THE COMMON PAST

- Cracow-Munich (MPI) cooperation and workshops
  Zakopane Summer School (1976 ...)

- Preparations for LEP (1983 ...) and visit at MPI(1986/7)

  The Monte Carlo Program Tiptop For Heavy Fermion Production And Decay At Lep And Slc.

  Asymmetries In Heavy Fermion Production And Decay.

  QCD And QED Corrections To The Longitudinal Polarization Asymmetry.

- Analysis of τ-production and decay

  TAUOLA: A Library of Monte Carlo programs to simulate decays of polarized tau leptons. (250+)

  The tau decay library TAUOLA, update with exact $\mathcal{O}(\alpha)$ QED corrections in leptonic decay modes.

  The tau decay library TAUOLA: Version 2.4. (250+)

- Discussions on LEP, τ-physics, RADCOR

COMMON INTERESTS

$\Rightarrow$ **LHC**

HARD SCATTERING AND ELECTROWEAK CORRECTIONS AT THE LHC

J.H. Kühn

I. Introduction

II. Form Factors and Four-Fermion Scattering — Jantzen, J.H.K., Penin, Smirnov

III. Z, W and Photon Production — J.H.K., Kulesza, Pozzorini, Schulze

IV. Top Production — J.H.K., Scharf, Uwer

V. Conclusions

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SFB TR9

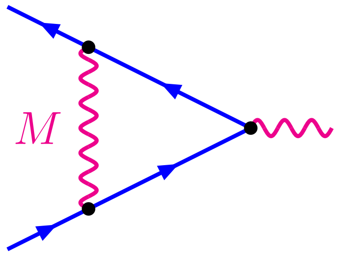
I. Introduction

"Typical" size of electroweak corrections: $\frac{\alpha_{\text{weak}}}{\pi} \approx 10^{-2}$

new aspects at LHC: $\sqrt{\hat{s}} \approx$ 1-2TeV $\gg M_{W,Z}$

strong enhancement of negative corrections

one-loop example: massive U(1)



$$\Rightarrow \quad \text{Born} * \left[1 + \frac{\alpha}{4\pi}\left(-\ln^2\frac{s}{M^2} + 3\ln\frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3}\right)\right]$$

$\frac{s}{M^2}$	$-\ln^2\frac{s}{M^2}$	$+3\ln\frac{s}{M^2}$	$-\frac{7}{2}+\frac{\pi^2}{3}$	Σ	$* 4\,\frac{\alpha_{\text{weak}}}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	$+15.15$	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	$+19.31$	-0.21	-22.3	-27%

(four-fermion cross section $\Rightarrow$ factor 4)

- leading $\log^2$ multiplied by (charge)^2 $= I(I+1) = \begin{cases} 3/4 & I = 1/2 \\ 2 & I = 1 \end{cases}$

- important subleading logarithms (NLL+...)

- two-loop terms may be relevant

- interplay between electroweak and QCD corrections

- important differences between fermions and electroweak gauge bosons

II. Form Factors & Four-Fermion Scattering at Two Loop

LL: Fadin et al. (2000)

NLL: J.H.K., Penin, Smirnov (2000)
Large (!) subleading corrections
important angular dependent terms

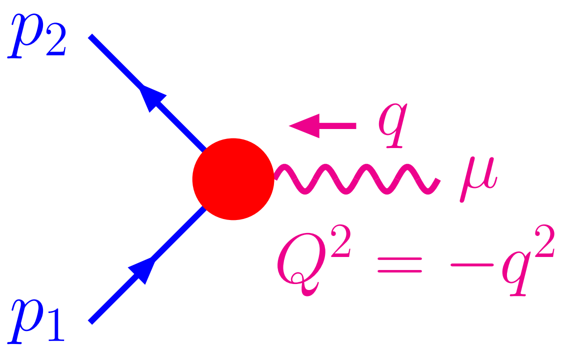
NNLL: J.H.K., Moch, Penin, Smirnov (2001)
Large (!) NNLL terms,
oscillating signs of LL, NLL, NNLL
$\Rightarrow$ compensations

N^3LL+N^4LL Jantzen, J.H.K., Penin, Smirnov (2003-2005)

Additional complication in SM: massless photon

$$|Q^2| \gg M^2_{W,Z} \gg m^2_\gamma$$

A) Form Factor and Evolution Equations



Born:

$$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow \mathcal{F} = \mathcal{F}_{\text{Born}}\, F_0(\alpha(M^2)) \exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: N^{4}LL $\quad\Rightarrow\quad$ corresponds to all terms of the form:

$$\alpha^n \left[\underset{\text{LL}}{\mathcal{L}^{2n}} + \underset{\text{NLL}}{\mathcal{L}^{2n-1}} + \underset{\text{NNLL}}{\mathcal{L}^{2n-2}} + \underset{\text{N}^3\text{LL}}{\mathcal{L}^{2n-3}} + \underset{\text{N}^4\text{LL}}{\mathcal{L}^{2n-4}} \right]$$

$$\mathcal{L} \equiv \ln(Q^2/M^2)$$

NNLL requires running of α (i.e. β_0 and β_1) and:

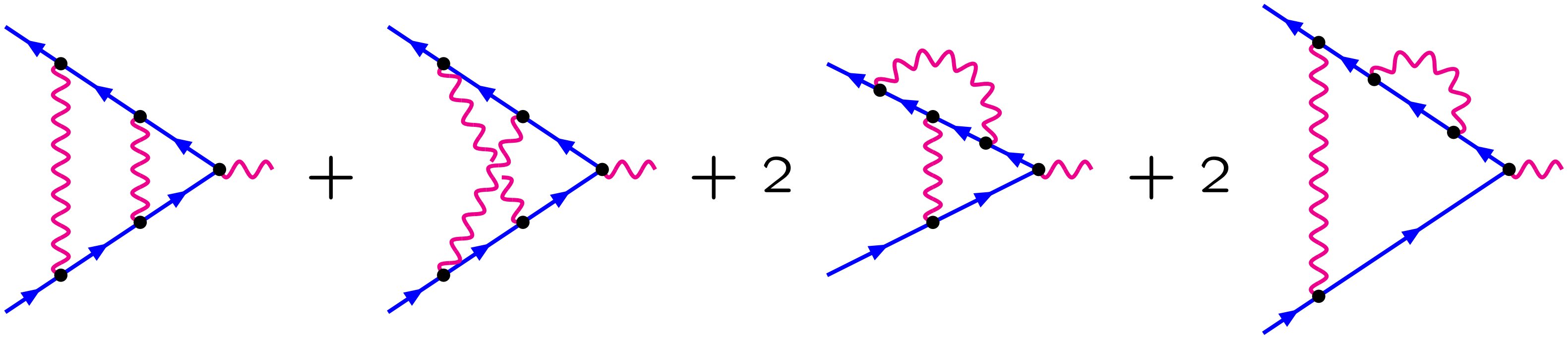
$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$	up to $\mathcal{O}(\alpha)$	(one-loop)
$\gamma(\alpha)$	up to $\mathcal{O}(\alpha^2)$	(massless two loop)

N^{3}LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

N^{4}LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

B) Two-Loop Results: Massive U(1) Model

$$\mathcal{F}_\alpha(M,Q) = \mathcal{F}_{\text{Born}}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \ldots\right]$$

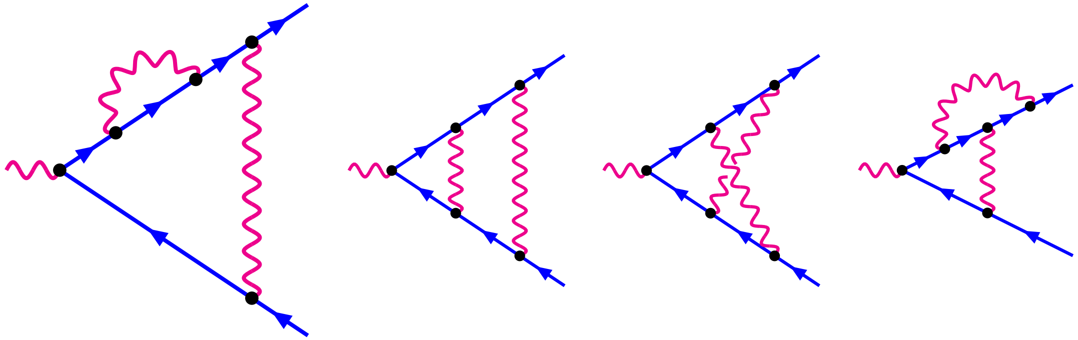


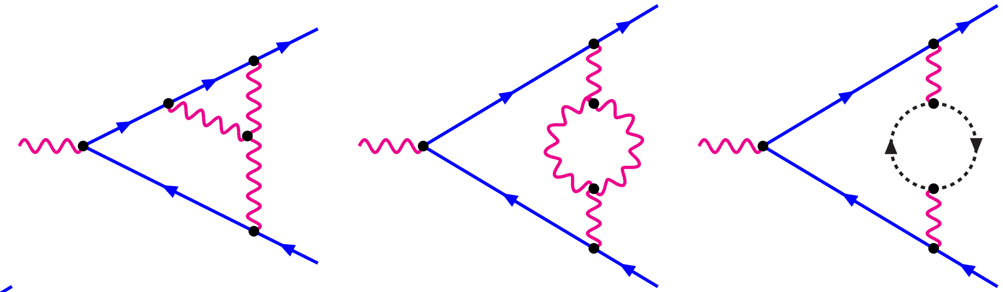
$$f^{(2)} = \frac{1}{2}\mathcal{L}^4 - 3\,\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right)\mathcal{L}$$
$$+\frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3}\ln^4 2 + 256\,\text{Li}_4\left(\frac{1}{2}\right)$$

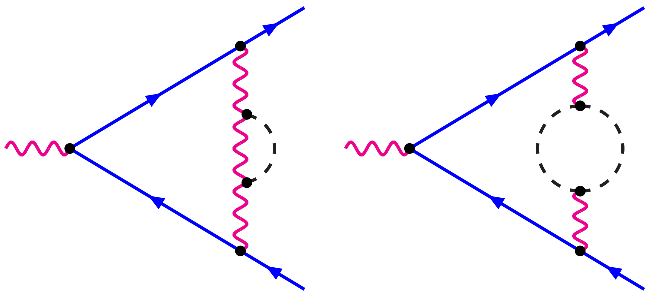
$\mathcal{L} \equiv \ln(Q^2/M^2)$

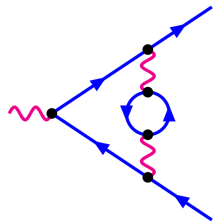
C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

Abelian (C_F^2): 

non-Abelian ($C_F C_A$): last 2 + 

Higgs: 

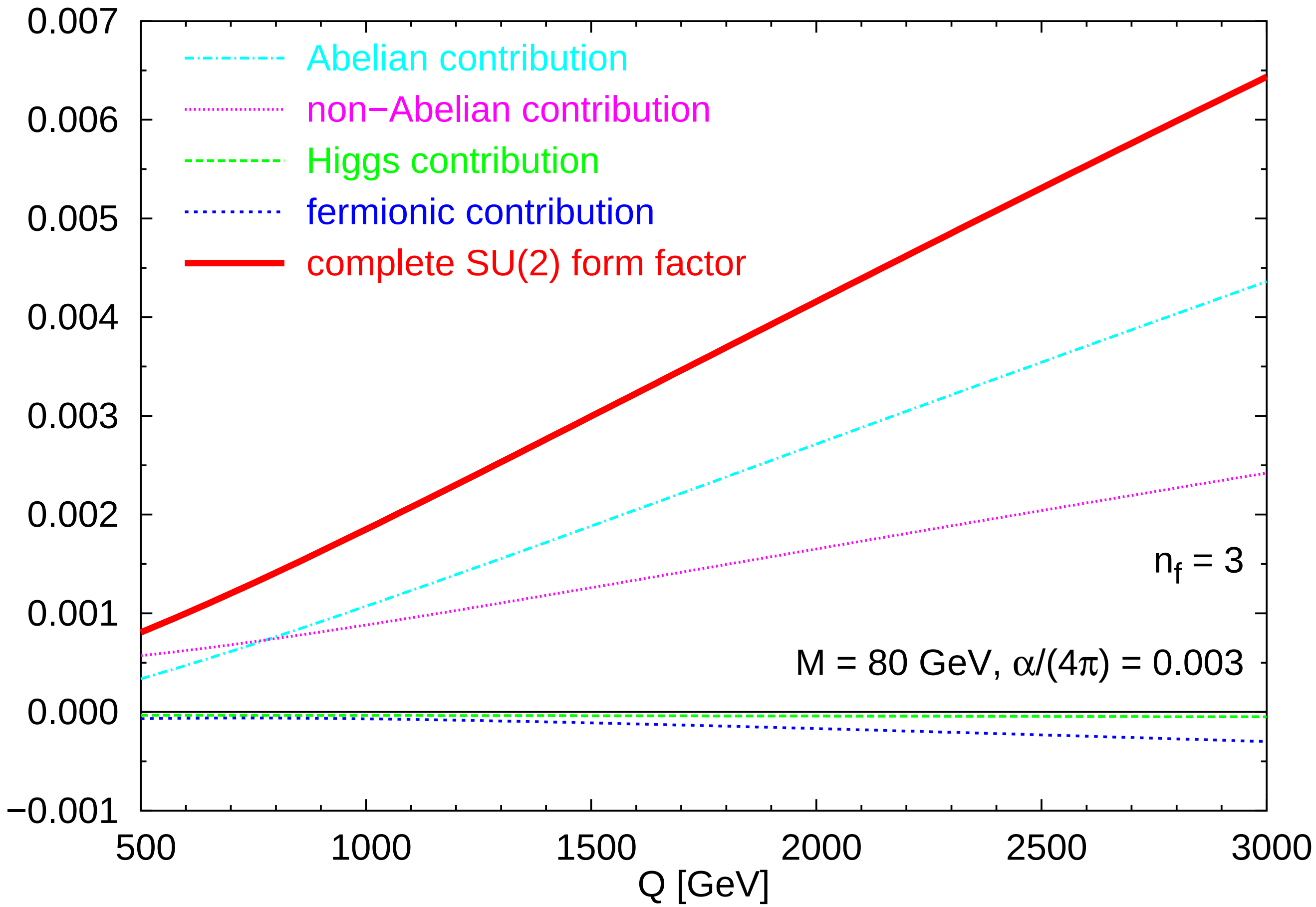
fermion ($C_F T_F n_f$): 

+ 1-loop×1-loop corrections + renormalization

$$f_2 = +\frac{9}{32}\mathcal{L}^4 - \frac{19}{48}\mathcal{L}^3 - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\mathcal{L}^2$$
$$+ \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\mathcal{L}$$

individual contributions

(N^{3}LL approximation, $M_{\text{Higgs}} = M$, $n_f = 3$, Feynman-'t Hooft gauge)



D) Four fermion scattering

Evaluation in the high energy limit

define

$$\begin{aligned}\mathcal{A}^{\lambda} &= \bar{\psi}_2 t^a \gamma_\mu \psi_1 \bar{\psi}_4 t^a \gamma_\mu \psi_3 \\ \mathcal{A}^{\lambda}_{LL} &= \bar{\psi}_{2L} t^a \gamma_\mu \psi_{1L} \bar{\psi}_{4L} t^a \gamma_\mu \psi_{3L} \\ \mathcal{A}^{d}_{LR} &= \bar{\psi}_{2L} \gamma_\mu \psi_{1L} \bar{\psi}_{4R} \gamma_\mu \psi_{3R}\end{aligned}$$

define "reduced" amplitude $\tilde{\mathcal{A}}$

$$\mathcal{A} = \frac{ig^2}{s}\mathcal{F}^2\tilde{\mathcal{A}}$$

evolution equation

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}$$

$\tilde{\mathcal{A}}$: vector in isospin/chiral basis
χ: matrix

N^{3}LL **requires:**

- form factor up to N^{3}LL
- χ up to two loop, as obtained from hard contribution to single pole part of 4-fermion scattering amplitude

e.g. pure massive SU(2) theory with SSB:

$$
\begin{aligned}
\sigma^{(2)} \;=\; & \left[\frac{9}{2}\mathcal{L}^4 - \frac{449}{6}\mathcal{L}^3 + \left(\frac{4855}{18} + \frac{37}{3}\pi^2\right)\mathcal{L}^2\right. \\
& \left. + \left(\frac{34441}{216} - \frac{1247}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\,\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\mathcal{L}\right]\sigma_B
\end{aligned}
$$

for identical isospin in initial and final state

Electroweak theory

- infrared logs must be separated

- NNLL
  - result insensitive to form of gauge-boson mass generation
  - term of order $1 - M_W^2/M_Z^2 = \sin^2\theta$ included

- N^{3}LL
  - sensitive to details of mass generation, gauge boson mixing
  - Approximation: terms of $\mathcal{O}(\sin^2\theta)$ neglected

Result for the correction factor

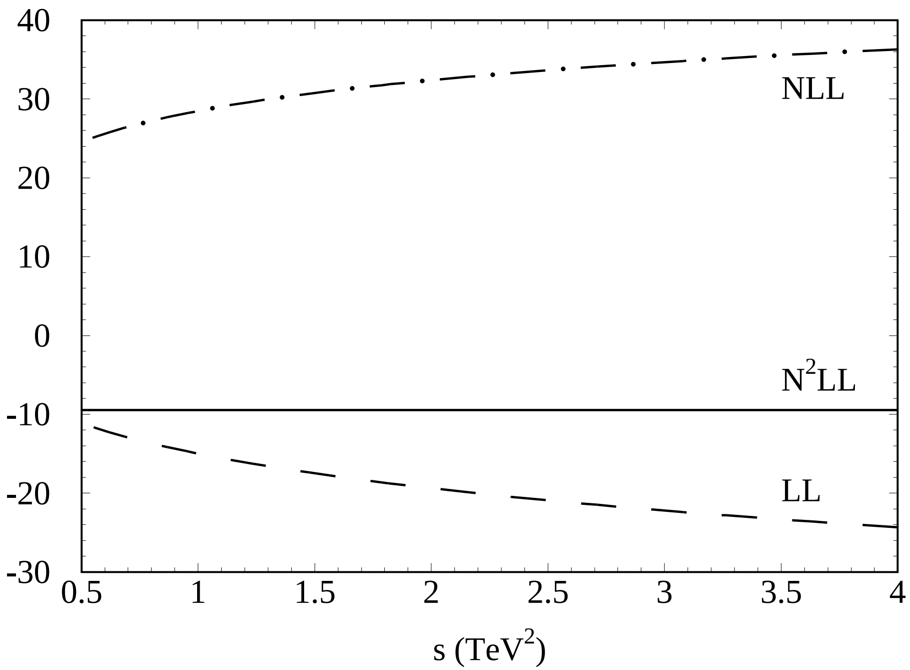
$$
\begin{aligned}
R(e^+e^- \to Q\bar{Q}) &= 1 - 1.66\,L(s) + 5.60\,l(s) - 8.39\,a + 1.93\,L^2(s) \\
&\quad -11.28\,L(s)\,l(s) + 33.79\,l^2(s) - 150.95\,l(s)\,a \\
R(e^+e^- \to q\bar{q}) &= 1 - 2.18\,L(s) + 20.94\,l(s) - 35.07\,a + 2.79\,L^2(s) \\
&\quad -51.98\,L(s)\,l(s) + 321.34\,l^2(s) - 603.43\,l(s)\,a \\
R(e^+e^- \to \mu^+\mu^-) &= 1 - 1.39\,L(s) + 10.12\,l(s) - 21.26\,a + 1.42\,L^2(s) \\
&\quad -20.33\,L(s)\,l(s) + 112.57\,l^2(s) - 260.15\,l(s)\,a
\end{aligned}
$$

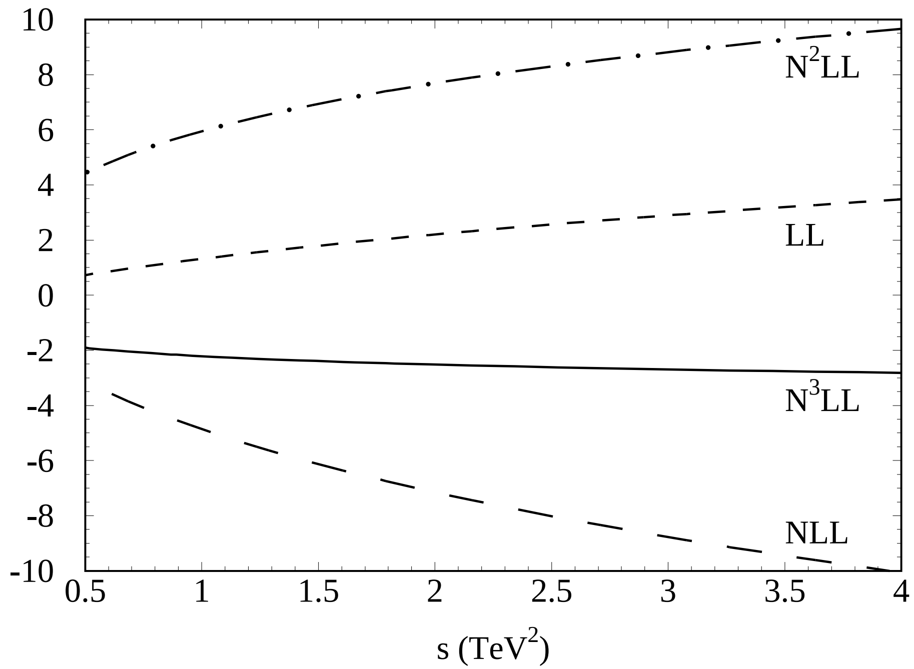
with

$$
\begin{aligned}
L(s) &= \frac{g^2}{16\pi^2}\ln^2\left(\frac{s}{M^2}\right) = 0.07 \quad (0.11) \\
l(s) &= \frac{g^2}{16\pi^2}\ln\left(\frac{s}{M^2}\right) = 0.014 \; (0.017) \\
a &= \frac{g^2}{16\pi^2} = 0.003
\end{aligned}
$$

for $\sqrt{s} = 1$ TeV (2 TeV)

Separate logarithmic contributions to $R(e^+e^- \to q\bar{q})$ in % to the Born approximation





one-loop LL $\left(\ln^2(s/M^2)\right)$, NLL $\left(\ln^1(s/M^2)\right)$ and N^{2}LL $\left(\ln^0(s/M^2)\right)$

two-loop LL $\left(\ln^4(s/M^2)\right)$, NLL $\left(\ln^3(s/M^2)\right)$, NNLL $\left(\ln^2(s/M^2)\right)$ and N^{3}LL $\left(\ln^1(s/M^2)\right)$

Large cancellations!

similar techniques

$$e^+e^- \to W^+W^-$$

- longitudinal vs. transverse Ws !
- impact of Yukawa coupling (top-quark!)

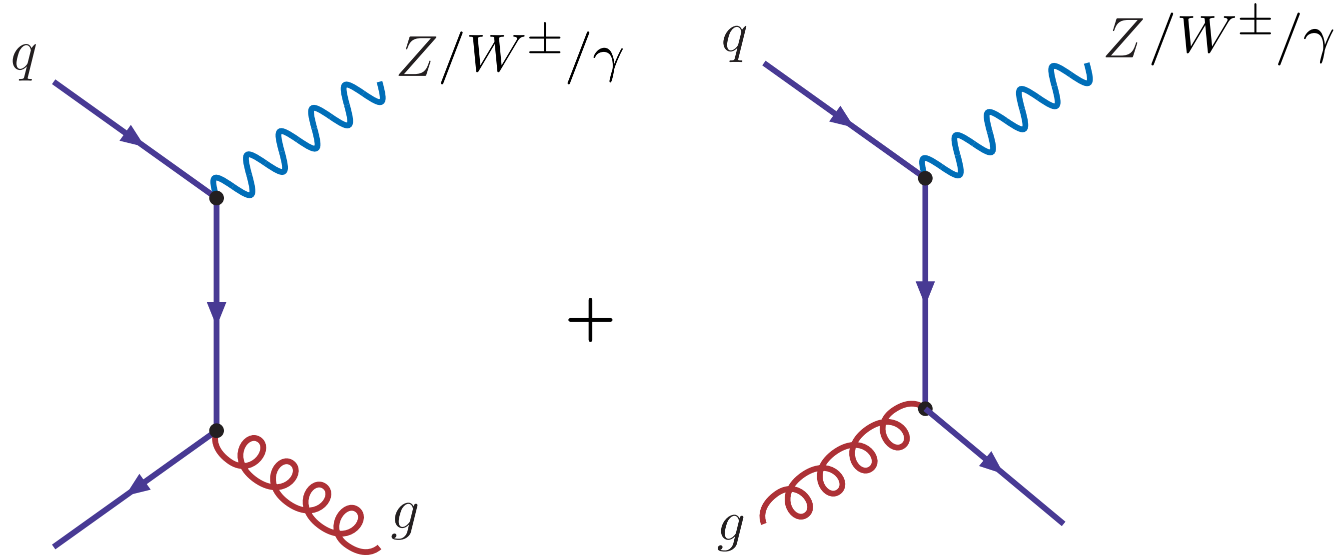
III. Z, W and Photon Production at LHC

J.H.K., Kulesza, Pozzorini, Schulze

Phys. Lett. B609(2005) 277, Nucl. Phys. B727(2005) 368,
JHEP 0603:059,2006, Phys.Lett. B651(2007), arXiv:0708.0476

sizable rate at large p_{T} (1-2 TeV)

Large electroweak corrections ($\hat{s} \gg M^2_{W,Z}$)



one-loop corrections:

Result for Z decomposed into "abelian" (A) and "non-abelian" (N) parts

$H_1^{\text{A,N}}$ plus counterterms $\delta C^{\text{A,N}}$ in closed analytical form:

kinematical functions of $(\hat{s}, \hat{t}, \hat{u})$ and 14 combinations of

$$1 \times A_0,\;\; 5 \times B_0,\;\; 5 \times C_0,\;\; 3 \times D_0$$

High energy limit

consider $q\bar{q} \to Zg$

NLL $\widehat{=}$ double + single logarithmic terms

$$H_1^{\mathrm{A}} \overset{\mathrm{NLL}}{=} -\left[\log^2\left(\frac{|\hat{s}|}{M_W^2}\right) - 3\log\left(\frac{|\hat{s}|}{M_W^2}\right)\right] H_0,$$
$$H_1^{\mathrm{N}} \overset{\mathrm{NLL}}{=} -\left[\log^2\left(\frac{|\hat{t}|}{M_W^2}\right) + \log^2\left(\frac{|\hat{u}|}{M_W^2}\right) - \log^2\left(\frac{|\hat{s}|}{M_W^2}\right)\right] H_0$$

- remaining subleading terms $\leq 2.5\%$
- NNLL includes non-enhanced terms (angular dependent)
- compact formulae

size of the correction:

$$\sqrt{\hat{s}} \;=\; 200\ \text{GeV}: \qquad \frac{\delta\sigma}{\sigma} \leq 0.3\%$$

$$\sqrt{\hat{s}} \;=\; 4000\text{GeV}: \qquad \frac{\delta\sigma}{\sigma} \approx 20 - 30\%$$

Result consistent with general considerations

one-loop:

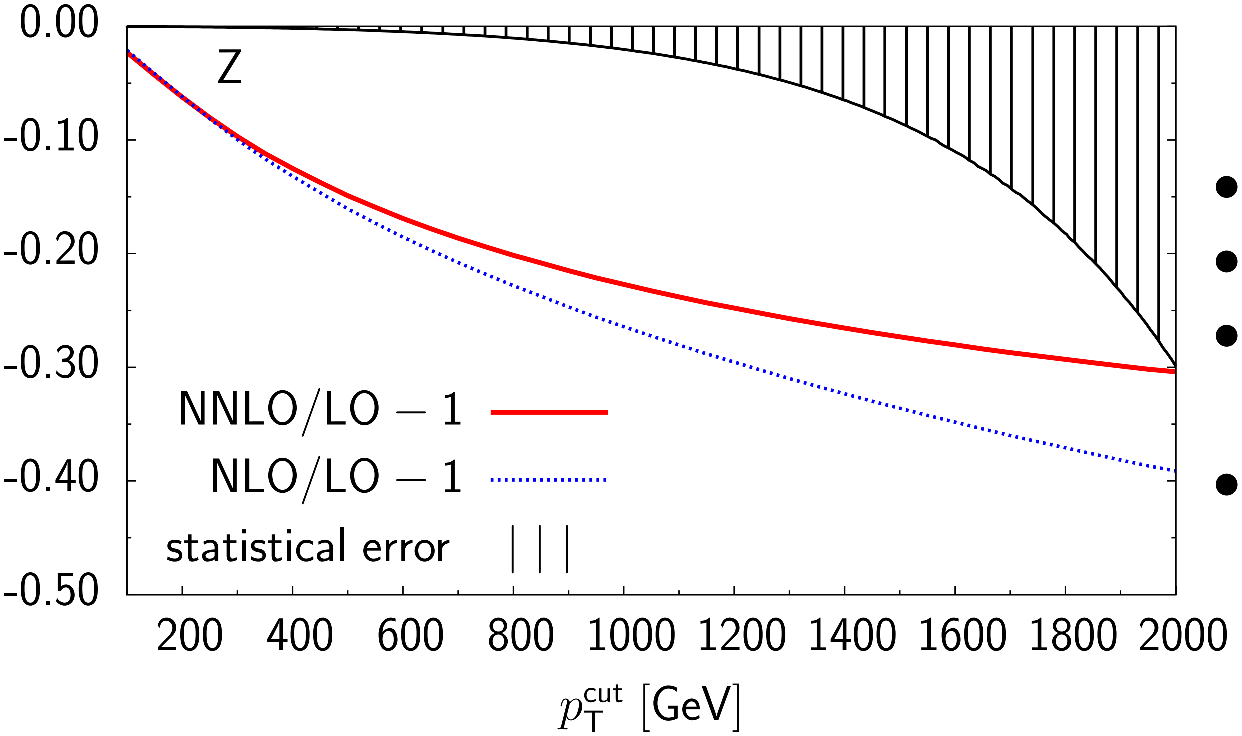
$$A^{(1)} = - \sum_{\lambda=\mathrm{L,R}} I^Z_{q_\lambda} \left[I^Z_{q_\lambda} C^{\mathrm{ew}}_{q_\lambda} \left(\mathsf{L}^2_{\hat{s}} - 3\mathsf{L}_{\hat{s}} \right) + \frac{c_\mathrm{W}}{s^3_\mathrm{W}} T^3_{q_\lambda} \left(\mathsf{L}^2_{\hat{t}} + \mathsf{L}^2_{\hat{u}} - \mathsf{L}^2_{\hat{s}} \right) \right]$$

two-loop (NLL):

$$\begin{aligned} A^{(2)} &= \sum_{\lambda=\mathrm{L,R}} \Bigg\{ \frac{1}{2} \left(I^Z_{q_\lambda} C^{\mathrm{ew}}_{q_\lambda} + \frac{c_\mathrm{W}}{s^3_\mathrm{W}} T^3_{q_\lambda} \right) \Bigg[I^Z_{q_\lambda} C^{\mathrm{ew}}_{q_\lambda} \left(\mathsf{L}^4_{\hat{s}} - 6\mathsf{L}^3_{\hat{s}} \right) \\ &\quad + \frac{c_\mathrm{W}}{s^3_\mathrm{W}} T^3_{q_\lambda} \left(\mathsf{L}^4_{\hat{t}} + \mathsf{L}^4_{\hat{u}} - \mathsf{L}^4_{\hat{s}} \right) \Bigg] - \frac{T^3_{q_\lambda} Y_{q_\lambda}}{8 s^4_\mathrm{W}} \left(\mathsf{L}^4_{\hat{t}} + \mathsf{L}^4_{\hat{u}} - \mathsf{L}^4_{\hat{s}} \right) \\ &\quad + \frac{1}{6} I^Z_{q_\lambda} \left[I^Z_{q_\lambda} \left(\frac{b_1}{c^2_\mathrm{W}} \left(\frac{Y_{q_\lambda}}{2} \right)^2 + \frac{b_2}{s^2_\mathrm{W}} C_{q_\lambda} \right) + \frac{c_\mathrm{W}}{s^3_\mathrm{W}} T^3_{q_\lambda} b_2 \right] \mathsf{L}^3_{\hat{s}} \Bigg\} \end{aligned}$$

with $\mathsf{L}^n_{\hat{r}} = \log^n \left(\frac{|\hat{r}|}{M^2_W} \right)$, $b_1 = -41/(6c^2_\mathrm{W})$ and $b_2 = 19/(6s^2_\mathrm{W})$

Complete one loop calculation
NLL approximation at two loops



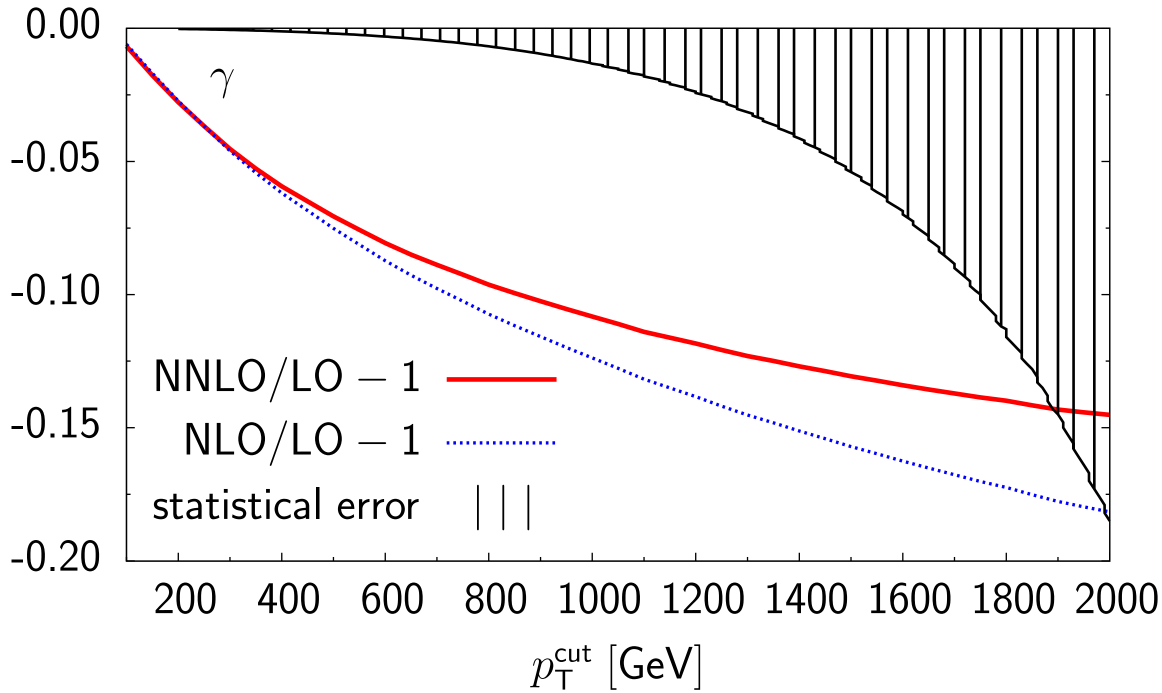
- one-loop $\sim 30\%$ at $p_T \sim 1$TeV
- two-loop relevant above 1 TeV
- important angular-dependent logarithmic terms
- experiment: p_T up to 2 TeV

Relative **NLO** and **NNLO** corrections w.r.t. the **LO** and statistical error for the unpolarized integrated cross section for $pp \to Zj$ at $\sqrt{s} = 14$ TeV.

Photon production

- full **NLO** and logarithmic approximations ($\log^2 + \log + \mathrm{const}$) available

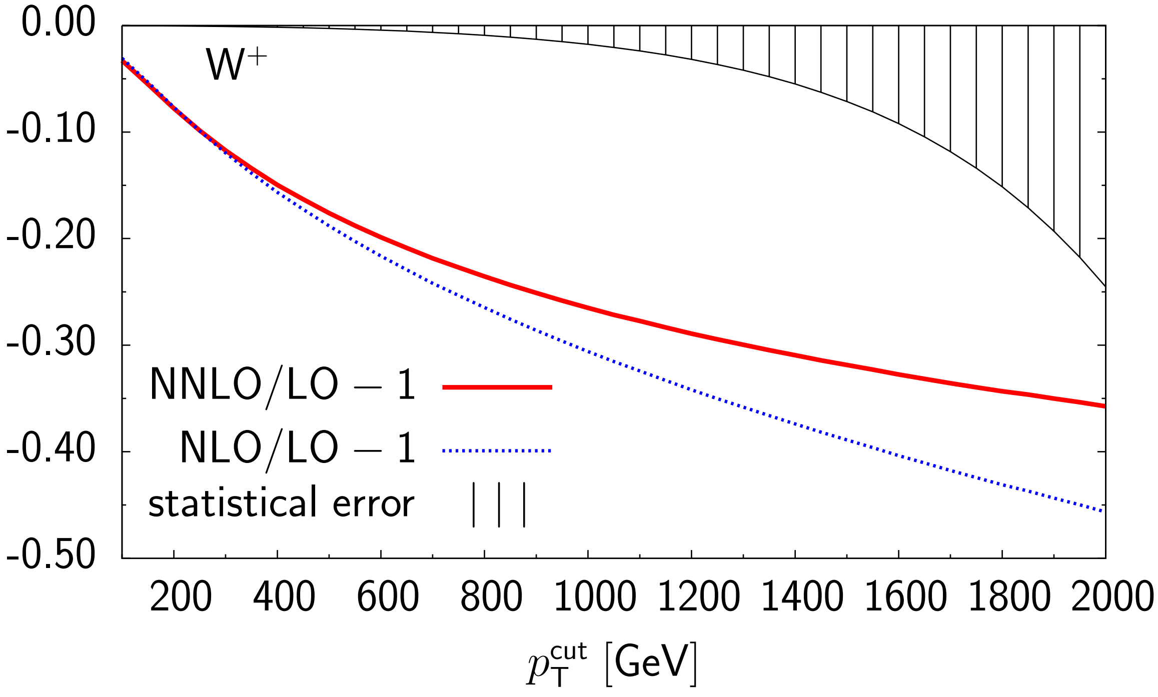
- dominant two-loop terms ($\log^4 + \log^3$) available

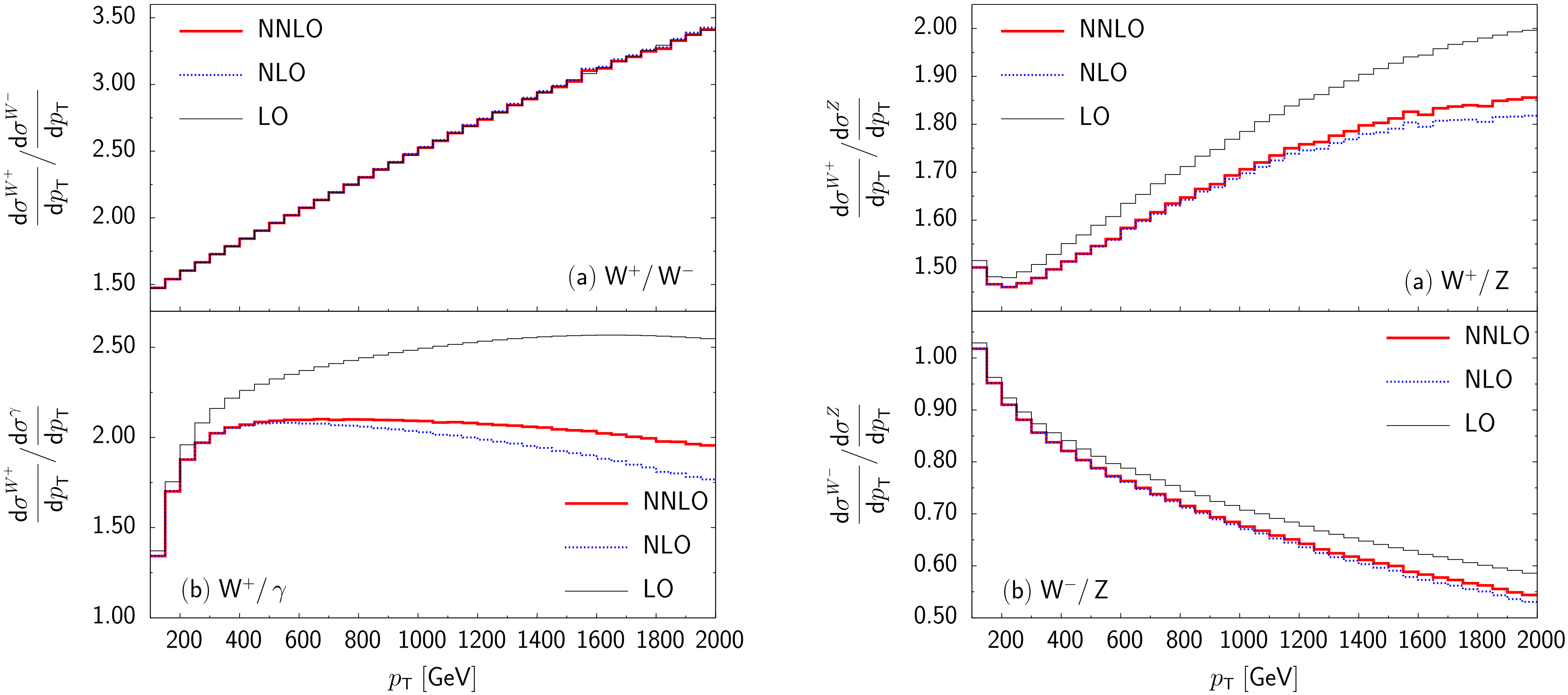


W production

additional complications:

- photon radiation as necessary part of virtual corrections (gauge invariance)
- IR singularities must be compensated by real radiation
- $p_\mathrm{T}(W) = p_\mathrm{T}(\mathrm{jet}) + p_\mathrm{T}(\gamma)$





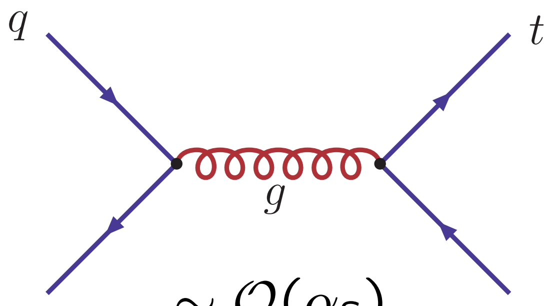
ratios are less sensitive to QCD corrections

III. Top Production:

J.H.K., Scharf, Uwer

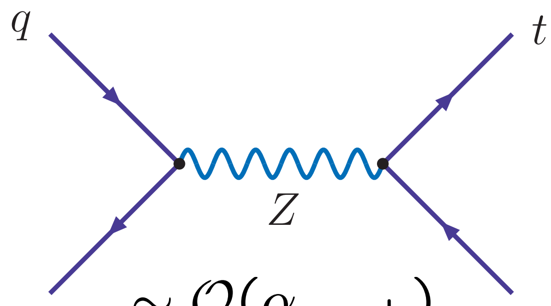
Eur. Phys. J. C45(2006) 139
Eur. Phys. J. C51(2007) 37

$q\,\bar{q} \longrightarrow t\,\bar{t}:$



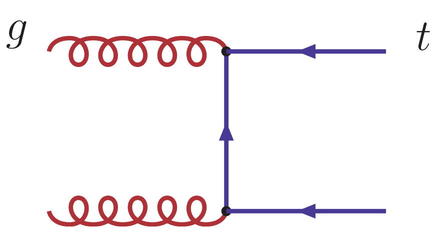
$\sim \mathcal{O}(\alpha_{\mathrm{s}})$

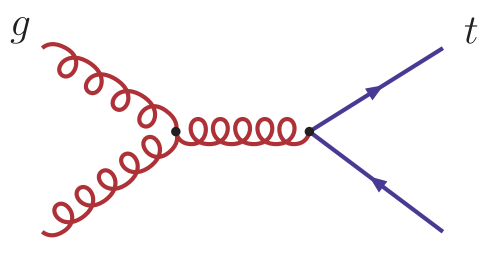
no interference with



$\sim \mathcal{O}(\alpha_{\mathrm{weak}})$

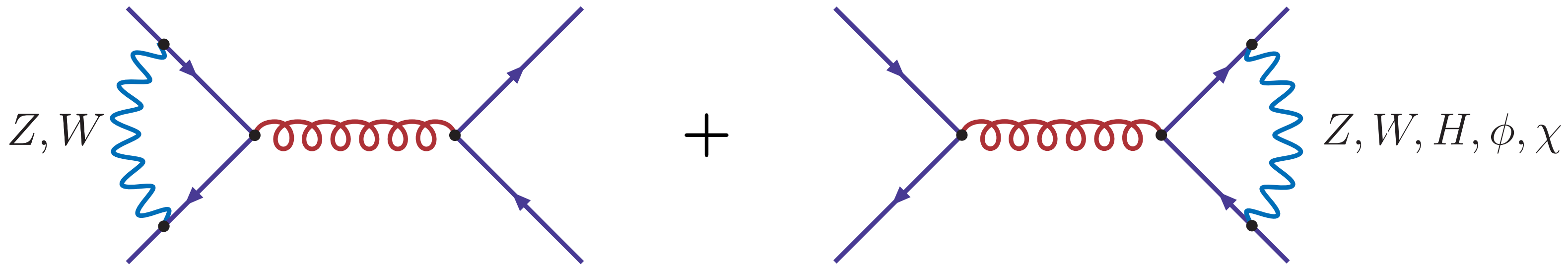
$g\,g \longrightarrow t\,\bar{t}:$

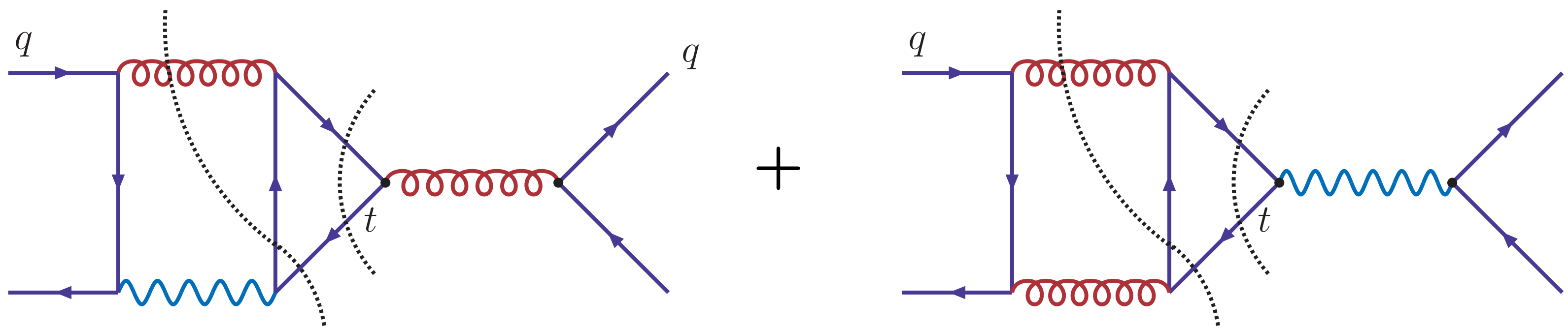




$\sim \mathcal{O}(\alpha_{\mathrm{s}})$

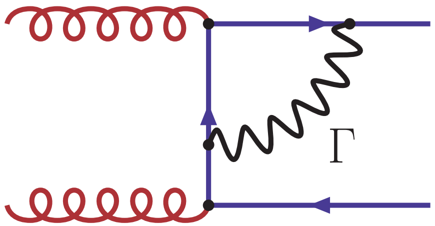
$\mathcal{O}(\alpha_s^2\alpha_{\text{weak}})$ weak corrections $(q\,\bar{q} \to t\,\bar{t}\,)$

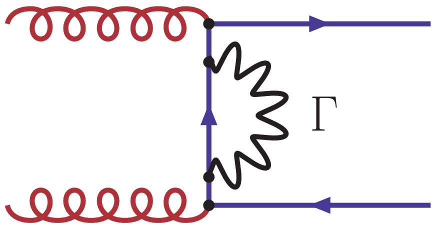


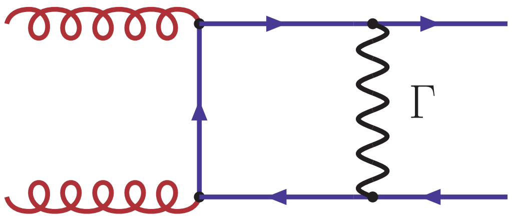


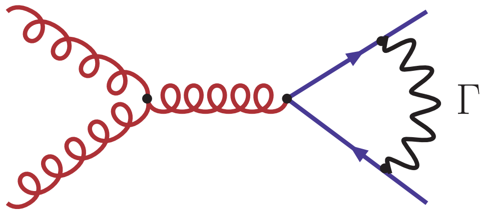
cuts of second group individually IR-divergent

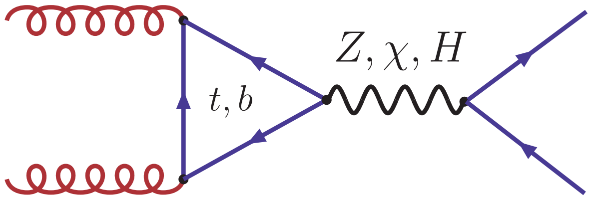
$\mathcal{O}(\alpha_s^2 \alpha_{\mathrm{weak}})$ weak corrections $(g\, g \to t\, \bar{t}\,)$











- analytical & numerical results available
  (earlier partial results by Beenakker *et al.*, some disagreements)
  independent evaluation by Bernreuther & Fücker

- $(\text{box contribution})_{\text{up-quark}} = -(\text{box contribution})_{\text{down-quark}}$
  $\Rightarrow$ suppression

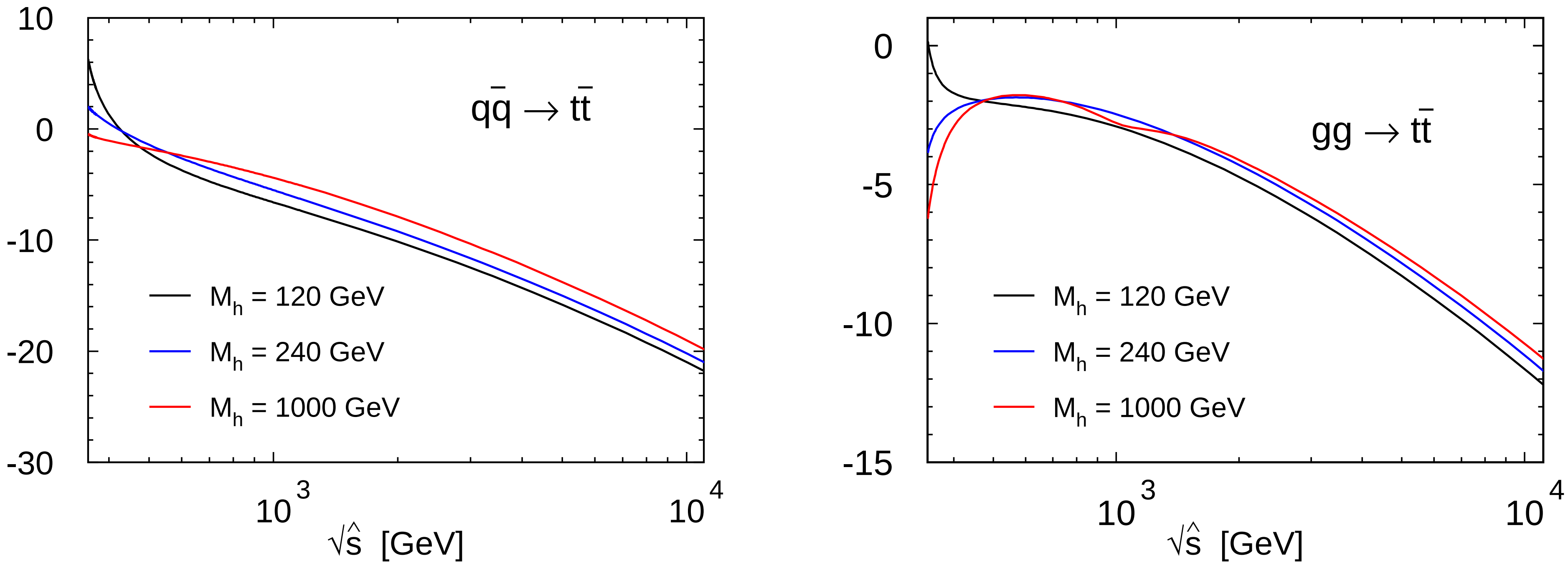
- box contribution moderately $\hat{s}$-dependent

- strong increase with $\hat{s}$

- sizable M_{h}-dependence, large effect close to threshold

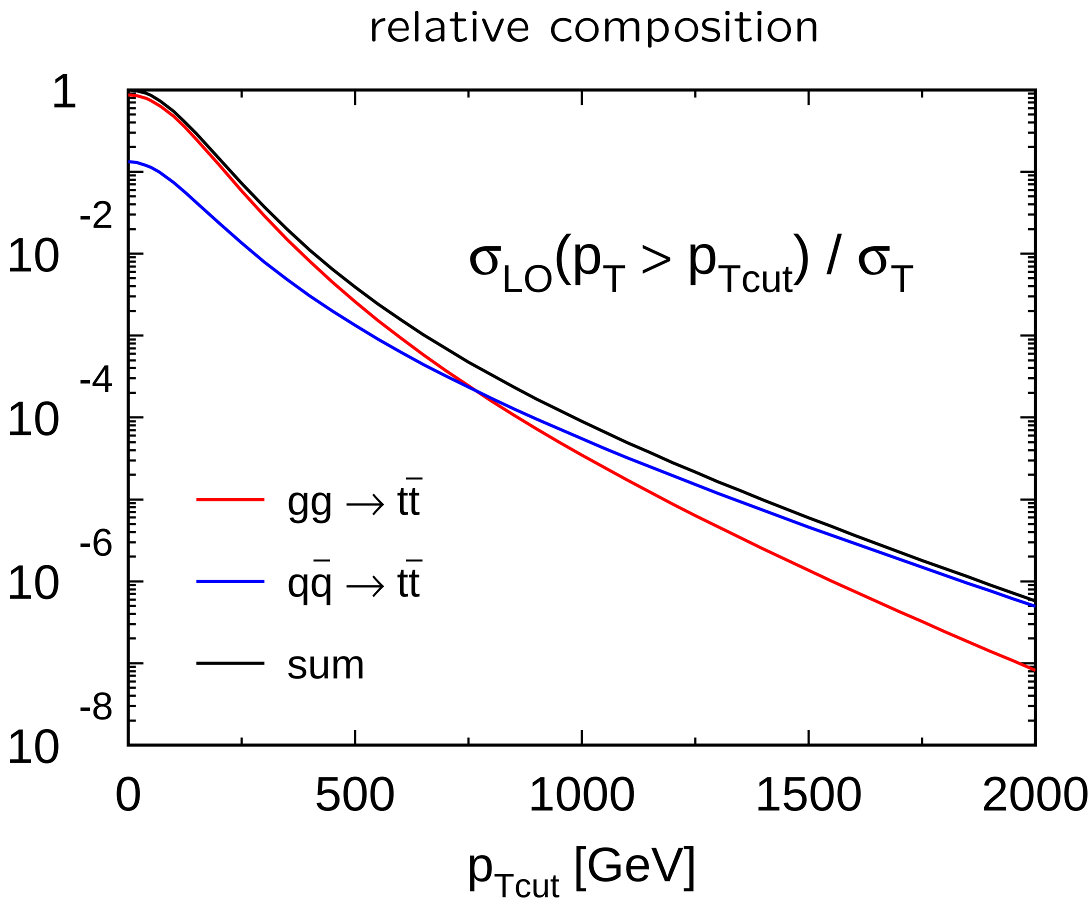
large corrections for large $\sqrt{\hat{s}}$

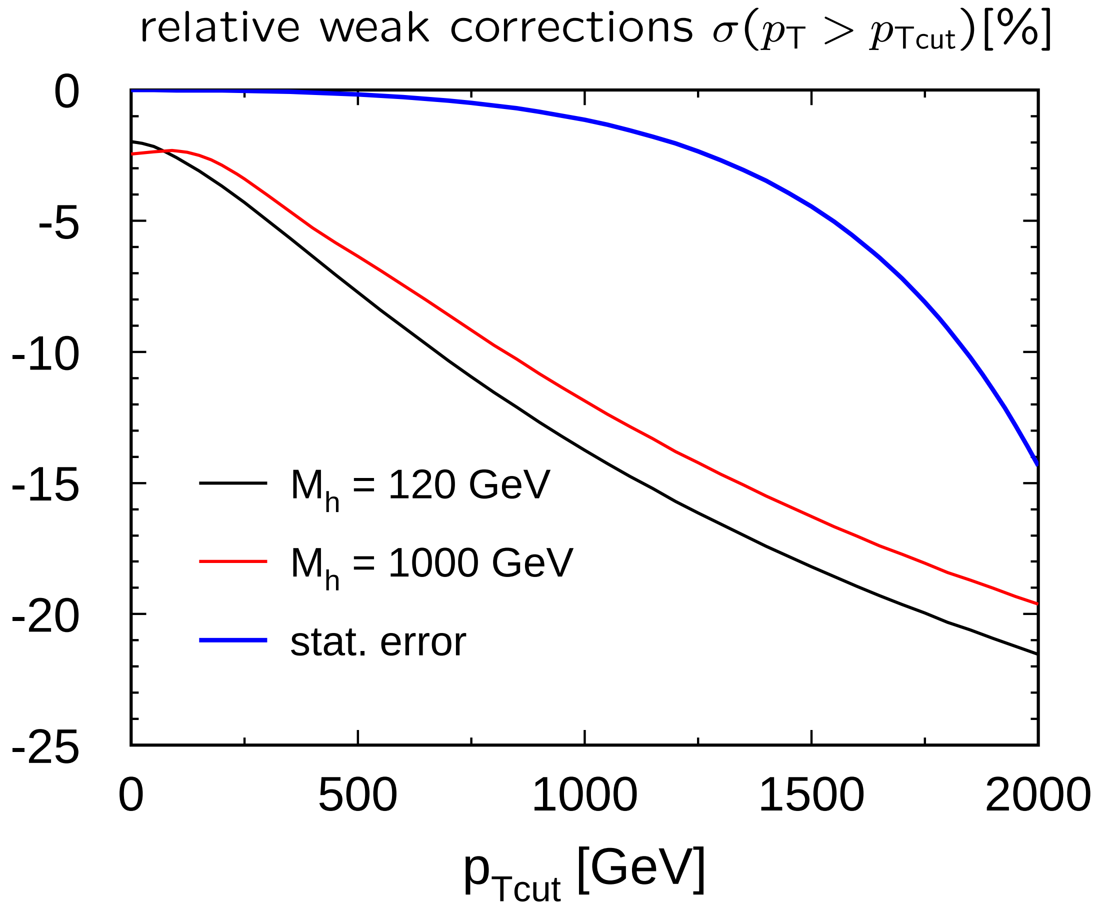
sizable M_h-dependence



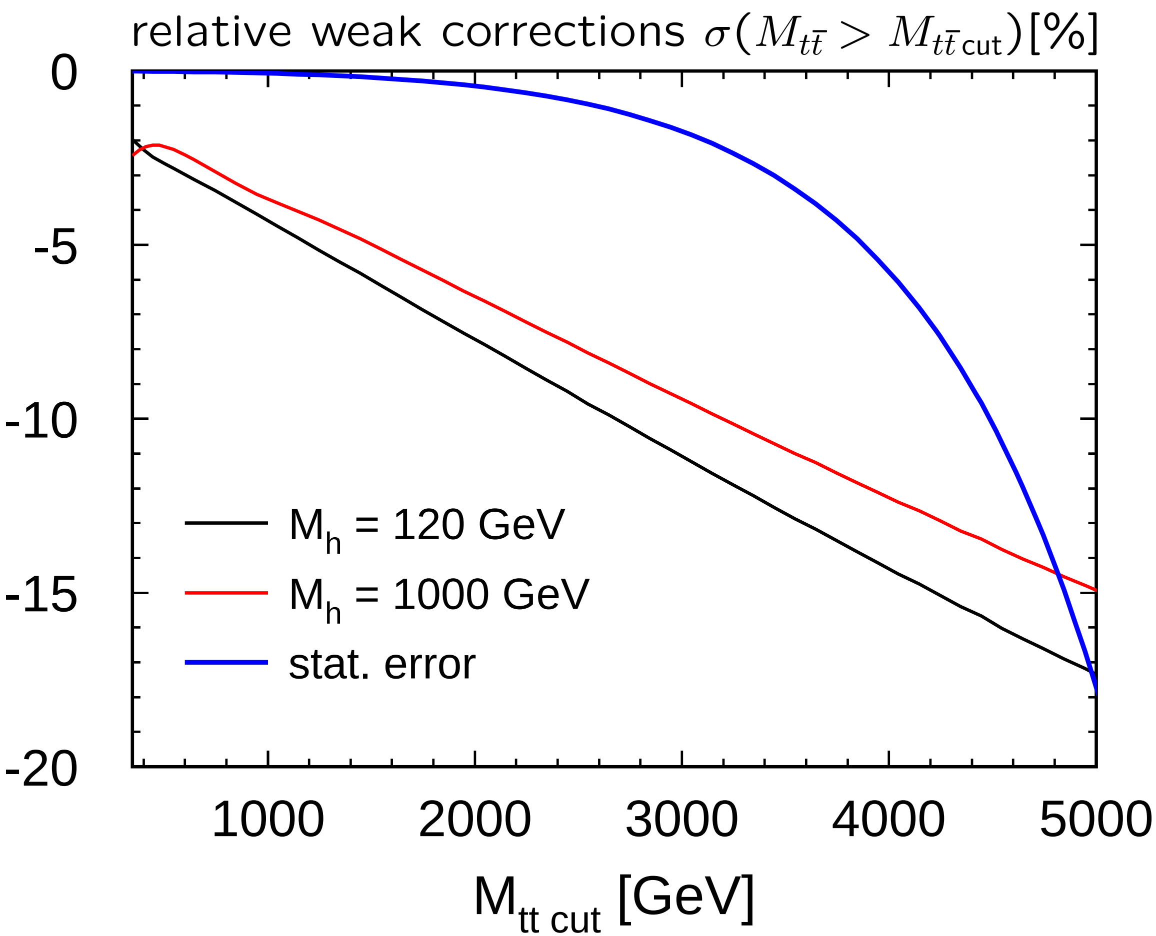
(relative weak corrections [%])

Transverse momentum dependence





$M_{t\bar{t}}$ -dependence



IV. Conclusions

- LHC will explore the TeV-region: $\hat{s}\big/M_W^2 \gg 1$

- electroweak corrections amount to $\mathcal{O}(10\% - 20\%)$ in the interesting kinematic region

- form factors and four-fermion scattering in two loop

- p_{T}-distributions of Z, W, γ and their ratios will be strongly affected

- two-loop terms might become relevant

- top-quark distributions at large $\hat{s}$ are strongly modified

- sizable M_{h}-dependence