	PDG value	model
m_{ρ_0}	773.37 ± 0.19	775.49 ± 0.34	input
Γ_{ρ_0}	147.1 ± 1.0	149.4 ± 1.0	input
m_ω	782.4 ± 0.5	782.41 ± 0.12	-
Γ_ω	8.33 ± 0.27	8.49 ± 0.08	-
m_{ρ_1}	1490 ± 11	1465 ± 25	1340
Γ_{ρ_1}	429 ± 27	400 ± 60	256
m_{ρ_2}	1870 ± 25	1720 ± 20	1730
Γ_{ρ_2}	357 ± 46	250 ± 100	330
m_{ρ_3}	2120	-	2047
Γ_{ρ_3}	300	-	391
m_{ρ_4}	model	-	2321
Γ_{ρ_4}	model	-	444
m_{ρ_5}	model	-	2567
Γ_{ρ_5}	model	-	491

Parameter	model(fit)	PDG value	model
β	2.148±0.003	-	input
$\vert c_\omega^\pi\vert$	(18.7±0.5)·10^{-4}	-	-
$Arg(c_\omega^\pi)$	0.106 ± 0.020	-	-
$\vert F_2\vert$	0.59 ± 0.10	-	-
$Arg(F_2)$	-2.20 ± 0.16	-	-
$\vert F_3\vert$	0.048 ± 0.056	-	-
$Arg(F_3)$	-2. ± 1.4	-	-
$\vert F_4\vert$	0.40 ± 0.07	-	-
$Arg(F_4)$	-2.9 ± 0.3	-	-
$\vert F_5\vert$	0.43 ± 0.05	-	-
$Arg(F_5)$	1.19 ± 0.18	-	-
$\chi^2/d.o.f.$	271/270	-	-



data point at 3.1 GeV ($J/\Psi \to \pi\pi$) leads to strong constraints

$e^+e^- \to K^+K^-\,,\ K^0\bar{K}^0$

isospin symmetry:

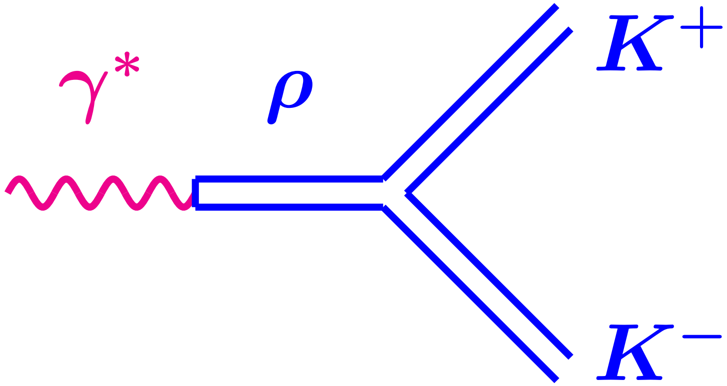
$$F_{K^+} = +F^{(I=1)} + F^{(I=0)}$$
$$F_{K^0} = -F^{(I=1)} + F^{(I=0)}$$

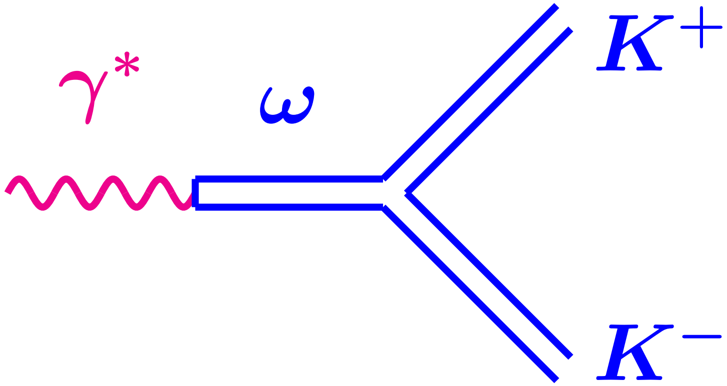
resonances:

$$\begin{aligned} F_{K^+}(s) = &+\tfrac{1}{2}\Big(c^K_\rho BW_\rho(s) + c^K_{\rho'} BW_{\rho'}(s) + c^K_{\rho''} BW_{\rho''}(s)\Big) \\ &+ \tfrac{1}{6}\Big(c^K_\omega BW_\omega(s) + c^K_{\omega'} BW_{\omega'}(s) + c^K_{\omega''} BW_{\omega''}(s)) \\ &+ \tfrac{1}{3}\Big(c_\phi BW_\phi(s) + c_{\phi'} BW_{\phi'}(s)\Big)\,, \end{aligned}$$

$$\begin{aligned} F_{K^0}(s) = &-\tfrac{1}{2}\Big(c^K_\rho BW_\rho(s) + c^K_{\rho'} BW_{\rho'}(s) + c^K_{\rho''} BW_{\rho''}(s)\Big) \\ &+ \tfrac{1}{6}\Big(c^K_\omega BW_\omega(s) + c^K_{\omega'} BW_{\omega'}(s) + c^K_{\omega''} BW_{\omega''}(s)\Big) \\ &+ \tfrac{1}{3}\Big(\eta_\phi c_\phi BW_\phi(s) + c_{\phi'} BW_{\phi'}(s)\Big) \end{aligned}$$

quark model:



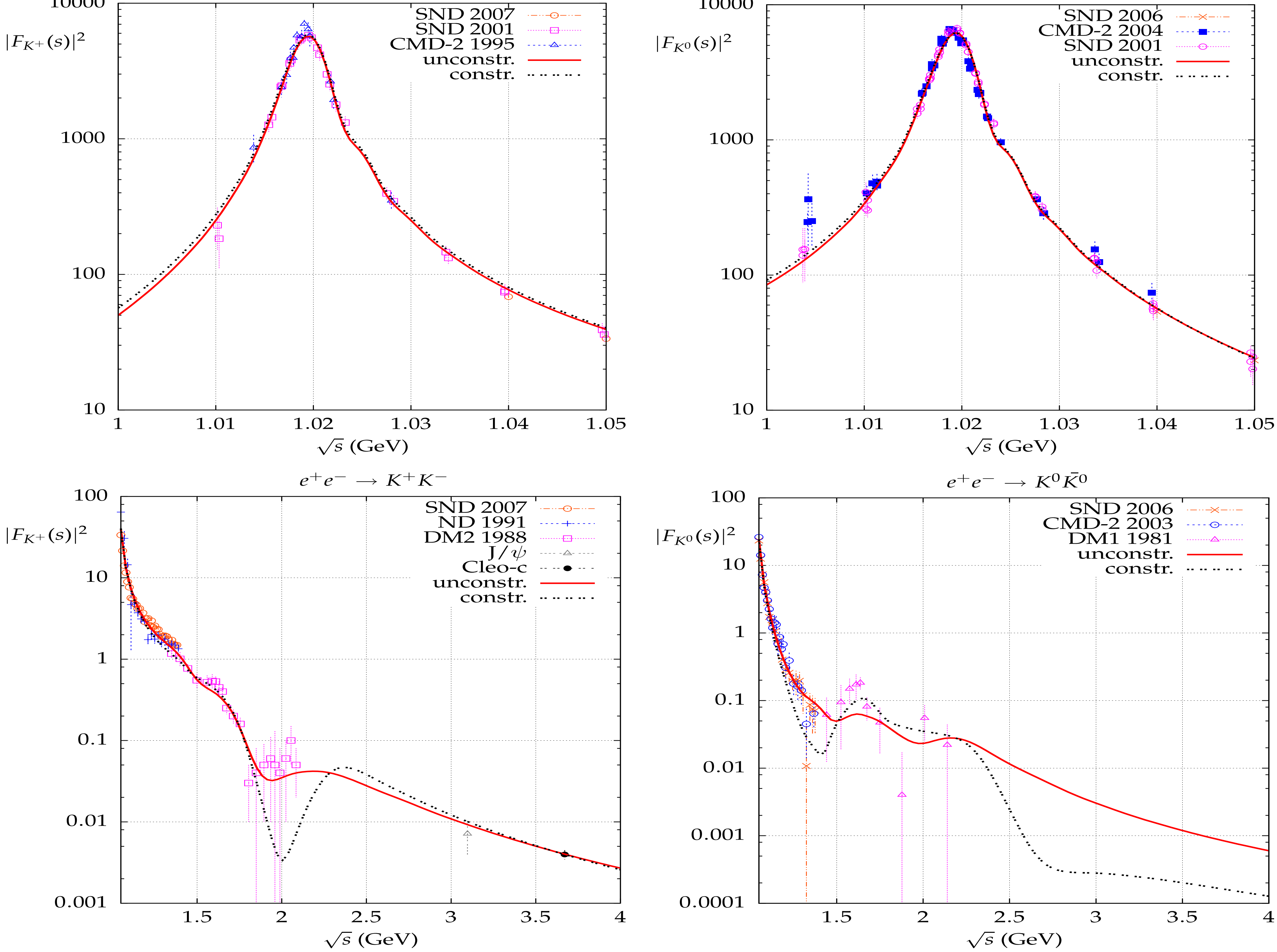


$$\frac{1}{\sqrt{2}} f_\rho \quad g_{\rho KK} \qquad\qquad \frac{1}{3\sqrt{2}} f_\omega \quad g_{\omega KK}$$

constraint: $f_\rho = f_\omega \,, \quad g_{\rho KK} = g_{\omega KK}$

$$\Rightarrow c_\rho = c_\omega$$

fit performed with **(solid curves)**
or without **(dashed curves)** this constraint



$|F_{K^+}(s)|^2$
SND 2007
SND 2001
CMD-2 1995
unconstr.
constr.
$\sqrt{s}$ (GeV)
$|F_{K^0}(s)|^2$
SND 2006
CMD-2 2004
SND 2001
unconstr.
constr.
$\sqrt{s}$ (GeV)
$e^+e^- \longrightarrow K^+K^-$
$|F_{K^+}(s)|^2$
SND 2007
ND 1991
DM2 1988
J/ψ
Cleo-c
unconstr.
constr.
$\sqrt{s}$ (GeV)
$e^+e^- \longrightarrow K^0\bar{K}^0$
$|F_{K^0}(s)|^2$
SND 2006
CMD-2 2003
DM1 1981
unconstr.
constr.
$\sqrt{s}$ (GeV)

$\tau \to K^- K^0 \nu$

Predictions based on isospin symmetry and $I = 1$ part of form factor:

$$\left(\frac{1}{BR(\tau \to \mu^- \bar{\nu}_\mu \nu_\tau)}\right) \frac{dBR(\tau \to K^- K^0 \nu_\tau)}{d\sqrt{Q^2}} =$$
$$\frac{|V_{ud}|^2}{2m_\tau^2} \left(1 + \frac{2Q^2}{m_\tau^2}\right) \left(1 - \frac{Q^2}{m_\tau^2}\right)^2 \left(1 - \frac{4m_K^2}{Q^2}\right)^{3/2}$$
$$\times \sqrt{Q^2}\, |F_{K^- K^0}(Q^2)|^2$$

and $F_{K^- K^0} = -F_{K^+} + F_{K^0}$

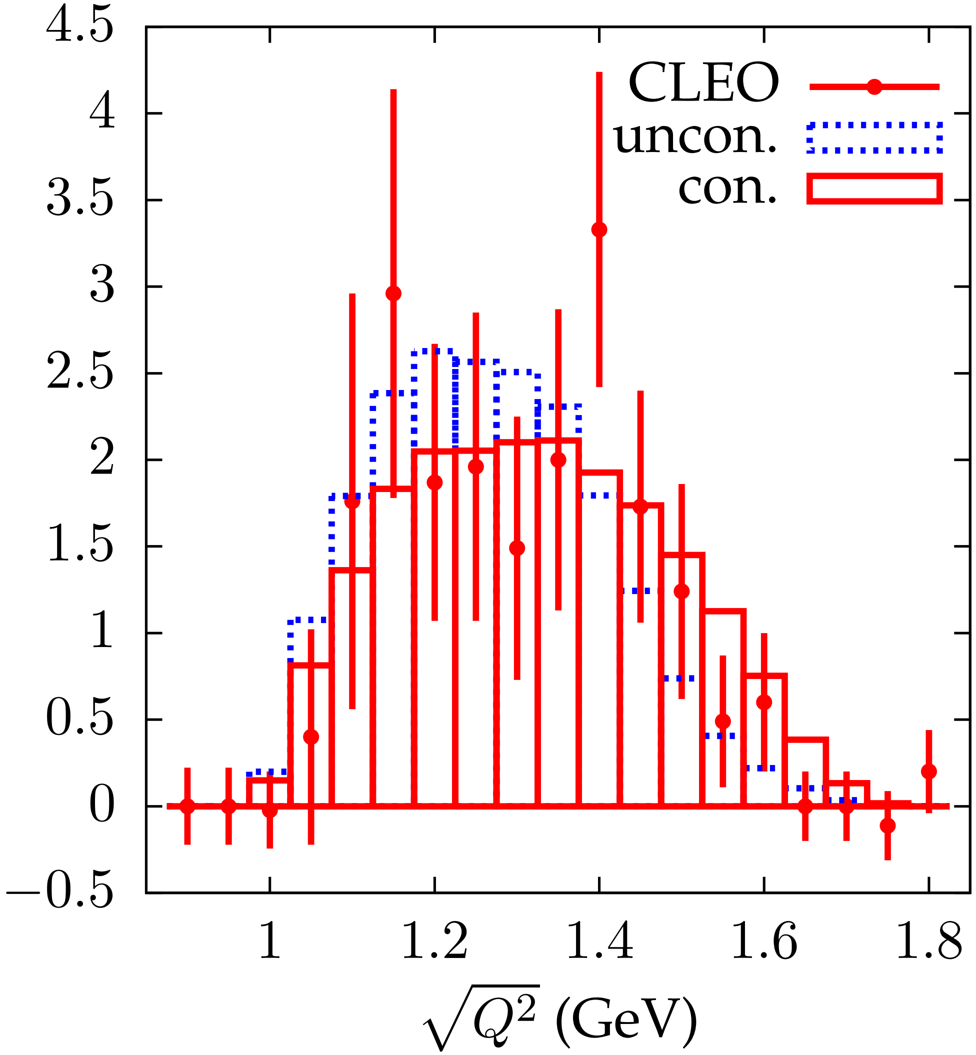
$\Rightarrow BR(\tau \to K^- K^0 \nu_\tau) = 0.135 - 0.190\%$

to be compared with

$$BR(\tau \to K^- K^0 \nu_\tau) = 0.158 \pm 0.016\%.$$

Q^2 distribution:

will provide further constraints!



(data from CLEO)

VI Experimental Results

KLOE

pion form factor, asymmetry

BABAR, BELLE

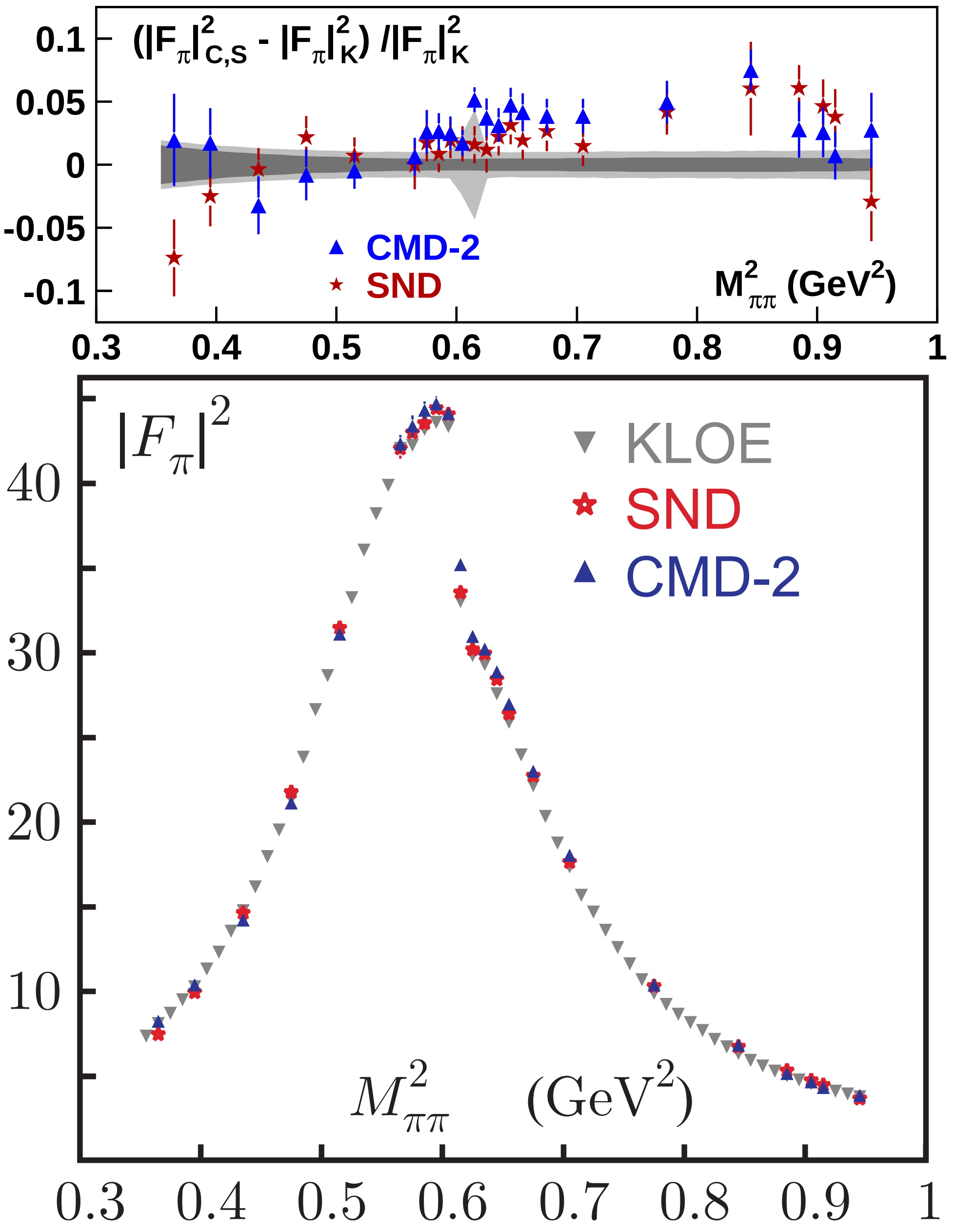
higher Q^2 available

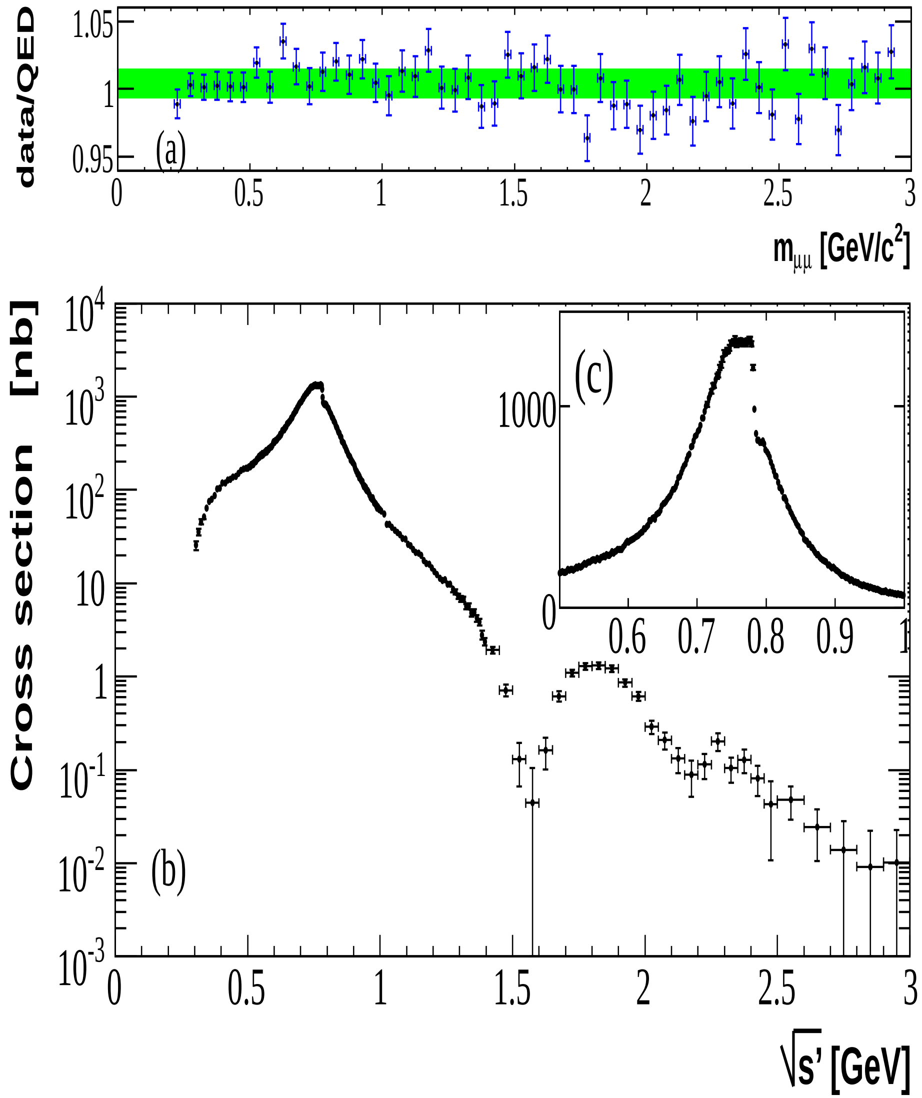
$\Rightarrow$ measurement of R(Q^2) from threshold up to at least 5 GeV.

Examples:

- $\pi\pi$
- $4\pi^{\pm}$, $\pi^+\pi^- 2\pi^0$
- 6π
- K K $\pi\pi$
- K K K K
- $p\bar{p}$, $\Lambda\bar{\Lambda}$, ...

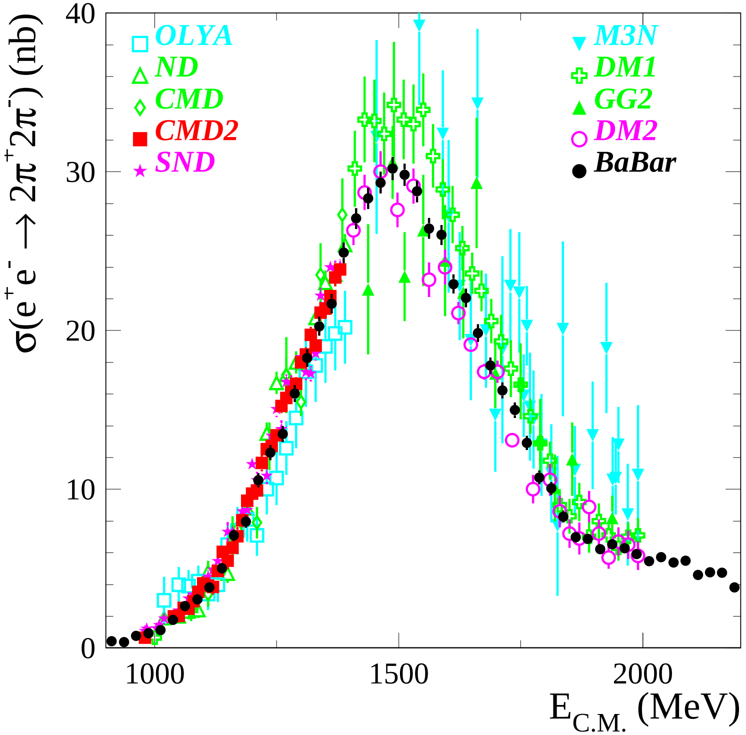
Pion Form Factor

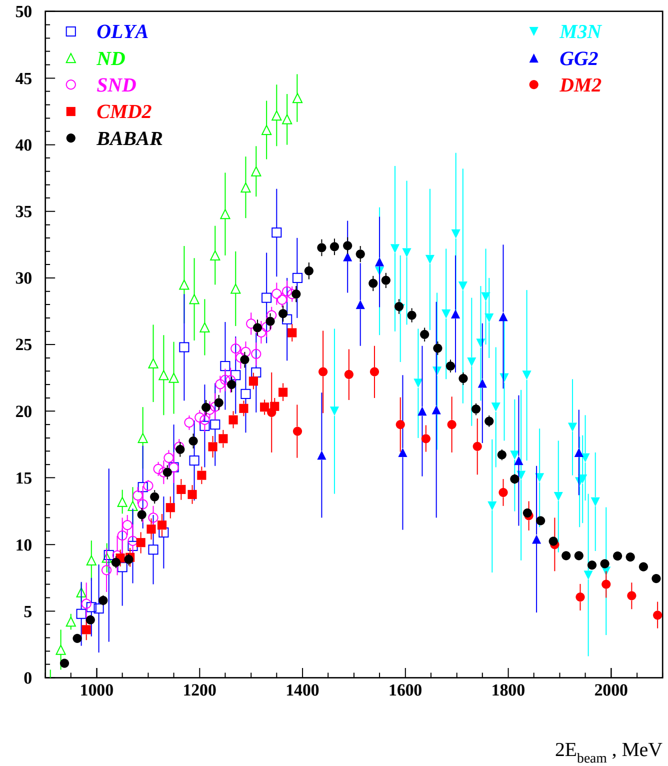




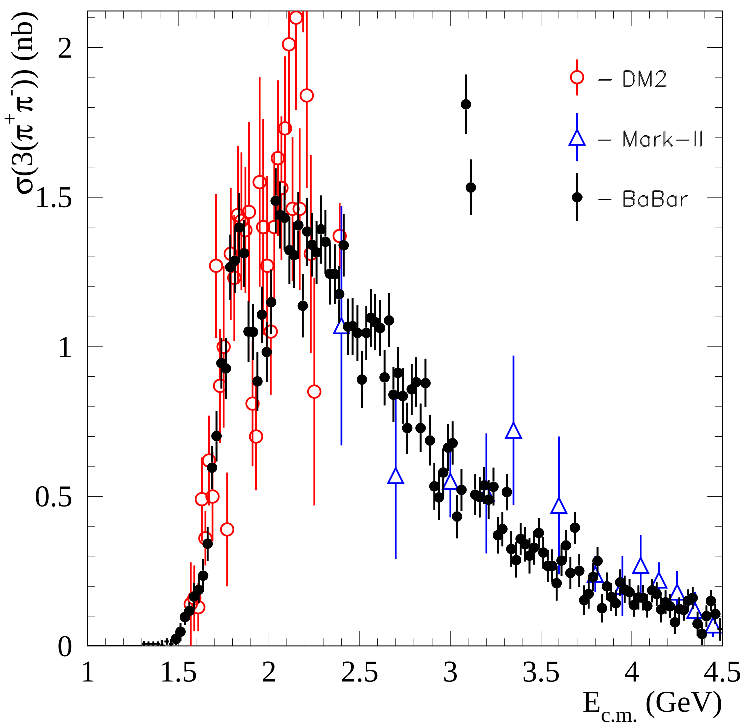
BABAR

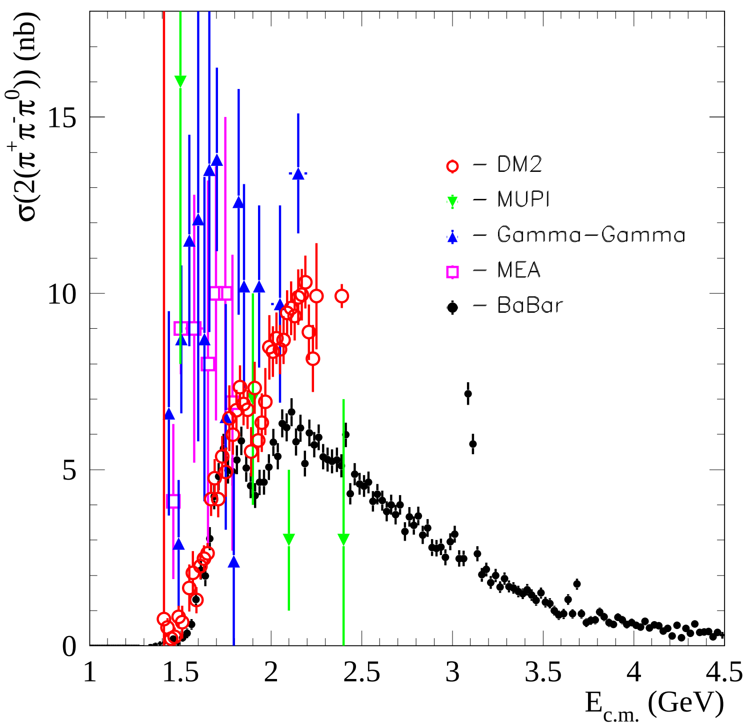
4π


OLYA
ND
CMD
CMD2
SND
M3N
DM1
GG2
DM2
BaBar
σ(e+e- → 2π+2π-) (nb)
E C.M. (MeV)


OLYA
ND
SND
CMD2
BABAR
M3N
GG2
DM2
2E beam , MeV

6π


– DM2
– Mark–II
– BaBar
σ(3(π+π-)) (nb)
E c.m. (GeV)


– DM2
– MUPI
– Gamma–Gamma
– MEA
– BaBar
σ(2(π+π-π0)) (nb)
E c.m. (GeV)

VII Conclusions

- continuous development of PHOKHARA

  ⇒ radiative corrections

  ⇒ more channels

  ⇒ cooperation between theory and experiment crucial

- charge asymmetry as analysis tool

- nucleon form factors:

  G_E and G_M can be measured for a wide range of Q^2

- pion form factor: structures at large Q^2

  kaon form factors: K^+K^- & $K^0\bar{K}^0 \Rightarrow K^-K^0$

  ⇒ prediction for $\tau \to \nu K^- K^0$

⇒ numerous experimental results during past 5 years