Charm and Bottom Quark Masses, the Strong Coupling Constant and Multi-Loop Results

Johann H. Kühn

I. Recent Multi-Loop Results

II. Quark Masses from Low Moments

III. The Strong Coupling

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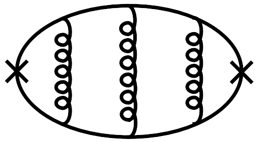
Computational Theoretical Particle Physics
SFB TR9

I. Recent Multi-Loop Results

- Massive tadpole integrals to 4-loops

One-scale integrals: M(Propagator) $= 0$ or m

e.g.  $\Rightarrow$ reduction to master integrals
(Laporta-algorithm)
analytic results for masters

applications:

non-diagonal current $\Rightarrow$ ρ-parameter

diagonal current $\Rightarrow$ m_b and m_c from sum rules

Four-loop results for vector, axial, scalar, pseudoscalar correlators at $q^2 = 0$ and increasingly higher derivatives wrt. q^2.

* Chetyrkin,JK,Sturm NPB744,121 (2006); Eur.Phys.J48,107 (2006)

* Chetyrkin,Faisst,Sturm,Tentyukov NPB742,208 (2006)

* Chetyrkin,Faisst,JK,Maierhöfer,Sturm PRL97 102003 (2006)

Boughezal,Czakon,Schutzmeier PRD74,074006 (2006)

Boughezal,Czakon NPB755,221 (2006)

Czakon,Schutzmeier JHEP0807,001 (2008)

* Maier,Maierhöfer,Marquard NPB797,218 (2008); arXiv:0806.3405[hep-ph]

* Sturm arXiv:0807.1646[hep-ph]

*Karlsruhe

see also P.Marquard; Formal theory, Talk 359

- Massless propagator integrals to 4-loops
(including finite part)

$\Rightarrow$ massless propagator integrals,

absorptive parts to 5-loops.

$\Rightarrow$ $R(s)$ in order α_s^4 !

*Baikov,Chetyrkin,JK PRL 101(2008) 012002;

*Karlsruhe

II. Quark Masses from Low Moments

WHY precise masses?

B-decays:

$\Gamma(B \to X_{\mathrm{u}} l \bar{\nu}) \sim G_{\mathrm{F}}^2 \, m_{\mathrm{b}}^5 \, |V_{\mathrm{ub}}|^2$

$\Gamma(B \to X_{\mathrm{c}} l \bar{\nu}) \sim G_{\mathrm{F}}^2 \, m_{\mathrm{b}}^5 \, f(m_{\mathrm{c}}^2/m_{\mathrm{b}}^2) \, |V_{\mathrm{cb}}|^2$

$B \to X_{\mathrm{s}} \gamma$

Υ-spectroscopy:

$$m\,(\Upsilon(1\mathrm{s})) = 2M_{\mathrm{b}} - \left(\frac{4}{3}\alpha_{\mathrm{s}}\right)^2 \frac{M_{\mathrm{b}}}{4} + \ldots$$

Higgs decay (ILC)

$$\Gamma(H \to b\bar{b}) = \frac{G_F\, M_H}{4\sqrt{2}\pi}\, m_b^2(M_H)\, \tilde{R}$$

$$\tilde{R} \;=\; 1 + 5.6667 a_s + 29.147 {a_s}^2 + 41.758 {a_s}^3 - 825.7 {a_s}^4 \qquad \left(a_s \equiv \frac{\alpha_s}{\pi}\right)$$

${a_s}^4$-term = 5-loop calculation [Baikov,...]

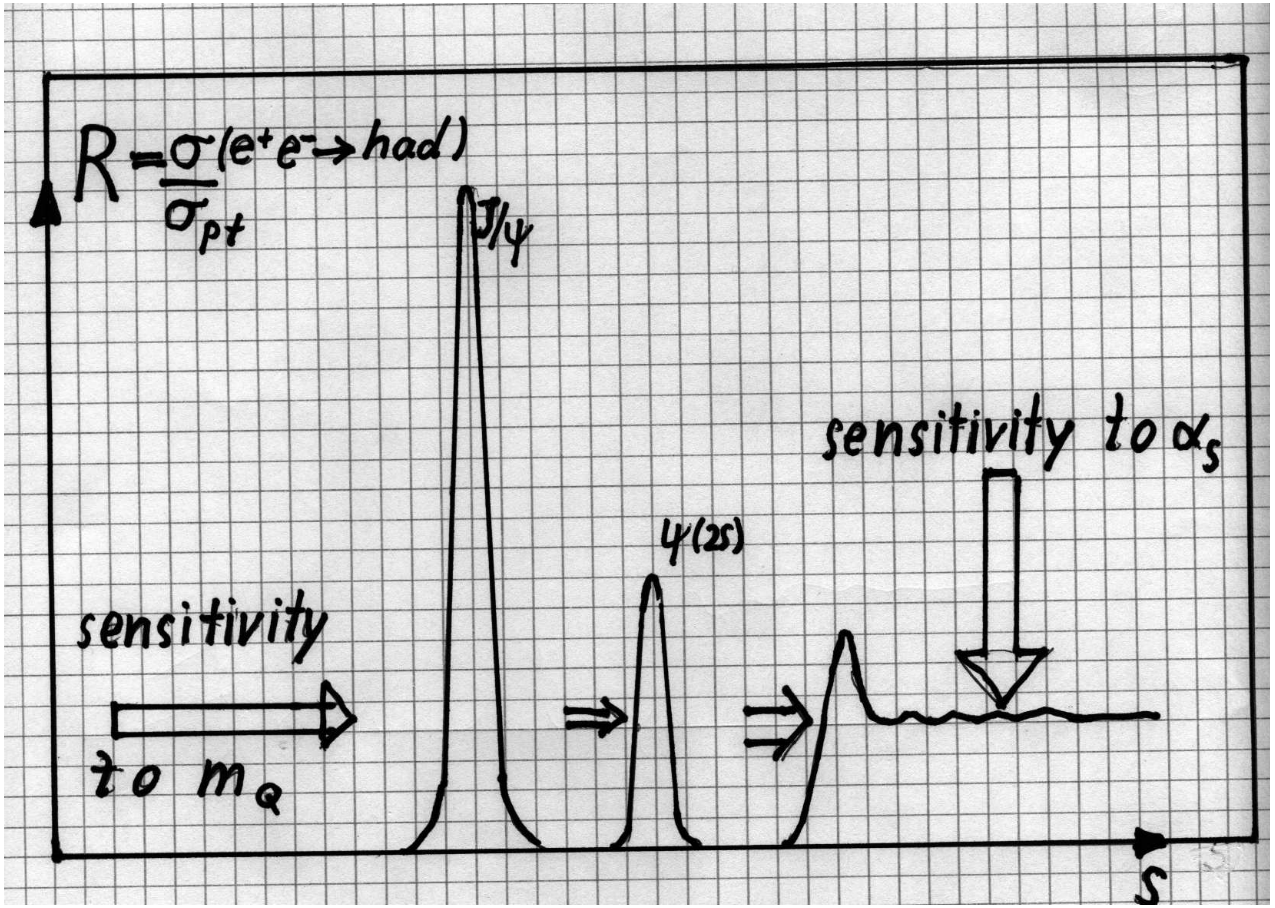
Yukawa Unification

$\lambda_\tau \sim \lambda_b$ or $\lambda_\tau \sim \lambda_b \sim \lambda_t$ at GUT scale

top-bottom $\to m_t / m_b \sim$ ratio of vacuum expectation values

request $\frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t} \Rightarrow \delta m_t \approx 1$ GeV $\Rightarrow \delta m_b \approx 25$ MeV

Main Idea (SVZ)



m_Q from

SVZ Sum Rules, Moments and Tadpoles

Some definitions:

$$R(s) \;=\; 12\pi\,\mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right)\,\Pi(q^2) \;\equiv\; i\int \mathrm{d}x\, e^{iqx}\langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) \;=\; Q_Q^2\,\frac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n\, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s}{\pi}\bar{C}_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}_n^{(3)} + \ldots$$

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2
[Chetyrkin, JK, Steinhauser, 1996]

up to high $n(\sim 30)$; VV, AA, PP, SS correlators
[Czakon et al., 2006], [Maierhöfer,Maier,Marquard, 2007]

Coefficients $\bar{C}_0$ and $\bar{C}_1$ in order α_s^3 (four-loop)
➪ reduction to master integrals through Laporta algorithm
[Chetyrkin,JK,Sturm], [Boughezal,Czakon,Schutzmeier] 2006

evaluation of master integrals numerically through difference equations (30 digits) or Padé method or analytically in terms of transcendentals
[Schröder+Vuorinen, Chetyrkin et al., Schröder+Steinhauser, Laporta,Broadhurst,Kniehl et al.]

$\bar{C}_2$ very recently [Maier,Maierhöfer,Marquard]

$\bar{C}_3$ would be desirable!

Using calculated moments

$$\mathcal{M}_n^{\mathrm{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_{\mathrm{c}}^2}\right)^n \bar{C}_n$$

and experiment

$$\mathcal{M}_n^{\mathrm{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint:

$$\mathcal{M}_n^{\mathrm{exp}} = \mathcal{M}_n^{\mathrm{th}}$$

⇨ m_{c}

Ingredients

experiment:

- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & Babar
- $\psi(3770)$ and $R(s)$ from BES
- $\alpha_s = 0.1187 \pm 0.0020$

theory:

- N^3LO for n=1 and n=2 (new)
- N^3LO - estimate for n =3,4
- include condensates

$$\delta \mathcal{M}_n^{\mathrm{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi} \bar{b}_n\right)$$

- estimate of non-perturbative terms (oscillations, based on Shifman)
- careful extrapolation of R_{uds}
- careful definition of R_c

Results (m_c)

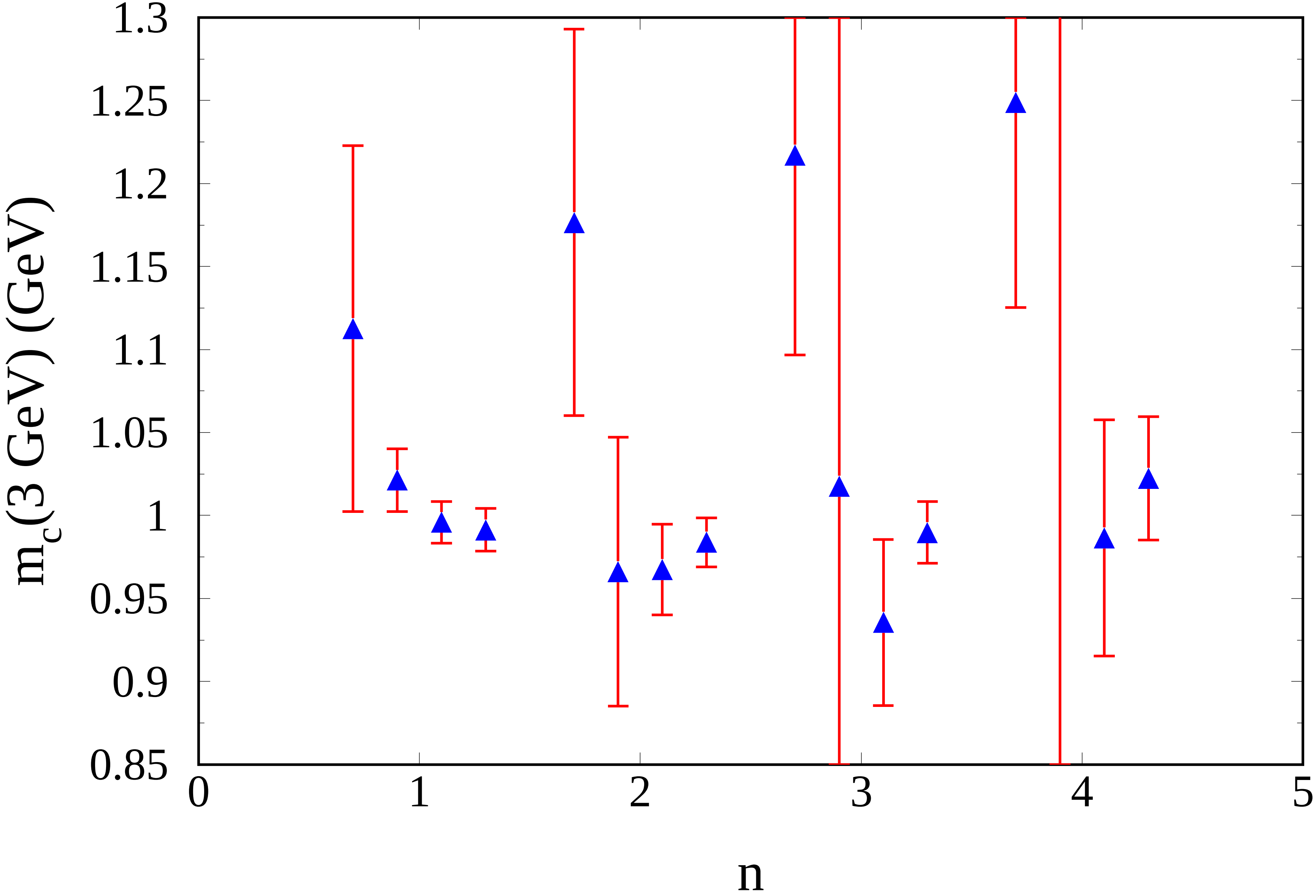
n	$m_c(3\ \text{GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.976	0.006	0.014	0.005	0.000	0.016	—	1.277
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$n = 1$:

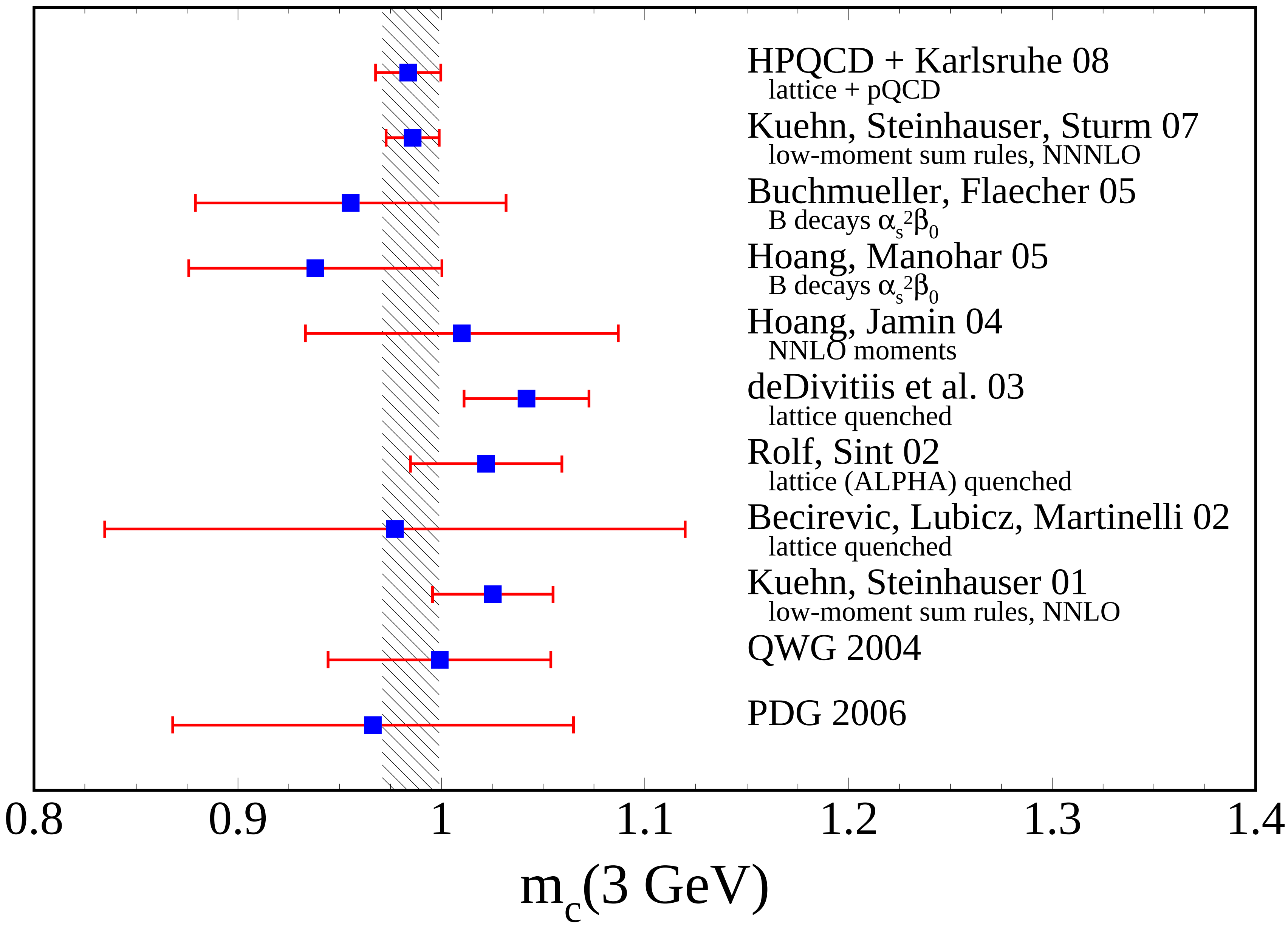
- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1286 \pm 13\,\text{MeV}$

$n = 2$:

- $m_c(3\,\text{GeV}) = \begin{cases} 979 \pm 25\,\text{MeV old, } C_2^{(3)}\text{estimated} \\ 976 \pm 16\,\text{MeV new, } C_2^{(3)}\text{calculated} \end{cases}$
- $m_c(m_c) = 1277 \pm 16\,\text{MeV}$



1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
n
m_c(3 GeV) (GeV)



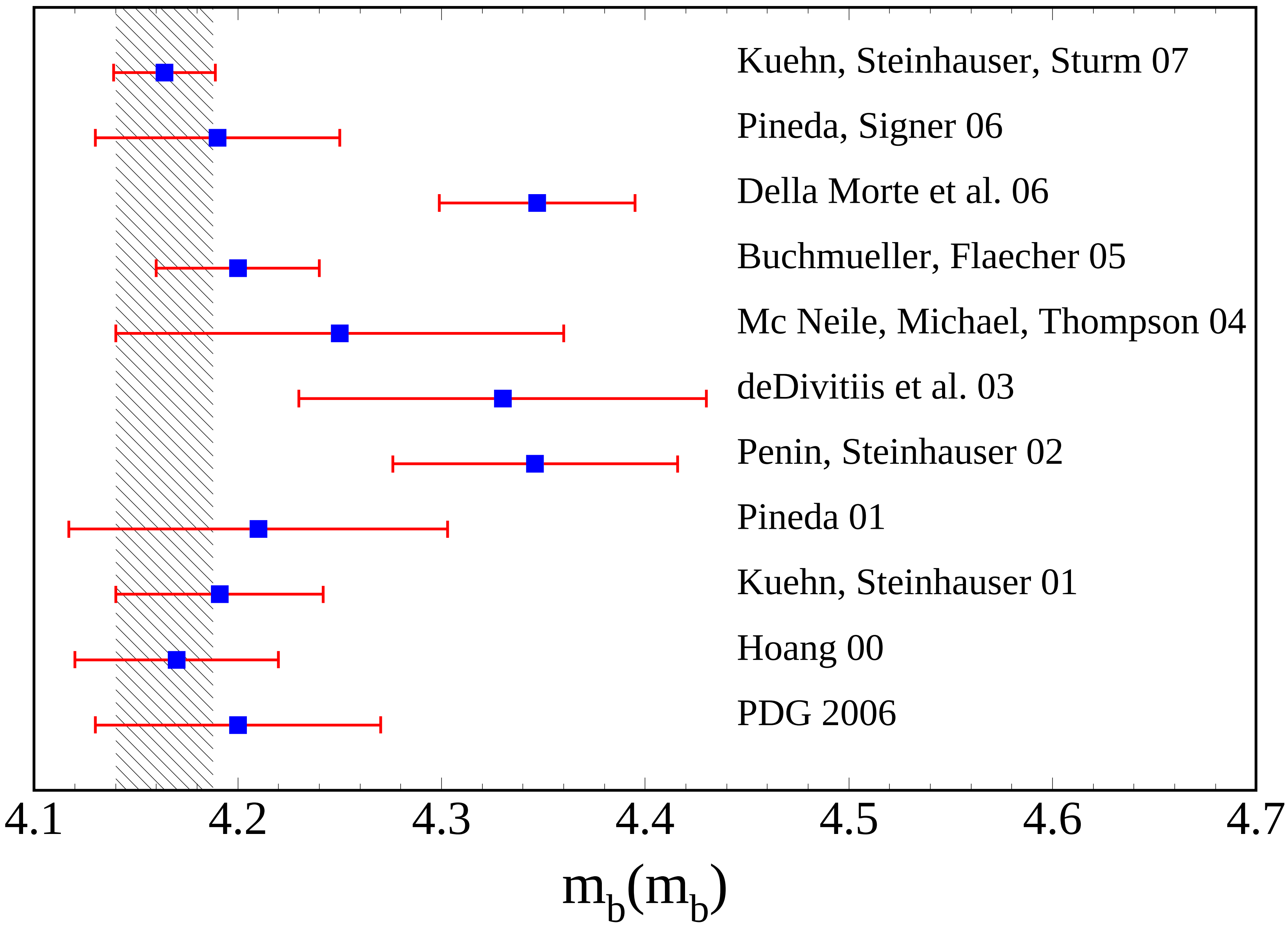
HPQCD + Karlsruhe 08
lattice + pQCD
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Hoang, Manohar 05
B decays $\alpha_s^2\beta_0$
Hoang, Jamin 04
NNLO moments
deDivitiis et al. 03
lattice quenched
Rolf, Sint 02
lattice (ALPHA) quenched
Becirevic, Lubicz, Martinelli 02
lattice quenched
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
QWG 2004
PDG 2006
0.8
0.9
1
1.1
1.2
1.3
1.4
m_c(3 GeV)

Results (m_b)

n	$m_b(10\ \text{GeV})$	exp	α_s	μ	total	$\delta\bar{C}_n^{30}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.607	0.014	0.012	0.003	0.019	—	4.162
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$n = 2$:

- $m_b(10\text{GeV}) = \begin{cases} 3609 \pm 25\,\text{MeV old, } C_2^{(3)}\text{estimated} \\ 3607 \pm 19\,\text{MeV new, } C_2^{(3)}\text{calculated} \end{cases}$
- $m_b(m_b) = 4162 \pm 19\,\text{MeV}$
- $m_b(m_t) = 2701 \pm 18 \pm 14\,\text{MeV}$
- $m_t/m_b = 59.8 \pm 1.3$


Kuehn, Steinhauser, Sturm 07
Pineda, Signer 06
Della Morte et al. 06
Buchmueller, Flaecher 05
Mc Neile, Michael, Thompson 04
deDivitiis et al. 03
Penin, Steinhauser 02
Pineda 01
Kuehn, Steinhauser 01
Hoang 00
PDG 2006
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$

recent analysis: lattice & pQCD

(HPQCD + Karlsruhe, arXiv:0805.2999[hep-lat])

⇒ C.Davies (Heavy Quarks, Talk 421)
lattice evaluation of pseudoscalar correlator

⇒ replace experimental moments by lattice simulation

pQCD for pseudoscalar correlator available:
"all" moments in $\mathcal{O}(\alpha_s^2)$
three lowest moments in $\mathcal{O}(\alpha_s^3)$.

lowest moment: dimensionless: $\sim \left(\bar{C}^{(0)} + \frac{\alpha_s}{\pi}\bar{C}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}^{(3)} + ...\right)$

⇒ $\alpha_s(3\text{GeV}) \Rightarrow \alpha_s(M_Z) = 0.113(4)$ arXiv:0805.2999[hep-lat]
0.1174(12) new, preliminary

higher moments: $\sim m_c^2 \times \left(1 + ...\frac{\alpha_s}{\pi}...\right)$

⇒ $m_c(3\text{GeV}) = 984(16)$ MeV arXiv:0805.2999[hep-lat]
986(10) MeV new, preliminary

to be compared with 986(13) MeV from e^+e^- !

$$m_{\mathrm{c}}(3\ \mathrm{GeV}) = 0.986(13)\ \mathrm{GeV}$$

$e^+e^- +$ pQCD

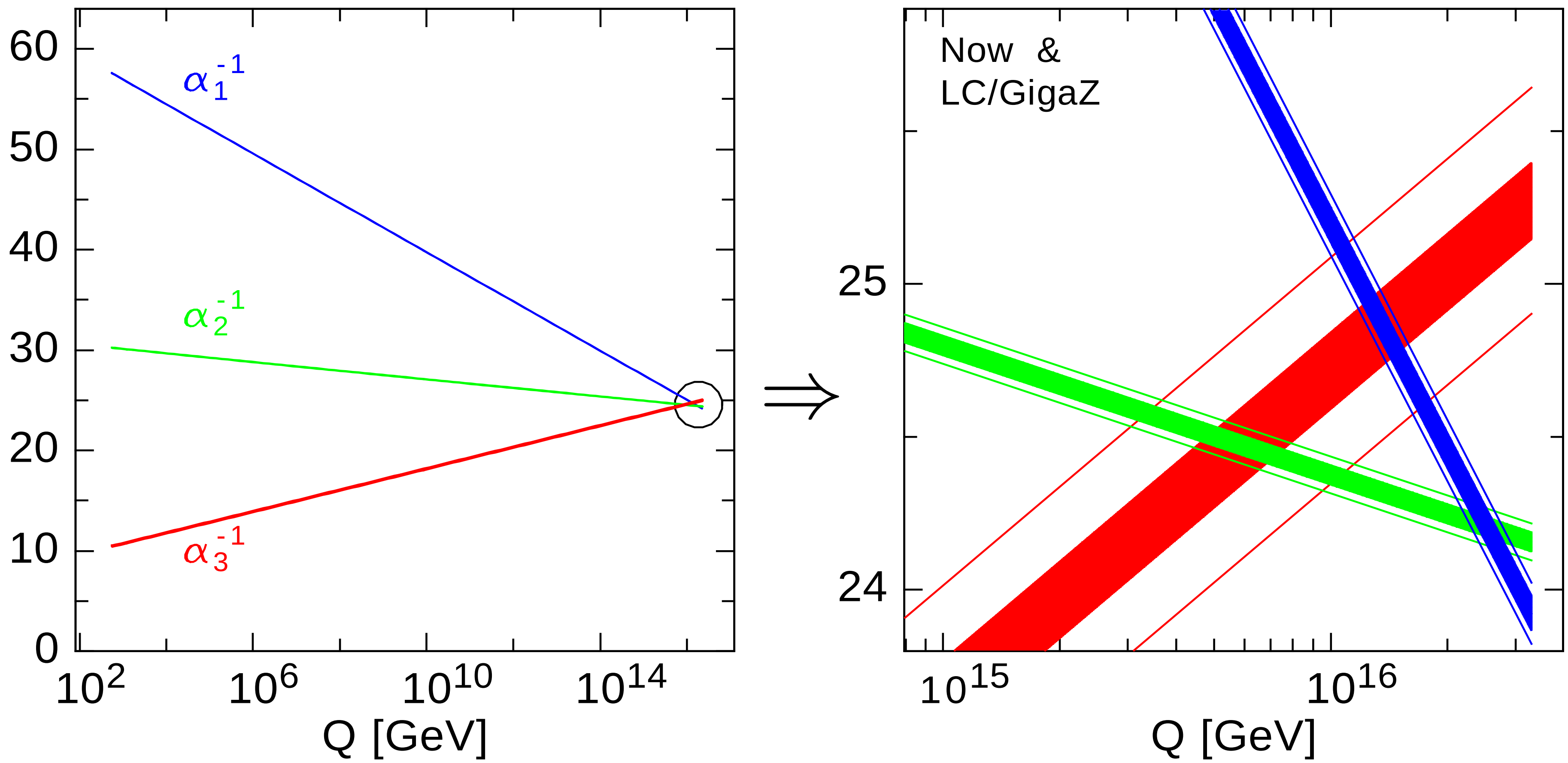
$$m_{\mathrm{c}}(3\ \mathrm{GeV}) = 0.986(10)\ \mathrm{GeV}$$

lattice + pQCD

$$m_{\mathrm{b}}(10\ \mathrm{GeV}) = 3.607(19)\ \mathrm{GeV}$$

$e^+e^- +$ pQCD $+ \bar{C}_2^{(3)}$

III. The Strong Coupling Constant in N^3LO



$$\alpha_s = 0.1183 \pm 0.0027 \quad \text{vs.} \quad \pm 0.0009$$

α_s from LEP

SM-fit: $\alpha_\mathrm{s} = 0.1185 \pm 0.0026$ (2007)

based on $\sim 10^7$ Z-events/experiment

GIGA-Z: 10^9 events

$\Rightarrow \delta\alpha_\mathrm{s} = 0.0009$

α_s based on

$$\Gamma_\mathrm{had} \;=\; \Gamma_0 \left(1 + \; a_s + 1.409\; a_s^2 - 12.767\; a_s^3\right)$$

$$+ \text{ small corrections}$$

$\left(a_s \equiv \frac{\alpha_\mathrm{s}}{\pi}\right)$

α_{s} from τ-decays

one of the most precise results for α_{s}

$$\frac{\Gamma(\tau \to h_{s=0}\nu)}{\Gamma(\tau \to l\overline{\nu}\nu)} = | V_{ud} |^2 S_{\mathsf{EW}} R_\tau = 3.471 \pm 0.011$$

$$R_\tau = 3\,(1 + \delta_P + \underbrace{\delta_{\mathsf{EW}}}_{\text{small}} + \underbrace{\delta_{\mathsf{NP}}}_{0.003 \pm 0.003})$$

$$\delta_P = 0.1998 \pm 0.043 \text{ (exp)}$$

- previous fixed order perturbation theory:

$$\delta_P = a_s + 5.202\, a_s^2 + 26.37\, a_s^3 + ?$$

- previous contour improved perturbation theory:

$$\delta_P = 1.364\, a_s + 2.54\, a_s^2 + 9.71\, a_s^3 + ?$$

previously:

estimates for α_s^4 (and α_s^5) terms only (FAC, PMS)

questions:

- are FAC/PMS supported by higher order calculations ?
- does the difference between fixed order (FOPT) and CIPT decrease upon inclusion of α_s^4 ?

aim: evaluate α_s^4

$\Rightarrow$ absorptive part of 5-loop correlators

Baikov,Chetyrkin,JK PRL 101(2008) 012002 consider $D_1(Q^2) \equiv -12\pi^2 Q^2 \frac{d}{dQ^2}\Pi_1 = \int_0^\infty ds \frac{Q^2}{(Q^2+s)^2} R(s)$

(Adler function, μ independent)

$$D_1(q^2) = 1 + a_s + a_s^2 (-0.1153\, n_f + 1.968)$$

$$+ a_s^3 (0.08621\, n_f^2 - 4.216\, n_f + 18.24)$$

$$+ a_s^4 (-0.010\, n_f^3 + 1.88\, n_f^2 - 34.4\, n_f + 135.8)$$

impact on α_{s} from Z-decays

$$R(s) = D(s) - \pi^2 \beta_0^2 \left\{ d_1/3 a_s^3 + \left(d_2 + \frac{5\beta_1}{6\beta_0} d_1 \right) a_s^4 \right\}$$

$$\Rightarrow \delta\alpha_{\mathrm{s}}(M_Z) = 0.0005$$

$$\alpha_{\mathrm{s}}(M_Z)^{\mathrm{NNNLO}} = 0.1190 \pm 0.0026$$

(Baikov,Chetyrkin,JK)

impact on α_s from τ-decays

uncertainty from higher orders dominant

$$\frac{\Gamma(\tau\to h_{s=0}\nu)}{\Gamma(\tau\to l\bar{\nu}\nu)} = |V_{ud}|^2 S_{EW}\, 3\,(1+ \delta_P + \underbrace{\delta_{EW}}_{\text{small}} + \underbrace{\delta_{NP}}_{0.003\pm 0.003})$$

$R_\tau = 3.471 \pm 0.011$

$\delta_P = 0.1998 \pm 0.043$ (exp)

	$\alpha_s^{FO}(m_\tau)$	$\alpha_s^{CI}(m_\tau)$
no α_s^4	$0.337 \pm 0.004 \pm 0.03$	$0.354 \pm 0.006 \pm 0.002$
$d_4 = 25$	$0.325 \pm 0.004 \pm 0.02$	$0.347 \pm 0.006 \pm 0.009$
$d_4 = 49.08$	$0.322 \pm 0.004 \pm 0.02$	$0.342 \pm 0.005 \pm 0.01$

scale $\mu^2/m_\tau^2 = 0.4 - 2$

use mean value between FOPT and CIPT

$$\alpha_s(m_\tau) = 0.332 \pm 0.005_{\text{exp}} \pm 0.015_{\text{theo}}$$

four-loop running (β_0, β_1, β_2, β_3)
four-loop matching at quark thresholds
($m_c(m_c) = 1.286(13)$ GeV, $m_b(m_b) = 4.164(25)$ GeV)

$$\alpha_s(M_Z) = 0.1202 \pm 0.006_{\text{exp}} \pm 0.018_{\text{theo}} \pm 0.0003_{\text{evol}}$$
$$= 0.1202 \pm 0.0019$$

consistent with α_s from Z

$\delta\alpha_s$ from τ dominated by theory.
$\delta\alpha_s$ from Z dominated by statistics.

significant difference between FO and CI

also in higher orders

(anticipated in Baikov,Chetyrkin,JK PRD67(2003)074026)

$\Rightarrow$ three recent studies of τ-decays,

using the new exact result for d_4.

■ Davier,Descotes-Genon,Hoecker,Malaescu,Zhang (arXiv 0803.0979[hep-ph])

starting point $\delta_P = 0.2066 \pm 0.0070$
(slight shift due to inclusion of recent BABAR data)

contour-improved analysis only
(claim: FO "exhibits problems of convergence")

partial resummation of higher orders;

μ-dependence artificially small
$\Rightarrow$small!! (probably underestimated) theory error

$\alpha_{\mathrm{s}}(m_\tau) = 0.344 \pm 0.005 \pm 0.007$

problems:

- specific assumptions about higher order behaviour of Adler function
- perturbative series is asymptotic anyhow, exclusion of FO not convincing
- probable underestimate of theory error (μ-dependence)
- dominance of π^2-terms doubtful

■ Beneke,Jamin (arXiv 0806.3156[hep-ph])

starting point $\delta_P = 0.2042 \pm 0.0050$
(slight differences in non-perturbative terms)

- careful analysis of differences between FO and CI
- model for higher order terms of Adler function (renormalon series,...)
- large cancelations between π^2-terms and Adler function terms

⇒ FO provides optimal description

$$\alpha_s(m_\tau) = \begin{cases} 0.320^{+0.012}_{-0.007} & \text{FO up to "}\alpha_s^5\text{"} \\ 0.3156 \pm 0.0059 & \text{FO+model for higher orders} \end{cases}$$

- Maltman,Yavin (arXiv 0807.0650[hep-ph])

  sum rules for spectral functions with
  weights to suppress higher dimensional operators

  - important contributions from $D = 10 - 16$?

    (comparison of vector and axial spectral function!)

  - use $w_N(y) = 1 - \frac{N}{N-1} y + \frac{1}{N-1} y^N$

    and contour improvement

$$\alpha_{\mathsf{S}}(m_\tau) = 0.3209 \pm 0.0046 \pm 0.0118$$

(shift relative to Davier et al. attributed to $D > 8$)

compilation of $\alpha_s(M_Z)$ from Z and τ

0.1190 ± 0.0026	BCK	(Z-decays)
0.1202 ± 0.0019	BCK	(τ-decays, FO& CI)
0.1212 ± 0.0011	Davier+...	(τ-decays, CI)
0.1180 ± 0.0008	BJ	(τ-decays, FO+...)
0.1187 ± 0.0016	MY	(τ-decays, CI, $D > 8$)

our evaluation: Z & $\tau \Rightarrow 0.1198 \pm 0.0015$ at NNNLO

comparison: α_s from lattice

$\alpha_s(M_Z) = 0.1170 \pm 0.0012$ (HPQCD & UKQCD, PRL95(2005) 052002)

update using finer lattices

$\alpha_s(M_Z) = 0.1183 \pm 0.0007$ (HPQCD, arXiv 0807.1687[hep-lat])

alternative perturb. analysis (Maltman et al.: 2008, arXiv 0807.2020[hep-lat])

$\alpha_s(M_Z) = 0.1192 \pm 0.0011$

to be compared with

$\alpha_s(M_Z) = 0.1174 \pm 0.0012$ (HPQCD & Karlsruhe 2008, preliminary)

Summary on α_s

- Adler function, $R(s)$, R_τ available to $\mathcal{O}(\alpha_s^4)$
- First and only N^{3}LO results
- α_s^4 terms move Z and τ closer together

combined (τ & Z): $\alpha_s(M_Z) = 0.1198 \pm 0.0015$

consistent with lattice + pQCD:

$$\alpha_s(M_Z) = 0.1174 \pm 0.0012$$