PRECISE QUARK MASSES: WHY and HOW

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SFB TR-9

The Puzzle

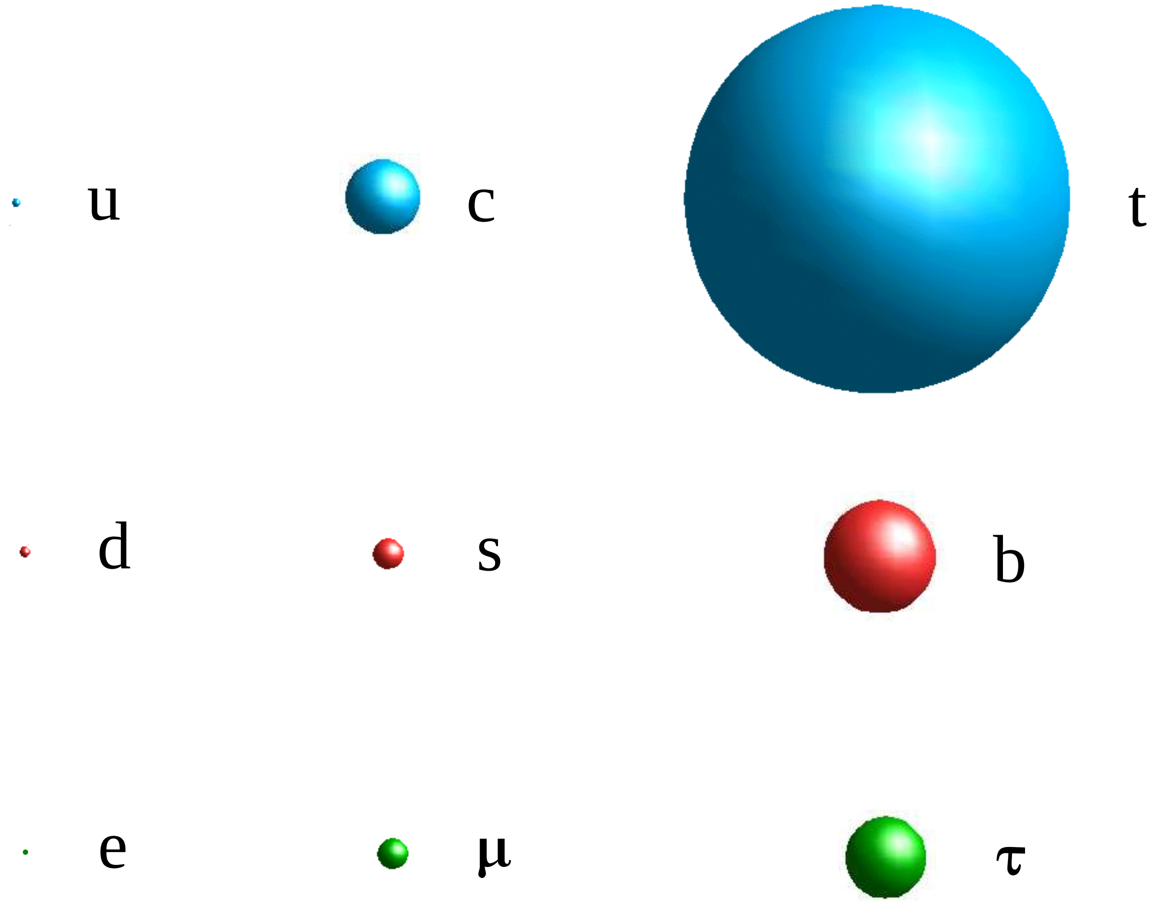
$m_{\mathrm{u}} = 1.5 - 3\mathrm{MeV}$ $\quad m_{\mathrm{c}} = 1.250 \pm 90\mathrm{MeV}$ $\quad m_{\mathrm{t}} = 171 \pm 2\mathrm{GeV}$

(Tevatron)

$m_{\mathrm{d}} = 3 - 7\mathrm{MeV}$ $\quad m_{\mathrm{s}} = 95 \pm 25\mathrm{MeV}$ $\quad m_{\mathrm{b}} = 4.20 \pm 0.07\mathrm{GeV}$

$m_{\mathrm{e}} = 0.511\mathrm{MeV}$ $\quad m_{\mu} = 106\mathrm{MeV}$ $\quad m_{\tau} = 1.777\mathrm{GeV}$

PDG


u
c
t
d
s
b
e
μ
τ

I Generalities

WHY precise masses?

B-decays:

$$\Gamma(B \to X_u l \bar{\nu}) \sim G_F^2 \, m_b^5 \, |V_{ub}|^2$$

$$\Gamma(B \to X_c l \bar{\nu}) \sim G_F^2 \, m_b^5 \, f(m_c^2/m_b^2) \, |V_{cb}|^2$$

moments of $\frac{dN}{dE_l}$, $\frac{dN}{dm(l\bar{\nu})}$,

$B \to X_s \gamma$: moments of m_{had}^2

Υ-spectroscopy:

$$m\left(\Upsilon(1\mathrm{s})\right) = 2M_{\mathrm{b}} - \left(\tfrac{4}{3}\alpha_{\mathrm{s}}\right)^2 \frac{M_{\mathrm{b}}}{4} + \ldots$$

sum rules:

$$\int \frac{\mathrm{d}s}{s^{n+1}}\, R_{\mathrm{Q}}(s) \sim \frac{1}{m_{\mathrm{Q}}^{2n}}$$

Higgs decay (ILC)

$$\Gamma(H \to b\bar{b}) = \frac{G_F \, M_H}{4\sqrt{2}\pi} \, m_b^2(M_H) \, \tilde{R}$$

$$\tilde{R} \;=\; 1 + 5.6667 a_s + 29.147 a_s{}^2 + 41.758 a_s{}^3 - 825.7 a_s{}^4$$

$$1 + 0.2075 \quad + 0.0391 \quad + 0.0020 \quad - 0.0015$$

$(m_H = 120\text{GeV})$

rapidly increasing coefficients! $\left(a_s \equiv \frac{\alpha_s}{\pi}\right)$

$a_s{}^4$-term = 5-loop calculation (Baikov,...)

(not yet known for $e^+e^- \to$ had)

perturbative vs. lattice:

$m_{\mathrm{D_s}} \Leftrightarrow m_{\mathrm{c}}$ (quenched)

1301 ± 34 (Rolf, Sint)

$m_{\mathrm{B}} \Leftrightarrow m_{\mathrm{b}}$ (quenched)

4301 ± 70 (ALPHA-Coll.)

Yukawa Unification

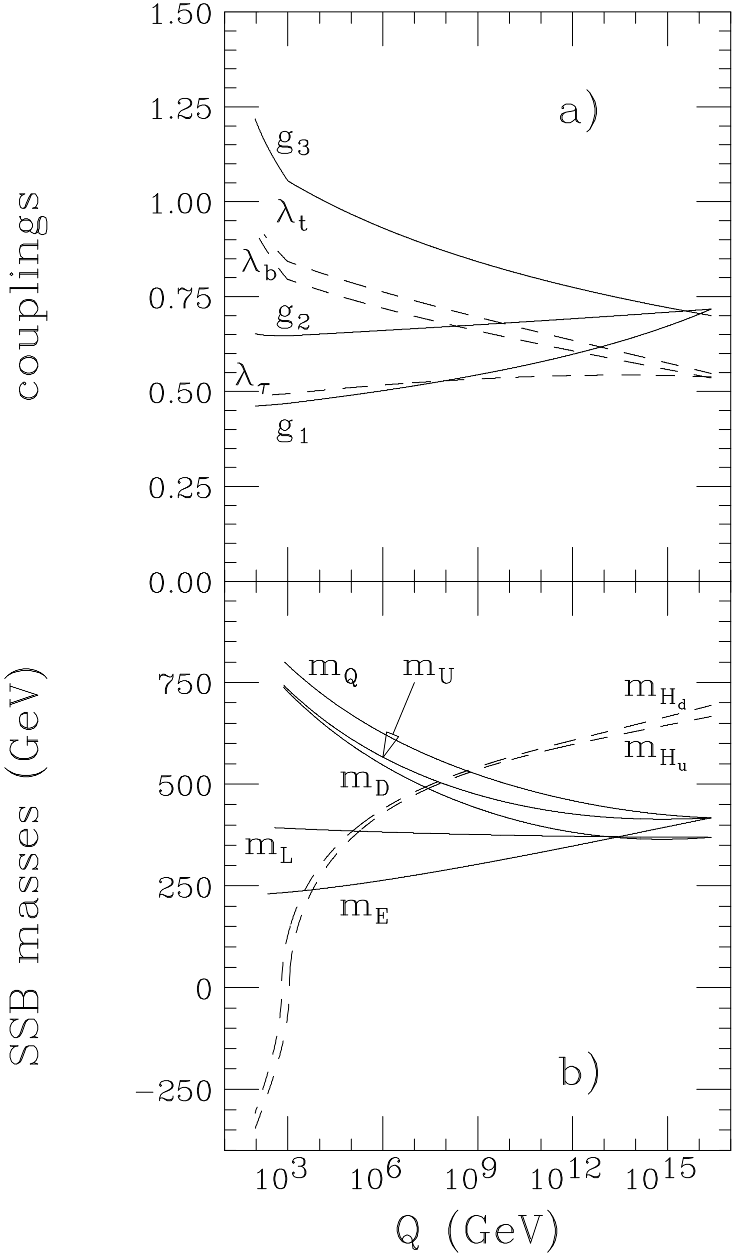
$$\lambda_\tau = \lambda_b \quad \text{or} \quad \lambda_\tau = \lambda_b = \lambda_t$$

identical coupling to Higgs boson(s) at GUT scale

top-bottom $\rightarrow m_t \big/ m_b \sim$ ratio of vacuum expectation values

request $\frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t}$

$\delta m_t \approx 1$ GeV $\Rightarrow \delta m_b \approx 25$ MeV



Baer *et al.*

Phys.Rev.D61,2000

defining the quark mass

leptons:

e^-: stable particle

M_e = pole of propagator

= kinematic mass (classic QED calculations)

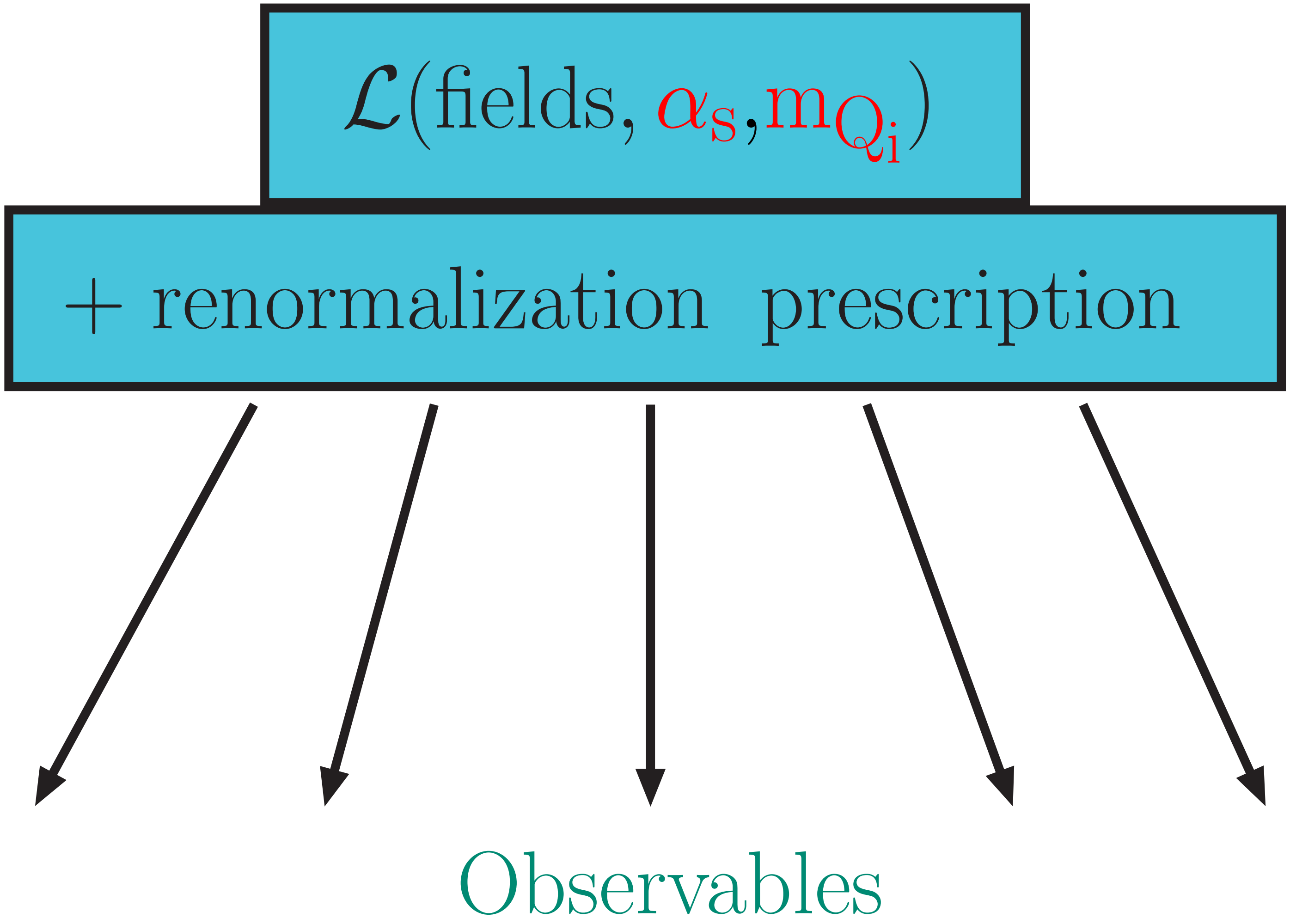
$M_\tau = E_{\text{threshold}}/2$ in $e^+e^- \to \tau^+\tau^-$

quarks:

no free quark exists

complicated "bound states"

strategy: m_Q = parameter of theory


$\mathcal{L}(\text{fields}, \alpha_S, m_{Q_i})$
+ renormalization prescription
Observables

$\overline{\text{MS}}$- vs. Pole-Mass

Pole-Mass (M_{pole}): close to intuition

- t $\rightarrow$ b W

  $M_{\text{pole}}(\text{b W}) = (171.4 \pm 2.1)\ \text{GeV} \pm \mathcal{O}(\Lambda?)$

- $e^+e^- \rightarrow \text{t}\,\bar{\text{t}}$

  "peak" at $2M_{\text{pole}} + \mathcal{O}(\alpha_s^2)$

- $M_{\text{B}} \approx M_{\text{pole}} + \mathcal{O}(\Lambda)$

  5280 MeV $\approx$ (4820 + 460)MeV

But: large corrections for observables
involving large momentum transfers

examples:

- running $\bar{m}(\mu)$ absorbs often large corrections

$$\Gamma(\mathrm{H} \to \mathrm{b\bar{b}}) \sim M_\mathrm{b}^2 \left(1 - 2\, a_s \ln\left(\frac{M_\mathrm{H}^2}{M_\mathrm{b}^2}\right) + \ldots\right)$$

- improvement even if scales are comparable

$$\delta\rho = 3\,\frac{G_\mathrm{F}\, M_\mathrm{t}^2}{8\sqrt{2}\pi^2} \left(1 - 2.8599 a_\mathrm{s} - 14.594 {a_\mathrm{s}}^2 - 93.1501 {a_\mathrm{s}}^3\right)$$

$$\delta\rho = 3\,\frac{G_\mathrm{F}\, m_\mathrm{t}^2(m_\mathrm{t})}{8\sqrt{2}\pi^2} \left(1 - 0.19325 a_\mathrm{s} - 3.9696 {a_\mathrm{s}}^2 - 1.6799 {a_\mathrm{s}}^3\right)$$

conversions: $M \Leftrightarrow \overline{m_{\mathrm{b}}}(\mu)$

$$\overline{m_{\mathrm{b}}}(\mu) = M\left\{1 - a_s\left[\frac{4}{3} + \ln\frac{\mu^2}{M^2}\right] - a_s{}^2\left[\# + \ln + \ln^2\right] + a_s{}^3\left[\# + ...\right]\right\}$$

$a_s{}^3$: Chetyrkin+Steinhauser; Melnikov+Ritbergen

examples: $M_{\mathrm{t}} = 171\,\mathrm{GeV} \quad \Rightarrow \quad m_{\mathrm{t}}(m_{\mathrm{t}}) = 161\,\mathrm{GeV}$

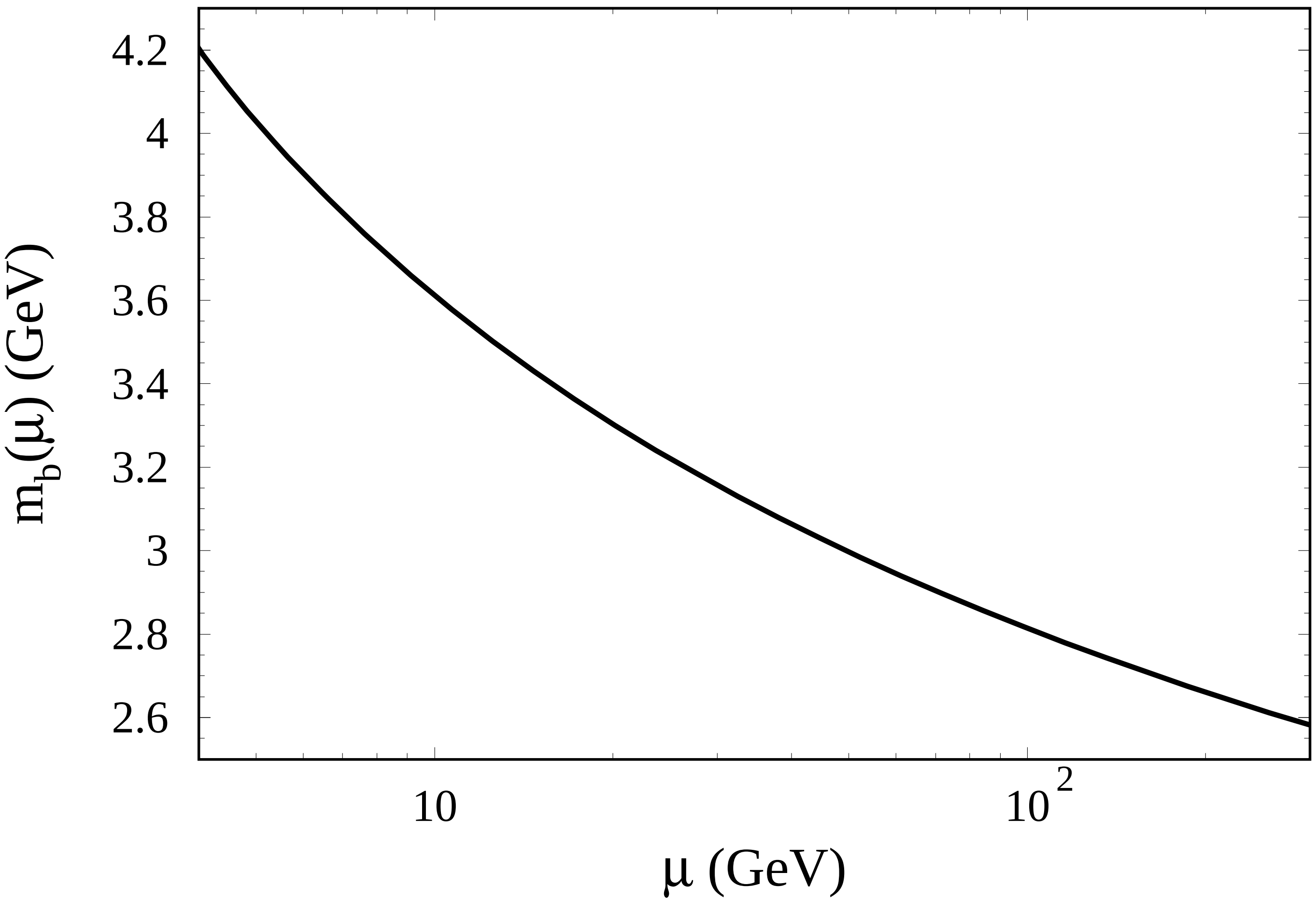
$m_{\mathrm{b}}(m_{\mathrm{b}}) = 4165\,\mathrm{MeV} \quad \Rightarrow \quad M_{\mathrm{b}} = 4796\,\mathrm{MeV}$

large logarithms for $\mu^2 \gg M^2$ $\rightarrow$ renormalization group

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\bar{m}(\mu) = \bar{m}(\mu)\,\gamma(\alpha_s)$$

$\gamma(\alpha_s) = -\sum_{i\geq 0}\gamma_i a_s{}^{i+1}$, (known up to γ_3, Chetyrkin; Larin+...)

+matching

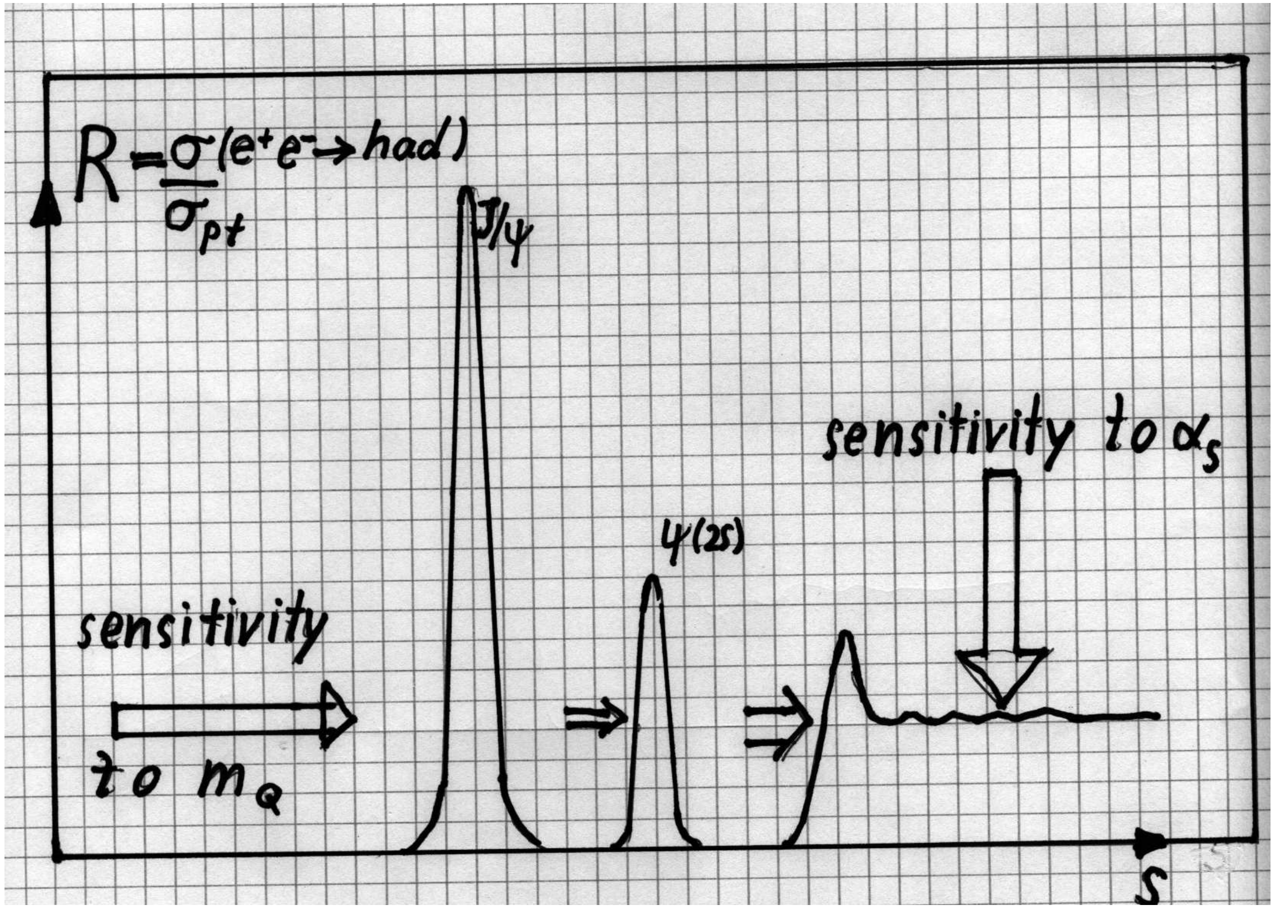


$$m_b(m_b) = 4165 \text{ MeV}$$
$$m_b(10\text{GeV}) = 3610 \text{ MeV}$$
$$m_b(M_Z) = 2836 \text{ MeV}$$
$$m_b(161\text{GeV}) = 2706 \text{ MeV}$$

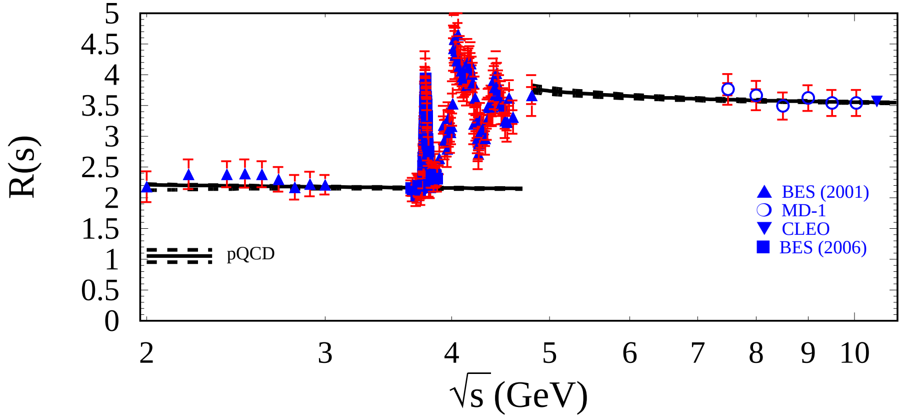
II Sum Rules with Charm and Bottom Quarks

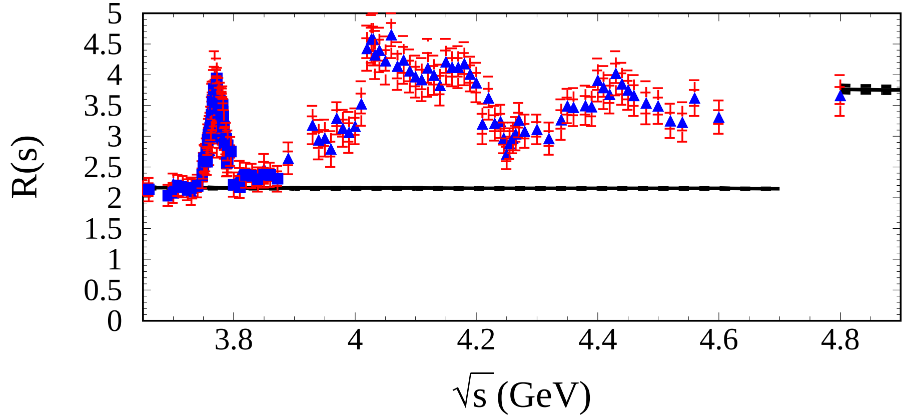
(Chetyrkin, JK, Steinhauser, Sturm)

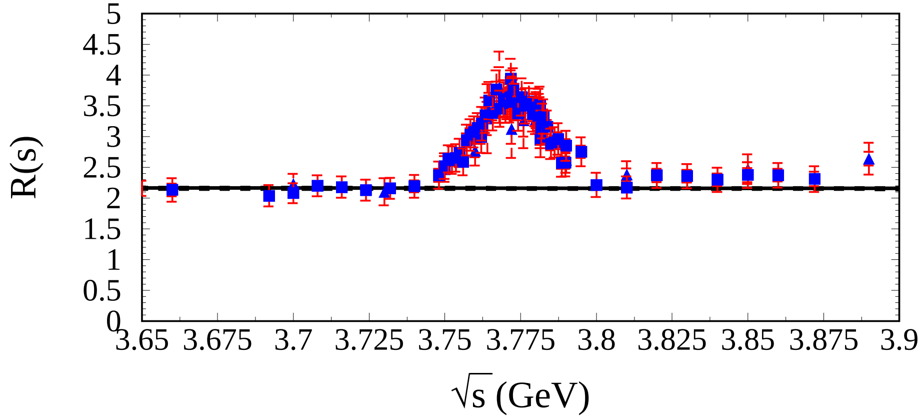
Main Idea (SVZ)



Data







pQCD and data agree well in the regions
2 – 3.73 GeV and 5 – 10.52 GeV

experiment	energy [GeV]	date	systematic error
BES	2 — 5	2001	4%
MD-1	7.2 — 10.34	1996	4%
CLEO	10.52	1998	2%
PDG	J/ψ		(7%) 2.5%
PDG	ψ'		(9%) 2.4%
PDG	ψ''		(15%)
BES	ψ'' region	2006	4%

m_Q from

SVZ Sum Rules, Moments and Tadpoles

Some definitions:

$$R(s) \ = \ 12\pi \, \mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu \, q_\nu\right) \Pi(q^2) \ \equiv \ i \int \mathrm{d}x \, e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) \ = \ Q_Q^2 \, \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n \, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2

(Chetyrkin, JK, Steinhauser, 1996)

recently up to $n = 30$! (Boughezal, Czakon, Schutzmeier)

recently also $\bar{C}_0$ and $\bar{C}_1$ in order α_s^3 (four loops!)

➪ reduction to master integrals through Laporta algorithm

(Chetyrkin, JK, Sturm; confirmed by Boughezal, Czakon, Schutzmeier)

evaluation of master integrals numerically through difference equations

(30 digits) or Padé method or analytially in terms of transcendentals

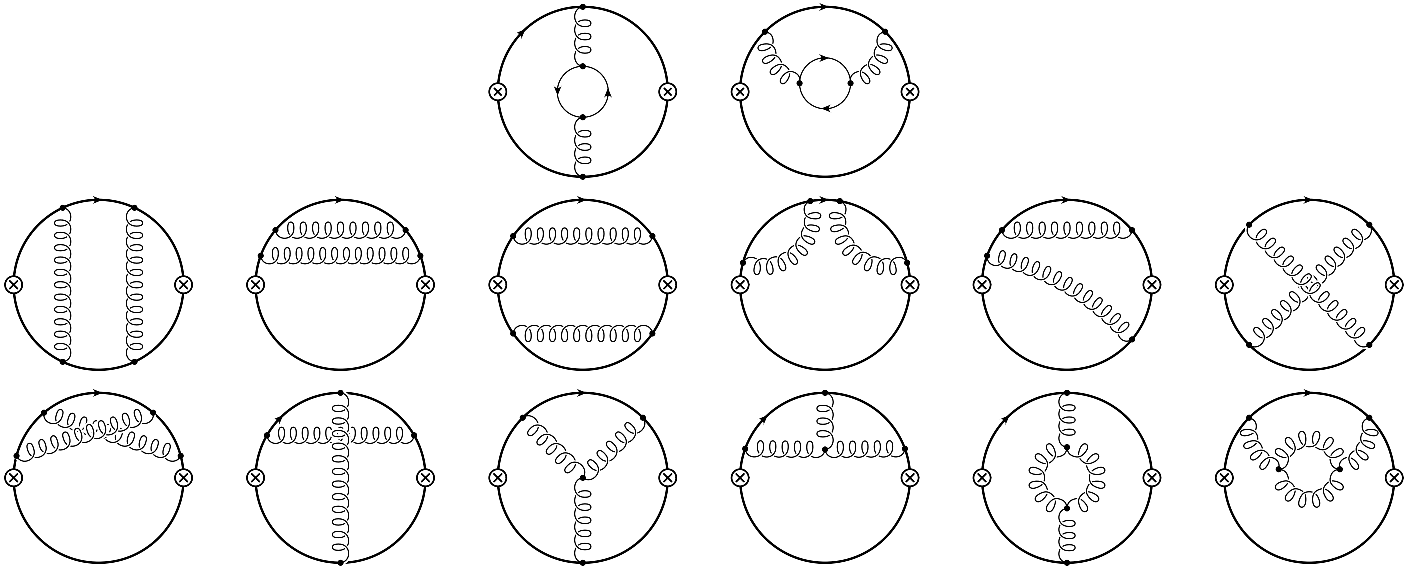
(Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser,

Laporta, Broadhurst, Kniehl et al.)

Analysis in NNLO

Coefficients $\bar{C}_n$ from three-loop one-scale tadpole amplitudes with "arbitrary" power of propagators;

FORM-program MATAD

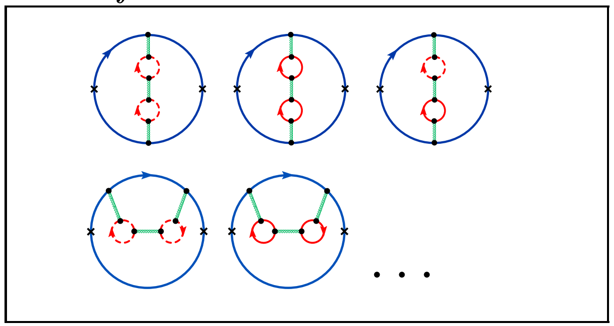


Analysis in N^{3}LO

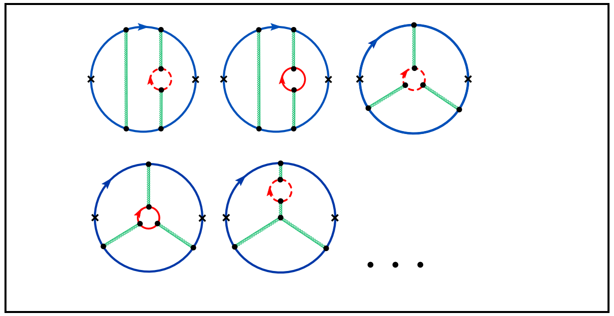
Algebraic reduction to 13 master integrals (Laporta algorithm);

numerical evaluation of master integrals

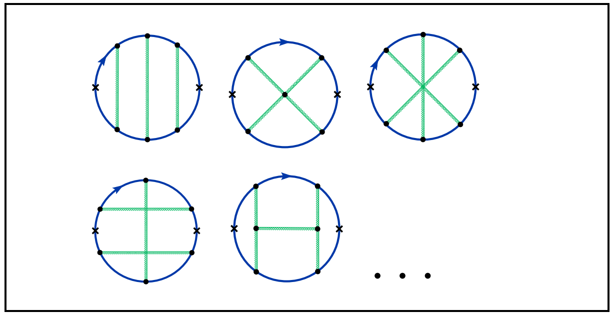
n_f^2-contributions



n_f^1-contributions



n_f^0-contributions



 : heavy quarks,  : light quarks,

n_f: number of active quarks

$\Longrightarrow$ About 700 Feynman-diagrams recall: $\Pi_Q(q^2) \;=\; Q_Q^2 \,\frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \, z^n$

$\bar{C}_n$ depend on the charm quark mass through $l_{mc} \equiv \ln(m_c^2(\mu)/\mu^2)$

$$\begin{aligned}
\bar{C}_n \;=\; & \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{mc}\right) \\
& + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{mc} + \bar{C}_n^{(22)} l_{mc}^2\right) \\
& + \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{mc} + \bar{C}_n^{(32)} l_{mc}^2 + \bar{C}_n^{(33)} l_{ms}^3\right)
\end{aligned}$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	−0.0889	−5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	—	6.7216	6.4916	−0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	—	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

estimate $-6 < C_n^{(30)} < 6$, $\quad n = 2, 3, 4$

Define the moments

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\bigg|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

Perturbation theory: $\bar{C}_n$ is function of α_s and $\ln\frac{m_c^2}{\mu^2}$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\,\frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

$$\Rightarrow \mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}}R_c(s)$$

constraint: $\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$

$\Rightarrow m_c$

SVZ:

$\mathcal{M}_n^{\text{th}}$ can be reliably calculated in pQCD:

low n: dominated by scales of $\mathcal{O}(2m_Q)$

- fixed order in α_s is sufficient, in particular no resummation of $1/v$ - terms from higher orders required
- condensates are unimportant
- pQCD in terms of short distance mass : $m_c(3\text{ GeV})$ ⇨ $m_c(m_c)$ stable expansion : no pole mass or closely related definition (1S-mass, potential-subtracted mass) involved
- moments available in NNLO
- and $\bar{C}_0$, $\bar{C}_1$ in N^{3}LO

update compared to NPB619 (2001)

experiment:

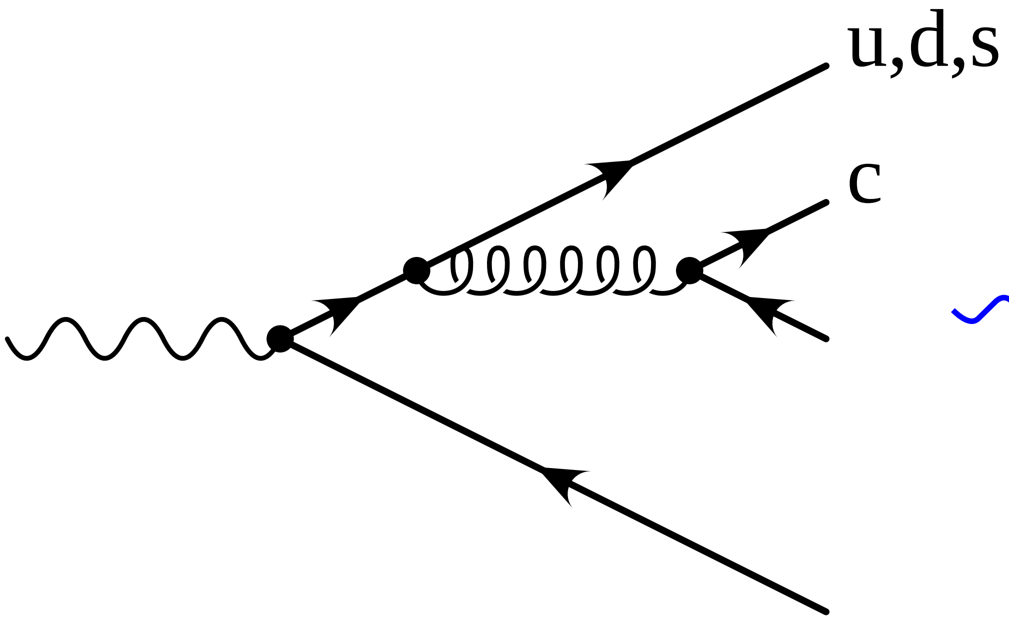
- $\alpha_s = 0.1187 \pm 0.0020$
- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & Babar
- $\psi(3770)$ from BES

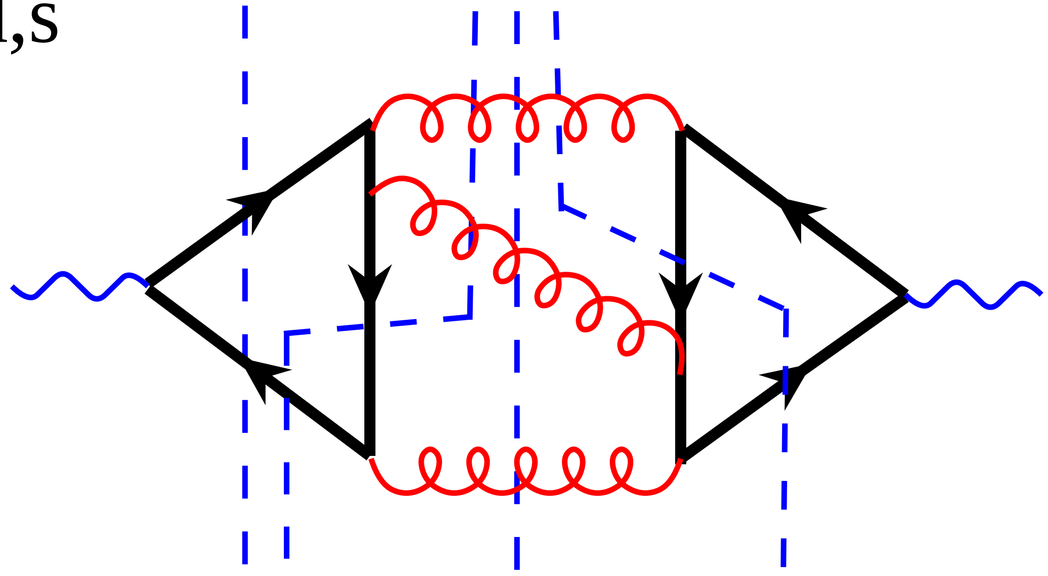
theory:

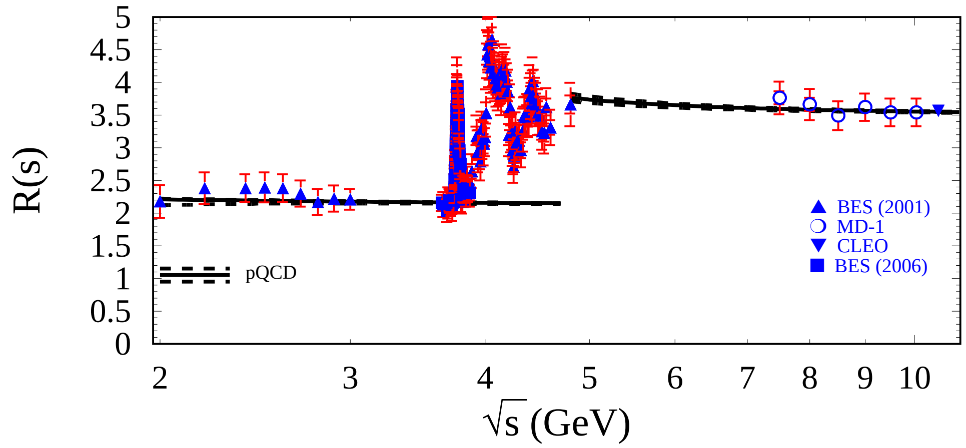
- N^3LO for n=1
- N^3LO - estimate for n =2,3,4
- include condensates

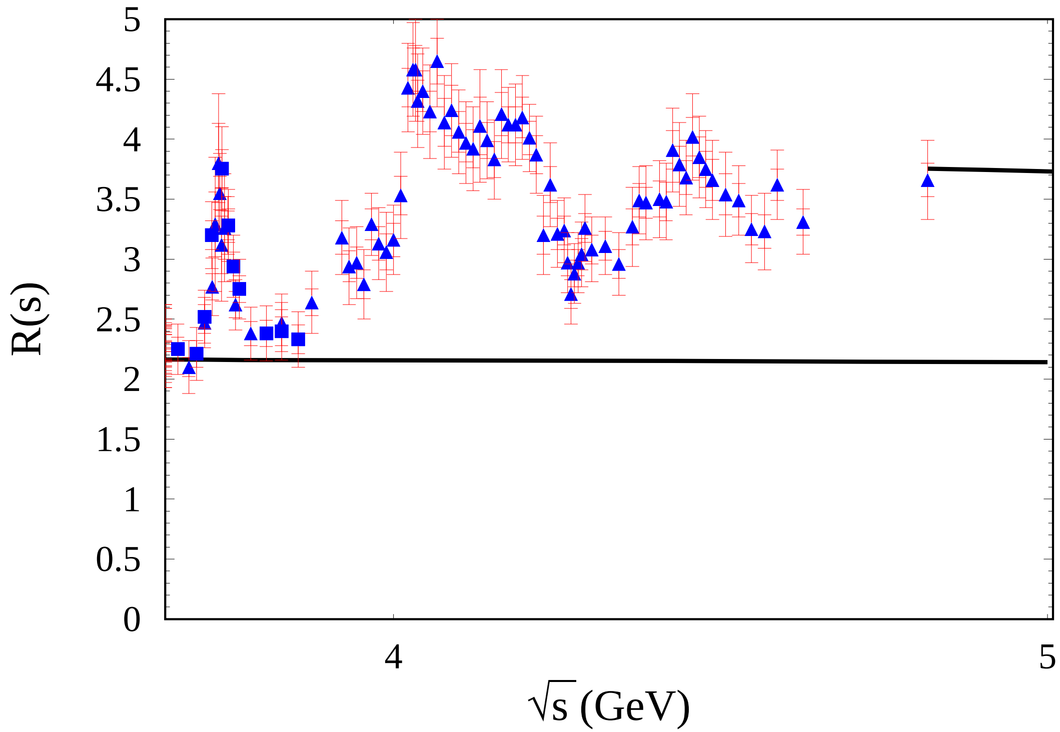
$$\delta\mathcal{M}_n^{\text{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi}\bar{b}_n\right)$$

- careful extrapolation of R_{uds}


u,d,s
c




R(s)
√s (GeV)
pQCD
BES (2001)
MD-1
CLEO
BES (2006)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
2
3
4
5
6
7
8
9
10


R(s)
√s (GeV)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
4
5

Contributions from

- narrow resonances: $R = \frac{9\,\Pi\, M_R \Gamma_e}{\alpha^2(s)}\, \delta(s - M_R^2)$
- threshold region ($2\,m_D$ – 4.8 GeV)
- perturbative continuum ($E \geq$ 4.8 GeV)

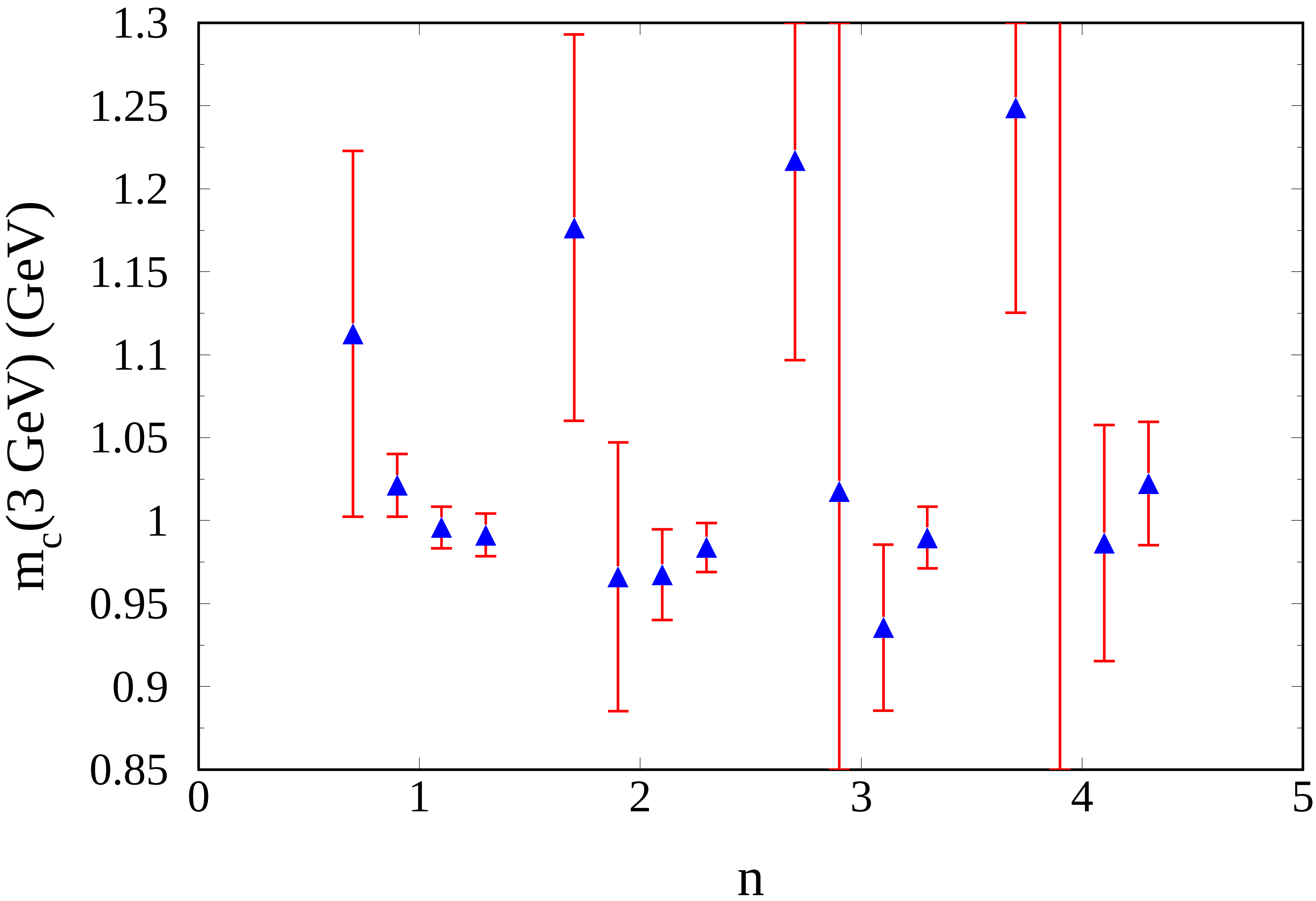
n	$\mathcal{M}_n^{\text{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{np}}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

Preliminary results (m_c)

n	m_c(3 **GeV**)	**exp**	α_s	μ	**np**	**total**	$\delta\bar{C}_n^{30}$	$m_c(m_c)$
1	0.988	0.009	0.008	0.001	0.001	0.013	—	1.287
2	0.983	0.006	0.013	0.003	0.000	0.015	0.006	1.283
3	0.989	0.005	0.013	0.012	0.002	0.019	0.010	1.289
4	1.022	0.003	0.007	0.036	0.007	0.037	0.014	1.318

$n = 1$:

- $m_c(3\,\text{GeV}) = 988 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1287 \pm 13\,\text{MeV}$

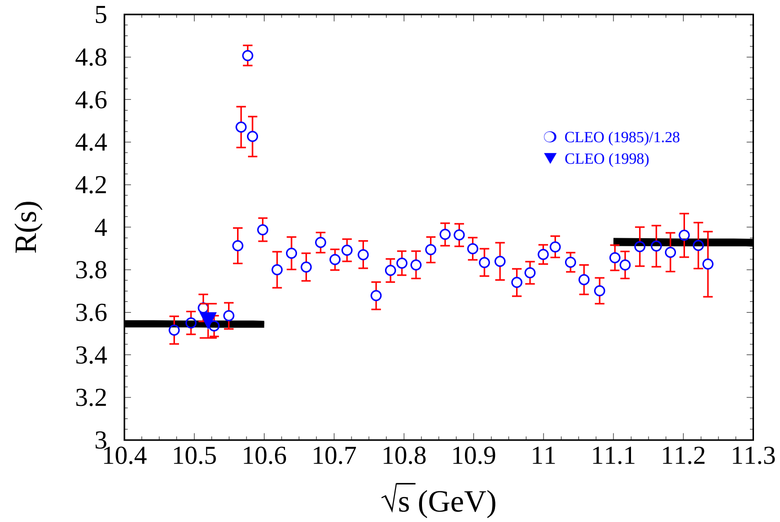


1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
n
m_c(3 GeV) (GeV)

update on m_b

Contributions from

- narrow resonances ($\Upsilon(1S) - \Upsilon(4S)$)
- threshold region (10.618 GeV – 11.2 GeV)
- perturbative continuum ($E \geq 11.2$ GeV)



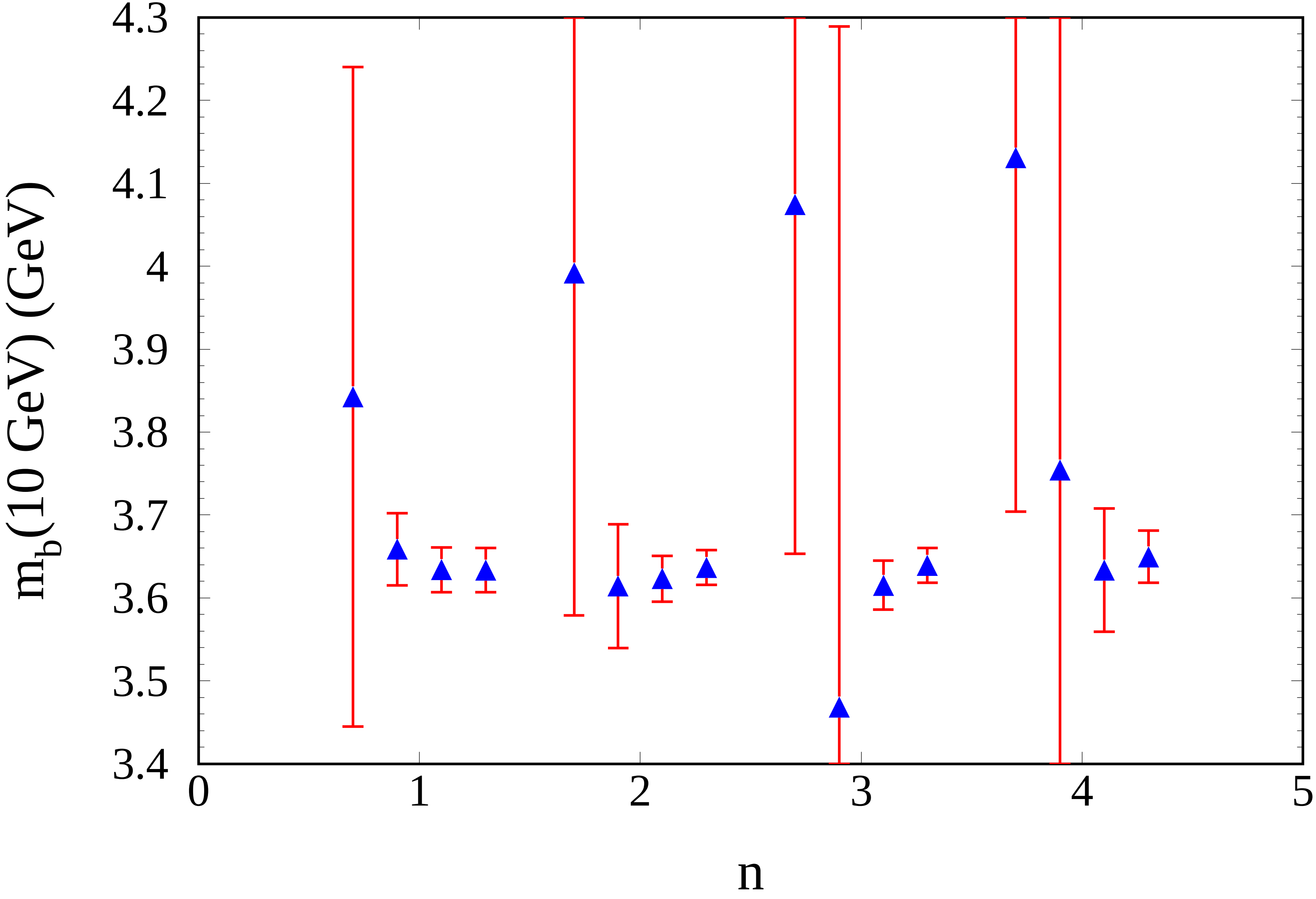
n	$\mathcal{M}_n^{\text{res},(1S-4S)}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(2n+1)}$
1	1.394(23)	0.296(32)	2.911(18)	4.601(43)
2	1.459(23)	0.248(27)	1.173(11)	2.880(37)
3	1.538(24)	0.208(22)	0.624(7)	2.370(34)
4	1.630(25)	0.175(19)	0.372(5)	2.177(32)

preliminary results (m_b)

n	m_b(10 GeV)	exp	α_s	μ	total	$\delta\bar{C}_n^{30}$	$m_b(m_b)$
1	3.594	0.020	0.007	0.001	0.021	—	4.150
2	3.612	0.014	0.012	0.001	0.018	0.005	4.167
3	3.622	0.010	0.014	0.010	0.020	0.008	4.177
4	3.637	0.008	0.014	0.026	0.031	0.012	4.192

$n = 2$:

- $m_b(10\text{GeV}) = 3612 \pm 23\,\text{MeV}$
- $m_b(m_b) = 4167 \pm 23\,\text{MeV}$



$m_b(10\ \mathrm{GeV})$ (GeV)
4.3
4.2
4.1
4
3.9
3.8
3.7
3.6
3.5
3.4
0
1
2
3
4
5
n

Summary on m_c and m_b

➪ drastic improvement in δm_c, δm_b from moments with low n in N^{2}LO

➪ direct determination of short-distance mass

- improved measurements of $\Gamma_e(J/\psi, \psi')$ and $\Gamma_e(\Upsilon, \Upsilon', \Upsilon'')$
- improved measurement of charm threshold region
- reanalysis of bottom threshold region
- new N^3LO results lead to significant improvements

preliminary results:

$$m_c(3\text{ GeV}) = 0.988(13)\text{ GeV}$$
$$m_c(m_c) = 1.287(13)\text{ GeV}$$

$$m_b(10\text{ GeV}) = 3.612(23)\text{ GeV}$$
$$m_b(m_b) = 4.167(23)\text{ GeV}$$

(old result: $m_c(m_c) = 1.304(27)$GeV, $m_b(m_b) = 4.191(51)$GeV)

III m_c and m_b: other characteristic results

no review

charm

- moments of B-decay distributions (hadron mass, lepton energy)

  HQE up to $\mathcal{O}(1/m_b^3)$, pQCD up to $\mathcal{O}(\alpha_s^2\beta_0)$

  (1240 ± 70)MeV — O. Buchmüller, Flächer

  $(1224 \pm 17 \pm 54)$MeV — Hoang, Manohar

- Lattice, from D_s (quenched ➪ $\pm(40 - 60)$MeV)

  $(1260 \pm 40 \pm 120)$MeV — Becirevic, Lubicz, Martinelli

  (1301 ± 34)MeV — Rolf, Sint

bottom

- moments of B-decay distributions (hadron mass, lepton energy)

  HQE up to $\mathcal{O}(1/m_b^3)$, pQCD up to $\mathcal{O}(\alpha_s^2\beta_0)$

  (4200 ± 40)MeV Buchmüller, Flächer

  (4170 ± 30)MeV Bauer et al.

- Υ-spectroscopy (1S-state), pNRQCD + nonperturbative effects

  (4346 ± 70)MeV Penin, Steinhauser (N^3LO)

  $(4210 \pm 90 \pm 25)$MeV Pineda (N^2LO)

- Lattice (HQET + $1/m_b$ terms, quenched)

  (4301 ± 70)MeV ALPHA-Coll. Dellta Morte et al.

IV m_s from τ-decays and sum rules

$$\boxed{\tau \longrightarrow \nu s\bar{d}}$$

input:

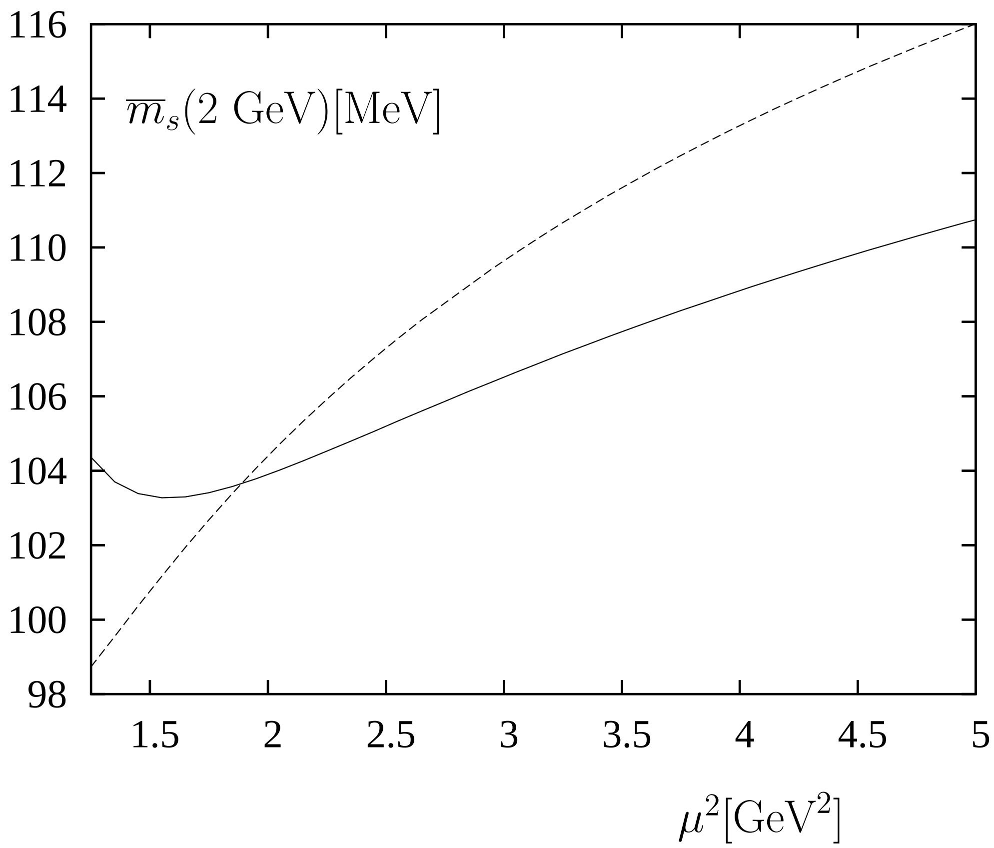
- moments of $m(s\bar{d})$ (ALEPH, OPAL)
- V_{us} (Czarnecki, Marciano, Sirlin)
- phenomenology (Gamiz et al)
- pQCD in $\mathcal{O}(\alpha^3)$ (Baikov, Chetyrkin, JK)

  (finite part of massless four-loop correlator)

➪ $m_s(M_\tau) = 100 \pm \left(\begin{smallmatrix}+5\\-3\end{smallmatrix}\right)_{\text{theo}} \pm \left(\begin{smallmatrix}+17\\-19\end{smallmatrix}\right)_{\text{rest}} \text{MeV}$

pseudoscalar sum rules in $\mathcal{O}(\alpha_s^4)$

$\bar{m}_s(2\text{ GeV}) = 105 \pm 6(\text{param.}) \pm 7(\text{hadr.})\text{MeV}$ (Chetyrkin, Khodjamirian)



Method	$\overline{m}_s(2\text{ GeV})$ [MeV]	Ref.
Pseudoscalar Borel sum rule	$105 \pm 6 \pm 7$	Chetyrkin
Pseudoscalar FESR	100 ± 12	Maltman
Scalar Borel sum rule	99 ± 16	Jamin
Vector FESR	139 ± 31	Eidemüller
Spectral function	> 77	Baikov
Hadronic τ decays	81 ± 22 96^{+5+16}_{-3-18} 104 ± 28	Gamiz Baikov Narison
τ decays $\oplus$ sum rules	99 ± 28	Narison
Lattice QCD ($n_f = 2$)	97 ± 22 $100 - 130$ $101 \pm 8^{+25}_{-0}$	Della Morte Gockeler Becirevic
Lattice QCD ($n_f = 3$)	$76 \pm 3 \pm 7$ 86.7 ± 5.9 $87 \pm 4 \pm 4$	Aubin Ishikawa Mason
PDG04 average	$80 - 130$	Eidelman

Summary

new multiloop results from pQCD + improved data (preliminary analysis)

$$m_c(3\ \text{GeV}) = 988 \pm 13\ \text{MeV} \qquad m_c(m_c) = 1287 \pm 13\ \text{MeV}$$

$$m_b(10\ \text{GeV}) = 3612 \pm 23\ \text{MeV} \qquad m_b(m_b) = 4167 \pm 23\ \text{MeV}$$

significantly reduced errors, consistent with other determinations,

but more precise

$$m_s(2\ \text{GeV}) = 105 \pm 10\ \text{MeV}$$

on the basis of N^3LO pseudoscalar sum rules