Quark Masses and the Strong Coupling Constant

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SFB TR9 Computational Theoretical Particle Physics

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I.

Quark Masses

based on Low Moments

WHY precise masses?

B-decays:

$$\Gamma(B \to X_u l\bar{\nu}) \sim G_F^2\, m_b^5\, |V_{ub}|^2$$

$$\Gamma(B \to X_c l\bar{\nu}) \sim G_F^2\, m_b^5\, f(m_c^2/m_b^2)\, |V_{cb}|^2$$

moments of $\frac{dN}{dE_l}$, $\frac{dN}{dm(l\bar{\nu})}$,

$B \to X_s\gamma$: moments of m_{had}^2

Υ-spectroscopy:

$$m\left(\Upsilon(1\mathrm{s})\right) = 2M_{\mathrm{b}} - \left(\tfrac{4}{3}\alpha_{\mathrm{s}}\right)^2 \frac{M_{\mathrm{b}}}{4} + \ldots$$

sum rules:

$$\int \frac{\mathrm{d}s}{s^{n+1}}\, R_{\mathrm{Q}}(s) \sim \frac{1}{m_{\mathrm{Q}}^{2n}}$$

Higgs decay (ILC)

$$\Gamma(H \to b\bar{b}) = \frac{G_F\; M_H}{4\sqrt{2}\pi}\, m_b^2(M_H)\; \tilde{R}$$

$$\tilde{R} \;=\; 1 + 5.6667 a_s + 29.147 {a_s}^2 + 41.758 {a_s}^3 - 825.7 {a_s}^4$$

rapidly increasing coefficients! $\left(a_s \equiv \frac{\alpha_s}{\pi}\right)$

${a_s}^4$-term = 5-loop calculation (Baikov,...)

Yukawa Unification

$$\lambda_\tau = \lambda_\mathrm{b} \quad \text{or} \quad \lambda_\tau = \lambda_\mathrm{b} = \lambda_\mathrm{t}$$

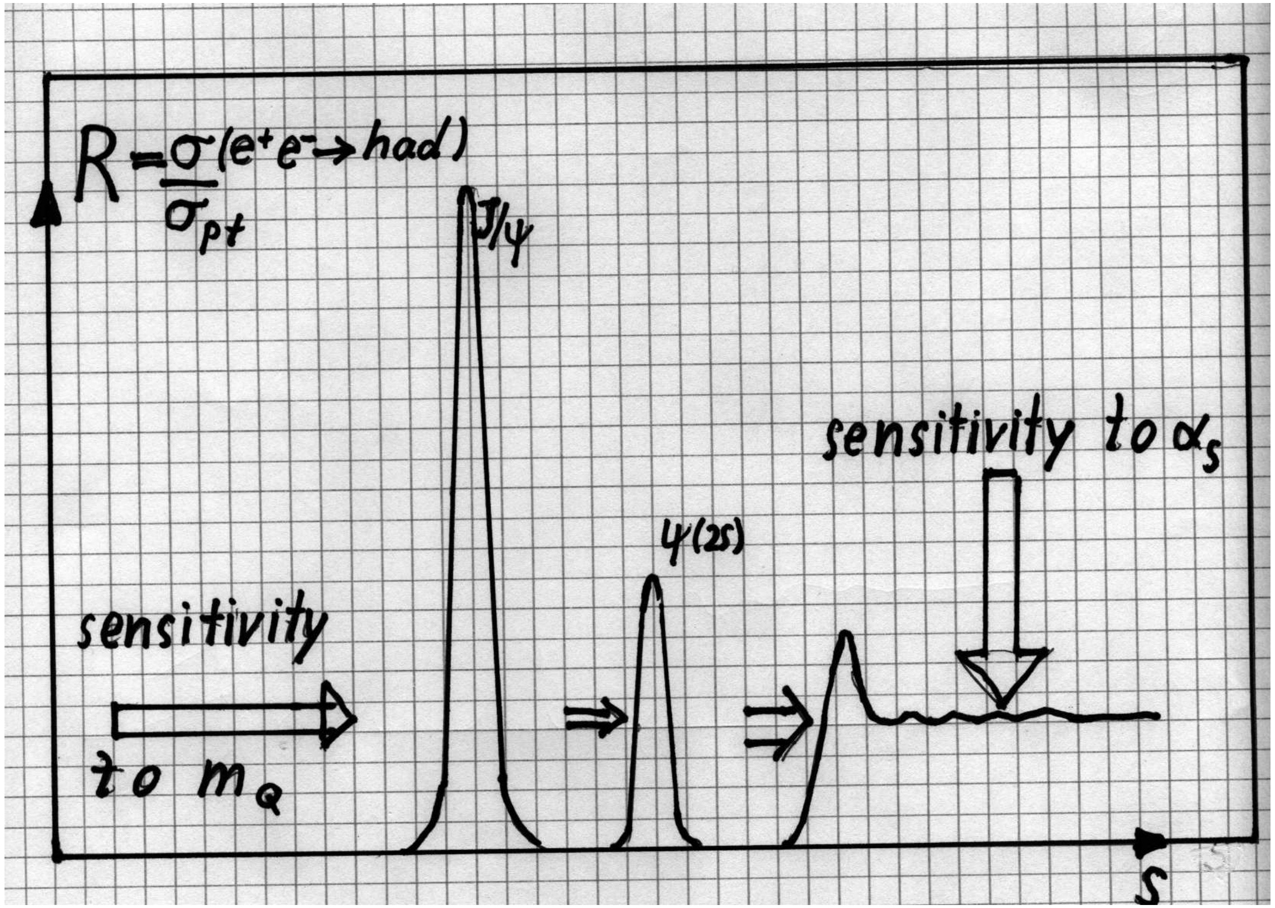
identical coupling to Higgs boson(s) at GUT scale

top-bottom $\rightarrow m_\mathrm{t} \big/ m_\mathrm{b} \sim$ ratio of vacuum expectation values

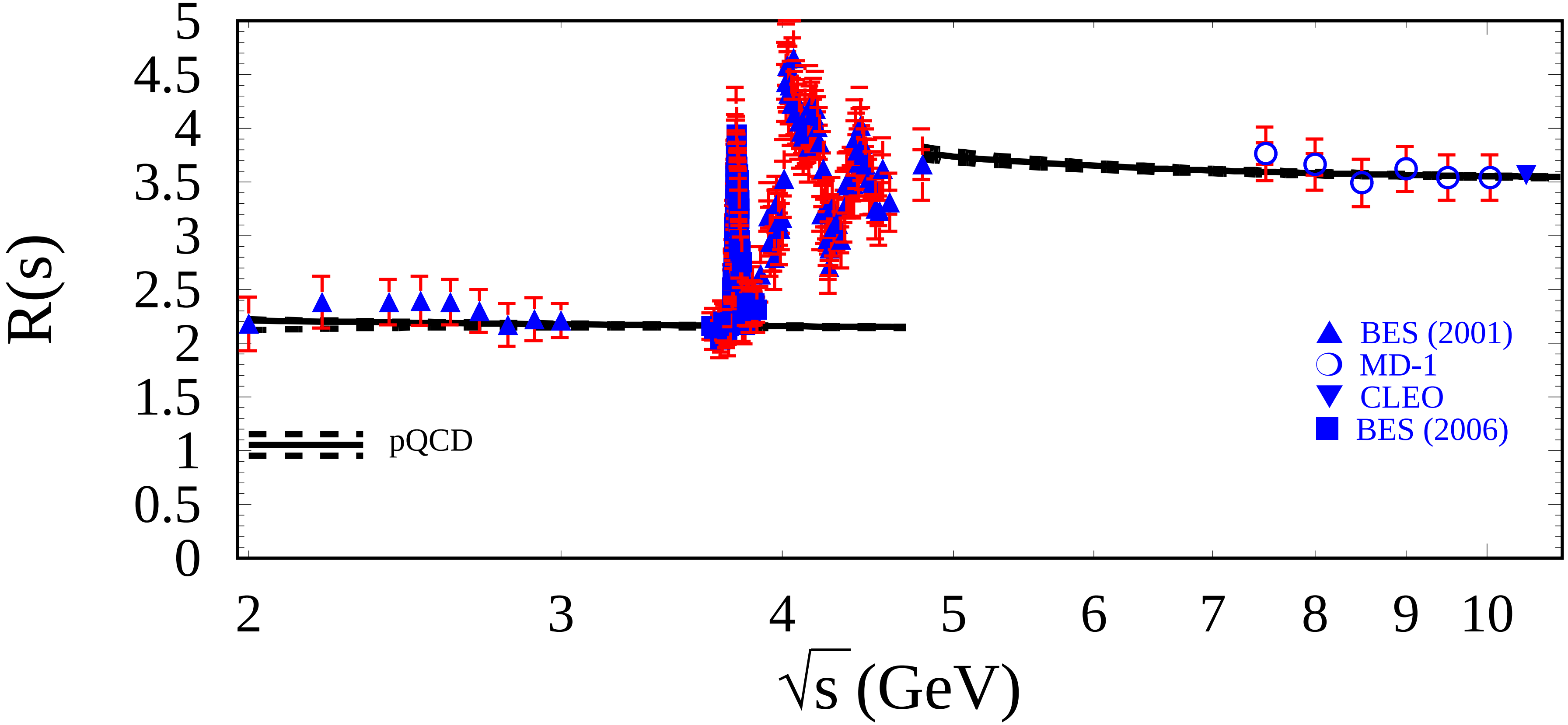
request $\frac{\delta m_\mathrm{b}}{m_\mathrm{b}} \sim \frac{\delta m_\mathrm{t}}{m_\mathrm{t}}$

$\delta m_\mathrm{t} \approx 1$ GeV $\Rightarrow \delta m_\mathrm{b} \approx 25$ MeV

Main Idea (SVZ)



Data



pQCD and data agree well in the regions
2 – 3.73 GeV and 5 – 10.52 GeV

m_Q from

SVZ Sum Rules, Moments and Tadpoles

Some definitions:

$$R(s) \;=\; 12\pi\,\mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right)\,\Pi(q^2) \;\equiv\; i\int \mathrm{d}x\, e^{iqx}\langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) \;=\; Q_Q^2\,\frac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n\, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2
[Chetyrkin, JK, Steinhauser, 1996]

up to high $n(\sim 30)$; VV, AA, PP, SS correlators
[Czakon et al., 2006], [Maierhöfer, Maier, Marquard, 2007]

➪ reduction to master integrals through Laporta algorithm
[Chetyrkin, JK, Sturm]; confirmed by [Boughezal, Czakon, Schutzmeier]

evaluation of master integrals numerically through difference equations

(30 digits) or Padé method or analytically in terms of transcendentals
[Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser,
Laporta, Broadhurst, Kniehl et al.]

$\bar{C}_2$ would be desirable!

Define the moments

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\bigg|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

$$\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint:

$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$$

➪ m_c

Ingredients

experiment:

- $\alpha_s = 0.1187 \pm 0.0020$
- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & Babar
- $\psi(3770)$ from BES

theory:

- N^3LO for n=1
- N^3LO - estimate for n =2,3,4
- include condensates

$$\delta\mathcal{M}_n^{\rm np} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi}\bar{b}_n\right)$$

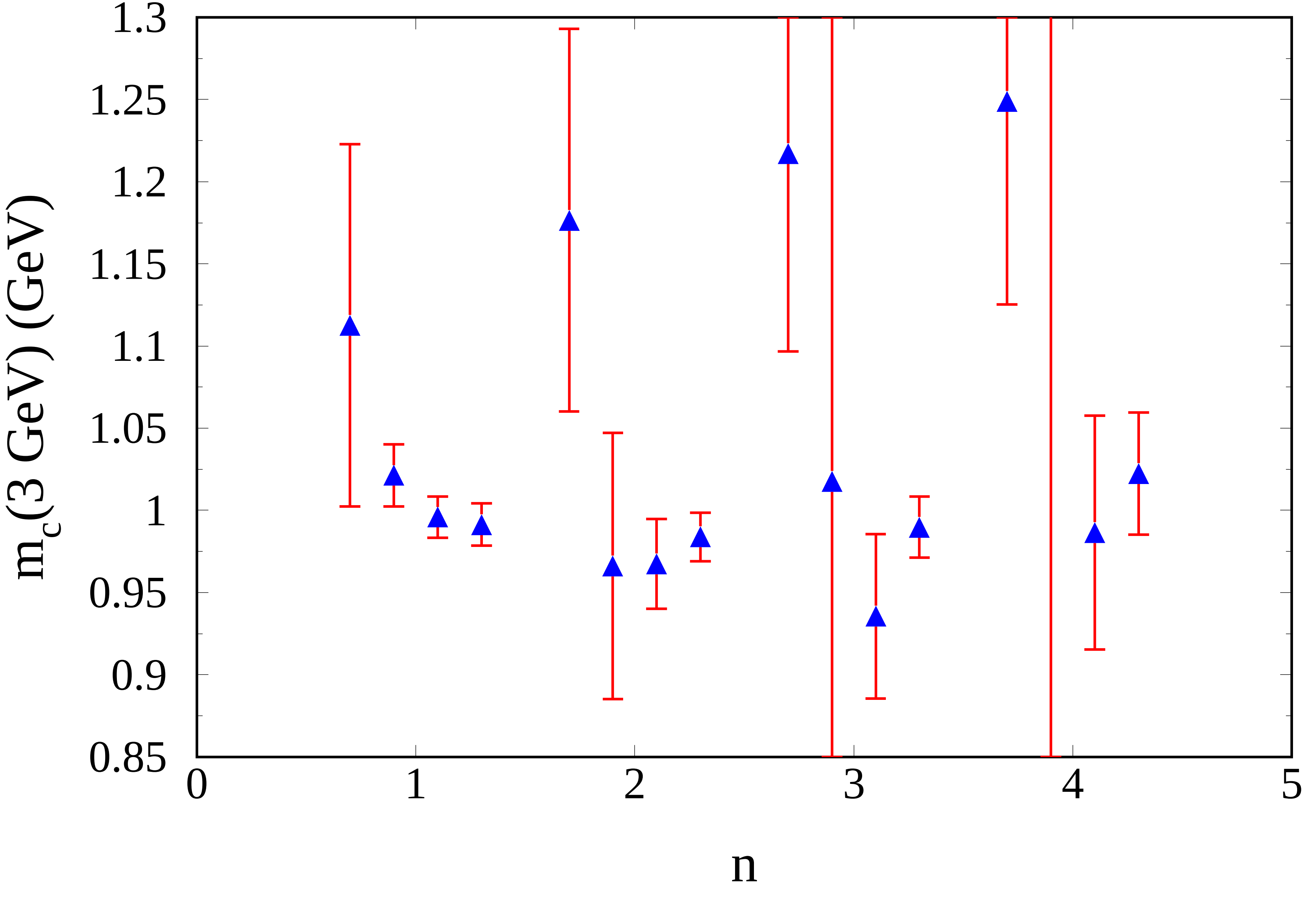
- estimate of non-perturbative terms (oscillations, based on Shifman)
- careful extrapolation of R_{uds}
- careful definition of R_c

Results (m_c)

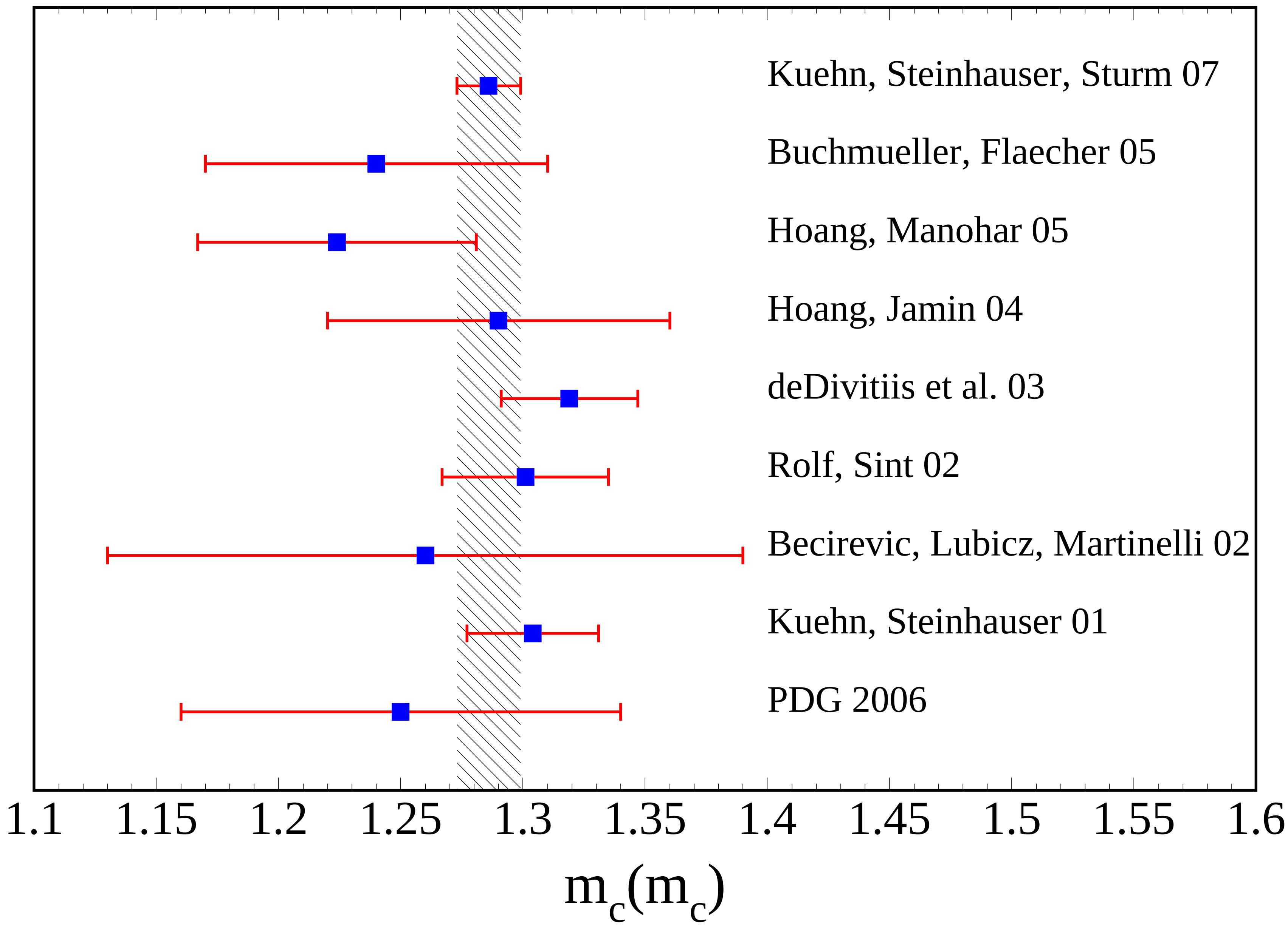
n	m_c(3 GeV)	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$n = 1$:

- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1286 \pm 13\,\text{MeV}$



m_c(3 GeV) (GeV)
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
n


Kuehn, Steinhauser, Sturm 07
Buchmueller, Flaecher 05
Hoang, Manohar 05
Hoang, Jamin 04
deDivitiis et al. 03
Rolf, Sint 02
Becirevic, Lubicz, Martinelli 02
Kuehn, Steinhauser 01
PDG 2006
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
1.55
1.6
$m_c(m_c)$

Results (m_b)

n	$m_b(10\ \text{GeV})$	exp	α_s	μ	total	$\delta\bar{C}_n^{30}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$n = 2$:

- $m_b(m_b) = 4164 \pm 25\,\text{MeV}$
- $m_b(10\text{GeV}) = 3609 \pm 25\,\text{MeV}$
- $m_b(m_t) = 2703 \pm 18 \pm 19\,\text{MeV}$
- $m_t/m_b = 59.8 \pm 1.3$

$$m_c(3\ \text{GeV}) = 0.986(13)\ \text{GeV}$$
$$m_c(m_c) = 1.286(13)\ \text{GeV}$$

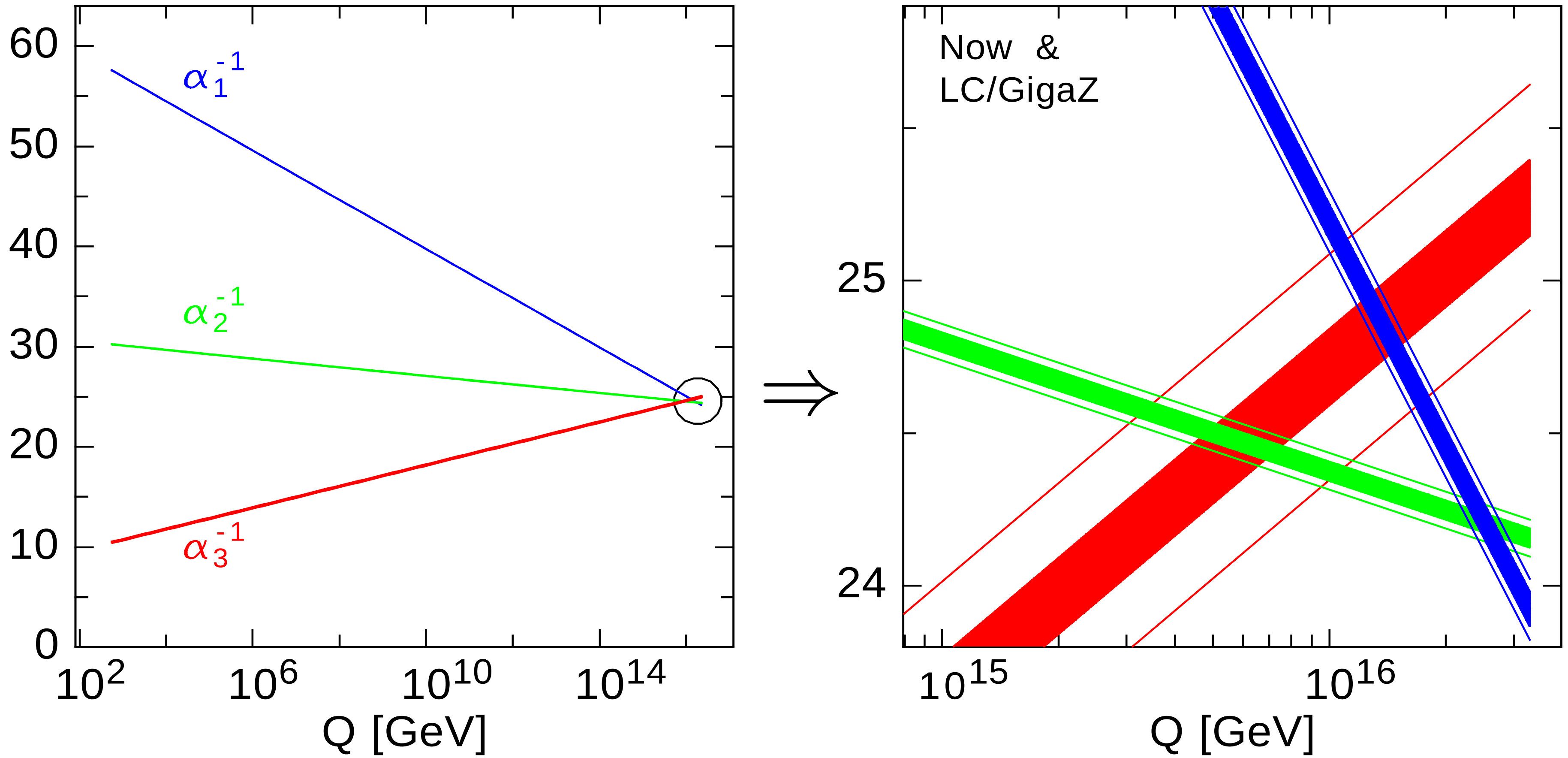
$$m_b(10\ \text{GeV}) = 3.609(25)\ \text{GeV}$$
$$m_b(m_b) = 4.164(25)\ \text{GeV}$$

II.

The Strong Coupling Constant

in N^3LO

from Zerwas



$$\alpha_s = 0.1183 \pm 0.0027 \quad \text{vs.} \quad \pm 0.0009$$

α_s from LEP

$R_\ell = \Gamma_h/\Gamma_\ell = 20.767 \pm 0.025$ (1.2 ‰ !)

(~ 10^6 leptonic events)

$$\alpha_s = 0.1226 \pm 0.0038 \;{}^{+0.0028}_{-0.0}\left(M_H = \;{}^{900}_{100}\mathrm{GeV}\right)$$

$\sigma_\ell \sim \frac{\Gamma_\ell^2}{\Gamma_{tot}^2} = 2.003 \pm 0.0027\mathrm{pb}$

(luminosity)

$$\alpha_s = 0.1183 \pm 0.0030 \;{}^{+0.0022}_{-0.0}\left(M_H = \;{}^{900}_{100}GeV\right)$$

SM-fit: $\alpha_s = 0.1185 \pm 0.0026$

based on $\sim 10^7$ Z-events/experiment

GIGA-Z: 10^9 events

$\Rightarrow \delta\alpha_s = 0.0009$ M. Winter

α_s based on

$$\Gamma_{\text{had}} = \Gamma_0 \left(1 + a_s + 1.409\, a_s^2 - 12.767\, a_s^3\right)$$

$$+ \text{ small corrections}$$

$$\left(a_s \equiv \frac{\alpha_s}{\pi}\right)$$

α_s from τ-decays

one of the most precise results for α_s

$$\frac{\Gamma(\tau\to h_{s=0}\nu)}{\Gamma(\tau\to l\bar{\nu}\nu)} = |V_{ud}|^2 S_{\mathrm{EW}} R_\tau = 3.471 \pm 0.011$$

$$R_\tau = 3\,(1 + \delta_P + \underbrace{\delta_{\mathrm{EW}}}_{\text{small}} + \underbrace{\delta_{\mathrm{NP}}}_{0.003\pm 0.003})$$

$$\delta_P = 0.1998 \pm 0.043 \text{ (exp)}$$

- previous fixed order perturbation theory:

$$\delta_P = a_s + 5.202\, a_s^2 + 26.37\, a_s^3 + ?$$

- previous contour improved perturbation theory:

$$\delta_P = 1.364\, a_s + 2.54\, a_s^2 + 9.71\, a_s^3 + ?$$

previously:

estimates for α_s^4 (and α_s^5) terms only (FAC, PMS)

questions:

- are FAC/PMS supported by higher order calculations

- does the difference between fixed order (FOPT) and CIPT decrease upon inclusion of α_s^4 ?

aim: evaluate α_s^4

$\Rightarrow$ absorptive part of 5-loop correlators

The long march towards α_s^4

consider $D_1(Q^2) \equiv -12\pi^2 Q^2 \frac{d}{dQ^2}\Pi_1 = \int_0^\infty ds \frac{Q^2}{(Q^2+s)^2} R(s)$

(Adler function, μ independent)

$$D_1(q^2) = 1 + a_s + a_s^2\,(-0.1153\,n_f + 1.968)$$

$$+ a_s^3\,(0.08621\,n_f^2 - 4.216\,n_f + 18.24)$$

$$+ a_s^4\,(-0.010\,n_f^3 + 1.88\,n_f^2 - 34.4\,n_f + 135.8)$$

relation to FAC/PMS

n_f	$d_4^{\text{FAC/PMS}}$	d_4^{exact}	$r_4^{\text{FAC/PMS}}$	r_4^{exact}
3	27 ± 16	49.08	-129 ± 16	-106.88
4	8 ± 28	27.39	112 ± 30	-92.90
5	-8 ± 44	9.21	97 ± 44	-79.98

impact on α_s from Z-decays

$$R(s) = D(s) - \pi^2\beta_0^2 \left\{ d_1/3 a_s^3 + \left(d_2 + \frac{5\beta_1}{6\beta_0} d_1 \right) a_s^4 \right\}$$

$$\Rightarrow \delta\alpha_s(M_Z) = 0.0005$$

$$\alpha_s(M_Z)^{\text{NNNLO}} = 0.1190 \pm 0.0026$$

impact on α_s from τ-decays

$$\frac{\Gamma(\tau\to h_{s=0}\nu)}{\Gamma(\tau\to l\overline{\nu}\nu)} = |V_{ud}|^2 S_{\text{EW}}\, 3\,(1+ \delta_P + \underbrace{\delta_{\text{EW}}}_{\text{small}} + \underbrace{\delta_{\text{NP}}}_{0.003\pm0.003})$$

$R_\tau = 3.471 \pm 0.011$

(Davier, Höcker, Zhang; ALEPH, OPAL, CLEO,. . .)

$\delta_P = 0.1998 \pm 0.043$ (exp) scale $\mu^2/M_\tau^2 = 0.4 - 2$

	$\alpha_s^{FO}(M_\tau)$	$\alpha_s^{CI}(M_\tau)$
no α_s^4	$0.337 \pm 0.004 \pm 0.03$	$0.354 \pm 0.006 \pm 0.002$
$d_4 = 25$	$0.325 \pm 0.004 \pm 0.02$	$0.347 \pm 0.006 \pm 0.009$
$d_4 = 49.08$	$0.322 \pm 0.004 \pm 0.02$	$0.342 \pm 0.005 \pm 0.01$

use mean value between FOPT and CIPT

$$\alpha_s(M_\tau) = 0.332 \pm 0.005_{\text{exp}} \pm 0.015_{\text{theo}}$$

four-loop running (β_0, β_1, β_2, β_3)
four-loop matching at quark thresholds
($m_c(m_c) = 1.286(13)$ GeV, $m_b(m_b) = 4.164(25)$ GeV)

$$\alpha_s(M_Z) = 0.1202 \pm 0.006_{\text{exp}} \pm 0.018_{\text{theo}} \pm 0.0003_{\text{evol}}$$

$$= 0.1202 \pm 0.0019$$

consistent with α_s from Z

$\delta\alpha_s$ from τ dominated by theory.
$\delta\alpha_s$ from Z dominated by statistics.

Summary

- Adler function, $R(s)$, R_τ available to $\mathcal{O}(\alpha_s^4)$

- First and only N^{3}LO results

$$\alpha_s(M_z) = \begin{cases} 0.1190 \pm 0.0026 & \text{from } Z \\ 0.1202 \pm 0.0019 & \text{from } \tau \end{cases}$$

- α_s^4 terms move Z and τ closer together

combined

$$\alpha_s(M_Z) = 0.1198 \pm 0.0015$$