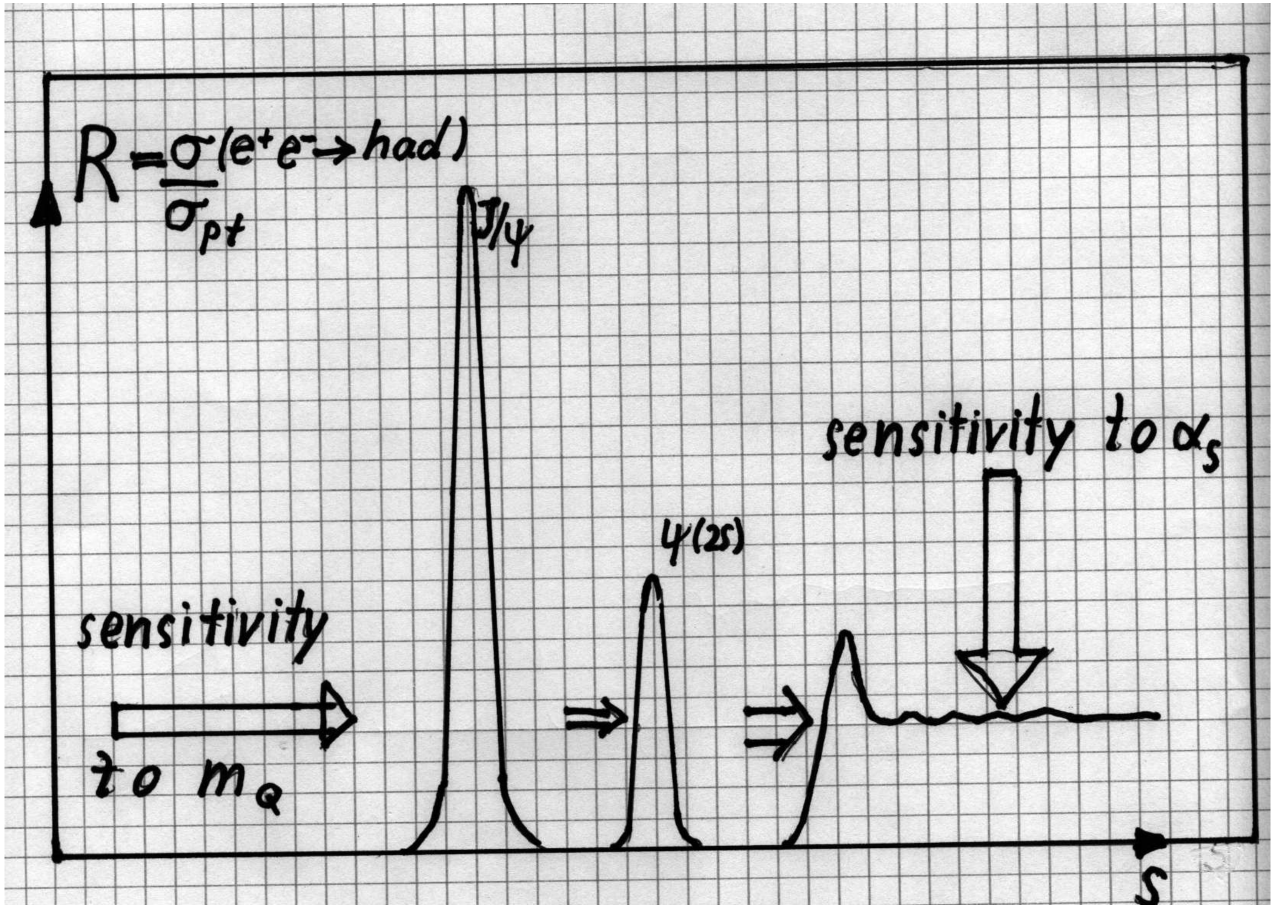
QUARK MASSES AND α_s

FROM $\sigma(e^+e^- \to \textbf{had})$

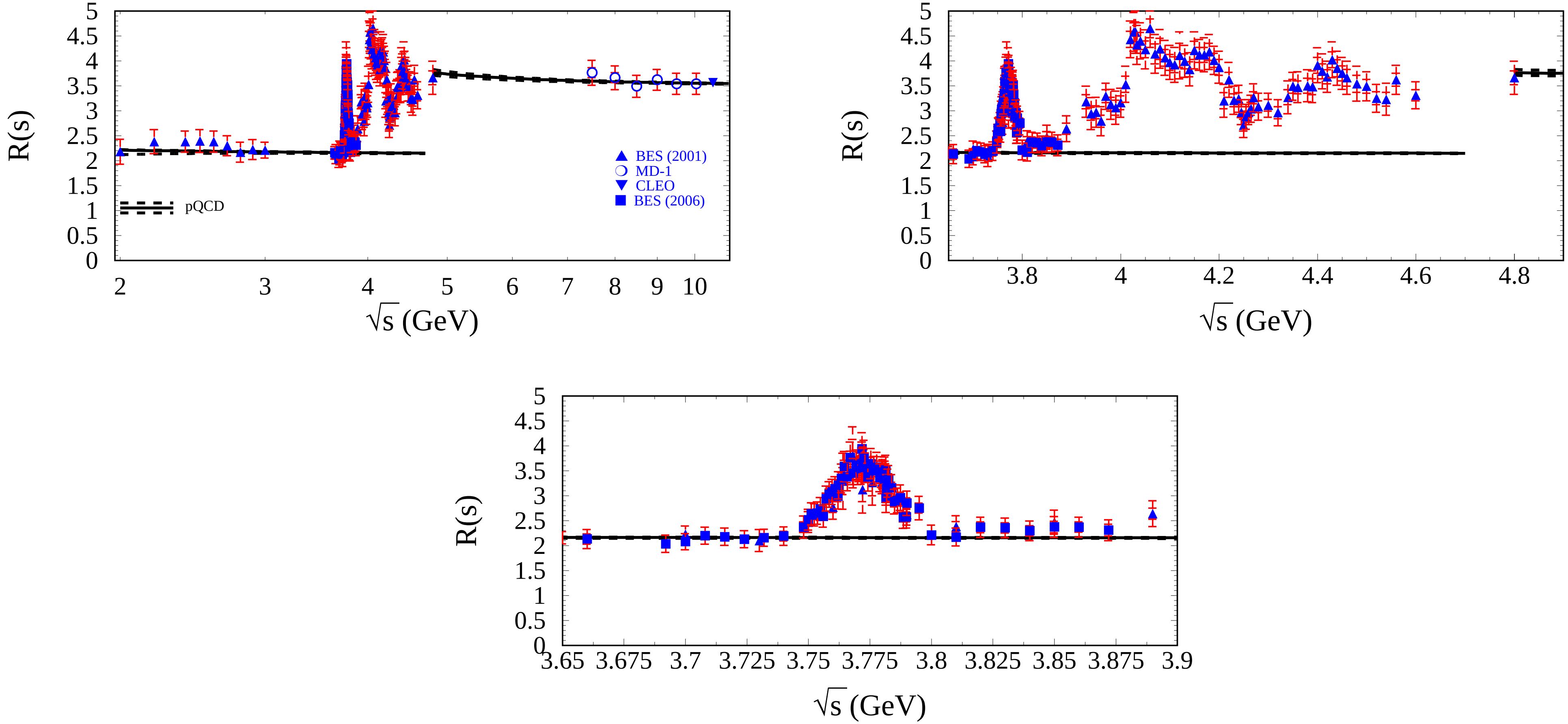
JK, Steinhauser, Sturm NPB
JK, Steinhauser, Teubner PRD

Karlsruhe Institute of Technology
Universität Karlsruhe (TH)
Forschungsuniversität • gegründet 1825
Computational Theoretical Particle Physics SFB TR9

Main Idea (SVZ)



Data



pQCD and data agree well in the regions
2 – 3.73 GeV and 5 – 10.52 GeV

experiment	energy [GeV]	date	systematic error
BES	2 — 5	2001	4 %
MD-1	7.2 — 10.34	1996	4 %
CLEO	10.52	1998	2 %
PDG	J/ψ		(7 %) 2.5 %
PDG	ψ'		(9 %) 2.4 %
PDG	ψ''		(15 %)
BES	ψ'' region	2006	4 %

Future improvements:

charm region (CLEO) 3%

bottom region ?? (CLEO)

$m_Q$from

SVZ Sum Rules, Moments and Tadpoles

Some definitions:

$$R(s) \;=\; 12\pi\,\mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right)\,\Pi(q^2) \;\equiv\; i\int \mathrm{d}x\, e^{iqx}\langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) \;=\; Q_Q^2\,\dfrac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n\, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2
[Chetyrkin, JK, Steinhauser, 1996]

up to high $n(\sim 30)$; VV, AA, PP, SS correlators
[Czakon et al., 2006], [Maierhöfer, Maier, Marquard, 2007]

➪ reduction to master integrals through Laporta algorithm
[Chetyrkin, JK, Sturm]; confirmed by [Boughezal, Czakon, Schutzmeier]

evaluation of master integrals numerically through difference equations

(30 digits) or Padé method or analytically in terms of transcendentals
[Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser,
Laporta, Broadhurst, Kniehl et al.]

$\bar{C}_2$ would be desirable!

Define the moments

$$\mathcal{M}_n^{\mathrm{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

$$\mathcal{M}_n^{\mathrm{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint:

$$\mathcal{M}_n^{\mathrm{exp}} = \mathcal{M}_n^{\mathrm{th}}$$

➪ m_c

update compared to NPB619 (2001)

experiment:

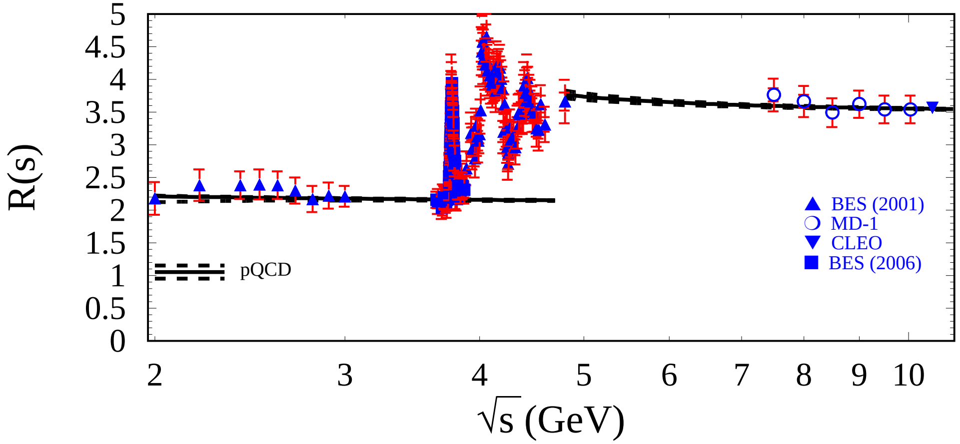
- $\alpha_s = 0.1187 \pm 0.0020$
- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & Babar
- $\psi(3770)$ from BES

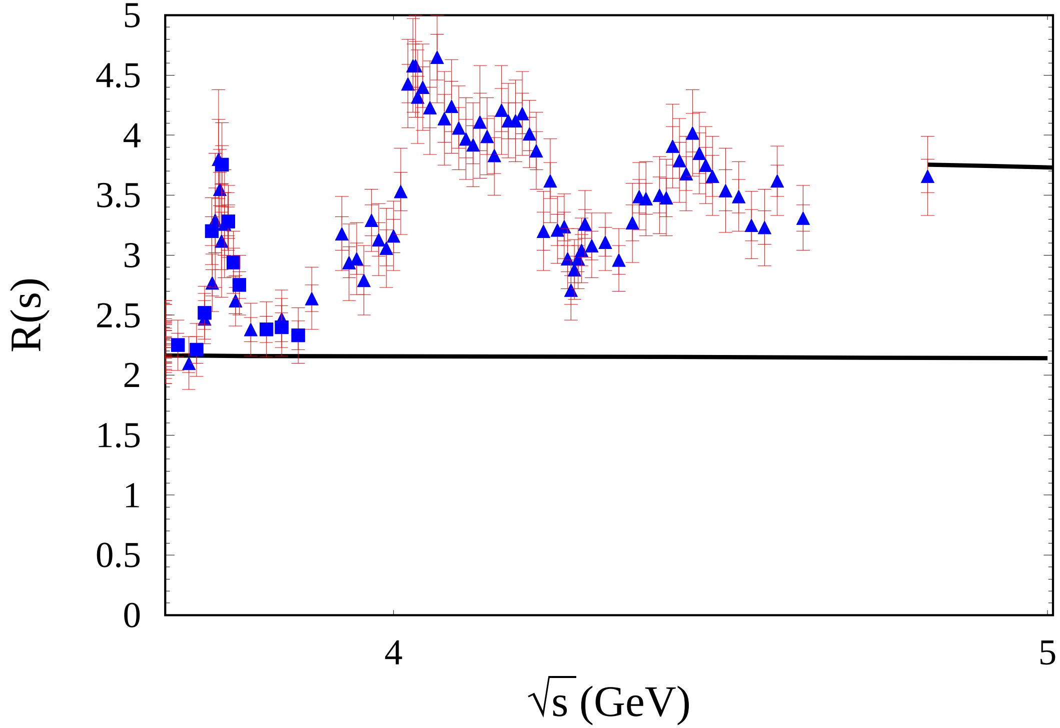
theory:

- N^3LO for n=1
- N^3LO - estimate for n =2,3,4
- include condensates

$$\delta\mathcal{M}_n^{\text{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi}\bar{b}_n\right)$$

- estimate of non-perturbative terms (oscillations, based on Shifman)
- careful extrapolation of R_{uds}
- careful definition of R_c


R(s)
√s (GeV)
BES (2001)
MD-1
CLEO
BES (2006)
pQCD


R(s)
√s (GeV)

Contributions from

- narrow resonances: $R = \frac{9\,\Pi\,M_R\,\Gamma_e}{\alpha^2(s)}\,\delta(s - M_R^2)$
- threshold region ($2\,m_D$ – 4.8 GeV)
- perturbative continuum ($E \geq 4.8$ GeV)

Results (m_c)

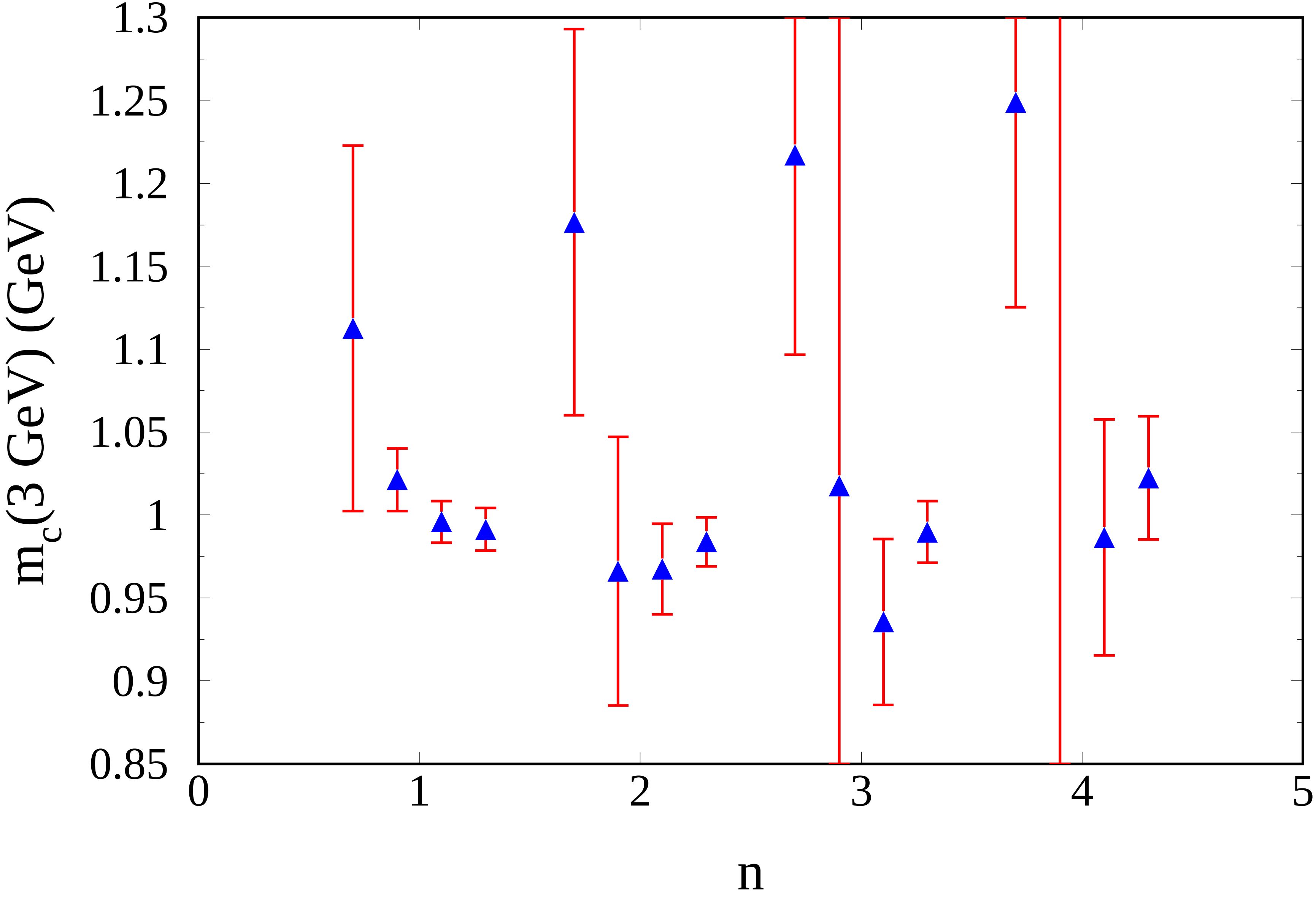
n	m_c(3 GeV)	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$n = 1$:

- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1286 \pm 13\,\text{MeV}$

Knowledge of C_n^{30} for $n = 2, 3$!?

other ("experimental") determinations of $\mathcal{M}_n$?



m_c(3 GeV) (GeV)
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
n

Results (m_b)

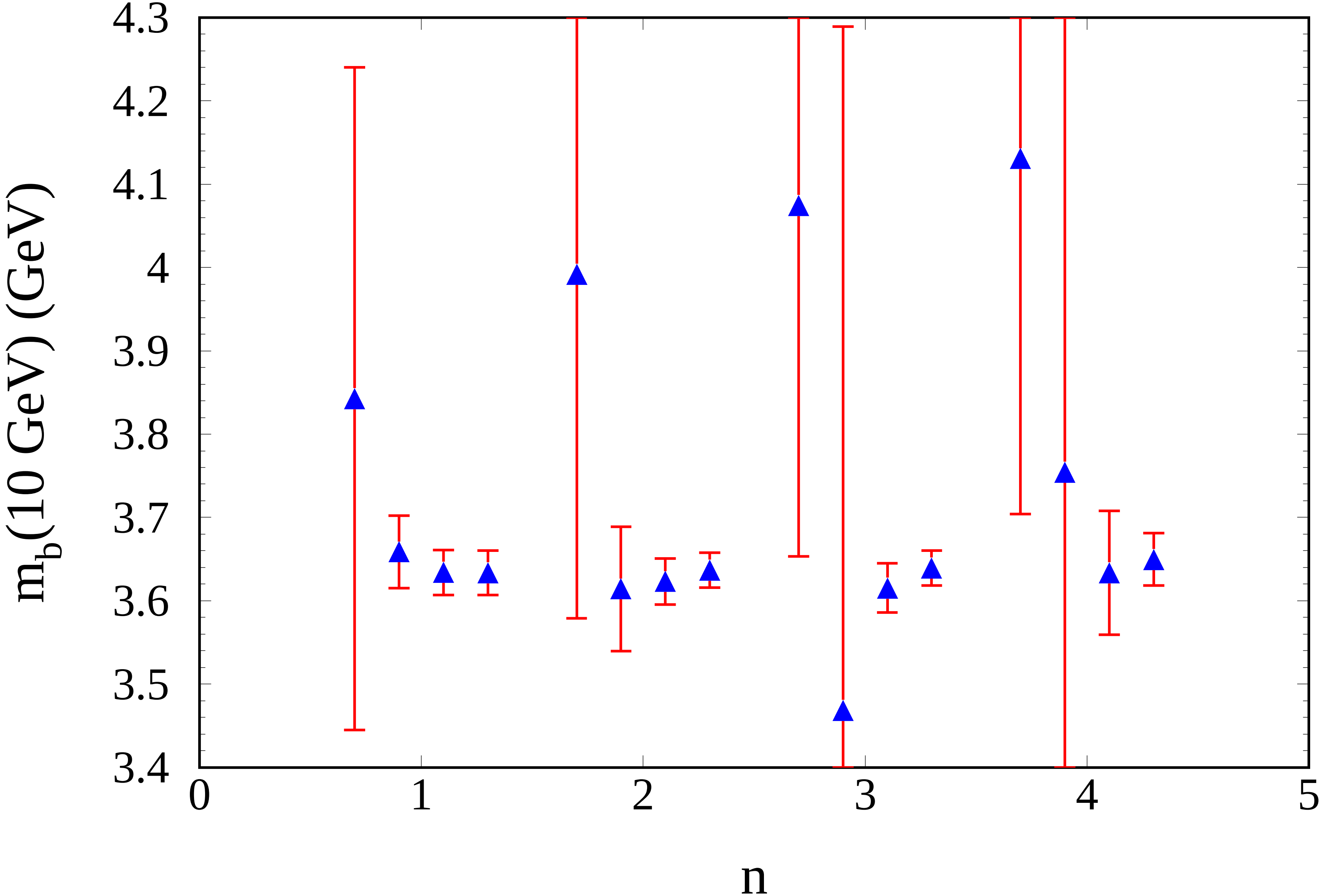
n	m_b(10 GeV)	exp	α_s	μ	total	$\delta\bar{C}_n^{30}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$n = 2$:

- $m_b(m_b) = 4164 \pm 25\,\text{MeV}$
- $m_b(10\text{GeV}) = 3609 \pm 25\,\text{MeV}$
- $m_b(m_t) = 2703 \pm 18 \pm 19\,\text{MeV}$
- $m_t/m_b = 59.8 \pm 1.3$

Knowledge of C_n^{30} for $n = 2, 3$ to confirm estimate!?

data above 11GeV?

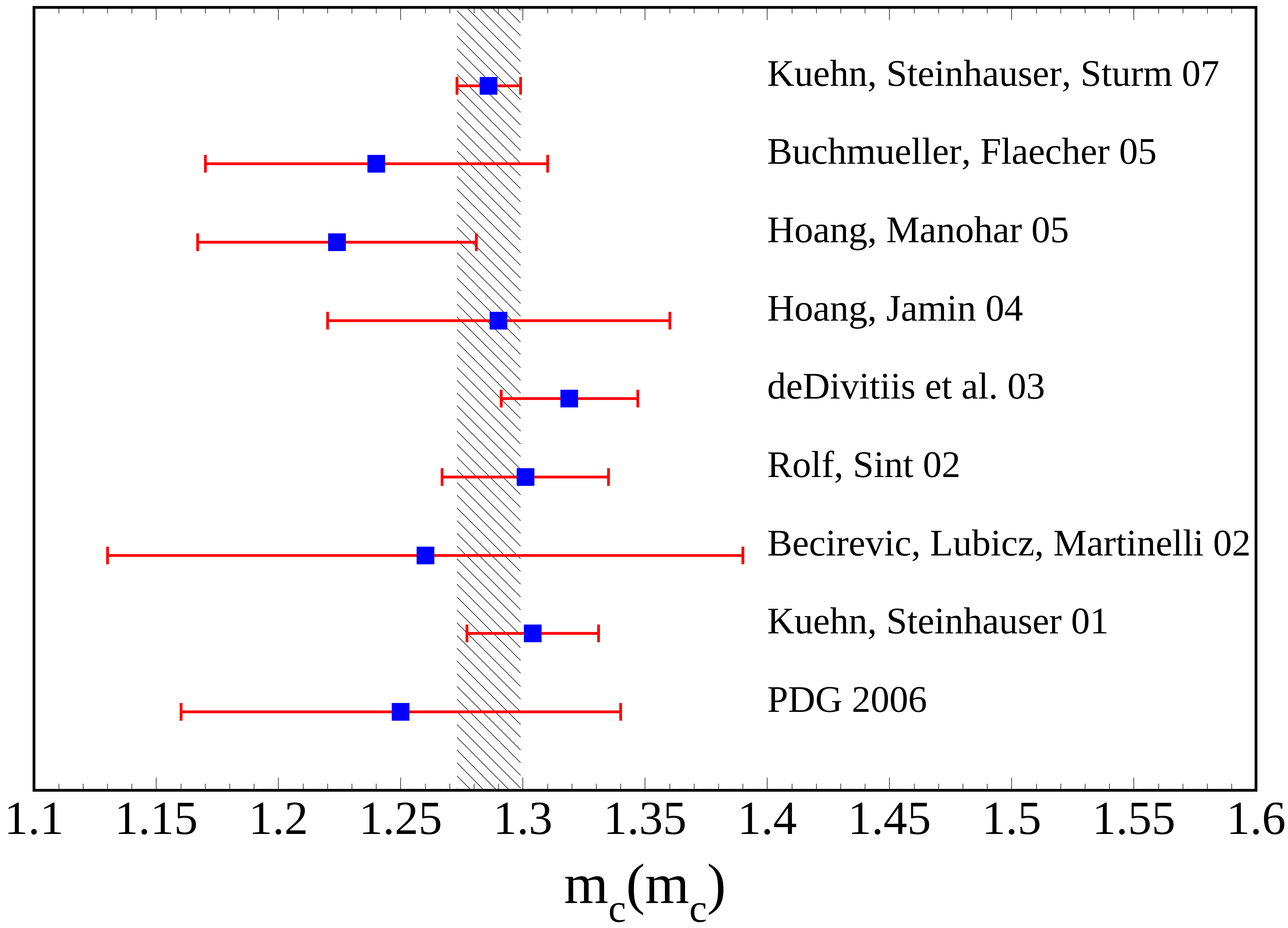


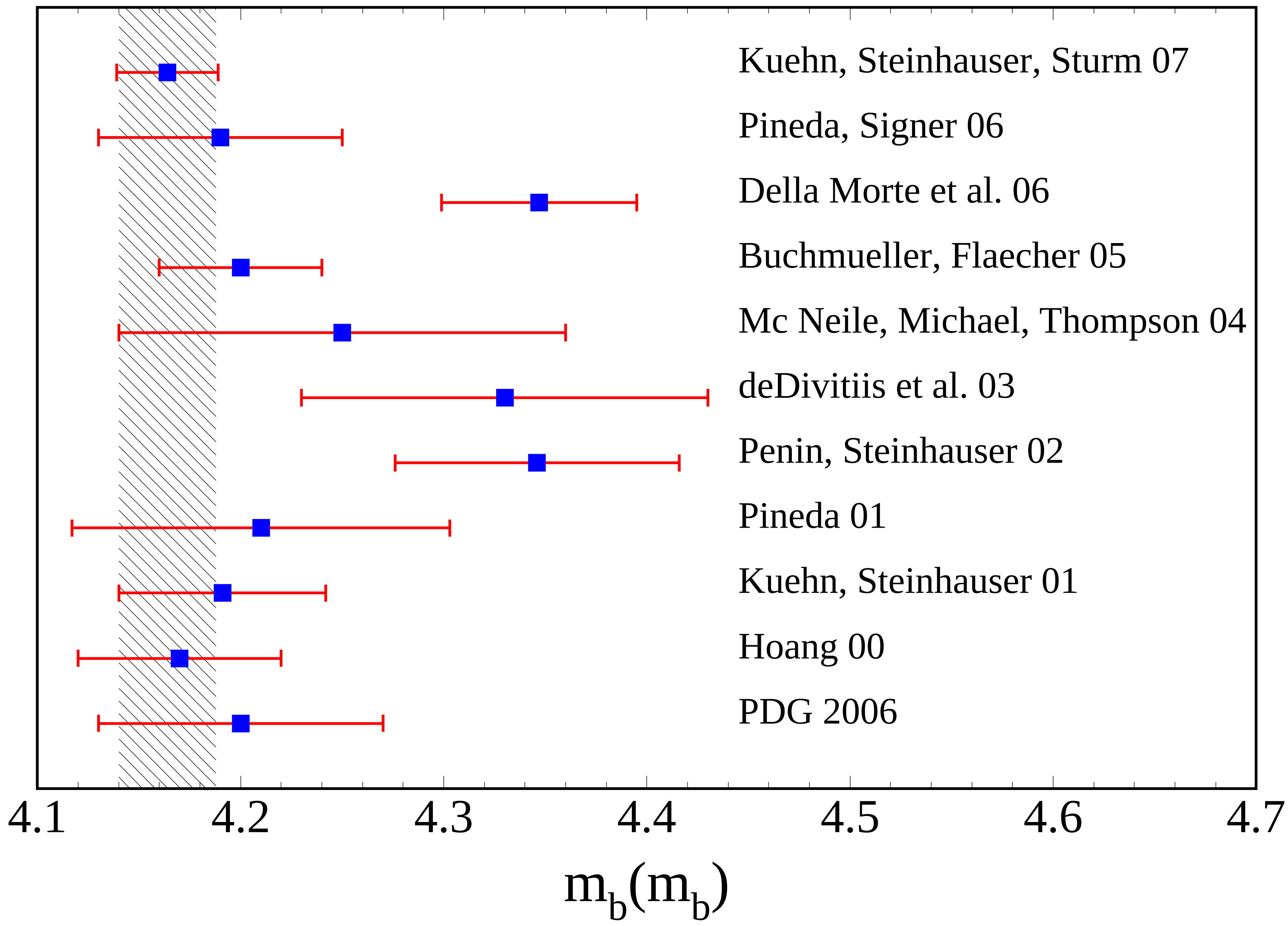
4.3
4.2
4.1
4
3.9
3.8
3.7
3.6
3.5
3.4
$m_b(10\ GeV)\ (GeV)$
0
1
2
3
4
5
n

$$m_c(3\ \text{GeV}) = 0.986(13)\ \text{GeV}$$
$$m_c(m_c) = 1.286(13)\ \text{GeV}$$

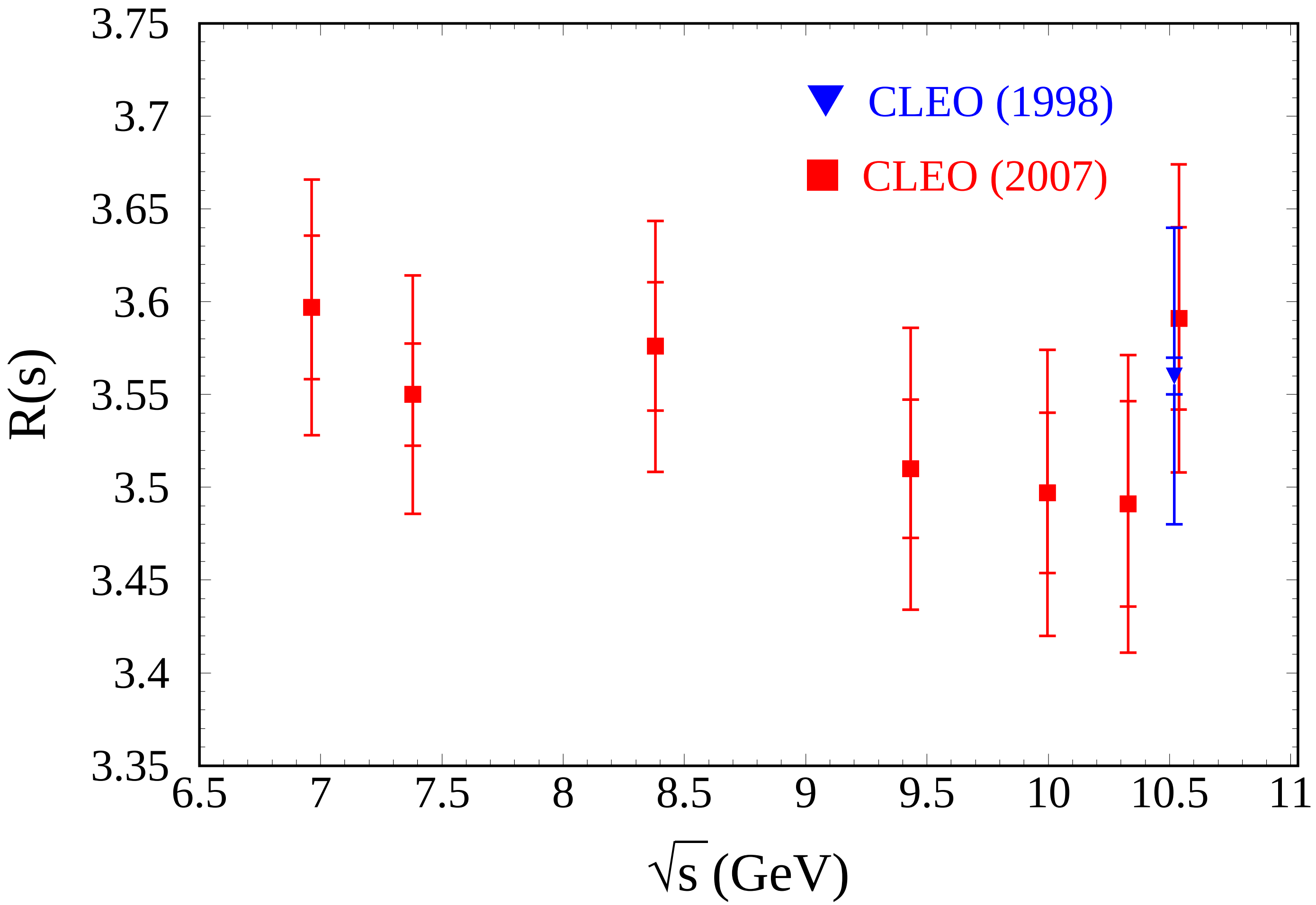
$$m_b(10\ \text{GeV}) = 3.609(25)\ \text{GeV}$$
$$m_b(m_b) = 4.164(25)\ \text{GeV}$$

(old result: $m_c(m_c) = 1.304(27)$GeV, $m_b(m_b) = 4.191(51)$GeV)


Kuehn, Steinhauser, Sturm 07
Buchmueller, Flaecher 05
Hoang, Manohar 05
Hoang, Jamin 04
deDivitiis et al. 03
Rolf, Sint 02
Becirevic, Lubicz, Martinelli 02
Kuehn, Steinhauser 01
PDG 2006
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
1.55
1.6
$m_c(m_c)$


Kuehn, Steinhauser, Sturm 07
Pineda, Signer 06
Della Morte et al. 06
Buchmueller, Flaecher 05
Mc Neile, Michael, Thompson 04
deDivitiis et al. 03
Penin, Steinhauser 02
Pineda 01
Kuehn, Steinhauser 01
Hoang 00
PDG 2006
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$

R measurement and α_s



α_s and R

basic idea: $R^{\rm exp} = R^{\rm th}(\alpha_s, m_q) \Rightarrow \alpha_s$ (weak dependence on variation of m_q)

rhad: [Harlander,Steinhauser'02]

$R^{\rm th}(s)$:

- full quark mass dependence up to $\mathcal{O}(\alpha_s^2)$
- $\mathcal{O}(\alpha_s^3)$: $(m_q^2/s)^0$, $(m_q^2/s)^1$, $(m_q^2/s)^2$
- ...
- consistent running and decoupling of α_s

[v. Ritbergen,Larin,Vermaseren'97,Czakon'05]

[Chetyrkin,Kniehl,Steinhauser'97]

α_s and R

basic idea: $R^{\text{exp}} = R^{\text{th}}(\alpha_s, m_q) \Rightarrow \alpha_s$ (weak dependence on variation of m_q)

rhad: [Harlander,Steinhauser'02]

$R^{\text{exp}}(s) \Rightarrow \alpha_s^{(4)}(s)$ $(n_f = 4)$

$\sqrt{s}$ (GeV)	$\alpha_s^{(4)}(s)$	$\delta\alpha_s^{\text{stat}}$	$\delta\alpha_s^{\text{sys,cor}}$	$\delta\alpha_s^{\text{sys,uncor}}$	$\alpha_s^{(4)}(s)\vert_{\text{CLEO}}$
10.538	0.2113	0.0026	0.0618	0.0444	0.232
10.330	0.1280	0.0048	0.0469	0.0445	0.142
9.996	0.1321	0.0032	0.0516	0.0344	0.147
9.432	0.1408	0.0039	0.0526	0.0291	0.159
8.380	0.1868	0.0187	0.0461	0.0195	0.218
7.380	0.1604	0.0131	0.0404	0.0138	0.195
6.964	0.1881	0.0221	0.0386	0.0134	0.237

$\Uparrow$
massless
approx.!!!

α_s and R

basic idea: $R^{\text{exp}} = R^{\text{th}}(\alpha_s, m_q) \Rightarrow \alpha_s$ (weak dependence on variation of m_q)

rhad: [Harlander,Steinhauser'02]

$R^{\text{exp}}(s) \Rightarrow \alpha_s^{(4)}(s)$ $(n_f = 4)$

- Evolve to common scale and combine

  $\Rightarrow \alpha_s^{(4)}(9\ \text{GeV}) = 0.160 \pm 0.024 \pm 0.024$

α_s and R

basic idea: $R^{\text{exp}} = R^{\text{th}}(\alpha_s, m_q) \Rightarrow \alpha_s$ (weak dependence on variation of m_q)

rhad: [Harlander,Steinhauser'02]

$R^{\text{exp}}(s) \Rightarrow \alpha_s^{(4)}(s)$ $(n_f = 4)$

- Evolve to common scale and combine

  $\Rightarrow \alpha_s^{(4)}(9\ \text{GeV}) = 0.160 \pm 0.024 \pm 0.024$

- $\alpha_s^{(4)}(9\ \text{GeV}) \to \alpha_s^{(4)}(\mu_b^{\text{dec}}) \to \alpha_s^{(5)}(\mu_b^{\text{dec}}) \to \alpha_s^{(5)}(M_Z)$
  (practically) independent from μ_b^{dec} (4-loop running and 3-loop decoupling) RunDec: [Chetyrkin,JK,Steinhauser'00]

$\Rightarrow \alpha_s^{(5)}(M_Z) = 0.110^{+0.010+0.010}_{-0.012-0.011} = 0.110^{+0.014}_{-0.017}$ [JK,Steinhauser,Teubner'07]

α_s and R

basic idea: $R^{\text{exp}} = R^{\text{th}}(\alpha_s, m_q)$ ➪ α_s (weak dependence on variation of m_q)

rhad: [Harlander,Steinhauser'02]

$R^{\text{exp}}(s)$ ➪ $\alpha_s^{(4)}(s)$ $(n_f = 4)$

- Evolve to common scale and combine

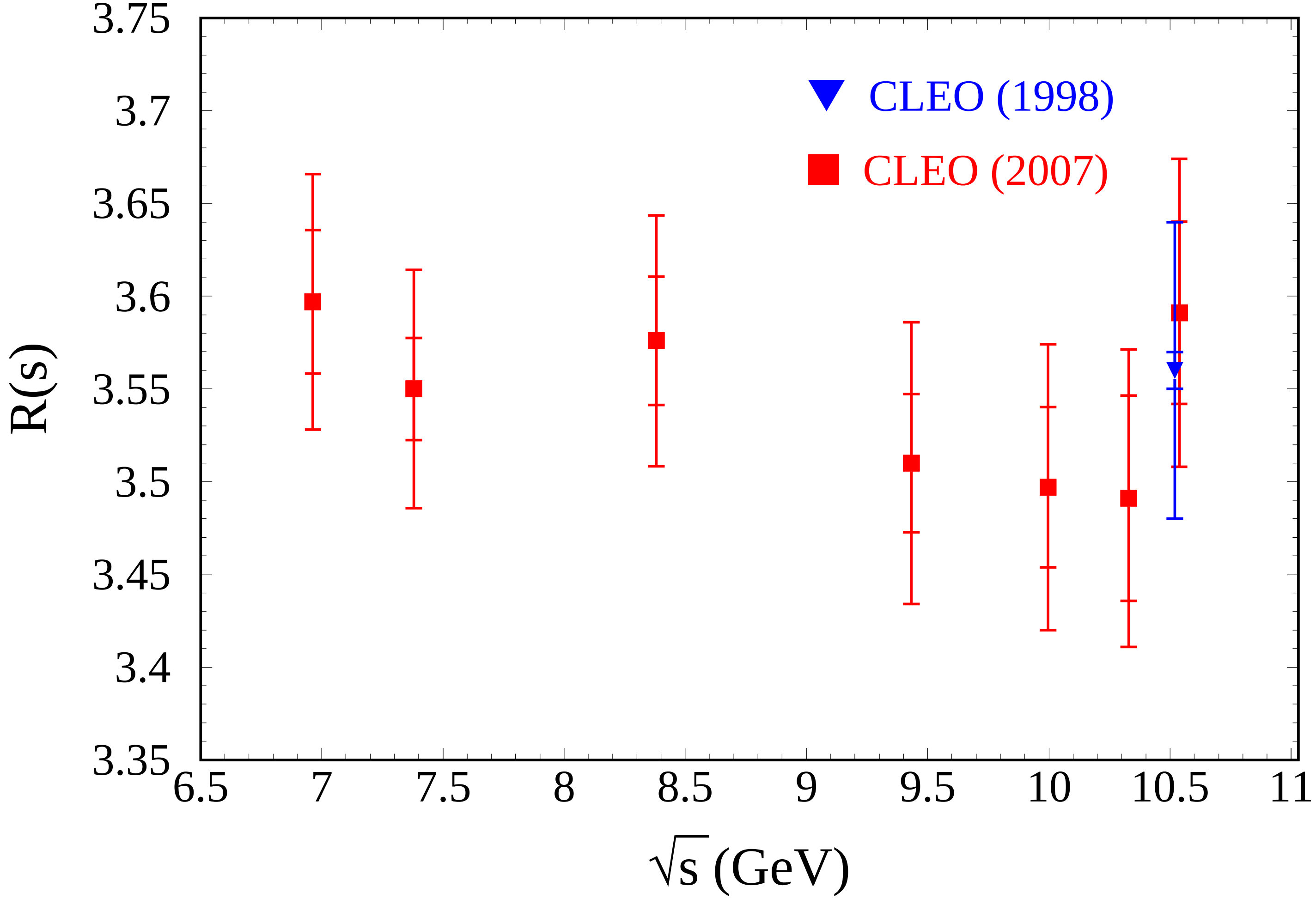
  ➪ $\alpha_s^{(4)}(9\ \text{GeV}) = 0.160 \pm 0.024 \pm 0.024$

- $\alpha_s^{(4)}(9\ \text{GeV}) \to \alpha_s^{(4)}(\mu_b^{\text{dec}}) \to \alpha_s^{(5)}(\mu_b^{\text{dec}}) \to \alpha_s^{(5)}(M_Z)$
  (practically) independent from μ_b^{dec} (4-loop running and 3-loop decoupling) RunDec: [Chetyrkin,JK,Steinhauser'00]

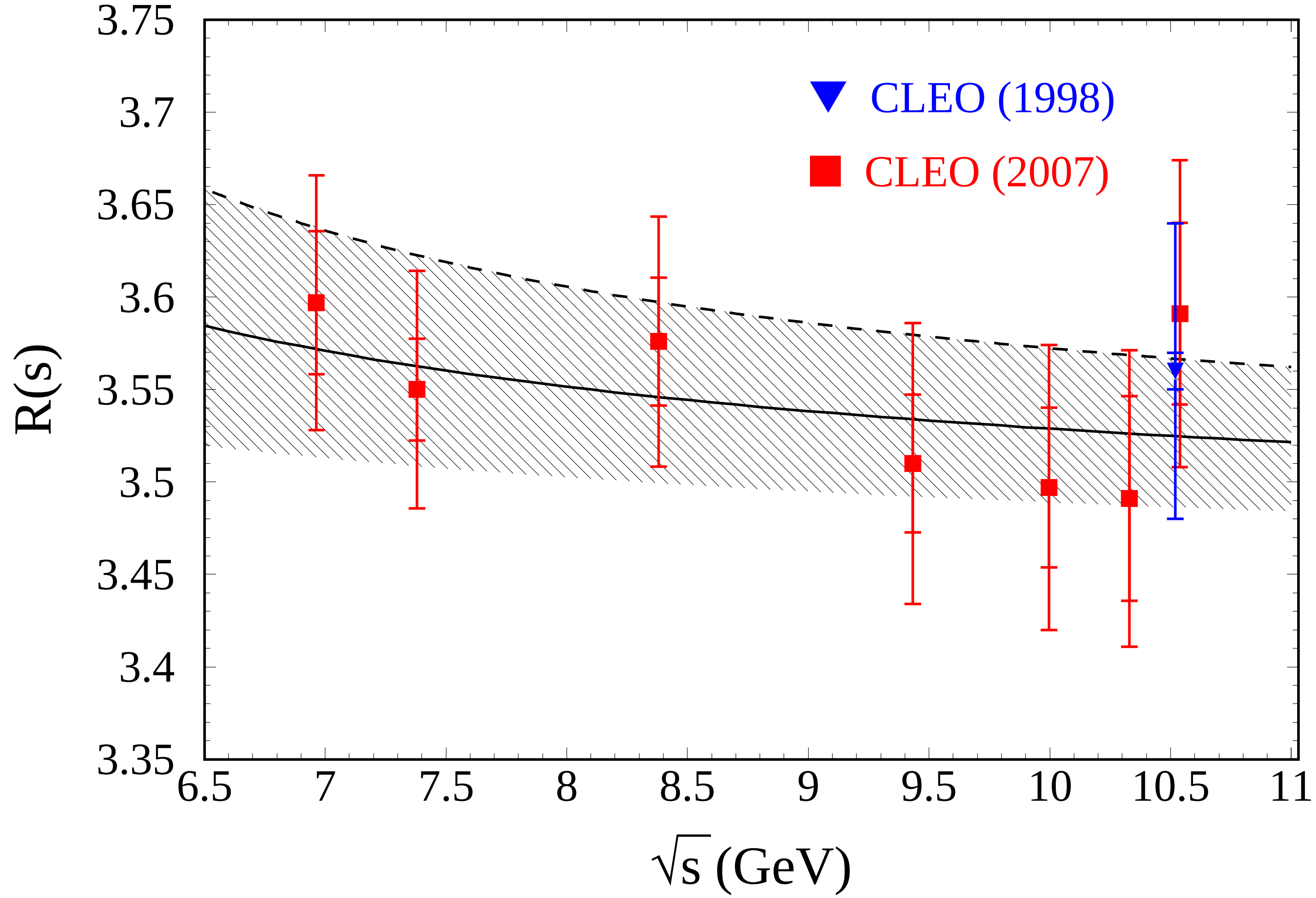
➪ $\alpha_s^{(5)}(M_Z) = 0.110^{+0.010+0.010}_{-0.012-0.011} = 0.110^{+0.014}_{-0.017}$ [JK,Steinhauser,Teubner'07]

- CLEO analysis: $\alpha_s^{(5)}(M_Z^2)|_{\text{CLEO}} = 0.126 \pm 0.005^{+0.015}_{-0.011}$
  massless approximation for $R(s)$, no decoupling of α_s

R: experiment + theory



R: experiment + theory



α_s **from** R

- $\alpha_s^{(5)}(M_Z) = 0.110^{+0.010}_{-0.012}{}^{+0.010}_{-0.011} = 0.110^{+0.014}_{-0.017}$ [JK,Steinhauser,Teubner'07]

- Combine with $\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$ [JK,Steinhauser'01]

  R measurements between 2 and 10.5 GeV from

  BES'01, MD-1'96, CLEO'97

➪ $\alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}$

- Compare: $\alpha_s^{(5)}(M_Z) = 0.1189 \pm 0.0010$ [Bethke'06]