PRECISE QUARK MASSES:

m_c and m_b

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SFB TR-9

I Generalities

WHY precise masses?

- B-decays
- Υ-spectroscopy
- sum rules
- perturbative vs. lattice

Higgs decay (ILC)

$$\Gamma(H \to b\bar{b}) = \frac{G_F\, M_H}{4\sqrt{2}\pi}\, m_b^2(M_H)\, \tilde{R}$$

$$\tilde{R} \;=\; 1 + 5.6667 a_s + 29.147 {a_s}^2 + 41.758 {a_s}^3 - 825.7 {a_s}^4$$

$$1 + 0.2075 \quad + 0.0391 \quad + 0.0020 \quad - 0.0015$$

$(m_H = 120\text{GeV})$

rapidly increasing coefficients! $\left(a_s \equiv \frac{\alpha_s}{\pi}\right)$

${a_s}^4$-term = 5-loop calculation (Baikov,...)

(not yet known for $e^+e^- \to$ had)

Yukawa Unification

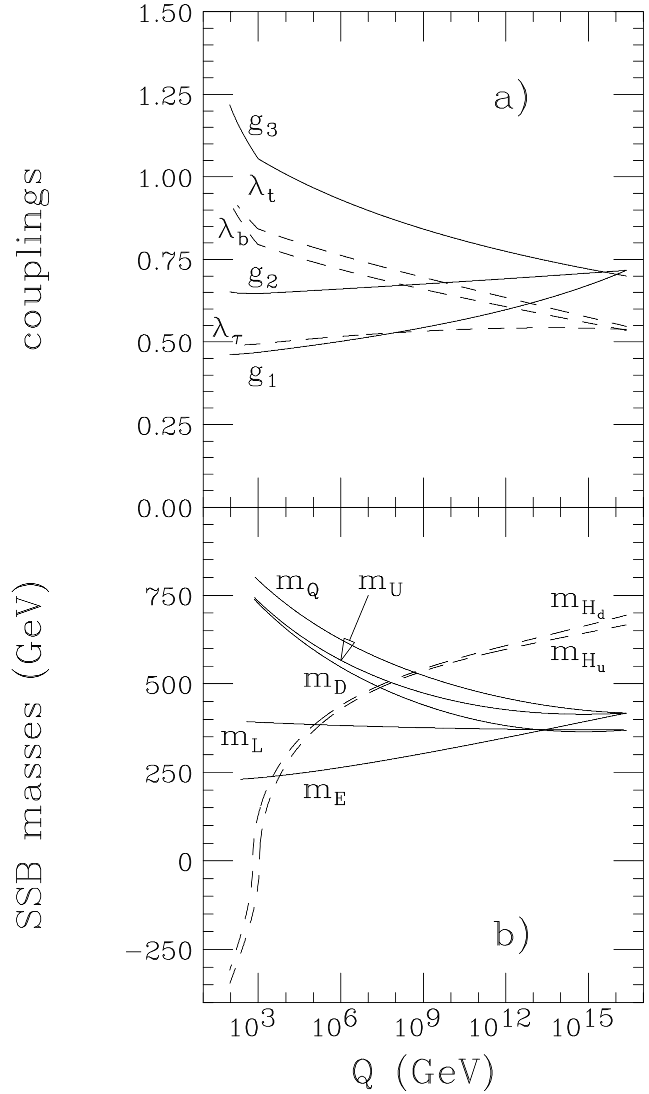
request $\frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t}$

$\delta m_t \approx 1$ GeV

$\Rightarrow \delta m_b \approx 25$ MeV

Baer *et al.*

Phys.Rev.D61,2000



$\overline{\text{MS}}$- vs. Pole-Mass

Pole-Mass (M_{pole}): close to intuition

- t $\rightarrow$ b W

  $M_{\text{pole}}(\text{b W}) = (171.4 \pm 2.1)\ \text{GeV} \pm \mathcal{O}(\Lambda?)$

- $e^+e^- \rightarrow \text{t}\,\bar{\text{t}}$

  "peak" at $2M_{\text{pole}} + \mathcal{O}(\alpha_s^2)$

- $M_{\text{B}} \approx M_{\text{pole}} + \mathcal{O}(\Lambda)$

  5280 MeV $\approx (4820 + 460)$MeV

But: large corrections for observables
involving large momentum transfers

examples:

- running $\bar{m}(\mu)$ absorbs often large corrections

$$\Gamma(\mathrm{H} \to \mathrm{b\bar{b}}) \sim M_\mathrm{b}^2 \left(1 - 2\, a_s \ln\left(\frac{M_\mathrm{H}^2}{M_\mathrm{b}^2}\right) + \ldots\right)$$

- improvement even if scales are comparable

$$\delta\rho = 3\frac{G_\mathrm{F}\, M_\mathrm{t}^2}{8\sqrt{2}\pi^2} \left(1 - 2.8599 a_\mathrm{s} - 14.594 a_\mathrm{s}{}^2 - 93.1501 a_\mathrm{s}{}^3\right)$$

$$\delta\rho = 3\frac{G_\mathrm{F}\, m_\mathrm{t}^2(m_\mathrm{t})}{8\sqrt{2}\pi^2} \left(1 - 0.19325 a_\mathrm{s} - 3.9696 a_\mathrm{s}{}^2 - 1.6799 a_\mathrm{s}{}^3\right)$$

conversions: $M \Leftrightarrow \overline{m_{\mathrm{b}}}(\mu)$

$$\overline{m_{\mathrm{b}}}(\mu) = M \left\{ 1 - a_{\mathrm{s}} \left[\frac{4}{3} + \ln \frac{\mu^2}{M^2} \right] - a_{\mathrm{s}}{}^2 \left[\# + \ln + \ln^2 \right] + a_{\mathrm{s}}{}^3 \left[\# + \ldots \right] \right\}$$

$a_{\mathrm{s}}{}^3$: Chetyrkin+Steinhauser; Melnikov+Ritbergen

examples: $M_{\mathrm{t}} = 171\mathrm{GeV} \Rightarrow m_{\mathrm{t}}(m_{\mathrm{t}}) = 161\mathrm{GeV}$

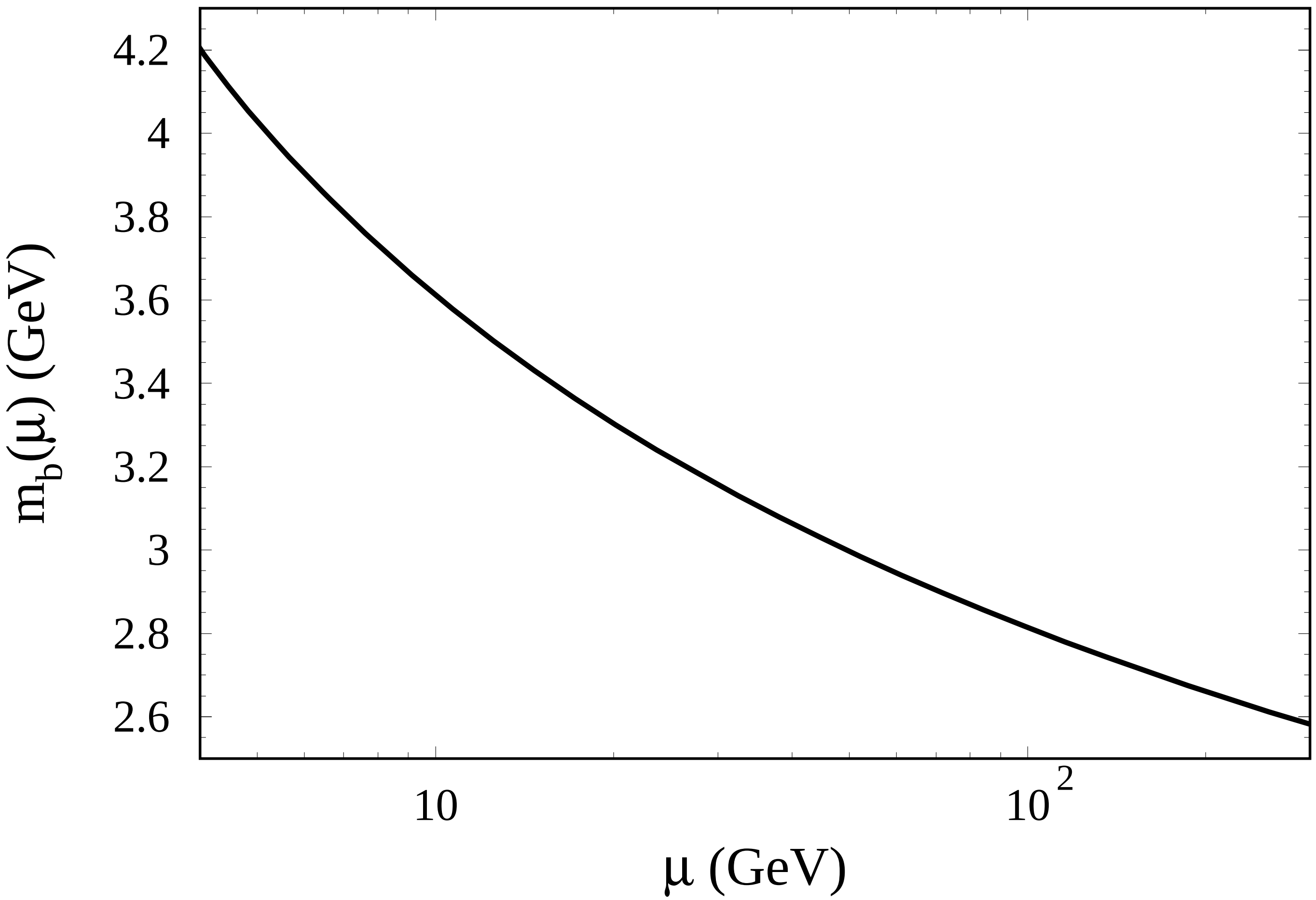
$m_{\mathrm{b}}(m_{\mathrm{b}}) = 4165\mathrm{MeV} \Rightarrow M_{\mathrm{b}} = 4796\mathrm{MeV}$

large logarithms for $\mu^2 \gg M^2$ $\rightarrow$ renormalization group

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \bar{m}(\mu) = \bar{m}(\mu)\, \gamma(\alpha_{\mathrm{s}})$$

$\gamma(\alpha_{\mathrm{s}}) = -\sum_{i \geq 0} \gamma_i a_{\mathrm{s}}{}^{i+1}$, (known up to γ_3, Chetyrkin; Larin+...)

+matching

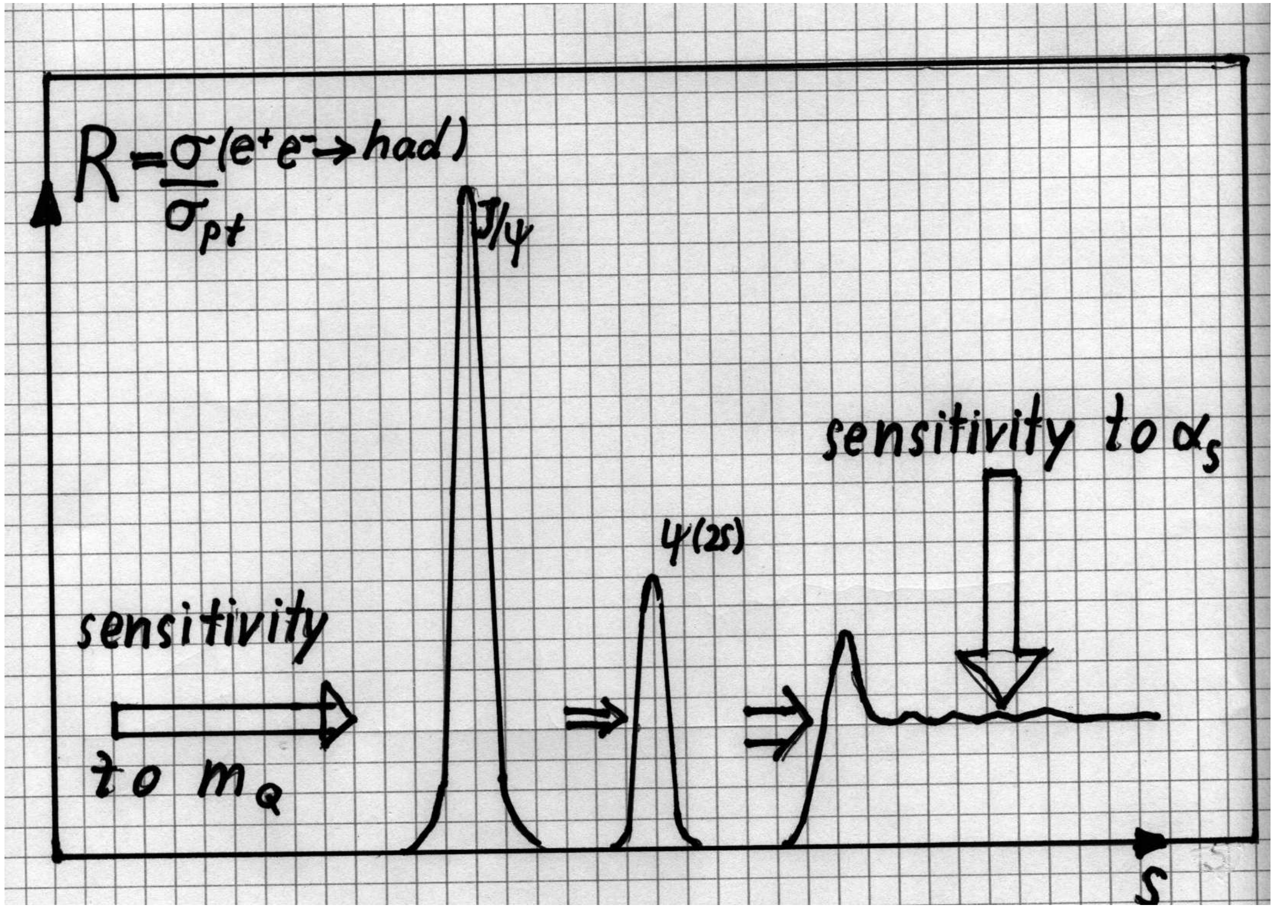


$$m_b(m_b) = 4164 \text{ MeV}$$
$$m_b(10\text{GeV}) = 3609 \text{ MeV}$$
$$m_b(M_Z) = 2834 \text{ MeV}$$
$$m_b(161.8\text{GeV}) = 2703 \text{ MeV}$$

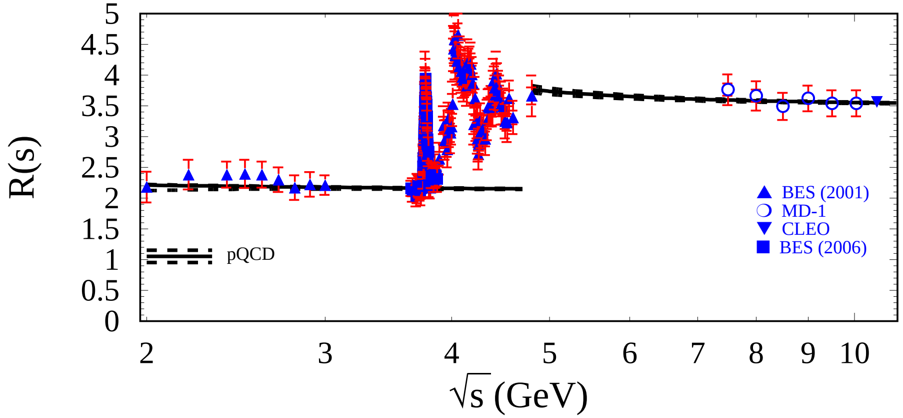
II Sum Rules with Charm and Bottom Quarks

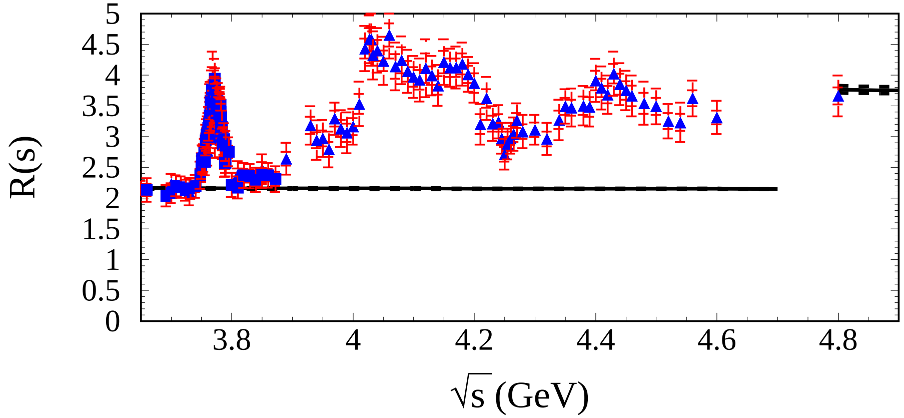
(Chetyrkin, JK, Steinhauser, Sturm)

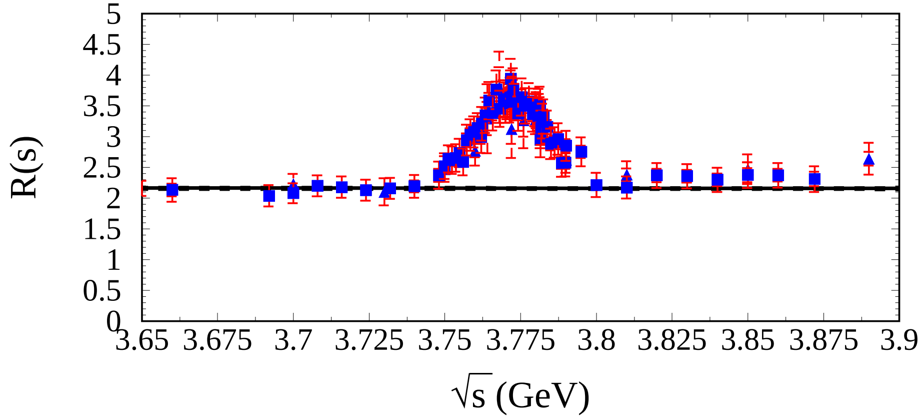
Main Idea (SVZ)



Data







pQCD and data agree well in the regions
2 – 3.73 GeV and 5 – 10.52 GeV

m_Q from

SVZ Sum Rules, Moments and Tadpoles

Some definitions:

$$R(s) \;=\; 12\pi\,\mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right)\,\Pi(q^2) \;\equiv\; i\int \mathrm{d}x\, e^{iqx}\langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) \;=\; Q_Q^2\,\frac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n\, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2

(Chetyrkin, JK, Steinhauser, 1996)

recently also $\bar{C}_0$ and $\bar{C}_1$ in order α_s^3 (four loops!)

➪ reduction to master integrals through Laporta algorithm

(Chetyrkin, JK, Sturm; confirmed by Boughezal, Czakon, Schutzmeier)

evaluation of master integrals numerically through difference equations

(30 digits) or Padé method or analytially in terms of transcendentals

(Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser,

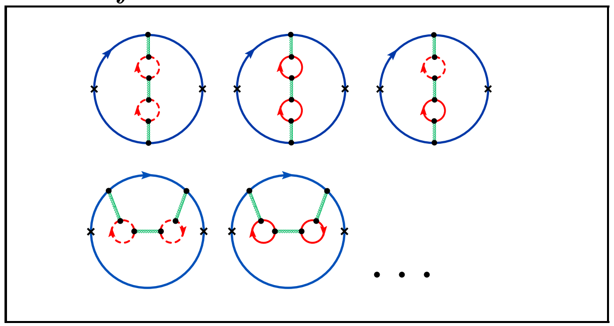
Laporta, Broadhurst, Kniehl et al.)

Analysis in N^{3}LO

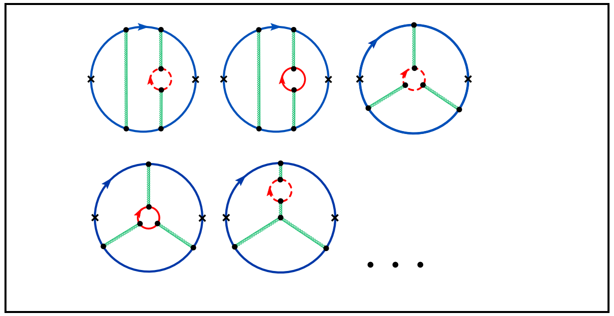
Algebraic reduction to 13 master integrals (Laporta algorithm);

numerical evaluation of master integrals

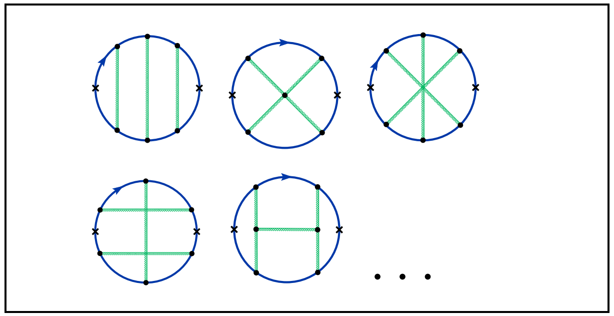
n_f^2-contributions



n_f^1-contributions



n_f^0-contributions



 : heavy quarks,  : light quarks,

n_f: number of active quarks

$\Longrightarrow$ About 700 Feynman-diagrams recall: $\Pi_Q(q^2) \;=\; Q_Q^2 \,\frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \, z^n$

$\bar{C}_n$ depend on the charm quark mass through $l_{m_c} \equiv \ln(m_c^2(\mu)/\mu^2)$

$$\begin{aligned}
\bar{C}_n \;=\; & \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right) \\
& + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right) \\
& + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{mc} + \bar{C}_n^{(32)} l_{mc}^2 + \bar{C}_n^{(33)} l_{ms}^3\right)
\end{aligned}$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	− 0.0889	−5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	—	6.7216	6.4916	−0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	—	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

estimate $-6 < C_n^{(30)} < 6$, $\quad n = 2, 3, 4$

Define the moments

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Bigg|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

$$\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint:

$$\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$$

➪ m_c

update compared to NPB619 (2001)

experiment:

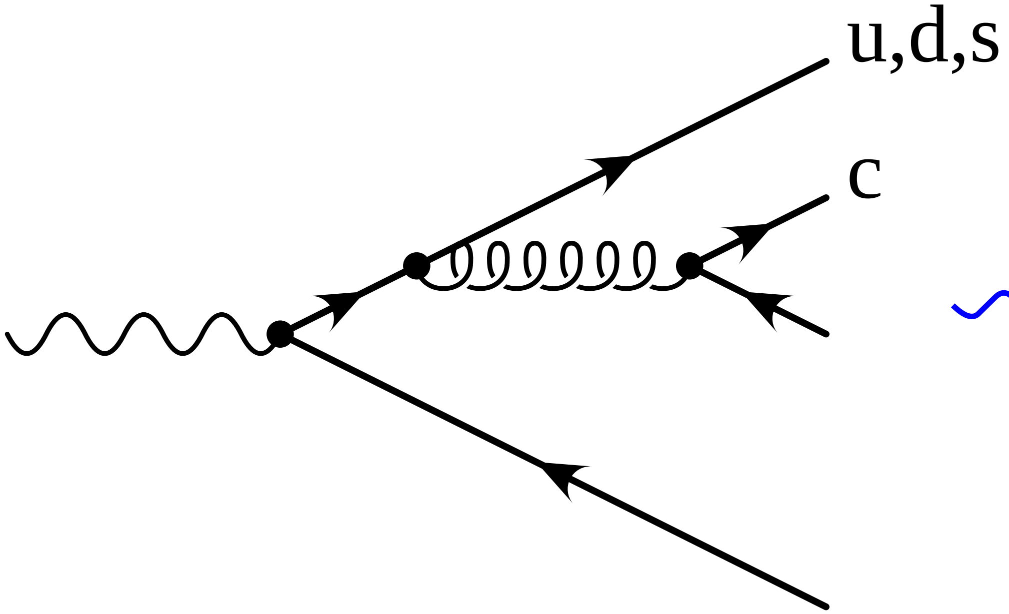
- $\alpha_s = 0.1187 \pm 0.0020$
- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & Babar
- $\psi(3770)$ from BES

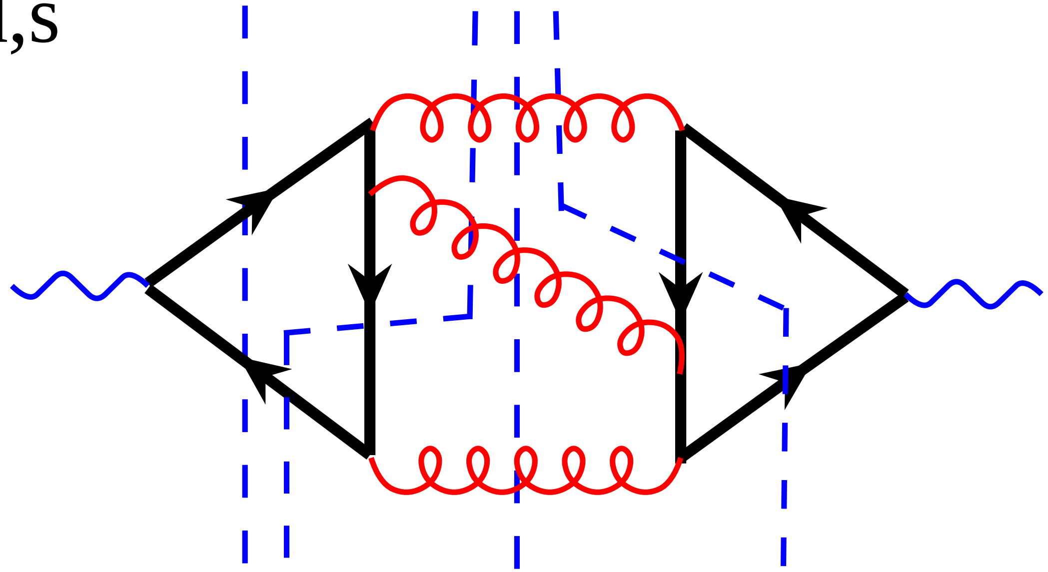
theory:

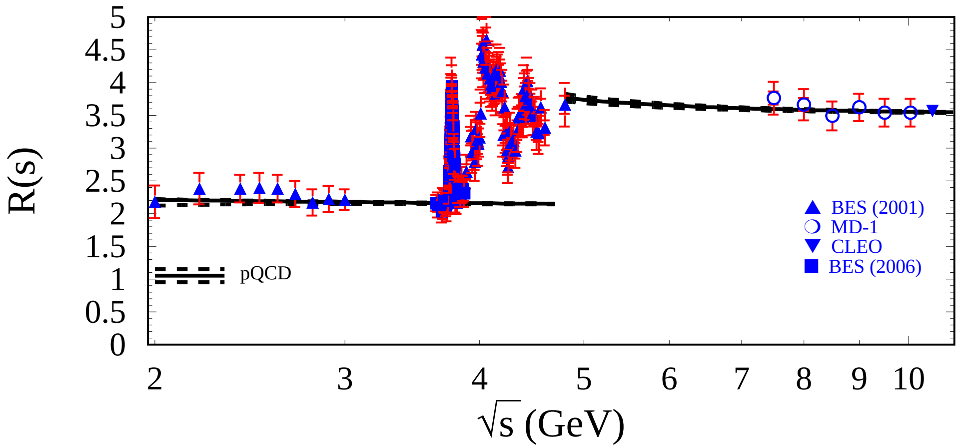
- N^3LO for n=1
- N^3LO - estimate for n =2,3,4
- include condensates

$$\delta\mathcal{M}_n^{\text{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi}\bar{b}_n\right)$$

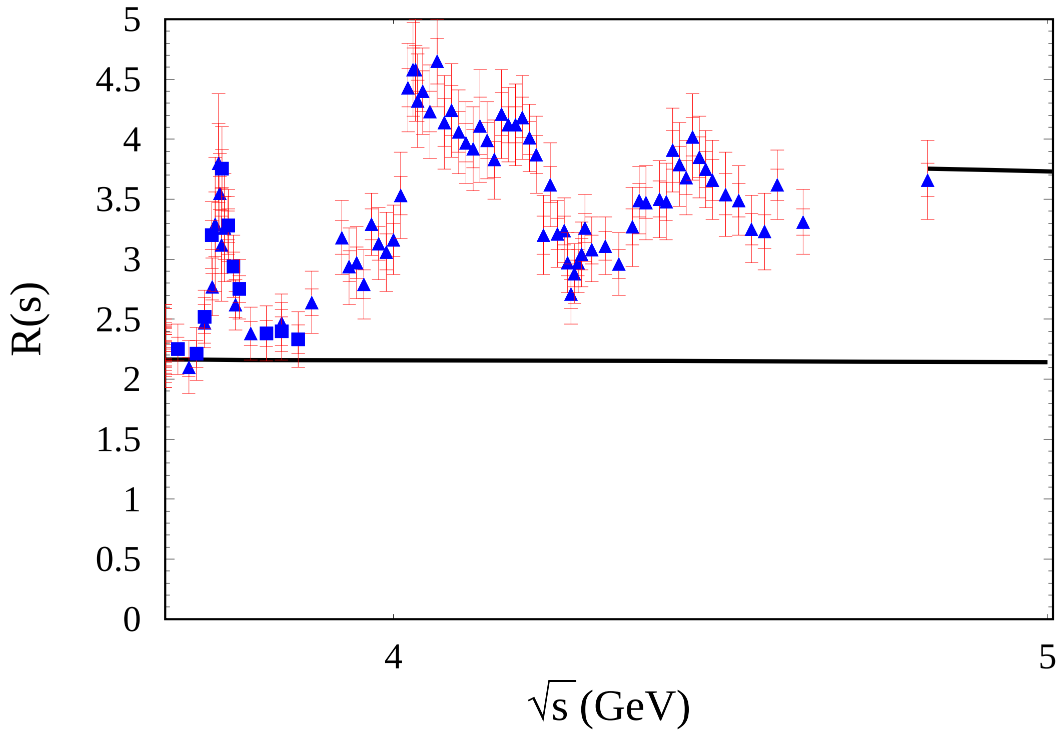
- careful extrapolation of R_{uds}
- estimate of non-perturbative terms (oscillations)


u,d,s
c





R(s)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
2
3
4
5
6
7
8
9
10
√s (GeV)
pQCD
BES (2001)
MD-1
CLEO
BES (2006)



R(s)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
4
5
√s (GeV)

Contributions from

- narrow resonances: $R = \frac{9\,\Pi\,M_R \Gamma_e}{\alpha^2(s)}\,\delta(s - M_R^2)$
- threshold region $(2\,m_D - 4.8\,\text{GeV})$
- perturbative continuum $(E \geq 4.8\,\text{GeV})$

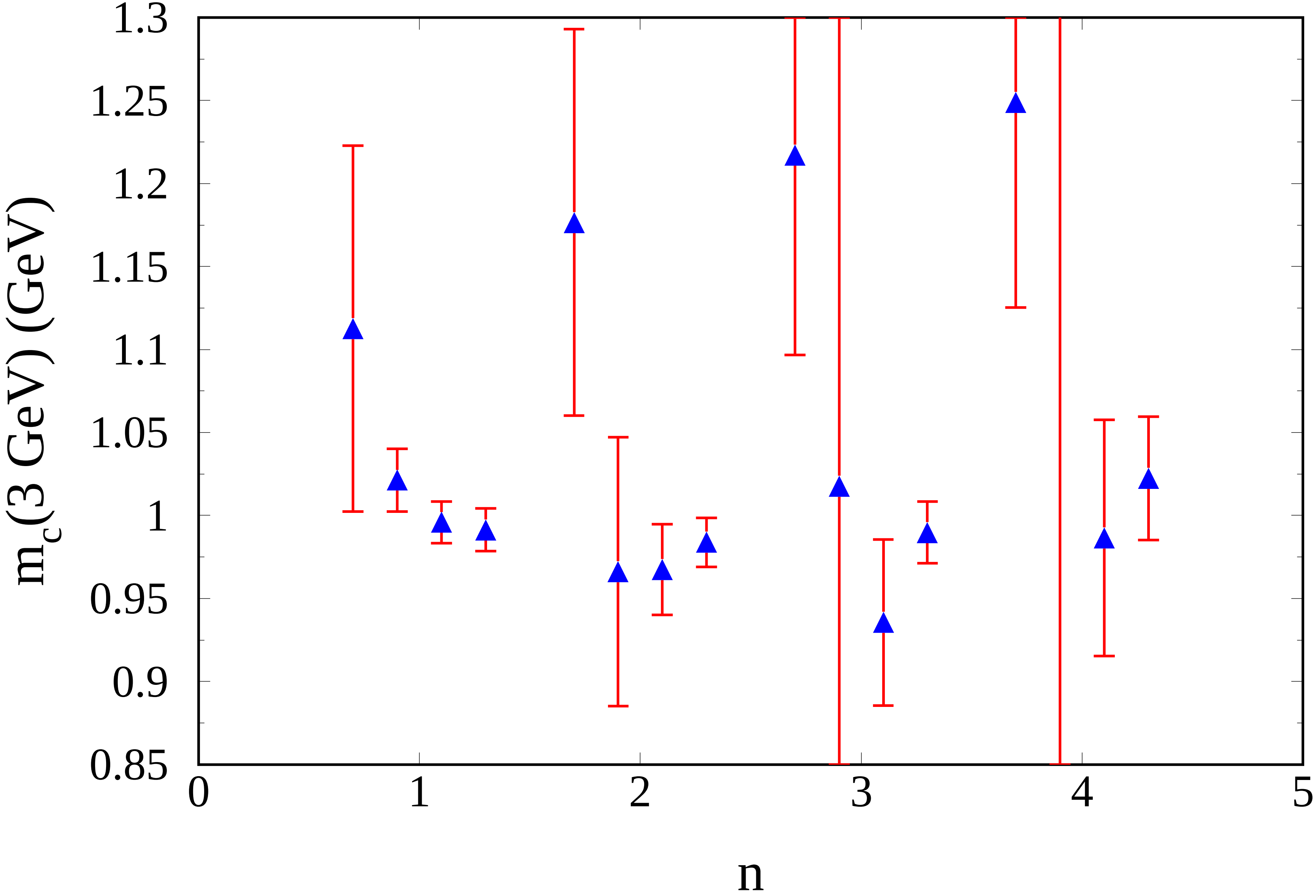
n	$\mathcal{M}_n^{\text{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{np}}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

Results (m_c)

n	m_c(3 GeV)	exp	α_s	μ	np	total	$\delta\bar{C}_n^{30}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$n = 1$:

- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1286 \pm 13\,\text{MeV}$



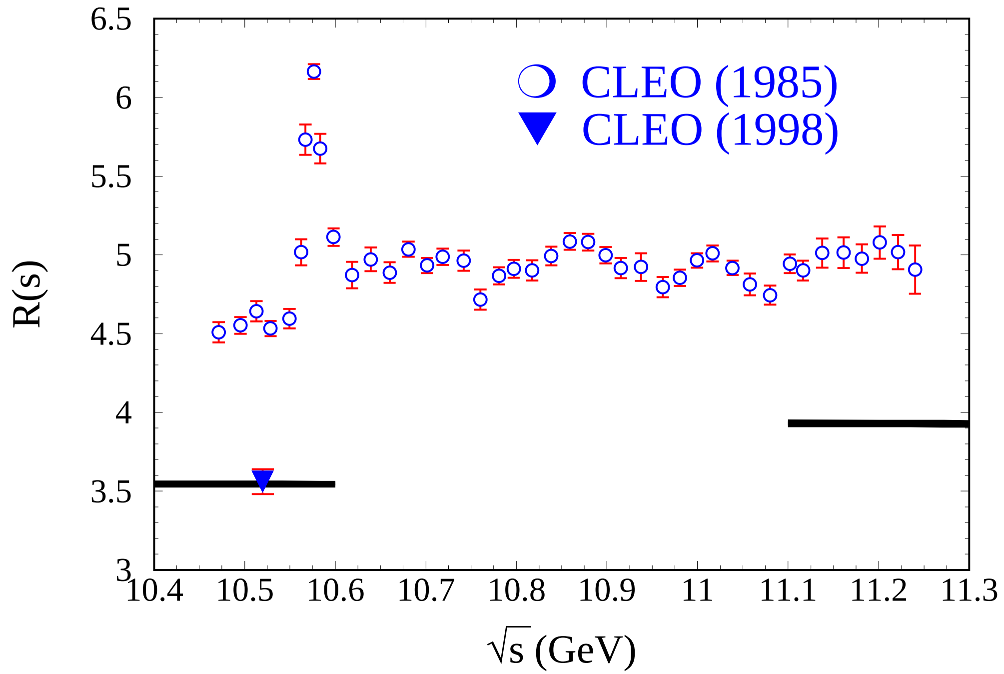
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
m_c(3 GeV) (GeV)
n

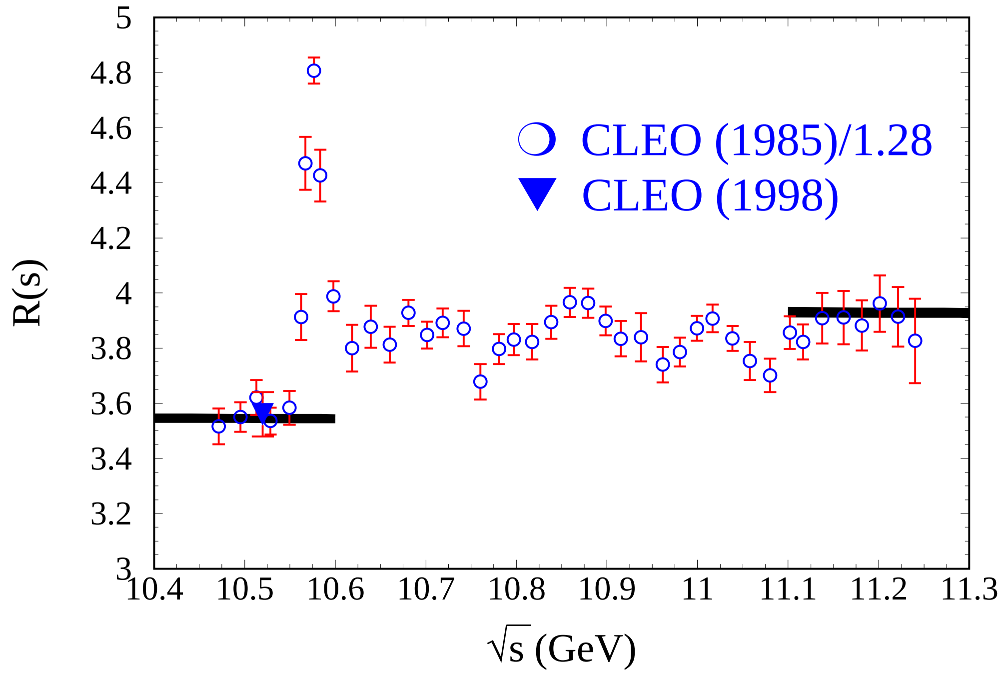
update on m_b

Contributions from

- narrow resonances ($\Upsilon(1S) - \Upsilon(4S)$)
- threshold region (10.618 GeV − 11.2 GeV)
- perturbative continuum ($E \geq 11.2$ GeV)

n	$\mathcal{M}_n^{\text{res},(1\text{S}-4\text{S})} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{thresh}} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{cont}} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{exp}} \times 10^{(2n+1)}$
1	1.394(23)	0.296(32)	2.911(18)	4.601(43)
2	1.459(23)	0.249(27)	1.173(11)	2.881(37)
3	1.538(24)	0.209(22)	0.624(7)	2.370(34)
4	1.630(25)	0.175(19)	0.372(5)	2.178(32)


CLEO (1985)
CLEO (1998)
R(s)
√s (GeV)

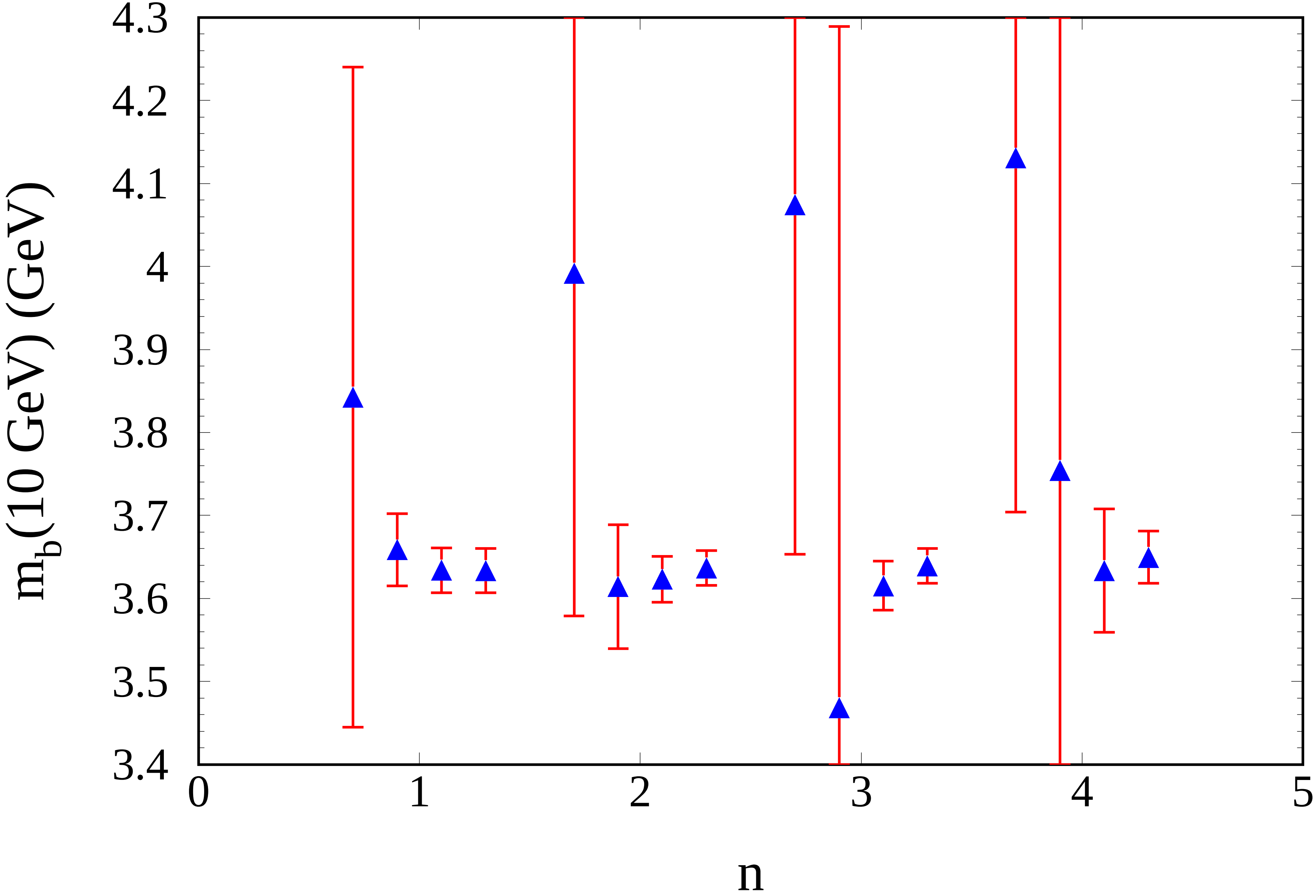

CLEO (1985)/1.28
CLEO (1998)
R(s)
√s (GeV)

Results (m_b)

n	m_b(10 GeV)	exp	α_s	μ	total	$\delta\bar{C}_n^{30}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$n = 2$:

- $m_b(m_b) = 4164 \pm 25\,\text{MeV}$
- $m_b(10\text{GeV}) = 3609 \pm 25\,\text{MeV}$
- $m_b(m_t) = 2703 \pm 18 \pm 19\,\text{MeV}$
- $m_t/m_b = 59.8 \pm 1.3$



4.3
4.2
4.1
4
3.9
3.8
3.7
3.6
3.5
3.4
0
1
2
3
4
5
n
m_b(10 GeV) (GeV)

Summary

➪ drastic improvement in δm_c, δm_b from moments with low n in N^{2}LO

➪ direct determination of short-distance mass

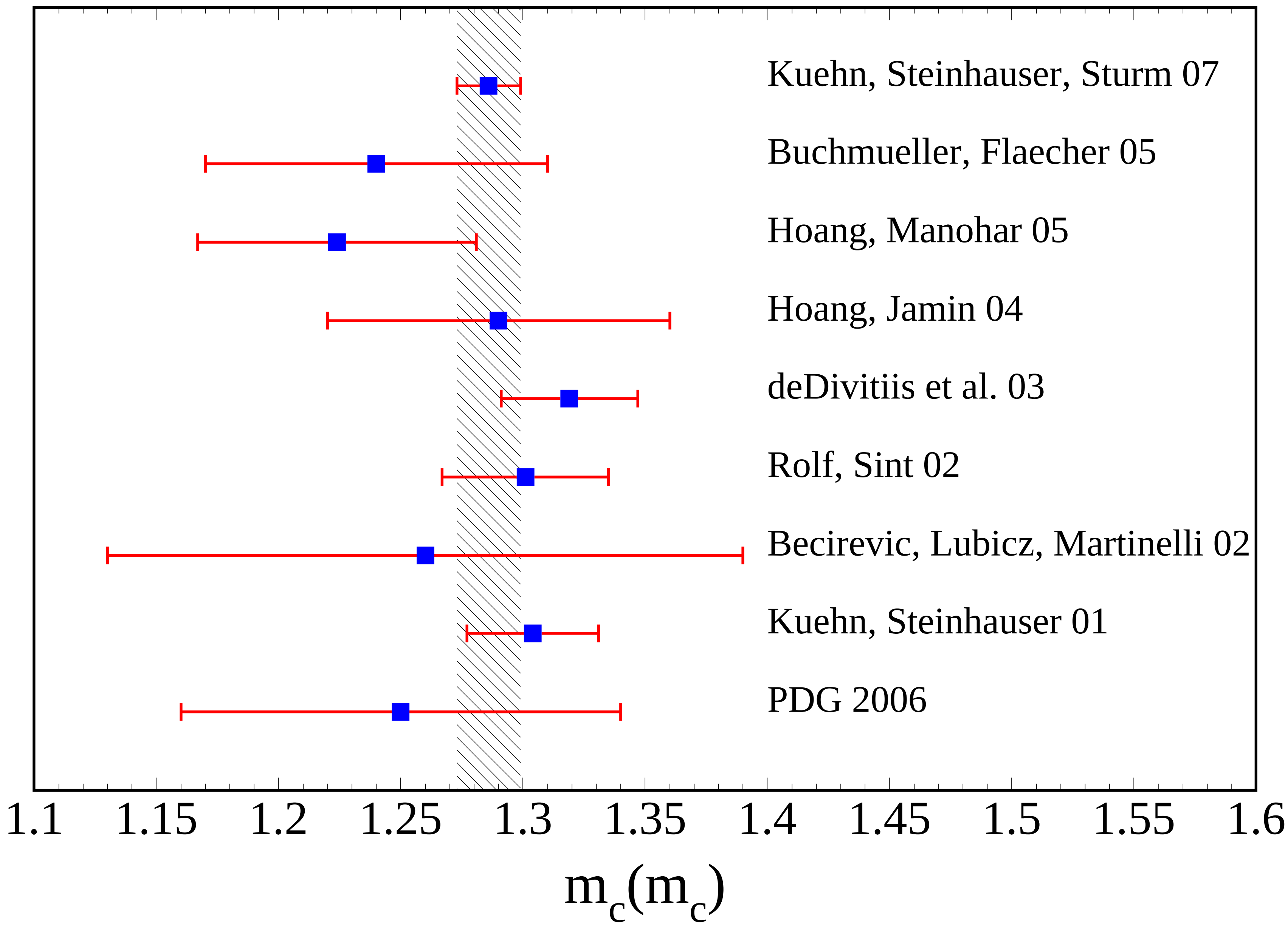
- improved measurements of $\Gamma_e(J/\psi, \psi')$ and $\Gamma_e(\Upsilon, \Upsilon', \Upsilon'')$
- improved measurement of charm threshold region
- reanalysis of bottom threshold region
- new N^3LO results lead to significant improvements

$$m_c(3\ \text{GeV}) = 0.986(13)\ \text{GeV}$$
$$m_c(m_c) = 1.286(13)\ \text{GeV}$$

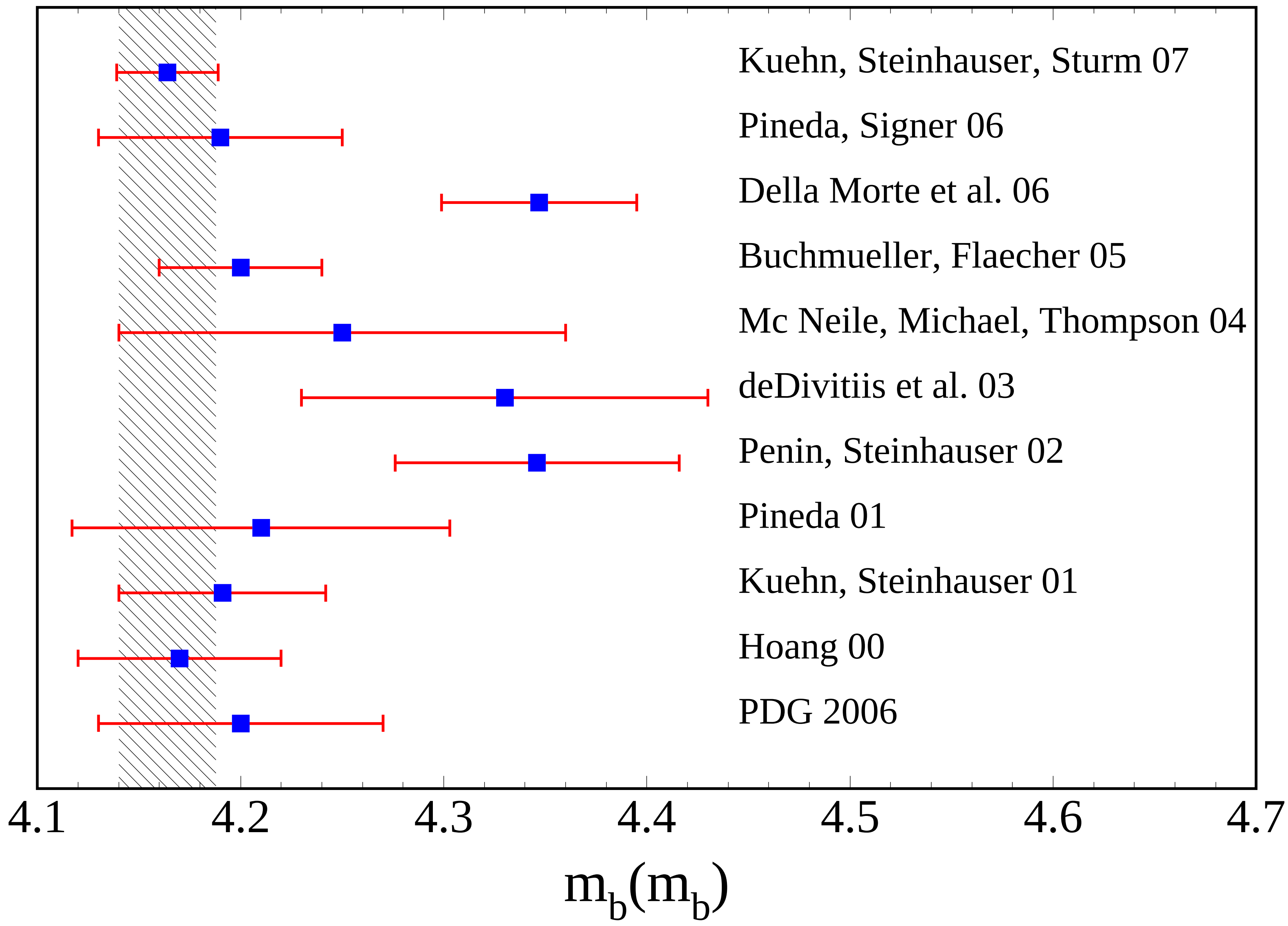
$$m_b(10\ \text{GeV}) = 3.609(25)\ \text{GeV}$$
$$m_b(m_b) = 4.164(25)\ \text{GeV}$$

(old result: $m_c(m_c) = 1.304(27)$GeV, $m_b(m_b) = 4.191(51)$GeV)

$m_c(m_c)$ (GeV)	Method
1.286 ± 0.013	low-moment sum rules, NNNLO
1.24 ± 0.07	fit to B decay distribution, $\alpha_s^2\beta_0$
$1.224 \pm 0.017 \pm 0.054$	fit to B decay data, $\alpha_s^2\beta_0$
1.29 ± 0.07	NNLO moments
1.319 ± 0.028	lattice, quenched
$1.26 \pm 0.04 \pm 0.12$	lattice, quenched
1.301 ± 0.034	lattice (ALPHA), quenched
1.304 ± 0.027	low-moment sum rules, NNLO
1.25 ± 0.09	PDG 2006


Kuehn, Steinhauser, Sturm 07
Buchmueller, Flaecher 05
Hoang, Manohar 05
Hoang, Jamin 04
deDivitiis et al. 03
Rolf, Sint 02
Becirevic, Lubicz, Martinelli 02
Kuehn, Steinhauser 01
PDG 2006
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
1.55
1.6
$m_c(m_c)$

$m_b(m_b)$ (GeV)	Method
4.164 ± 0.025	low-moment sum rules, NNNLO
$4.19 \quad \pm 0.06$	Υ sum rules, NNLL (not complete)
4.347 ± 0.048	lattice (ALPHA), quenched
$4.20 \quad \pm 0.04$	fit to B decay distribution, $\alpha_s^2\beta_0$
$4.25 \quad \pm 0.02 \pm 0.11$	lattice (UKQCD)
$4.33 \quad \pm 0.10$	lattice, quenched
4.346 ± 0.070	$\Upsilon(1S)$, NNNLO
$4.210 \pm 0.090 \pm 0.025$	$\Upsilon(1S)$, NNLO
4.191 ± 0.051	low-moment sum rules, NNLO
$4.17 \quad \pm 0.05$	Υ sum rules, NNLO
$4.20 \quad \pm 0.07$	PDG



Kuehn, Steinhauser, Sturm 07
Pineda, Signer 06
Della Morte et al. 06
Buchmueller, Flaecher 05
Mc Neile, Michael, Thompson 04
deDivitiis et al. 03
Penin, Steinhauser 02
Pineda 01
Kuehn, Steinhauser 01
Hoang 00
PDG 2006
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$