

Inclusive and Exclusive Cross Sections at Low Energy e^+e^- Colliders

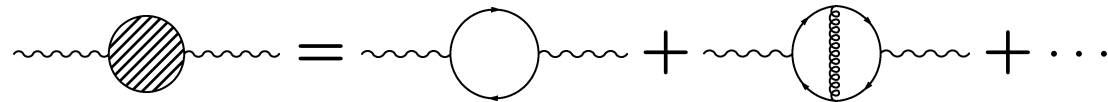
Johann H. Kühn

- I. Perturbative Predictions for $R(s)$
- II. α_s from $R(s)$
- III. m_c and m_b from $R(s)$
- IV. Baryon Form Factors
- V. Inclusive J/ψ Production
and Glue Balls
- VI. Summary

I. Perturbative Predictions for $R(s)$

remember: $R(s) = 12\pi \text{Im}\Pi(q^2 = s + i\epsilon)$

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu) \Pi(q^2) \equiv i \int dx e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$



$\Pi(q^2)$ sums all intermediate states

status of theory:

- massless limit

$$R = 3 \sum_i Q_i^2 \left(1 + \frac{\alpha_s}{\pi} + \# \left(\frac{\alpha_s}{\pi} \right)^2 + \# \left(\frac{\alpha_s}{\pi} \right)^3 + \# \left(\frac{\alpha_s}{\pi} \right)^4 + \dots \right)$$

parton
model

QED
Källen+
Sabry
1955

Chetyrkin+...
Dine+...
Celmaster
1979

Gorishny,
Kataev, Larin;
Surguladze,
Samuel
1991

Baikov, Chetyrkin, JK
2008

- full quark-mass dependence up to $\mathcal{O}(\alpha_s^2)$

$$R_q = 3Q_q^2 \left[\frac{v(3-v^2)}{2} + \frac{\alpha_s}{\pi} f_1 \left(\frac{m_q^2}{s} \right) + \left(\frac{\alpha_s}{\pi} \right)^2 f_2 \left(\frac{m_q^2}{s} \right) + \dots \right]$$

Källen,
Sabry
1955

Chetyrkin, JK,
Steinhauser
1993

- expansion in m_q^2/s :

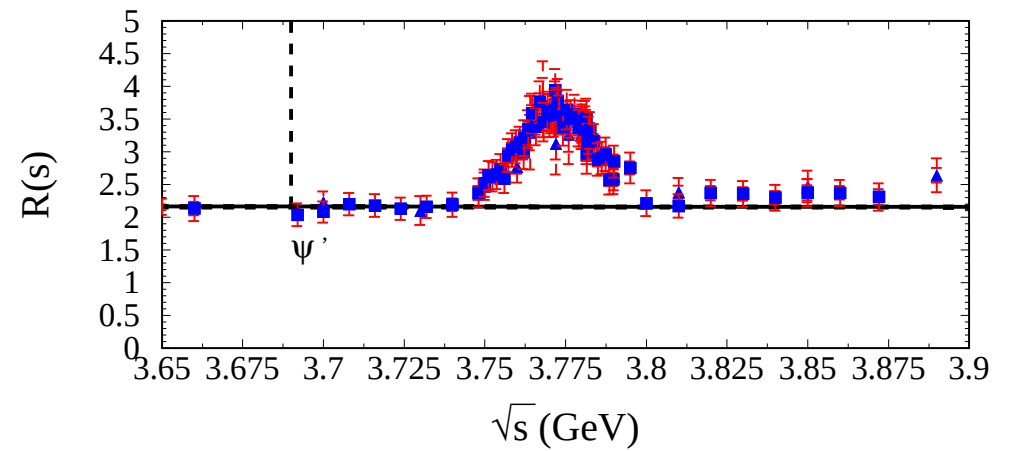
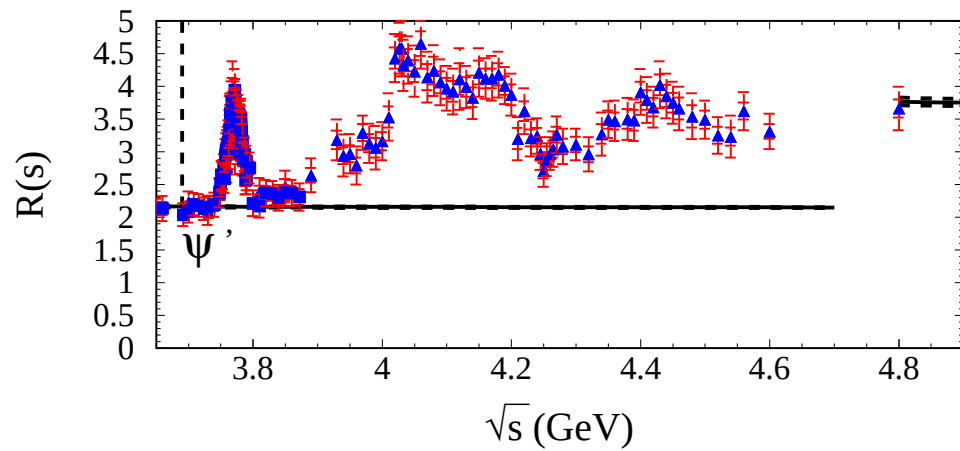
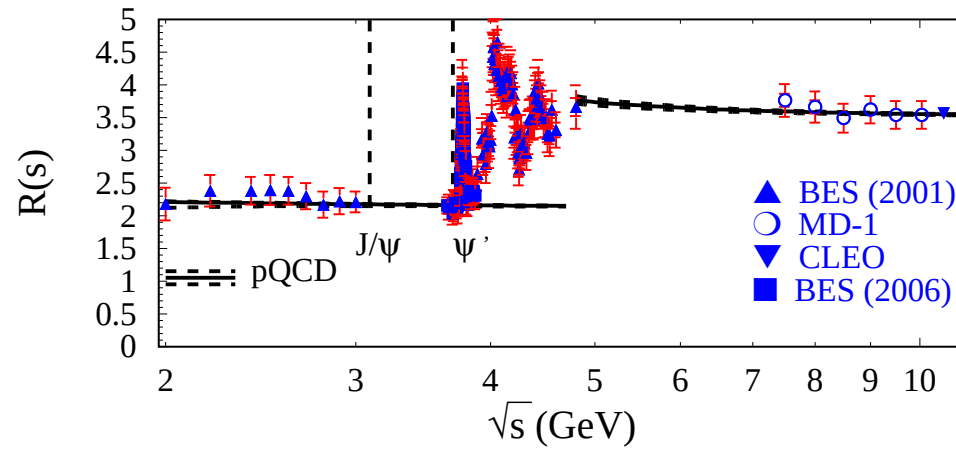
$\left(\frac{\alpha_s}{\pi} \right)^3$	$\frac{m_q^2}{s}$	Chetyrkin, JK	1990
$\left(\frac{\alpha_s}{\pi} \right)^3$	$\left(\frac{m_q^2}{s} \right)^2$	Chetyrkin, JK, Harlander	2000
$\left(\frac{\alpha_s}{\pi} \right)^4$	$\frac{m_q^2}{s}$	Baikov, Chetyrkin, JK	2003

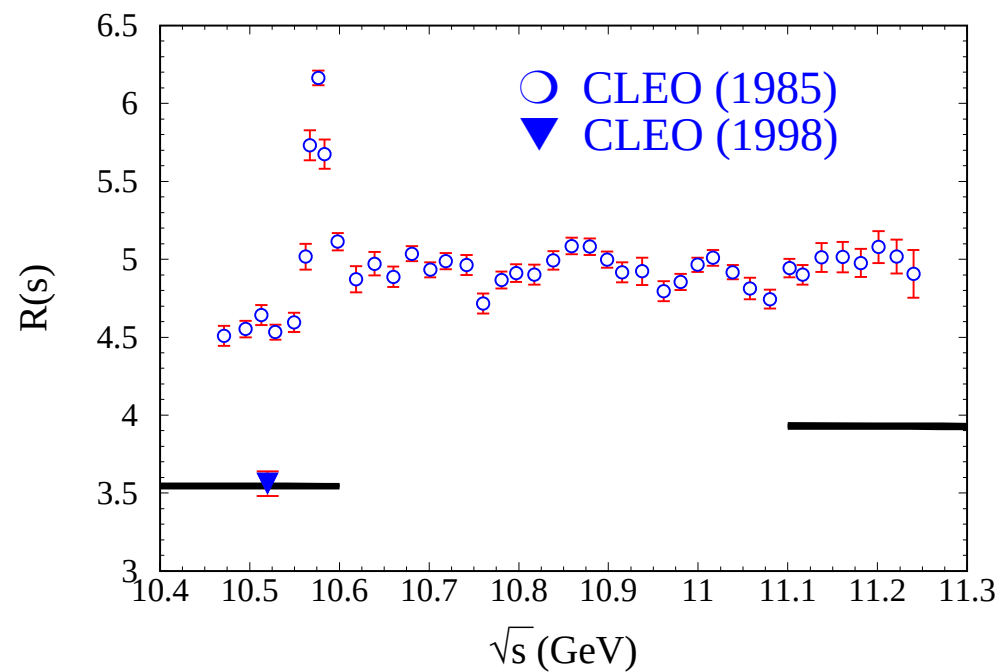
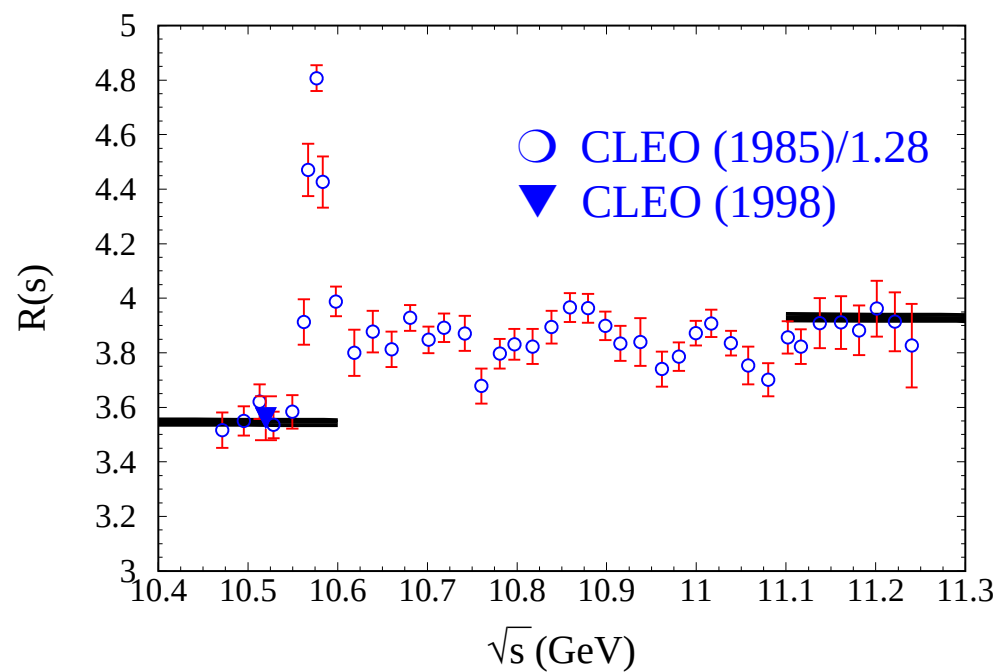
$\Rightarrow$ sufficient precision away from resonance region

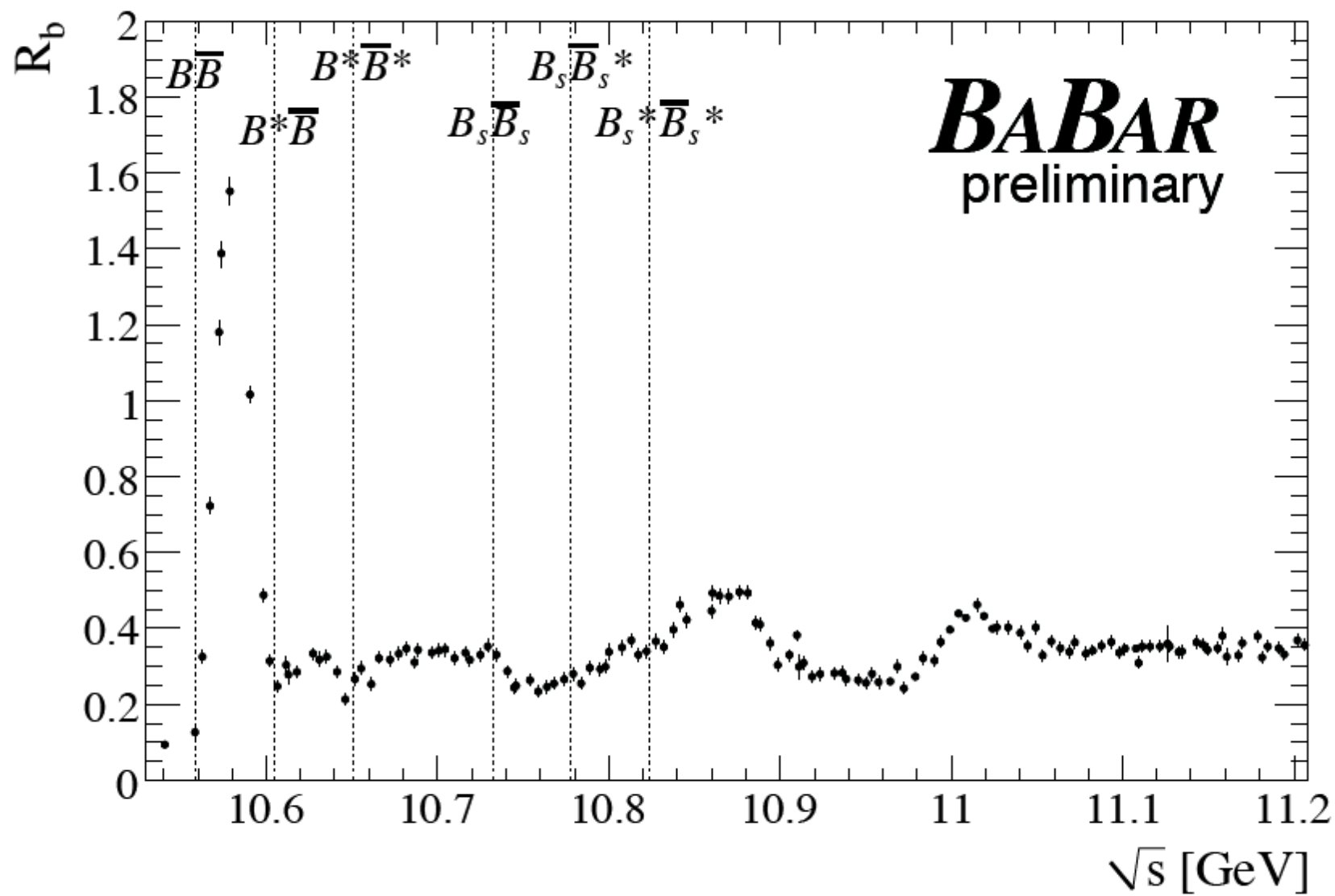
all formulas encoded in program rhad

by Harlander and Steinhauser (hep-ph/0212294)

II. α_s from $R(s)$, Theory vs Experiment







Theory: $R_b(11.2 \text{ GeV}) = 0.365 !$

Kinematic phase space threshold suppression $\sim v$

largely compensated by Coulomb enhancement:

Sommerfeld “rescattering” $\sim 4\pi/v$

- analysis from 2001 (Steinhauser, JK)

$$\alpha_s^{(3)}(3 \text{ GeV}) = 0.369^{+0.047}_{-0.046} {}^{+0.123}_{-0.130}$$

from BES(2000)
below 3.73 GeV

$$\alpha_s^{(4)}(4.8 \text{ GeV}) = 0.183^{+0.059}_{-0.064} {}^{+0.053}_{-0.057}$$

from BES(2000)
at 4.8 GeV

$$\alpha_s^{(4)}(8.9 \text{ GeV}) = 0.193^{+0.017}_{-0.017} {}^{+0.127}_{-0.107}$$

from MD1(1996)
below 7 – 10 GeV

$$\alpha_s^{(4)}(10.52 \text{ GeV}) = 0.186^{+0.008}_{-0.008} {}^{+0.061}_{-0.057}$$

from CLEO(1998)
10.52 GeV

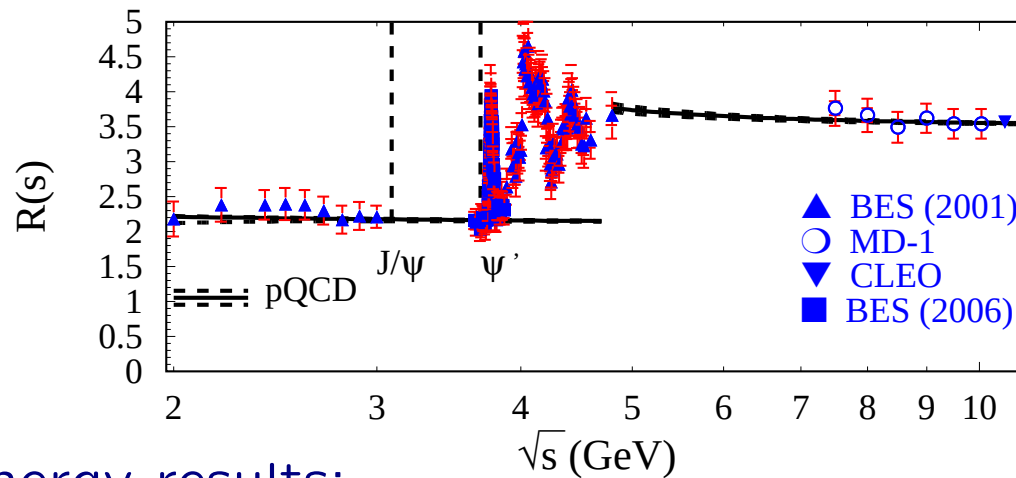
$$\Rightarrow \text{combine: } \alpha_s^{(4)}(5 \text{ GeV}) = 0.235^{+0.047}_{-0.047}$$

$$\Rightarrow \alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$$

- analysis from 2007 (JK, Steinhauser, Teubner)

$$\alpha_s^{(4)}(9 \text{ GeV}) = 0.160 \pm 0.024 \pm 0.024$$

(based on CLEO 2007
6.964 – 10.538 GeV)



combine all low energy results:

$$\Rightarrow \alpha_s^{(4)}(9 \text{ GeV}) = 0.182_{-0.025}^{+0.022} \Rightarrow \alpha_s^{(5)}(M_Z) = 0.119_{-0.011}^{+0.009}$$

to be compared with

$$\alpha_s^{(5)}(M_Z) = 0.1185 \pm 0.0026 \quad \text{from Z-decays}$$

aim: $\delta R/R < 0.5\%$!

New Result (BES II)

arXiv:0903.0900[hep-ex]

E_{cm} (GeV)	R	$\alpha_s^{(3)}(s)$	$\alpha_s^{(4)}(25 \text{ GeV}^2)$	$\alpha_s^{(5)}(M_Z^2)$
2.60	2.18	$0.266^{+0.030+0.125}_{-0.030-0.126}$	$0.209^{+0.044}_{-0.050}$	$0.117^{+0.012}_{-0.017}$
3.07	2.13	$0.192^{+0.029+0.103}_{-0.029-0.101}$		
3.65	2.14	$0.207^{+0.015+0.104}_{-0.015-0.104}$		

$\frac{\delta R}{R}$: sys $\sim 3.5\%$; stat $\sim 0.5 - 1\%$!

- perfect agreement with pQCD
- perfect agreement with

– α_s from Z decay rate: $\alpha_s^{(5)}(M_Z^2) = 0.1185 \pm 0.0026$

– τ -decays (Baikov, Chetyrkin, JK)

$$\alpha_s^{(3)}(m_\tau^2) = 0.332 \pm 0.005_{exp} \pm 0.015_{th} \Rightarrow \alpha_s^{(5)}(M_Z^2) = 0.120 \pm 0.019$$

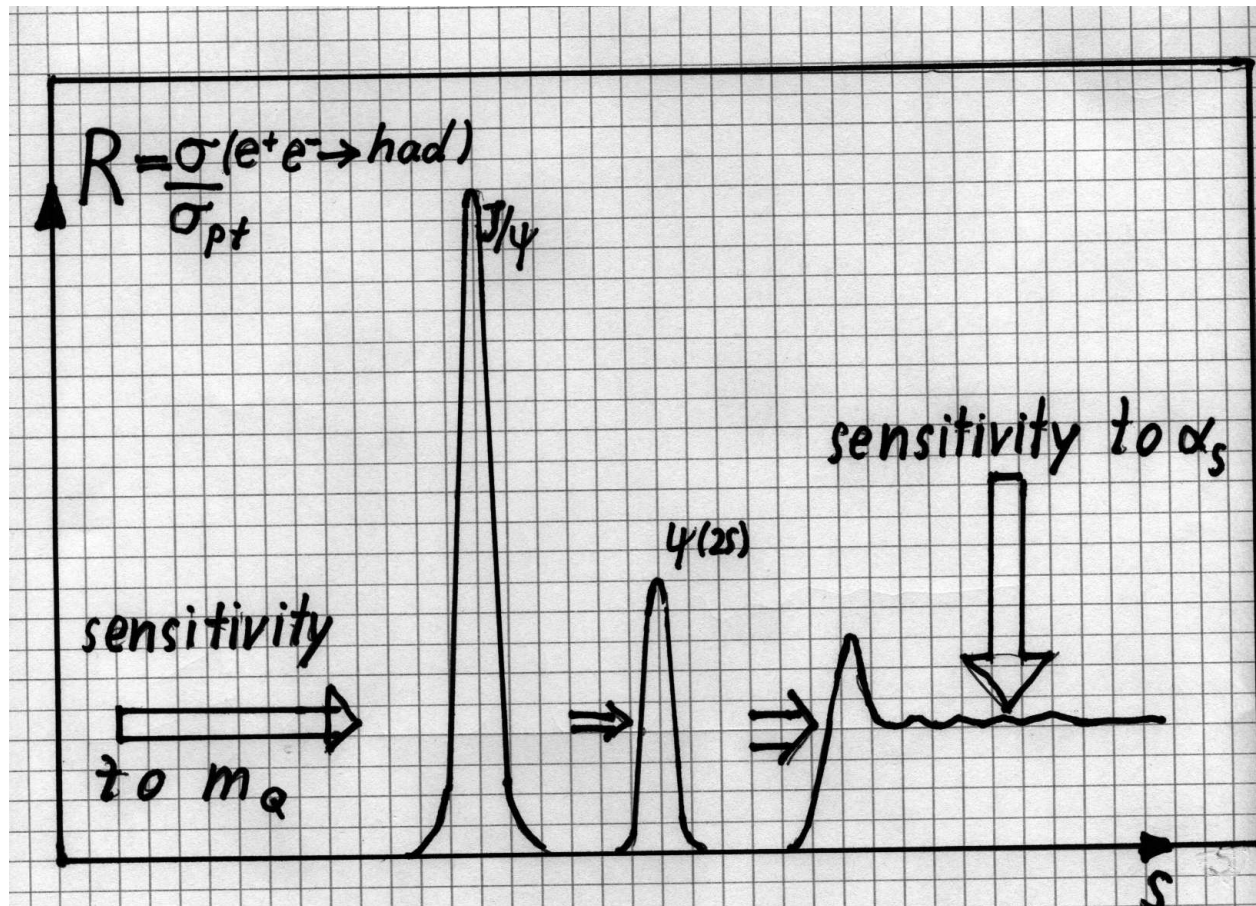
- relative importance of α_s^4 -terms for BES e.g. at 2.606 GeV:

$$0.266 \pm 0.030 \pm 0.120 \Rightarrow 0.286 \pm \dots$$

low energies (~ 2 GeV) of special interest

III. m_c and m_b from $R(s)$

Main Idea (SVZ)



m_Q from

SVZ Sum Rules, Moments and Tadpoles

(JK, Steinhauser 2001; JK, Steinhauser, Sturm 2007)

Some definitions:

$$R(s) = 12\pi \operatorname{Im} \left[\Pi(q^2 = s + i\epsilon) \right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu \right) \Pi(q^2) \equiv i \int dx e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\text{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_S}{\pi} \bar{C}_n^{(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_S}{\pi} \right)^3 \bar{C}_n^{(3)} + \dots$$

Using calculated moments

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!} \left(\frac{d}{dq^2} \right)^n \Pi_c(q^2) \Big|_{q^2=0} = \frac{9}{4} Q_c^2 \left(\frac{1}{4m_c^2} \right)^n \bar{C}_n$$

and experiment

$$\mathcal{M}_n^{\text{exp}} = \int \frac{ds}{s^{n+1}} R_c(s)$$

constraint:

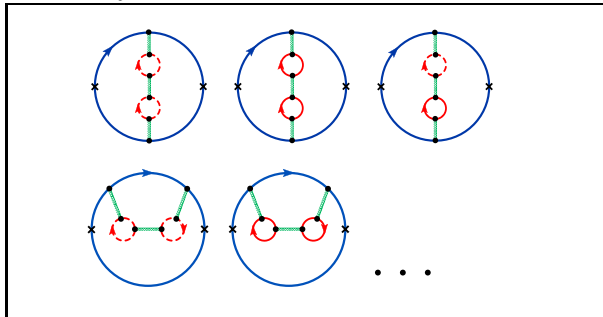
$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$$

$$\Leftrightarrow m_c$$

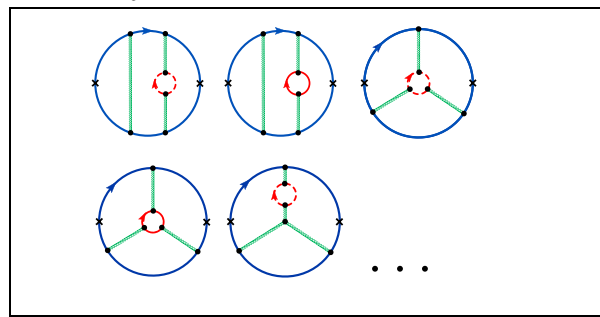
Analysis in N³LO

Algebraic reduction to 13 master integrals (Laporta algorithm);
numerical evaluation of master integrals

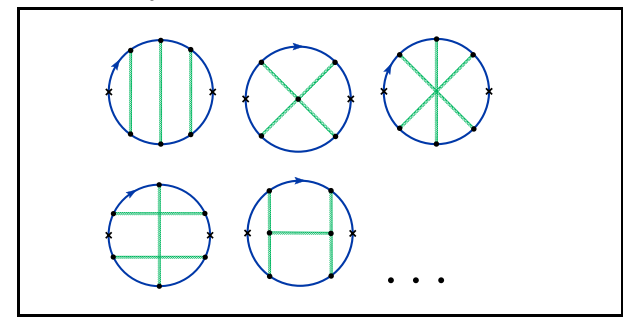
n_f^2 -contributions



n_f^1 -contributions



n_f^0 -contributions



 : heavy quarks,  : light quarks,

n_f : number of active quarks

$\Rightarrow$ About 700 Feynman-diagrams

recall: $\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n z^n$

$\bar{C}_n$ depend on the charm quark mass through $l_{m_c} \equiv \ln(m_c^2(\mu)/\mu^2)$

$$\begin{aligned} \bar{C}_n = & \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi} \left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c} \right) \\ & + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2 \right) \\ & + \left(\frac{\alpha_s(\mu)}{\pi} \right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_c} + \bar{C}_n^{(32)} l_{m_c}^2 + \bar{C}_n^{(33)} l_{m_c}^3 \right) \end{aligned}$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	−0.0889	−5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	−3.4937	6.7216	6.4916	−0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	−2.8395	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

estimate $C_4^{(30)} = -3.3 \pm 0.1$

results for $n = 2, 3, 4$ from Maier, Maierhöfer, Marquard

Experimental input: $\int \frac{ds}{s^{n+1}} R_c(s)$

Contributions from

- narrow resonances: $R = \frac{9 \Pi M_R \Gamma_e}{\alpha^2(s)} \delta(s - M_R^2)$
- threshold region ($2 m_D - 4.8 \text{ GeV}$)
- perturbative continuum ($E \geq 4.8 \text{ GeV}$)

n	$\mathcal{M}_n^{\text{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{np}}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	-0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

$\uparrow$
BES,
SLAC

$\uparrow$
BES

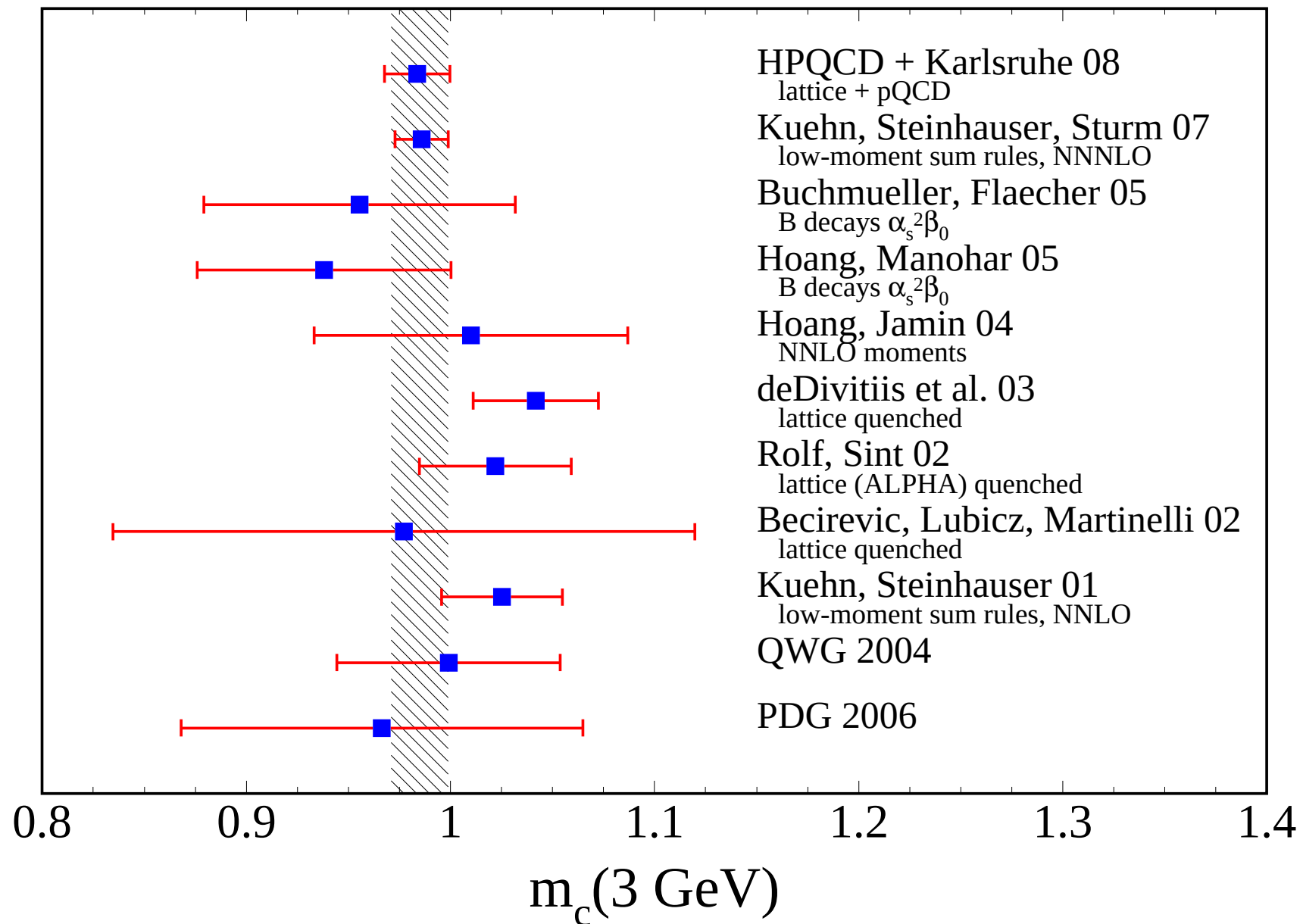
$\uparrow$
BES!
pQCD

Results (m_C)

n	$m_C(3 \text{ GeV})$	exp	α_S	μ	np	total	$m_C(m_C)$
1	0.986	0.009	0.009	0.002	0.001	0.013	1.286
2	0.976	0.006	0.014	0.005	0.000	0.016	1.277
3	0.978	0.005	0.014	0.007	0.002	0.016	1.278
4	1.004	0.003	0.008	0.030	0.007	0.032	1.302

results for $n = 1, 2, 3, 4$ nicely consistent

also consistent with Lattice (HPQCD & Karlsruhe)



Results (m_b)

n	$m_b(10 \text{ GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	4.149
2	3.607	0.014	0.012	0.003	0.019	4.162
3	3.617	0.010	0.014	0.006	0.019	4.172
4	3.629	0.008	0.015	0.021	0.027	4.183

$n = 1, 2, 3, 4$:

- $m_b(10\text{GeV}) = 3612 \pm 19 \text{ MeV}$
- $m_b(m_b) = 4167 \pm 19 \text{ MeV}$
- $m_b(m_t) = 2701 \pm 18 \pm 14 \text{ MeV}$
- $m_t/m_b = 59.8 \pm 1.3$

IV. Baryon Form Factors

matrix element of electromagnetic current

$$\langle N\bar{N}|J_\mu(0)|0\rangle \equiv -ie \cdot \bar{u}(q_2) \left(F_1^N(Q^2)\gamma_\mu - \frac{F_2^N(Q^2)}{4m_N} [\gamma_\mu, \not{Q}] \right) v(q_1)$$

$$F_1^N \hat{=} \text{Dirac FF};$$

$$F_2^N \hat{=} \text{Pauli FF}$$

$$G_M^N \equiv F_1^N + F_2^N \hat{=} \text{magnetic FF}; \quad G_E^N \equiv F_1^N + \frac{Q^2}{4m_N^2} F_2^N \hat{=} \text{electric FF}$$

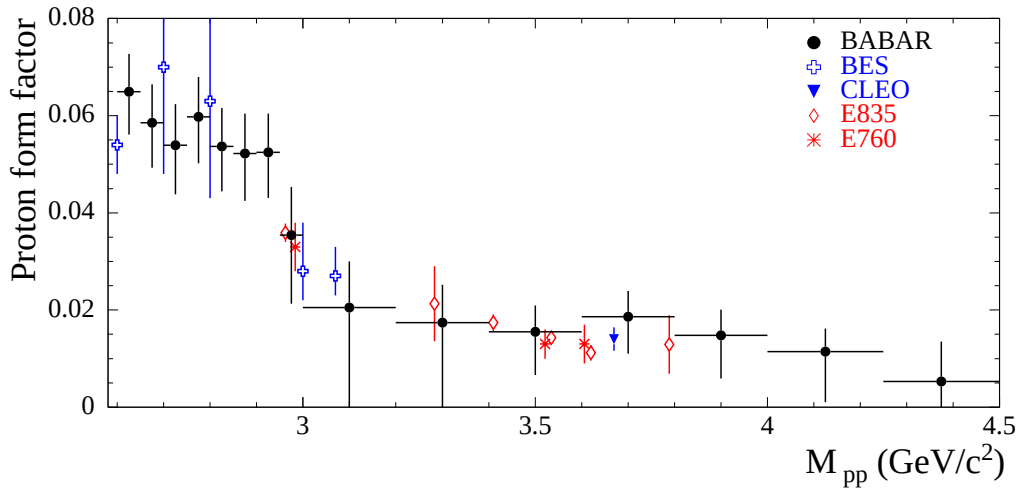
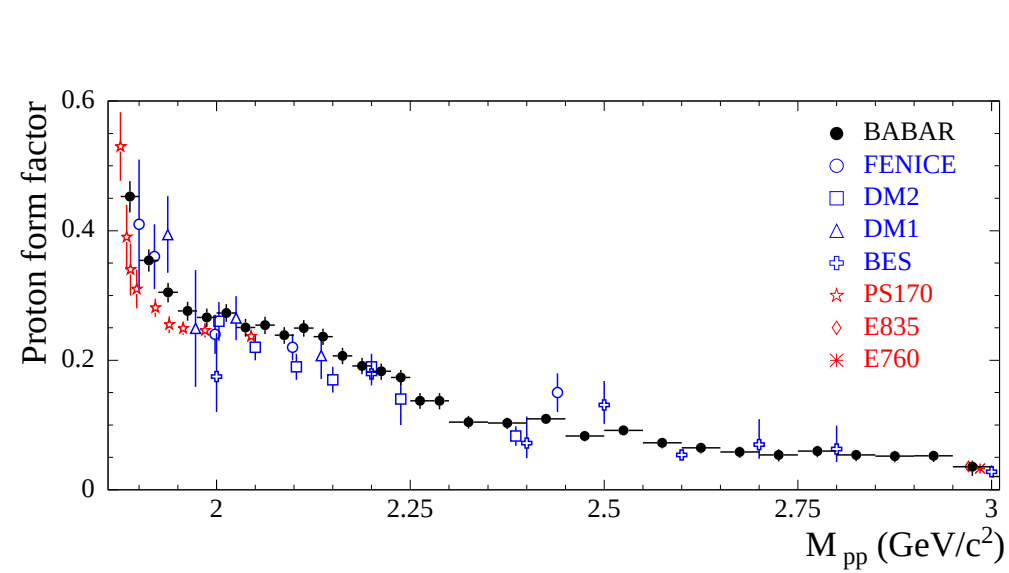
G_M and G_E can be separated via angular distributions

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta_N}{4Q^2} \left(|G_M^N|^2 (1 + \cos^2 \theta) + \frac{1}{\tau} |G_E^N|^2 \sin^2 \theta \right) .$$

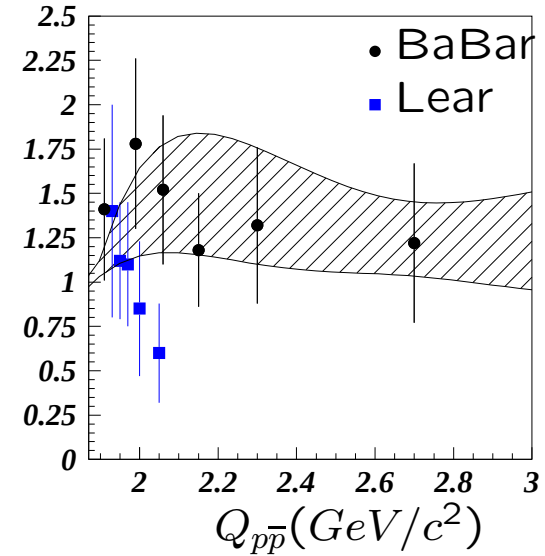
(conflicting results for $|G_E/G_M|$ at JLAB for spacelike q^2 :

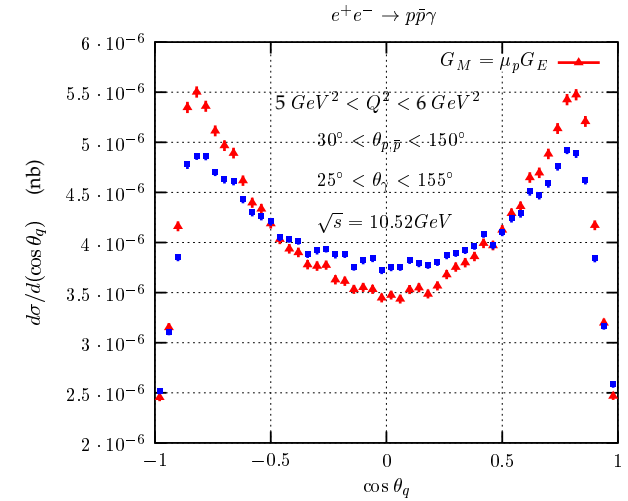
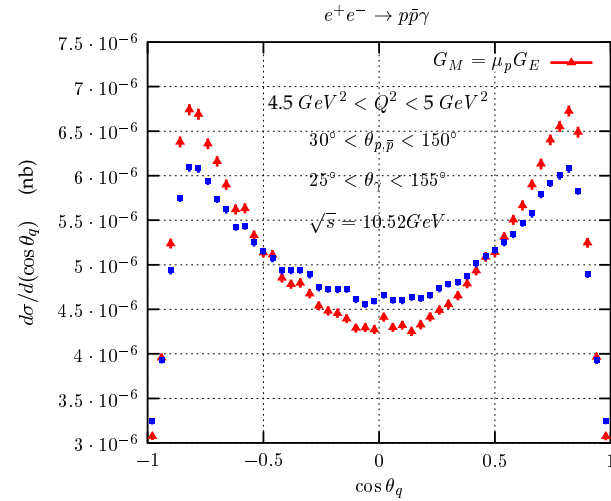
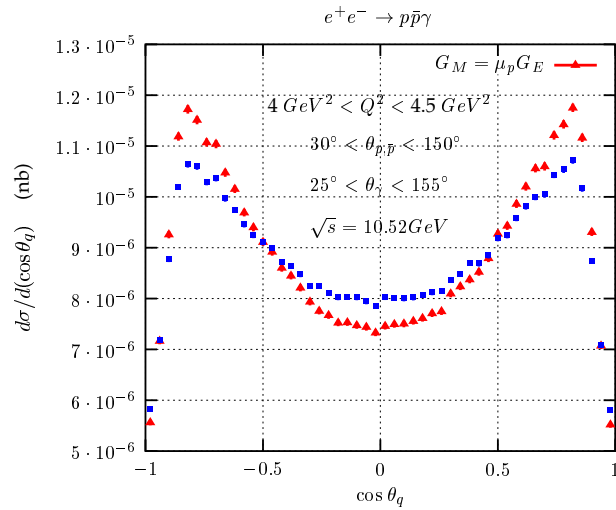
angular distribution (Rosenbluth separation) vs. polarization)

results for timelike q^2 (should lead to important constraints also for negative q^2) using radiative return (PHOKHARA, BaBar)



$$|G_E/G_M|$$





Prediction based on radiative return (PHOKHARA; Czyz, JK, Nowak, Rodrigo)

Interesting option for BESS or high luminosity B-factory

spacelike q^2 : scattering experiments

$\Rightarrow$ polarized electron beam and scattered proton

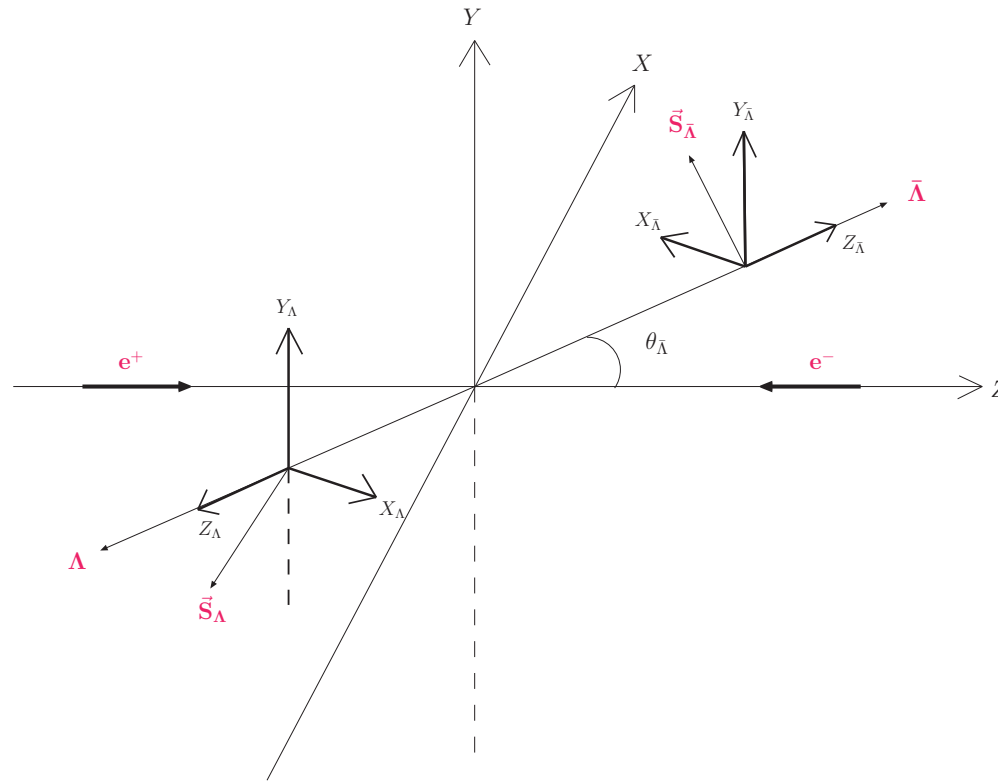
$$\vec{e}p \rightarrow e\vec{p}$$

timelike q^2 : $e^+e^- \rightarrow N\bar{N}$

no polarized beams at present! polarization of N and $\bar{N}$?

$N \Rightarrow \Lambda \rightarrow p\pi$; weak decay: analyser!

$$\begin{aligned} d\sigma \sim L_{\mu\nu}^0 H^{\mu\nu} = 4\pi^2 \alpha^2 & \left\{ |G_M|^2 (1 + \cos^2 \theta_{\bar{\Lambda}}) + \frac{1}{\tau} |G_E|^2 \sin^2 \theta_{\bar{\Lambda}} \right. \\ & + \frac{\text{Im}(G_M G_E^*)}{\sqrt{\tau}} \sin(2\theta_{\bar{\Lambda}}) (S_{\bar{\Lambda}}^y + S_{\bar{\Lambda}}^y) - \frac{\text{Re}(G_M G_E^*)}{\sqrt{\tau}} \sin(2\theta_{\bar{\Lambda}}) (S_{\bar{\Lambda}}^z S_{\bar{\Lambda}}^x + S_{\bar{\Lambda}}^z S_{\bar{\Lambda}}^x) \\ & + \left(\frac{1}{\tau} |G_E|^2 + |G_M|^2 \right) \sin^2 \theta_{\bar{\Lambda}} S_{\bar{\Lambda}}^x S_{\bar{\Lambda}}^x + \left(\frac{1}{\tau} |G_E|^2 - |G_M|^2 \right) \sin^2 \theta_{\bar{\Lambda}} S_{\bar{\Lambda}}^y S_{\bar{\Lambda}}^y \\ & \left. - \left(\frac{1}{\tau} |G_E|^2 \sin^2 \theta_{\bar{\Lambda}} - |G_M|^2 (1 + \cos^2 \theta_{\bar{\Lambda}}) \right) S_{\bar{\Lambda}}^z S_{\bar{\Lambda}}^z \right\} \end{aligned}$$



correlation between S_Λ and $\overline{S_\Lambda}$ is sensitive to G_E , G_M and relative phase

distributions: $\vec{S}_\Lambda \Rightarrow -0.642 \vec{n}_{\pi^-}$; $\vec{S}_{\overline{\Lambda}} \Rightarrow +0.642 \vec{n}_{\pi^+}$

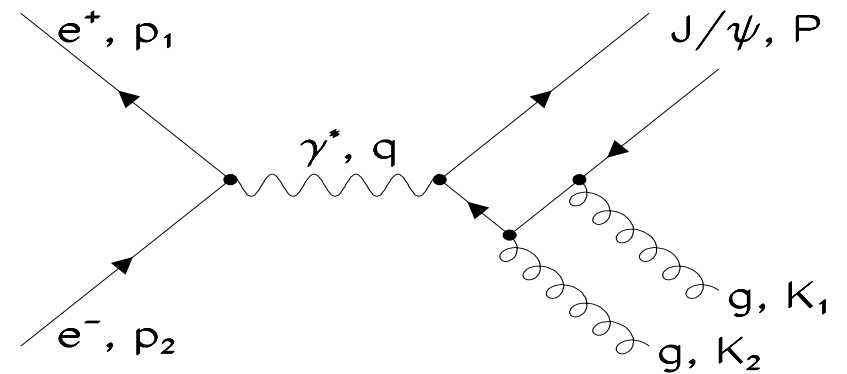
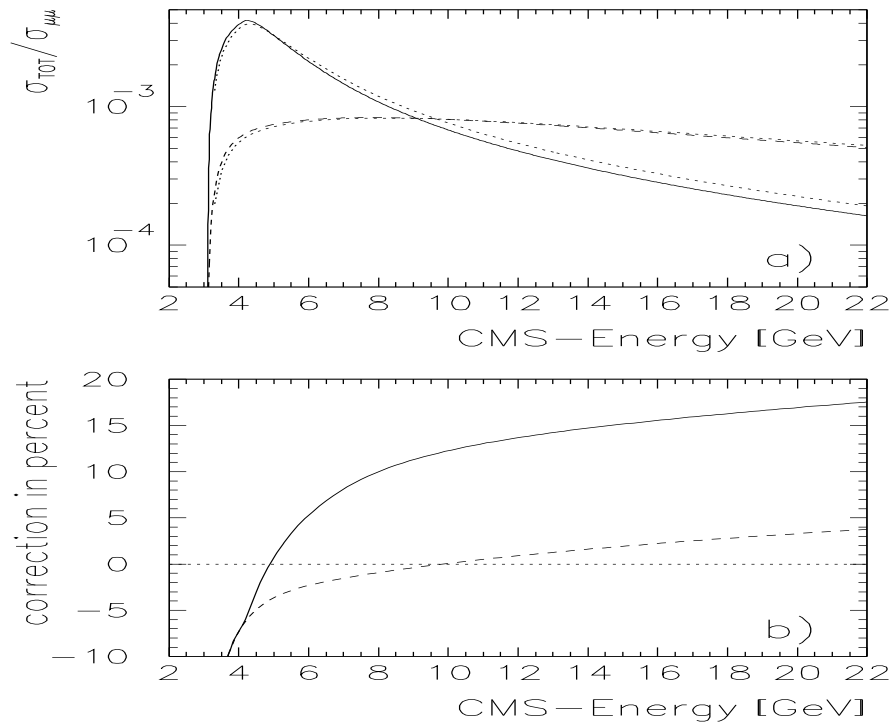
implemented in PHOKHARA (Czyz, Grzelinska, JK)

different final states: $p\overline{p}, \Lambda\overline{\Lambda}, \Sigma\overline{\Sigma}, \dots$

V. Exclusive Channels in $e^+e^- \rightarrow J/\psi + X$

High energies (inclusive J/ψ)

$$e^+e^- \rightarrow J/\psi + gg$$

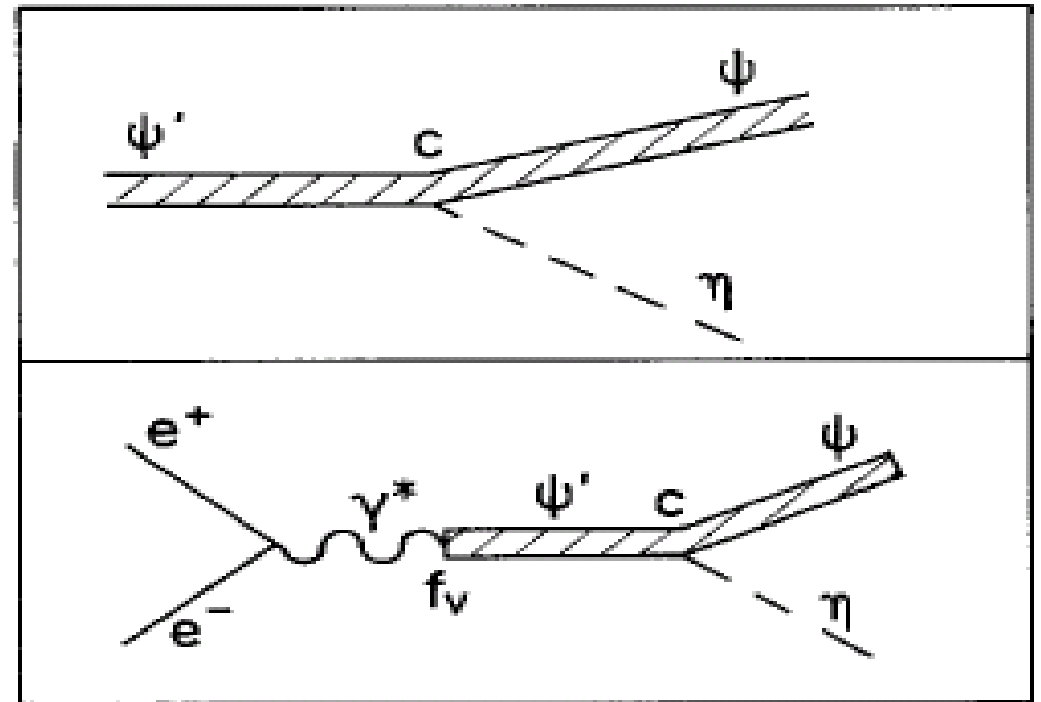
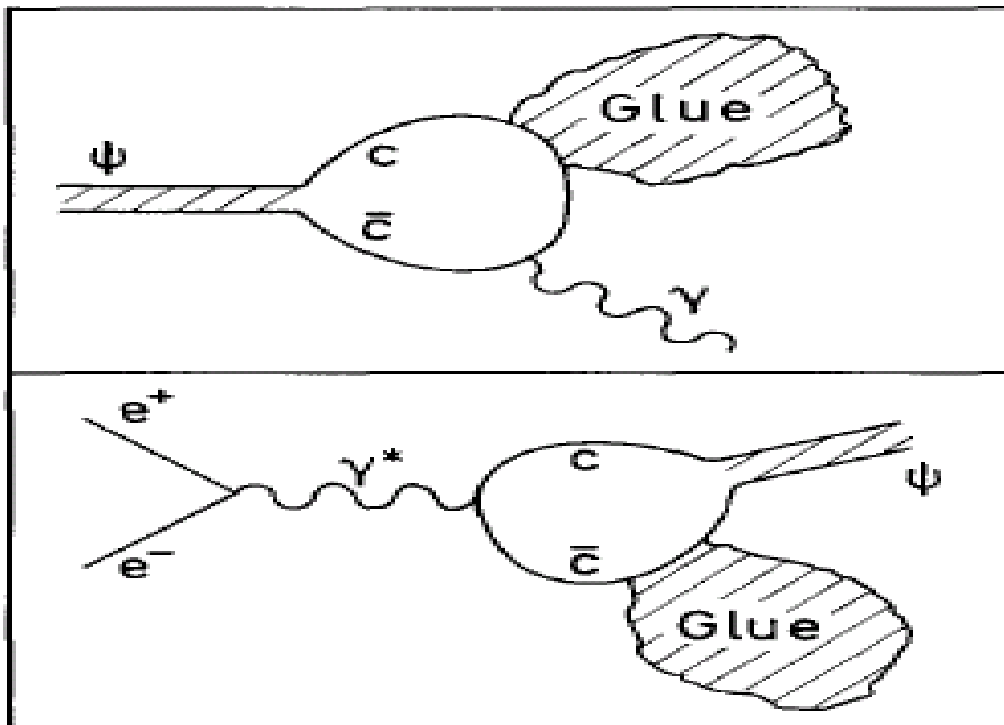


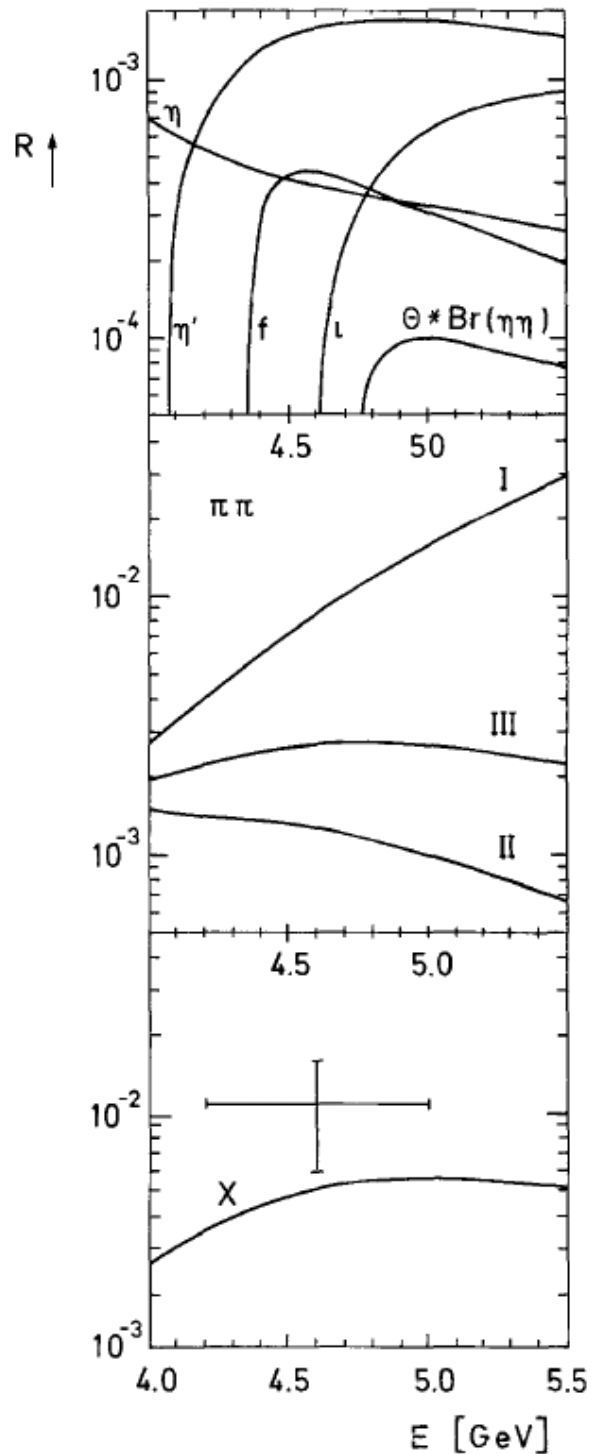
angular distributions, properties of gluon jets (Driesen, JK, Mirkes, 1993)

Low energies: nonperturbative physics! exclusive channels

$$J/\psi \rightarrow \gamma \text{ Resonance} \iff \gamma^* \rightarrow J/\psi + \text{Resonance}$$

Models





applicable to η , π^0 , η' , $\eta(1405/1475)$, glueballs, f_0 , f_2 ;
 dynamical assumptions required for $J/\psi \pi^+ \pi^-$!

- clean signature: $e^+e^- \rightarrow J/\psi(\rightarrow \mu^+\mu^-) + \text{had}$
 and $m(\text{had}) \lesssim 2 - 3 \text{ GeV}$: analysis of glueballs
- radiative return: similar measurement at B-factory

(from JK, 1982)

VI. SUMMARY

- cross section measurements between 2 and 5 GeV
 $\Rightarrow \alpha_s, m_c$ with high precision
- nucleon form factors G_E, G_M
 $\Rightarrow$ surprises for $q^2 < 0$; $q^2 > 0$?
 $\Rightarrow$ compare $p, \Lambda, \Sigma, \dots$
- $e^+e^- \rightarrow J/\psi + X$
 $\Rightarrow$ interesting hadron dynamics
 $X =$ glue ball?