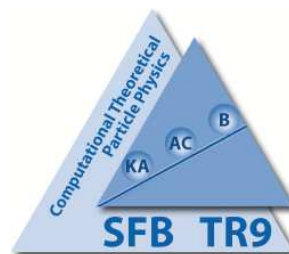


# ELECTROWEAK CORRECTIONS TO GAUGE BOSON AND HEAVY QUARK PRODUCTION AT HADRON COLLIDERS

J.H. Kühn



# Outline

## I. Introduction

## II. Z- and Photon Production

Phys. Lett. B609(2005) 277  
Nucl. Phys. B727(2005) 368  
JHEP 0603:059,2006

J.H.K., A.Kulesza, S.Pozzorini, M.Schulze

## III. Heavy Quark Production

Eur. Phys. J., C45, (2006) 139,  
+ work in preparation

J.H.K., A.Scharf, P.Uwer

## IV. Conclusions

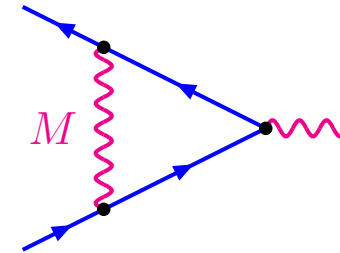
# I. Introduction

"Typical" size of electroweak corrections:  $\frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2}$

**new aspects at LHC:**  $\sqrt{\hat{s}} \approx 1\text{-}2\text{TeV} \gg M_{W,Z}^2$

strong enhancement of negative corrections

one-loop example: massive U(1)



$$\Rightarrow \text{Born} * \left[ 1 + \frac{\alpha}{4\pi} \left( -\ln^2 \frac{s}{M^2} + 3 \ln \frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3} \right) \right]$$

$\frac{s}{M^2}$	$-\ln^2 \frac{s}{M^2}$	$+3 \ln \frac{s}{M^2}$	$-\frac{7}{2} + \frac{\pi^2}{3}$	$\Sigma$	$* 4 \frac{\alpha_w}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	+15.15	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	+19.31	-0.21	-22.3	-27%

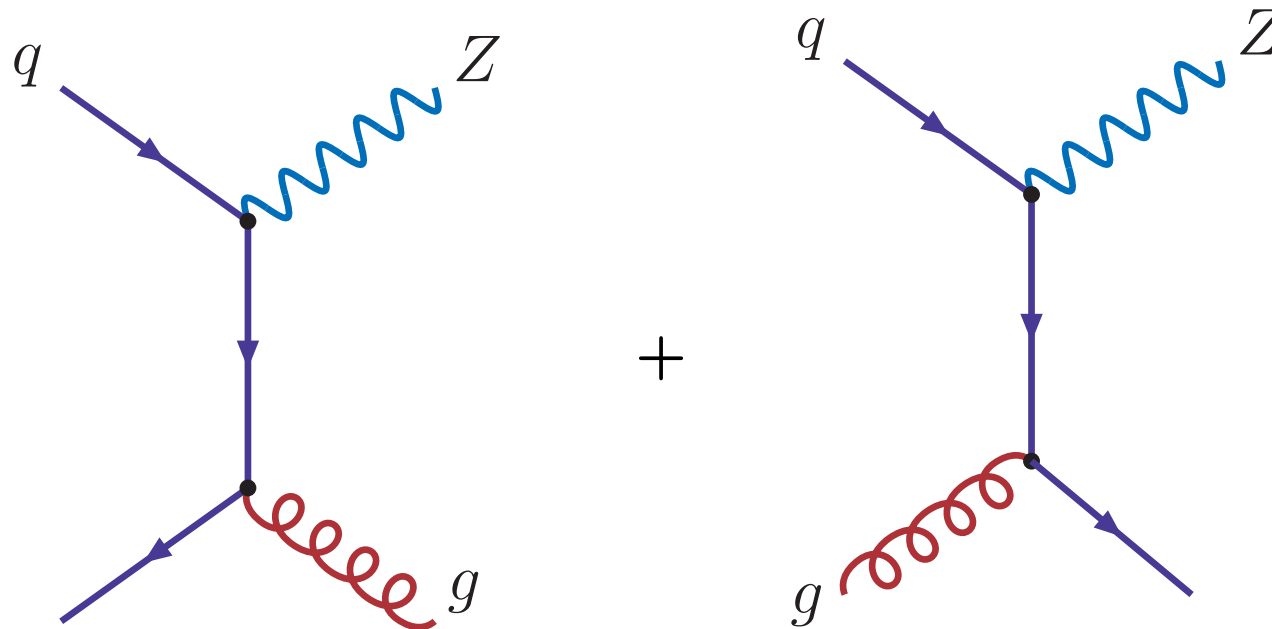
(four-fermion cross section  $\Rightarrow$  factor 4)

- leading  $\log^2$  multiplied by  $(\text{charge})^2 = I(I + 1) = \begin{cases} 3/4 & I = 1/2 \\ 2 & I = 1 \end{cases}$
- important subleading logarithms (NLL+...)
  - (  $\Rightarrow$  Penin:  $f\bar{f} \rightarrow f'\bar{f}'$  )
- two-loop terms may be relevant
- interplay between electroweak and QCD corrections
- important differences between fermions and electroweak gauge bosons

## II. Z and Photon Production

J.H.K., Kulesza, Pozzorini, Schulze

Large rate for Z-boson and photon production at LHC at large  $p_T$  (1-2 TeV)  
Large electroweak corrections ( $\hat{s} \gg M_{W,Z}^2$ )



## one-loop corrections:

Result decomposed into "abelian" (A) and "non-abelian" (N) parts

$H_1^{A,N}$  plus counterterms  $\delta C^{A,N}$  in closed analytical form:

kinematical functions of  $(\hat{s}, \hat{t}, \hat{u})$  and 14 combinations of

$$1 \times A_0, \quad 5 \times B_0, \quad 5 \times C_0, \quad 3 \times D_0$$

## High energy limit

consider  $q\bar{q} \rightarrow Zg$

**NLL**  $\hat{=}$  double + single logarithmic terms

$$H_1^A(M_V^2) \stackrel{\text{NLL}}{=} - \left[ \log^2 \left( \frac{|\hat{s}|}{M_W^2} \right) - 3 \log \left( \frac{|\hat{s}|}{M_W^2} \right) \right] H_0,$$
$$H_1^N(M_W^2) \stackrel{\text{NLL}}{=} - \left[ \log^2 \left( \frac{|\hat{t}|}{M_W^2} \right) + \log^2 \left( \frac{|\hat{u}|}{M_W^2} \right) - \log^2 \left( \frac{|\hat{s}|}{M_W^2} \right) \right] H_0$$

$$\delta C_{q\lambda}^A \stackrel{\text{NLL}}{=} \delta C_{q\lambda}^N \stackrel{\text{NLL}}{=} 0$$

(remaining subleading terms  $\leq 2.5\%$ )

**NNLL**: includes non-enhanced terms (angular dependent)

$$H_1^{A/N}(M_V^2) \stackrel{\text{NNLL}}{=} \text{Re} \left[ g_0^{A/N}(M_V^2) \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} + g_1^{A/N}(M_V^2) \frac{\hat{t}^2 - \hat{u}^2}{\hat{t}\hat{u}} + g_2^{A/N}(M_V^2) \right]$$

$$g_0^N(M_W^2) = 2\Delta_{UV}^- + \log^2 \left( \frac{-\hat{s}}{M_W^2} \right) - \log^2 \left( \frac{-\hat{t}}{M_W^2} \right) - \log^2 \left( \frac{-\hat{u}}{M_W^2} \right) + \log^2 \left( \frac{\hat{t}}{\hat{u}} \right) \\ - \frac{3}{2} \left[ \log^2 \left( \frac{\hat{t}}{\hat{s}} \right) + \log^2 \left( \frac{\hat{u}}{\hat{s}} \right) \right] - \frac{20\pi^2}{9} - \frac{2\pi}{\sqrt{3}} + 4,$$

$$g_1^N(M_W^2) = \frac{1}{2} \left[ \log^2 \left( \frac{\hat{u}}{\hat{s}} \right) - \log^2 \left( \frac{\hat{t}}{\hat{s}} \right) \right],$$

$$g_2^N(M_W^2) = -2 \left[ \log^2 \left( \frac{\hat{t}}{\hat{s}} \right) + \log^2 \left( \frac{\hat{u}}{\hat{s}} \right) + \log \left( \frac{\hat{t}}{\hat{s}} \right) + \log \left( \frac{\hat{u}}{\hat{s}} \right) \right] - 4\pi^2$$

$$g_0^A(M_V^2) = -\log^2 \left( \frac{-\hat{s}}{M_V^2} \right) + 3 \log \left( \frac{-\hat{s}}{M_V^2} \right) + \frac{3}{2} \left[ \log^2 \left( \frac{\hat{t}}{\hat{s}} \right) + \log^2 \left( \frac{\hat{u}}{\hat{s}} \right) \right. \\ \left. + \log \left( \frac{\hat{t}}{\hat{s}} \right) + \log \left( \frac{\hat{u}}{\hat{s}} \right) \right] + \frac{7\pi^2}{3} - \frac{5}{2},$$

$$g_1^A(M_V^2) = -g_1^N(M_W^2) + \frac{3}{2} \left[ \log \left( \frac{\hat{u}}{\hat{s}} \right) - \log \left( \frac{\hat{t}}{\hat{s}} \right) \right],$$

$$g_2^A(M_V^2) = -g_2^N(M_W^2)$$

+ simple approximations for finite parts of counter terms



## size of the correction:

$$\sqrt{\hat{s}} = 200 \text{ GeV} : \quad \frac{\delta\sigma}{\sigma} \leq 0.3\%$$

$$\sqrt{\hat{s}} = 4000\text{GeV} : \quad \frac{\delta\sigma}{\sigma} \approx 20 - 30\%$$

Result consistent with general considerations

one-loop:

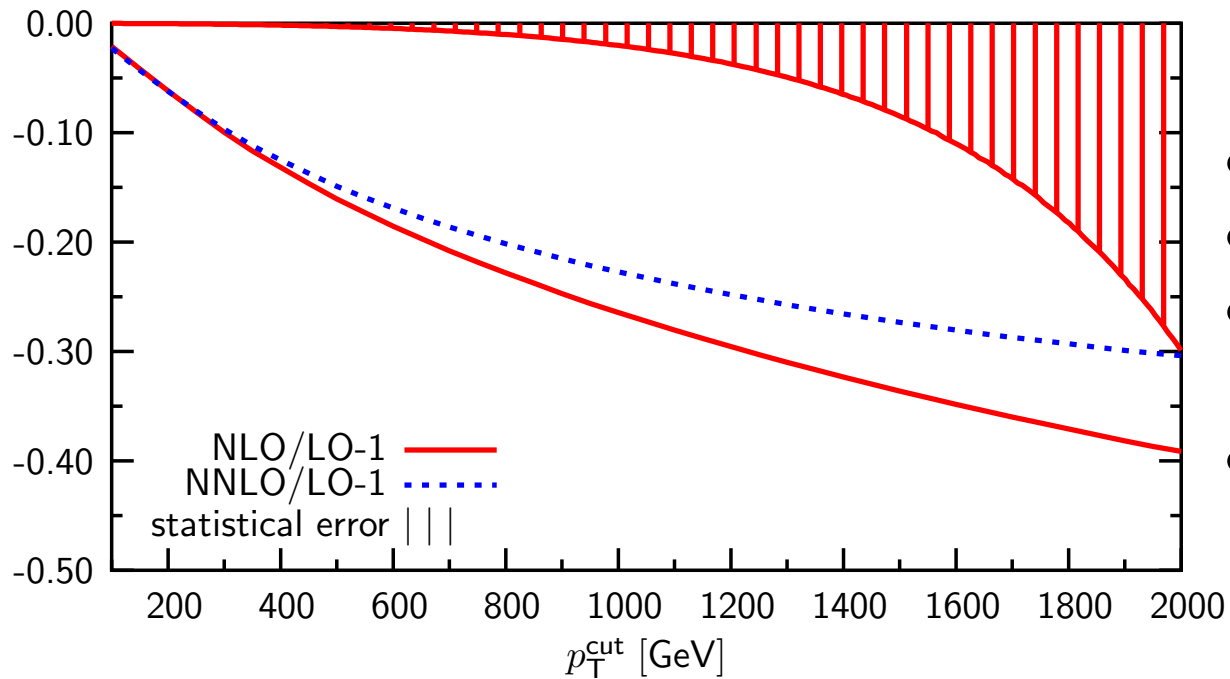
$$A^{(1)} = - \sum_{\lambda=L,R} I_{q\lambda}^Z \left[ I_{q\lambda}^Z C_{q\lambda}^{\text{ew}} \left( L_{\hat{s}}^2 - 3L_{\hat{s}} \right) + \frac{c_W}{s_W^3} T_{q\lambda}^3 \left( L_{\hat{t}}^2 + L_{\hat{u}}^2 - L_{\hat{s}}^2 \right) \right]$$

two-loop (NLL):

$$A^{(2)} = \sum_{\lambda=L,R} \left\{ \frac{1}{2} \left( I_{q\lambda}^Z C_{q\lambda}^{\text{ew}} + \frac{c_W}{s_W^3} T_{q\lambda}^3 \right) \left[ I_{q\lambda}^Z C_{q\lambda}^{\text{ew}} \left( L_{\hat{s}}^4 - 6L_{\hat{s}}^3 \right) + \frac{c_W}{s_W^3} T_{q\lambda}^3 \left( L_{\hat{t}}^4 + L_{\hat{u}}^4 - L_{\hat{s}}^4 \right) \right] - \frac{T_{q\lambda}^3 Y_{q\lambda}}{8s_W^4} \left( L_{\hat{t}}^4 + L_{\hat{u}}^4 - L_{\hat{s}}^4 \right) + \frac{1}{6} I_{q\lambda}^Z \left[ I_{q\lambda}^Z \left( \frac{b_1}{c_W^2} \left( \frac{Y_{q\lambda}}{2} \right)^2 + \frac{b_2}{s_W^2} C_{q\lambda} \right) + \frac{c_W}{s_W^3} T_{q\lambda}^3 b_2 \right] L_{\hat{s}}^3 \right\}$$

with  $L_{\hat{r}}^n = \log^n \left( \frac{|\hat{r}|}{M_W^2} \right)$ ,  $b_1 = -41/(6c_W^2)$  and  $b_2 = 19/(6s_W^2)$

# Complete **one loop** calculation NLL approximation at **two loops**



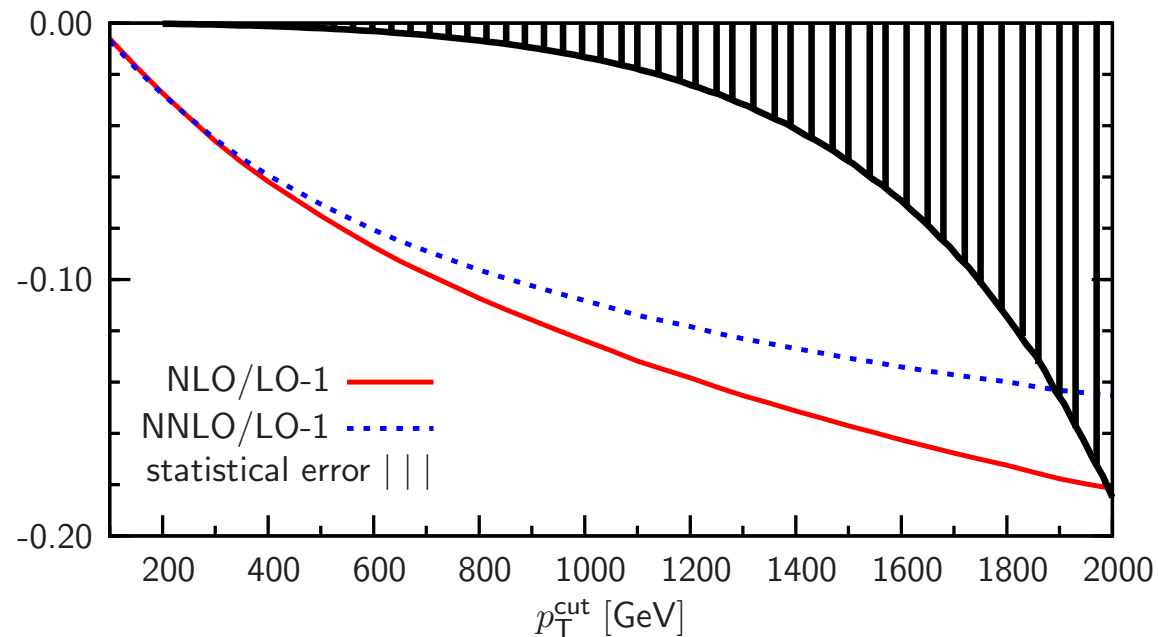
- **one-loop**  $\sim 30\%$  at  $p_T \sim 1\text{ TeV}$
- **two-loop** relevant above 1 TeV
- important angular-dependent logarithmic terms
- experiment:  $p_T$  up to 2 TeV

Relative **NLO** and **NNLO** corrections w.r.t. the **LO** and **statistical error** for the unpolarized integrated cross section for  $pp \rightarrow Zj$  at  $\sqrt{s} = 14\text{ TeV}$ .

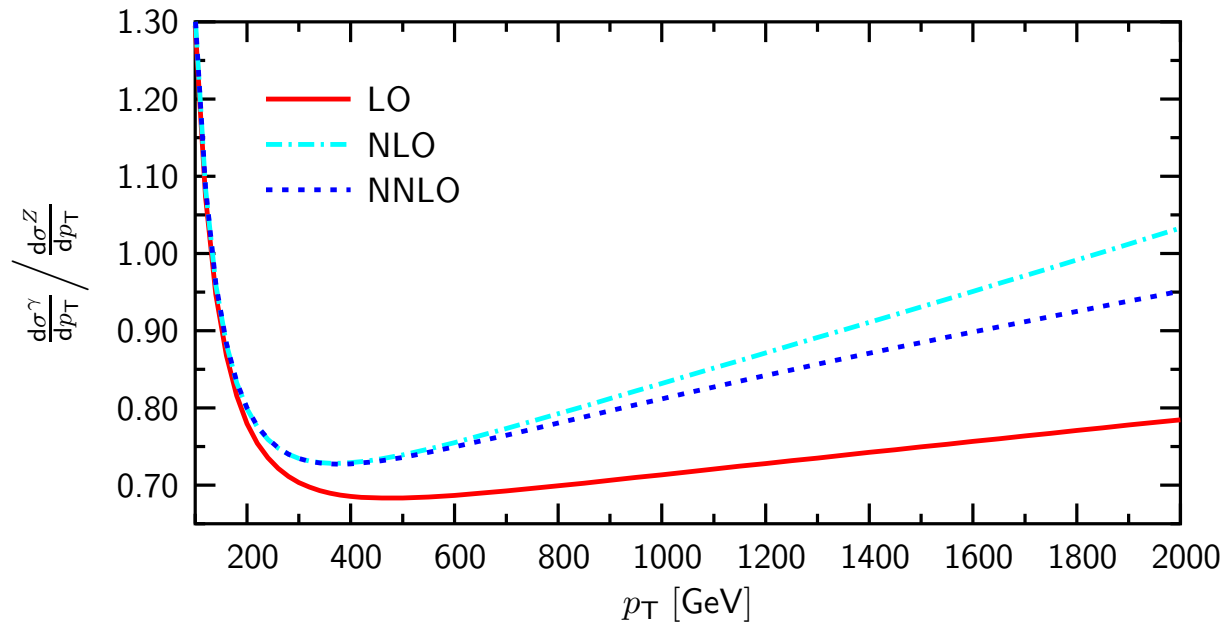
(related work: [Maina, Moretti, Ross](#))

# Photon production

- full **NLO** and logarithmic approximations ( $\log^2 + \log + \text{const}$ ) available
- dominant **two-loop** terms ( $\log^4 + \log^3$ ) available



## Photons vs. Z at large $p_T$

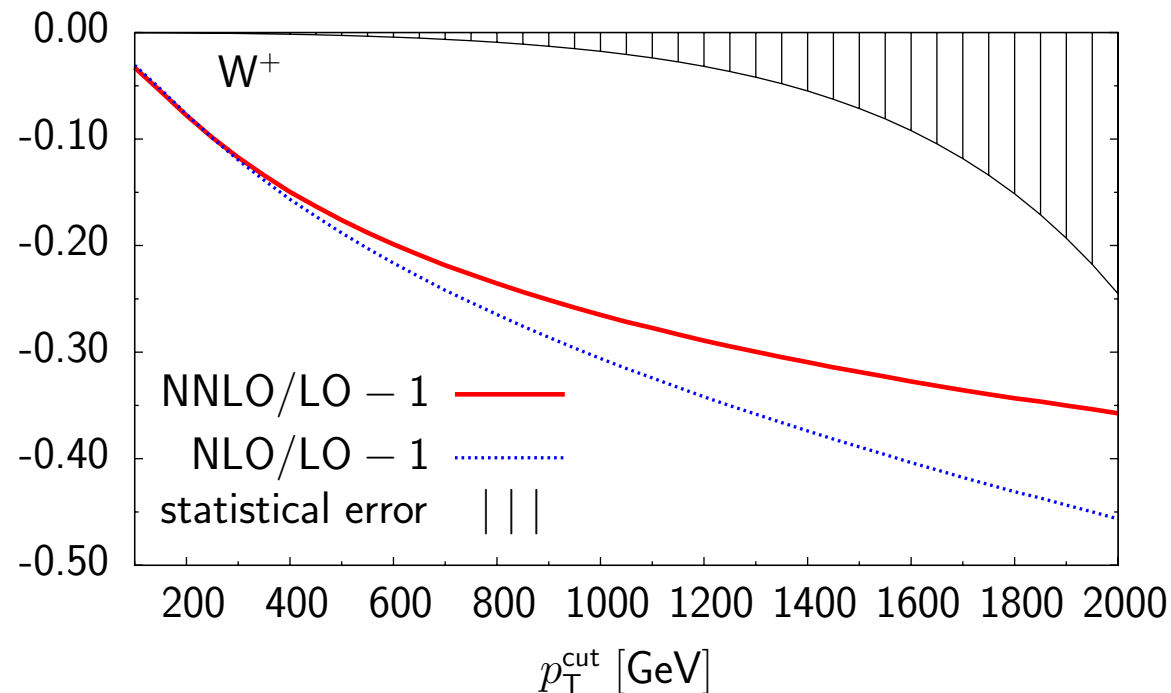


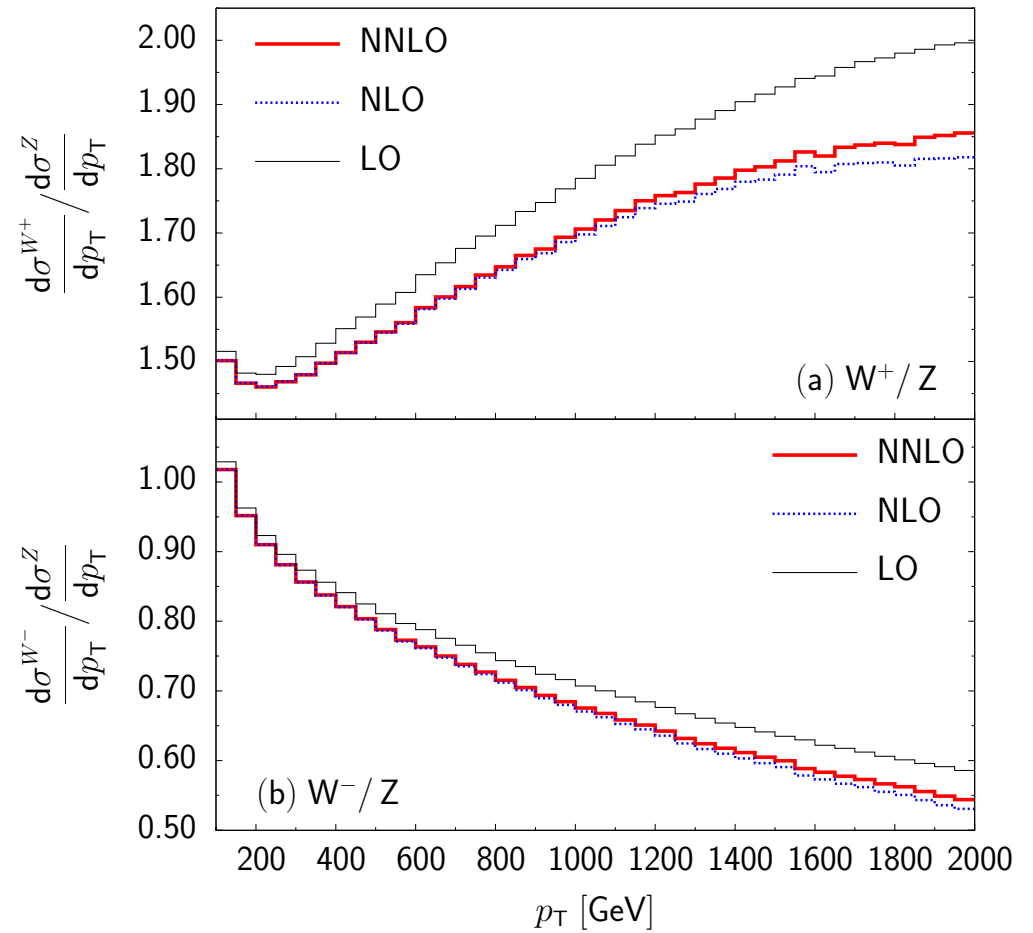
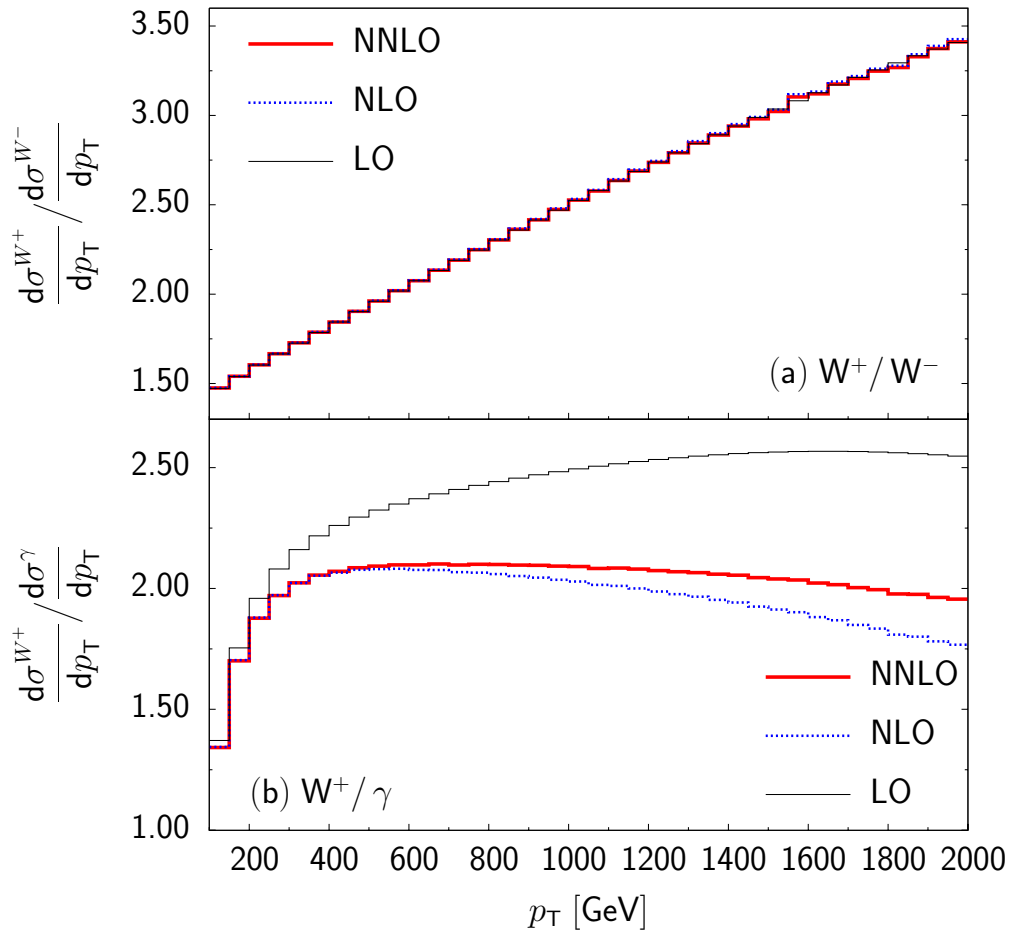
numerical results in qualitative agreement with [Maina, Moretti, Ross](#)

# W production

additional complications:

- photon radiation as necessary part of virtual corrections (gauge invariance)
- IR singularities must be compensated by real radiation
- $p_T(W) = p_T(\text{jet}) + p_T(\gamma)$





ratios are less sensitive to QCD corrections

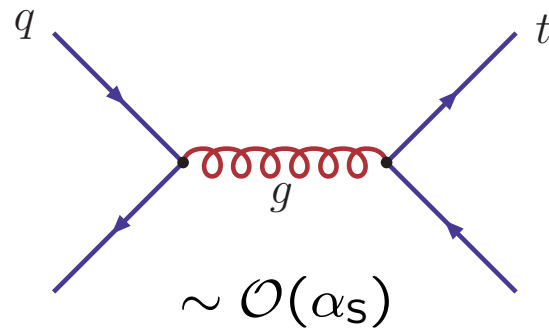
(related work on  $W$ 's: [Hollik](#), [Kniehl](#), [Kasparcik](#))

# III. Heavy Quark Production:

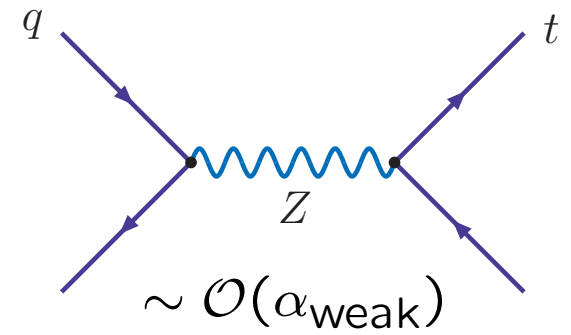
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## Top Quarks

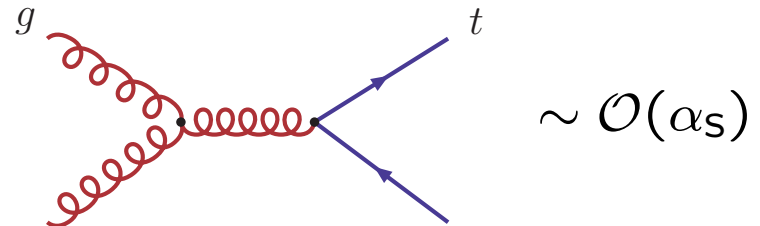
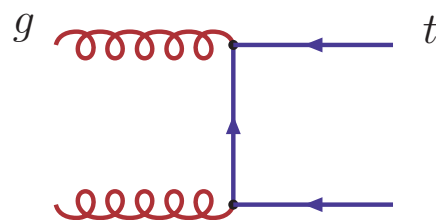
$$q\bar{q} \rightarrow t\bar{t}:$$



no  
interference  
with

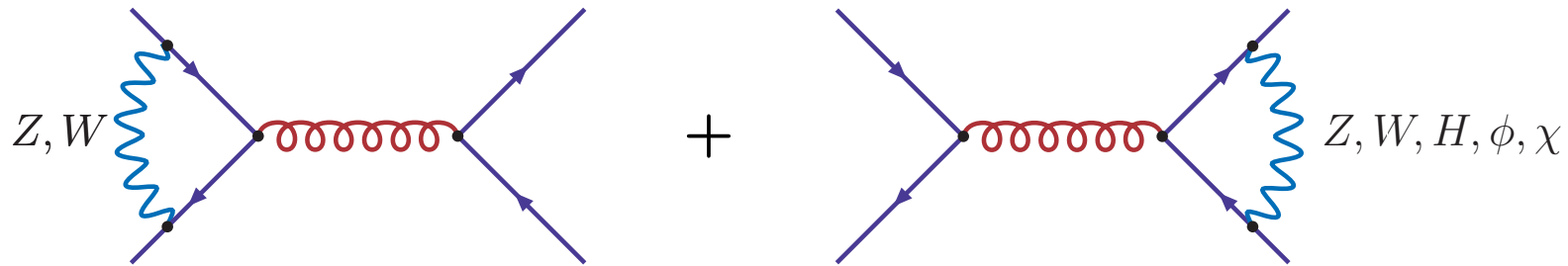


$$gg \rightarrow t\bar{t}:$$

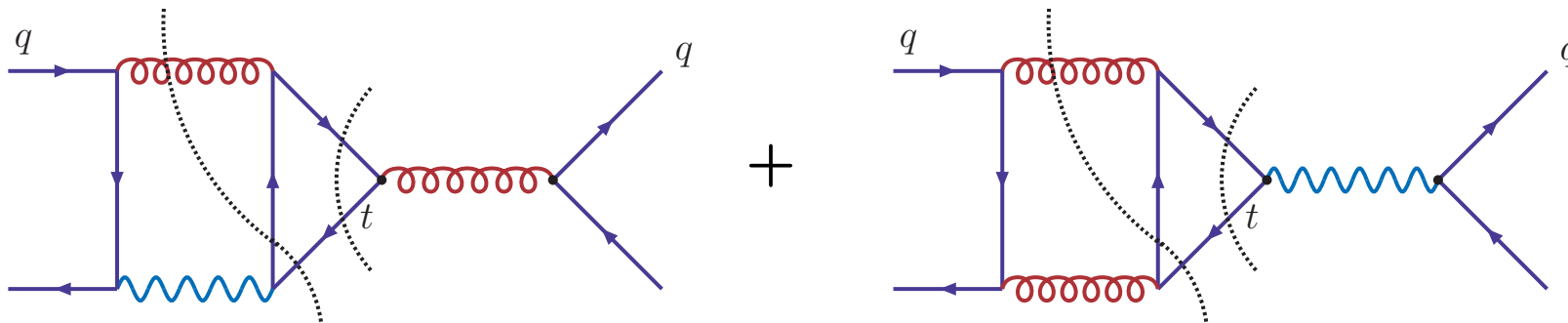




$O(\alpha_S^2 \alpha_{\text{weak}})$  weak corrections ( $q \bar{q} \rightarrow t \bar{t}$ )

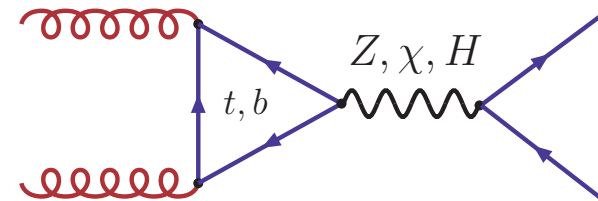
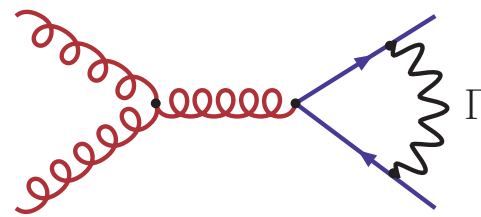
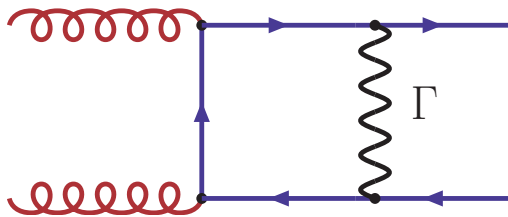
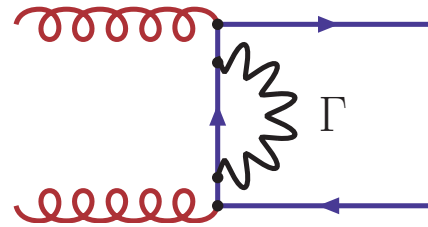
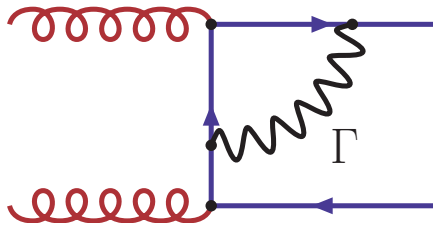


also Beenakker et. al  
Kao & Wackerath



cuts of second group individually IR-divergent

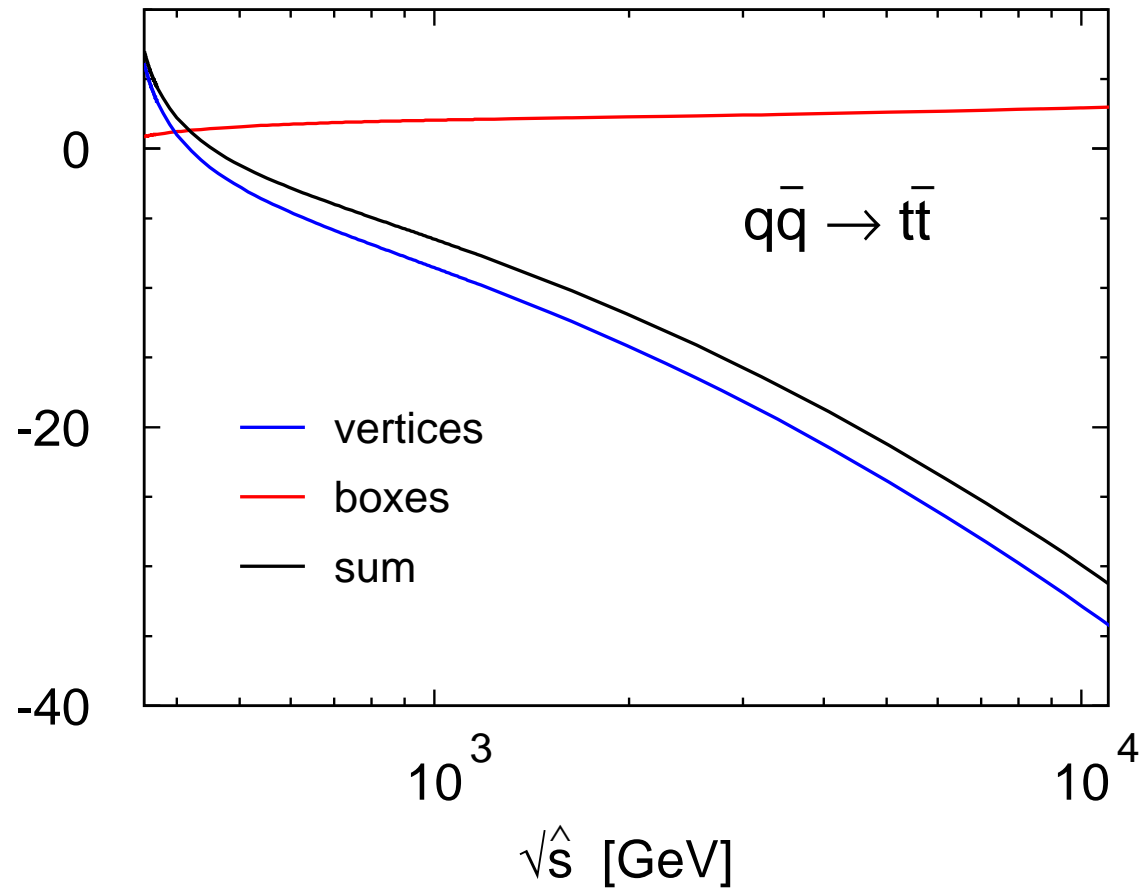
$\mathcal{O}(\alpha_S^2 \alpha_{\text{weak}})$  weak corrections ( $g g \rightarrow t \bar{t}$ )



- analytical & numerical results available  
(independent evaluation of Bernreuther & Fückler, many independent checks)
- $(\text{box contribution})_{\text{up-quark}} = -(\text{box contribution})_{\text{down-quark}}$   
 $\Rightarrow$  suppression
- box contribution moderately  $\hat{s}$ -dependent
- strong increase with  $\hat{s}$
- sizable  $M_h$ -dependence, large effect close to threshold

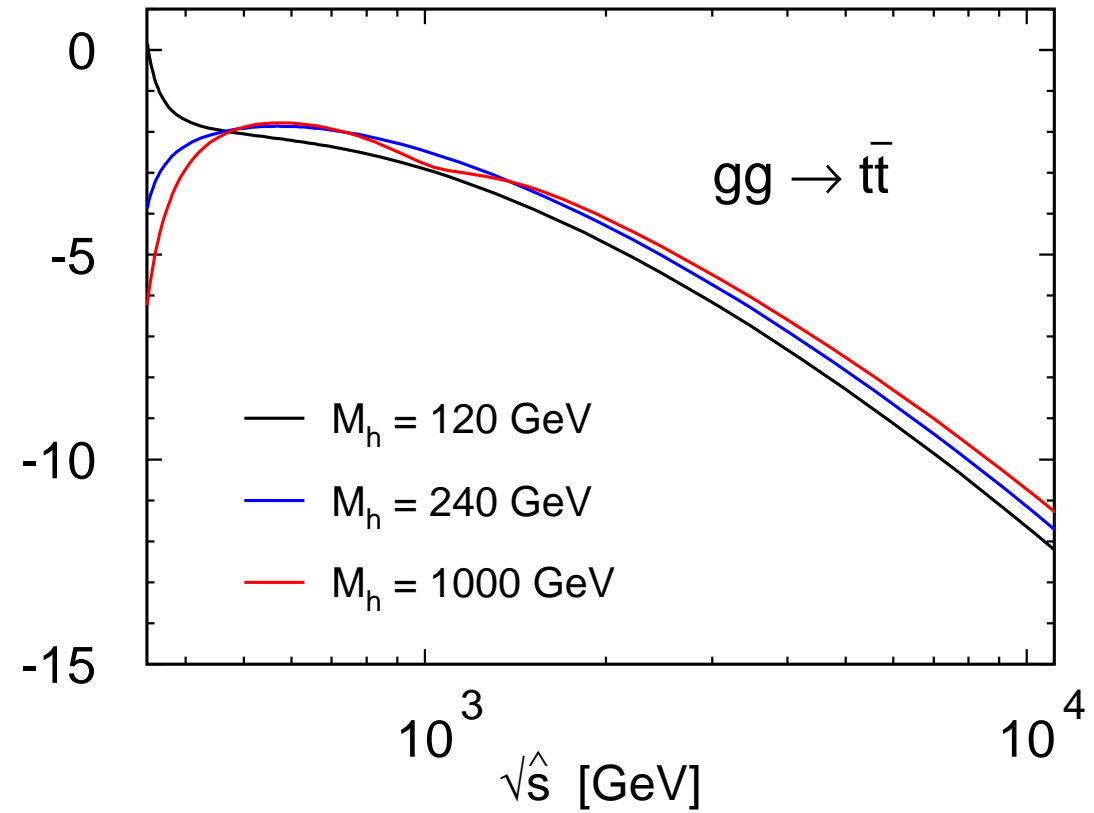
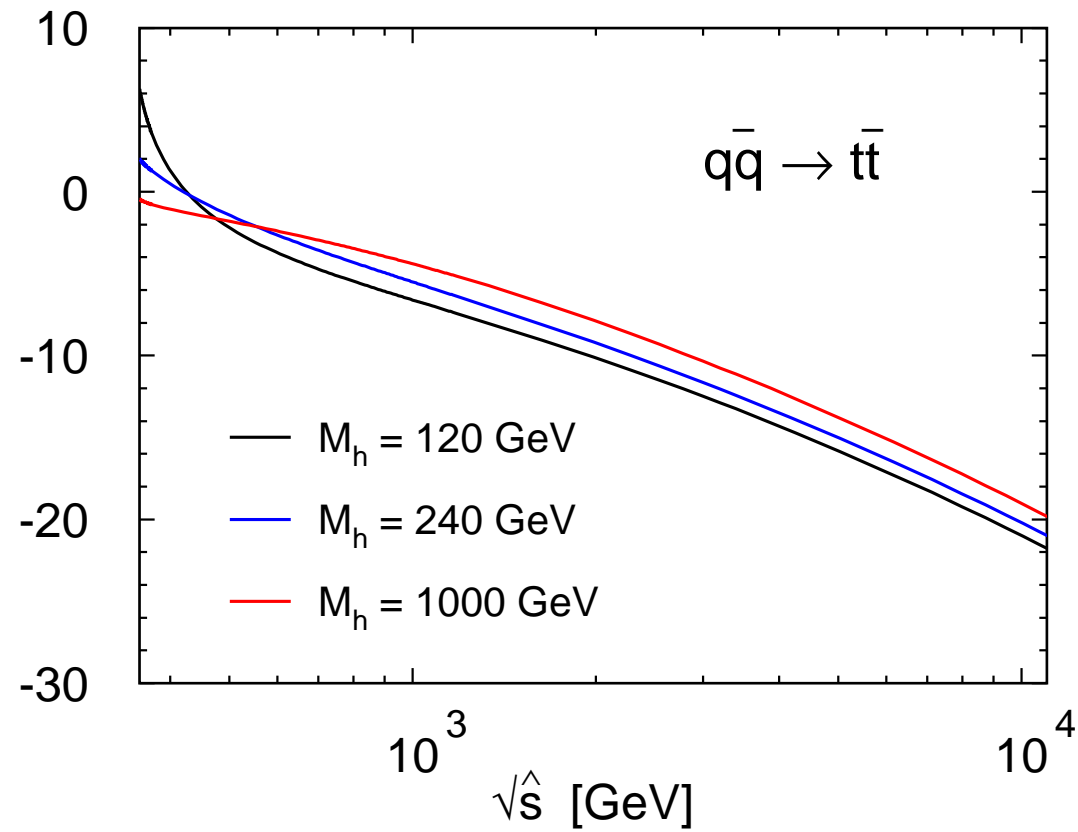
# large corrections for large $\sqrt{\hat{s}}$

(relative weak corrections [%])

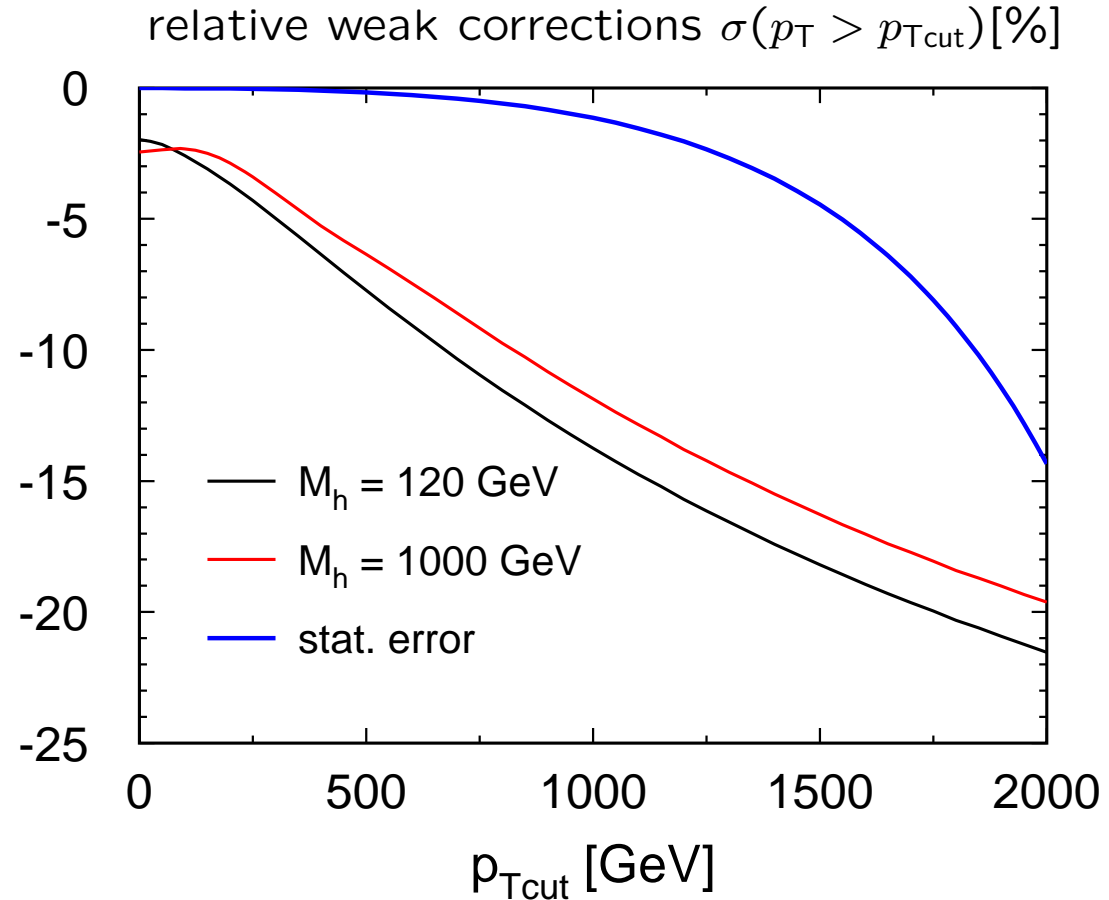
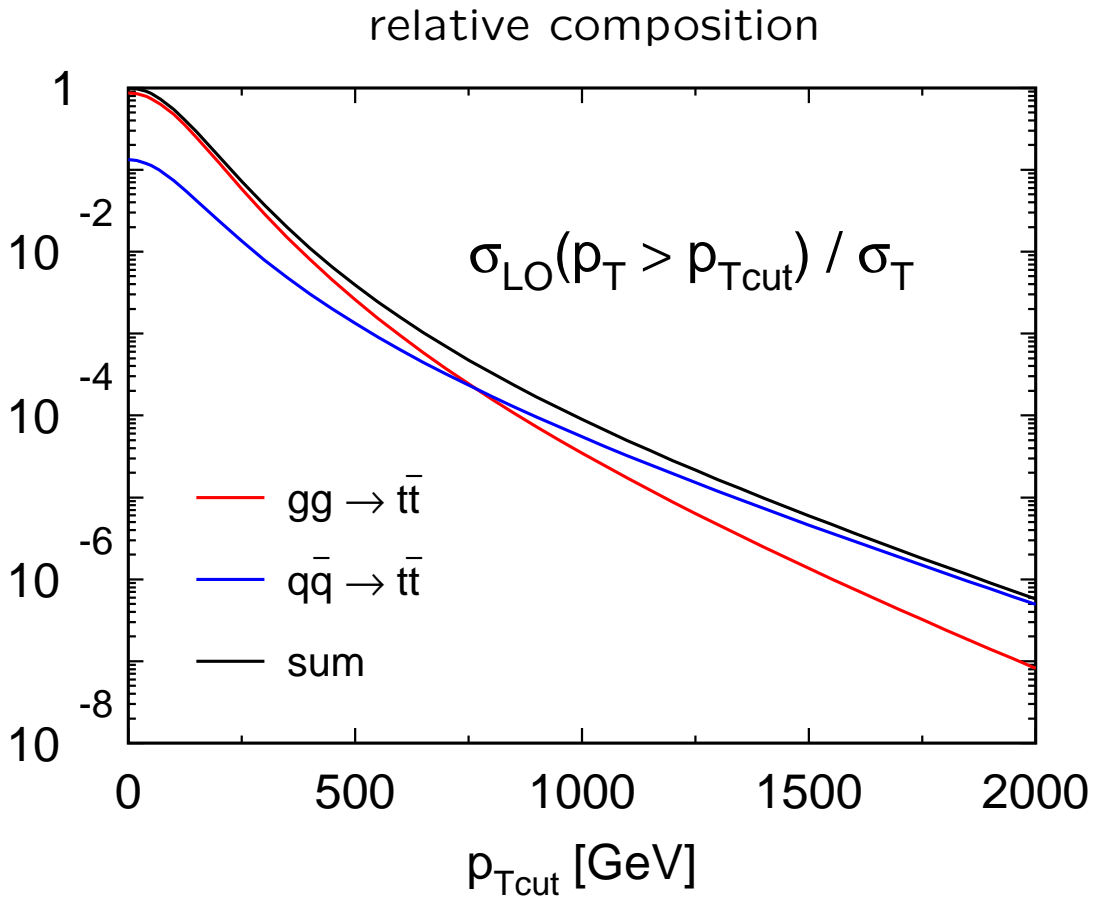


# sizeable $M_h$ -dependence

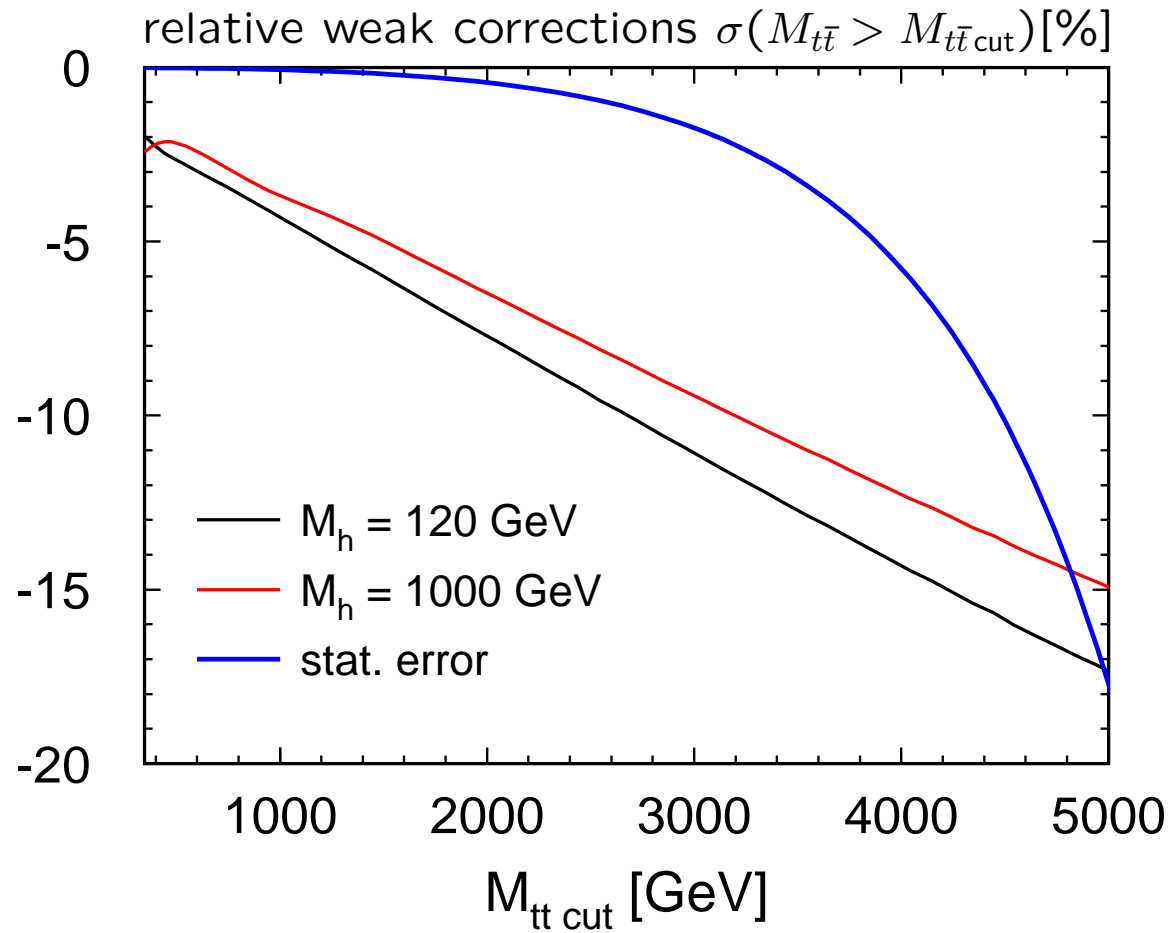
(relative weak corrections [%])



# Transverse momentum dependence



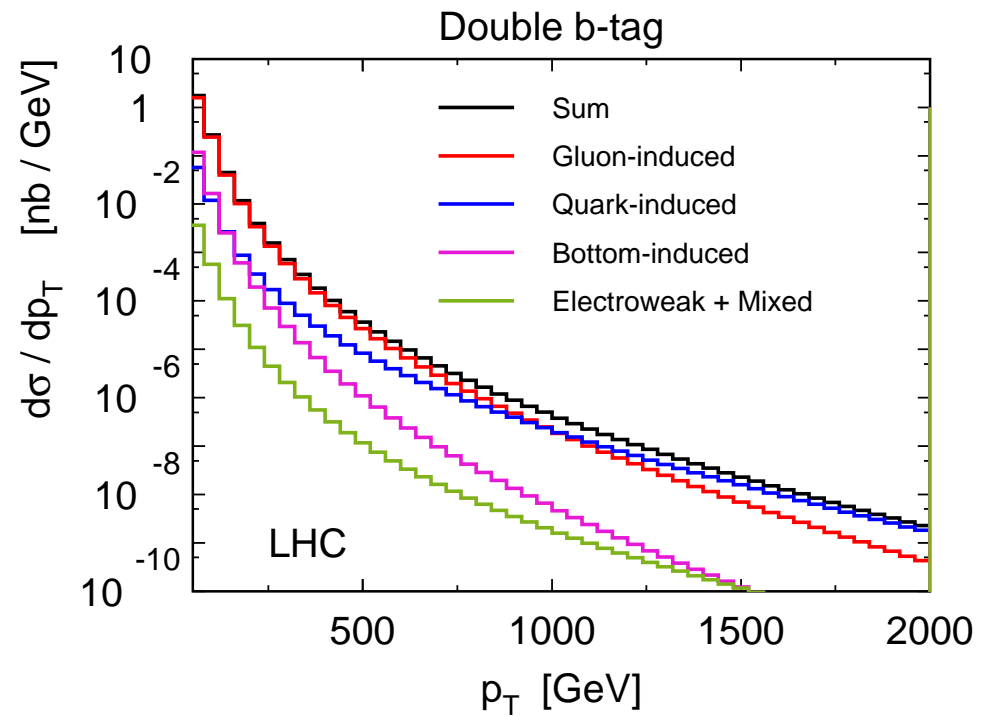
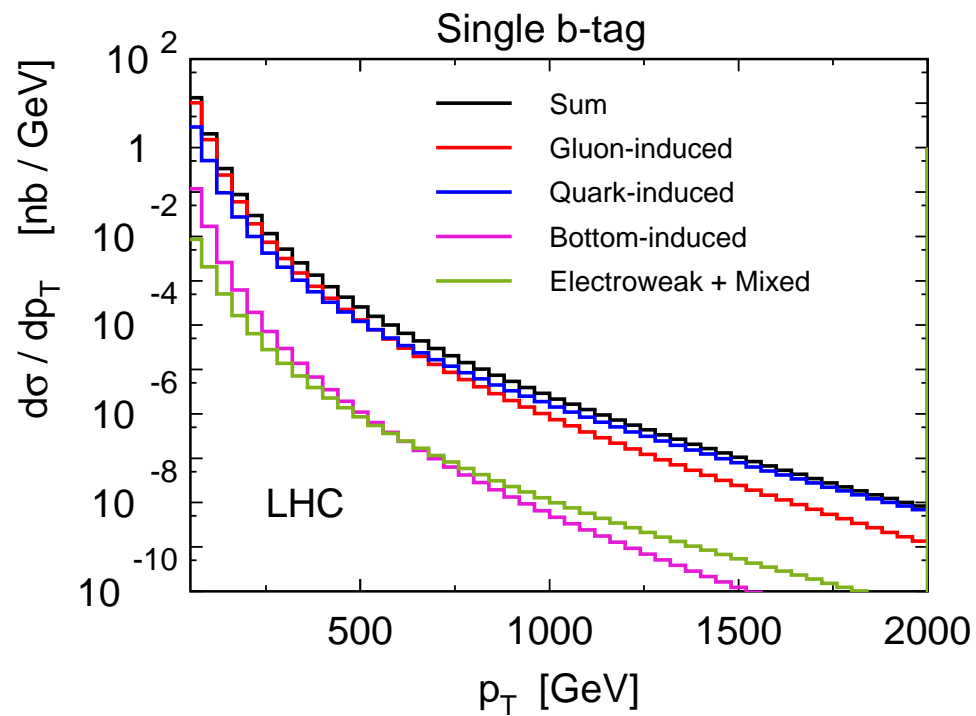
# $M_{t\bar{t}}$ -dependence



# Bottom-Quarks

preliminary results

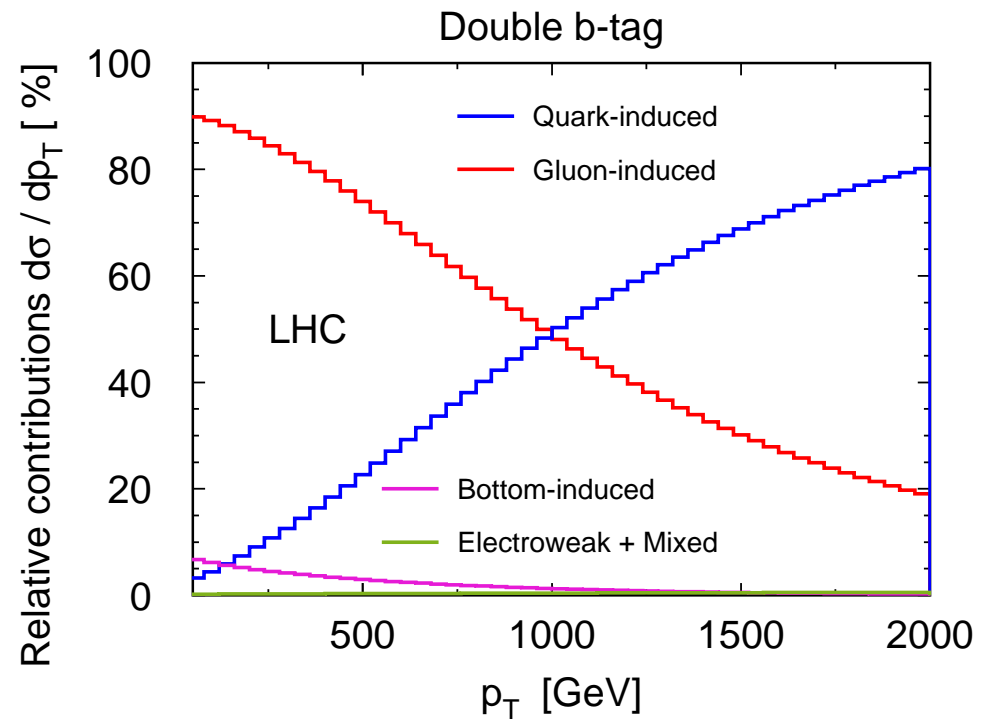
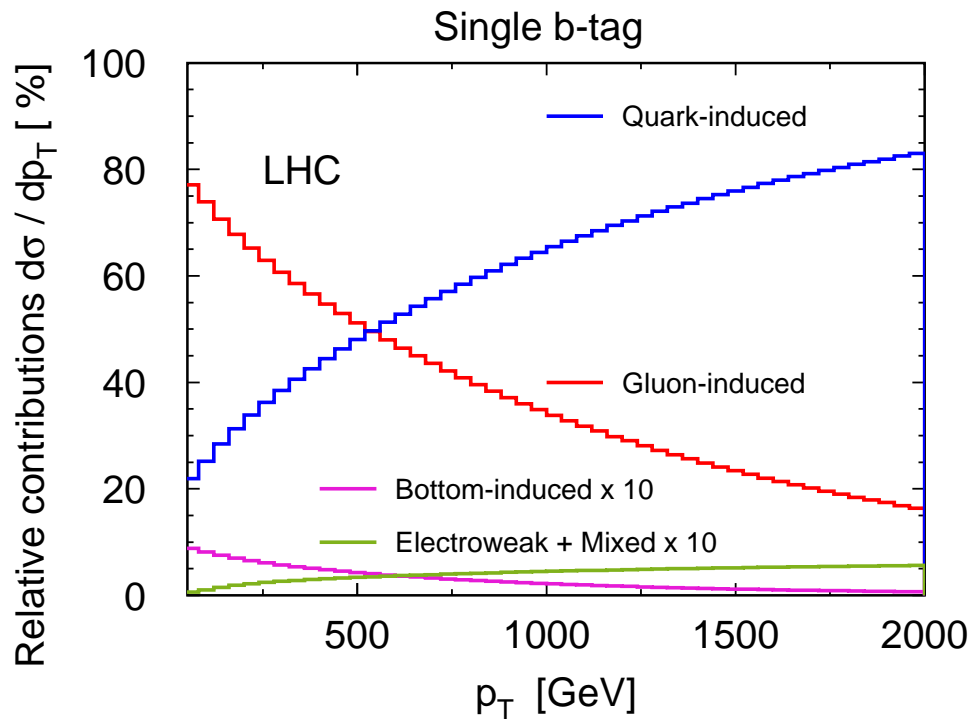
## Bottom-Jet Production





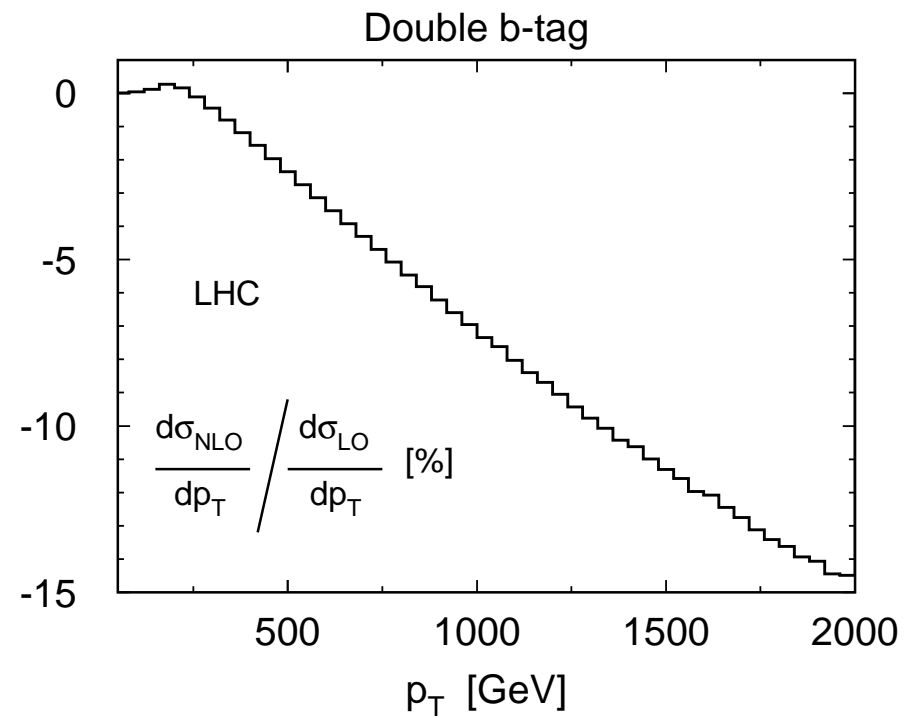
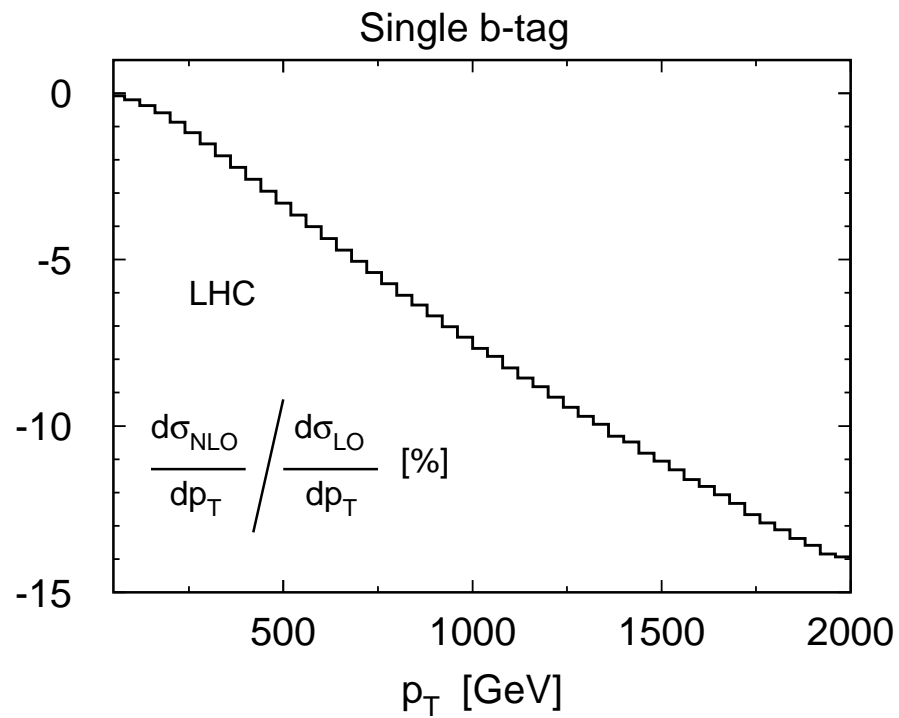
# Bottom-Jet Production

preliminary results



# Bottom-Jet Production

preliminary results



## IV. Conclusions

- LHC will explore the TeV-region:  $\hat{s}/M_W^2 \gg 1$
- electroweak corrections amount to  $\mathcal{O}(10\% - 20\%)$  in the interesting kinematic region
- $p_T$ -distributions of  $Z, \gamma$  and their ratio will be strongly affected
- two-loop terms might become relevant
- top-quark distributions at large  $\hat{s}$  are strongly modified
- sizable  $m_H$ -dependence
- interplay between electroweak and QCD effects

