Strange, Charm and Bottom masses at NNLO and N^{3}LO

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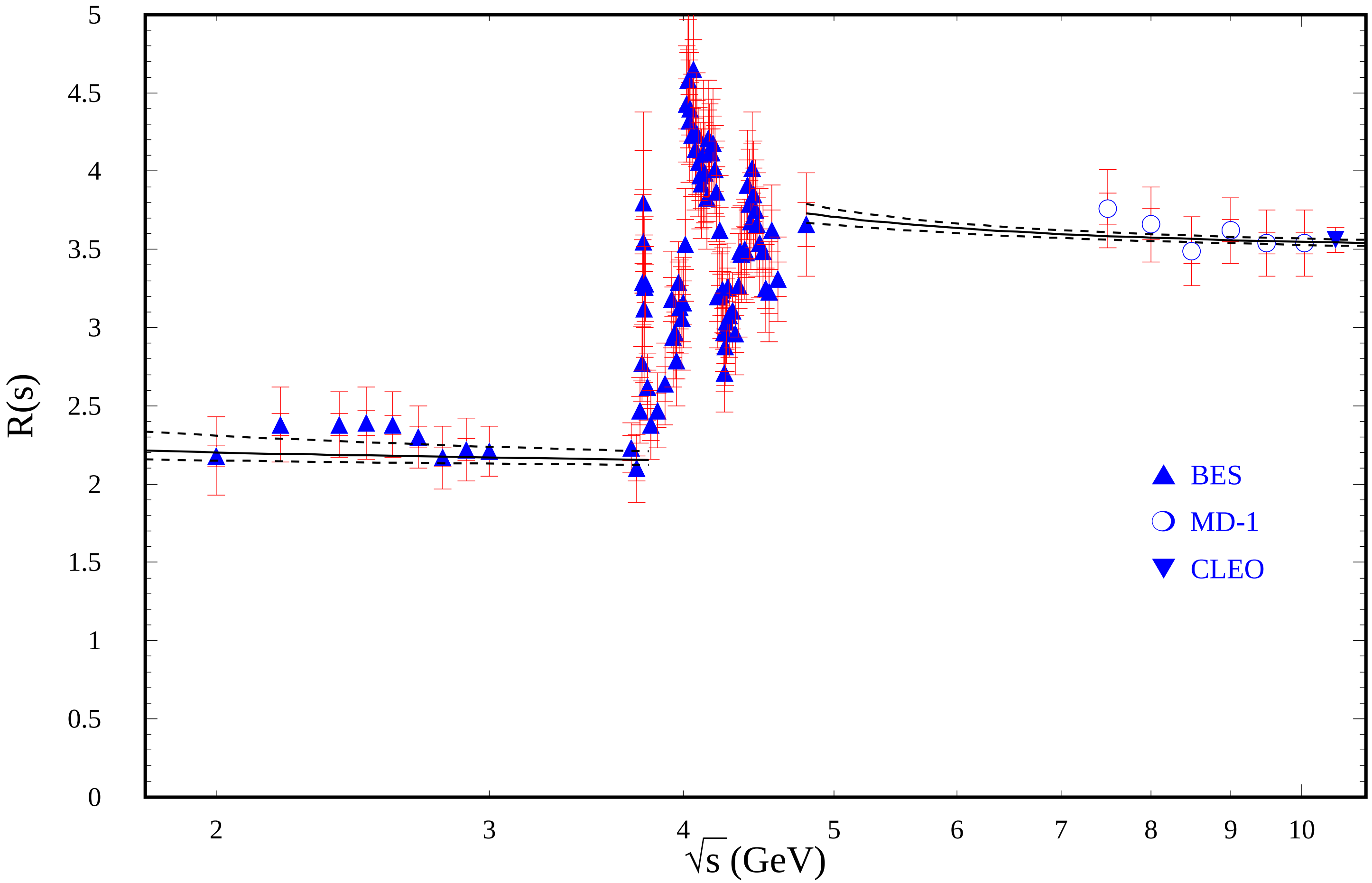
SFB TR-9

I. Sum Rules with Charm and Bottom Quarks: recent data and N^3LO calculations

II. m_c and m_b: overview

III. m_s from τ-decays and sum rules

I Sum Rules with Charm and Bottom Quarks



pQCD and data agree well in the regions

2 — 3.73 GeV and 5 — 10.52 GeV

experiment	energy [GeV]	date	systematic error
BES	2 — 5	2001	4%
MD-1	7.2 — 10.34	1996	4%
CLEO	10.52	1998	2%
PDG	J/ψ		(7%) 3%
PDG	ψ'		(9%) 5.7%
PDG	ψ''		15%

m_Q from
SVZ Sum Rules, Moments and Tadpoles

Some definitions

$$R(s) = 12\pi \, \mathrm{Im} \left[\Pi(q^2 = s + i\epsilon) \right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu \, q_\nu \right) \Pi(q^2) \equiv i \int \mathrm{d}x \, e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_Q(q^2) = Q_Q^2 \dfrac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n \, z^n$

with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2

(Chetyrkin, JK, Steinhauser)

recently up to $n = 30$!

(Boughezal, Czakon, Schutzmeier)

recently also $\bar{C}_0$ and $\bar{C}_1$ in order α_s^3 (four loops!)

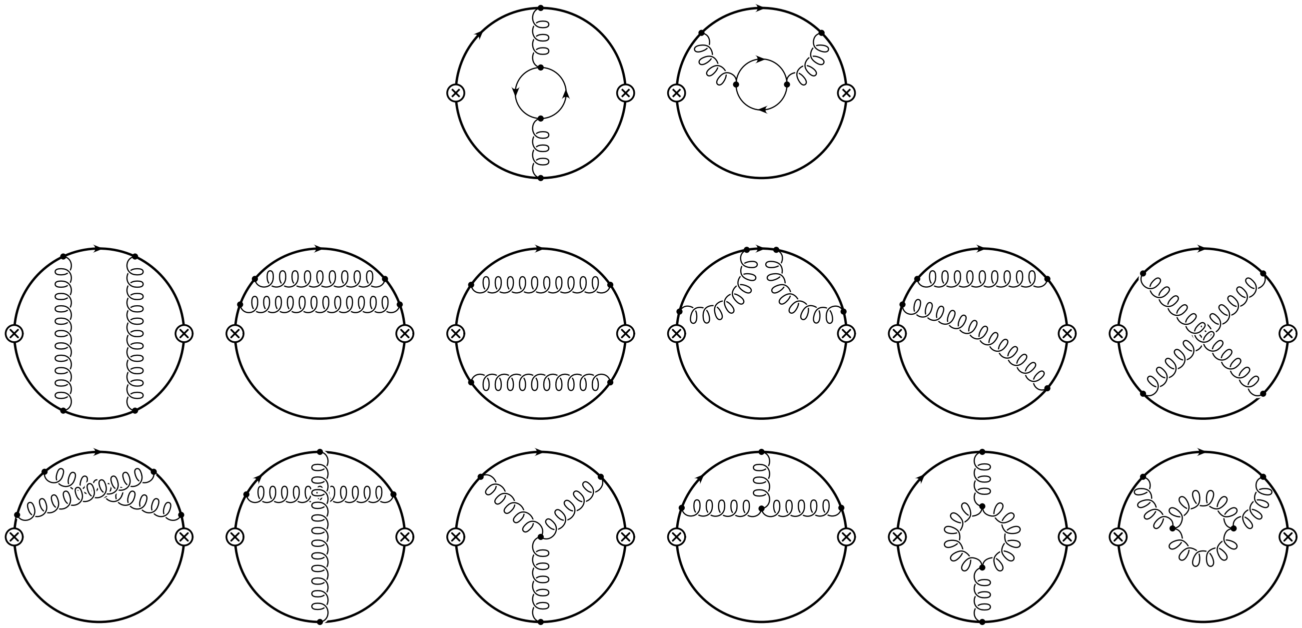
➪ talk at session V (QCD hard interactions)

(Chetyrkin, JK, Sturm; confirmed by Boughezal, Czakon, Schutzmeier)

Analysis in NNLO

Coefficients $\bar{C}_n$ from three-loop one-scale tadpole amplitudes with "arbitrary" power of propagators;

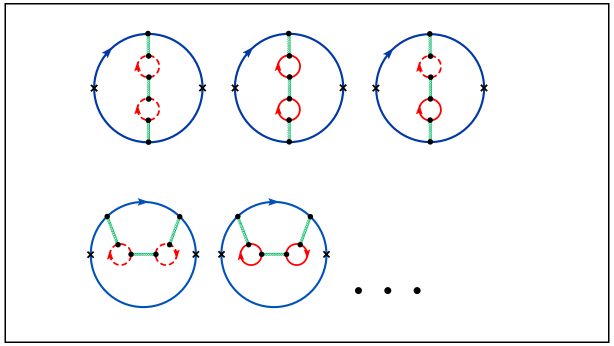
FORM-program MATAD



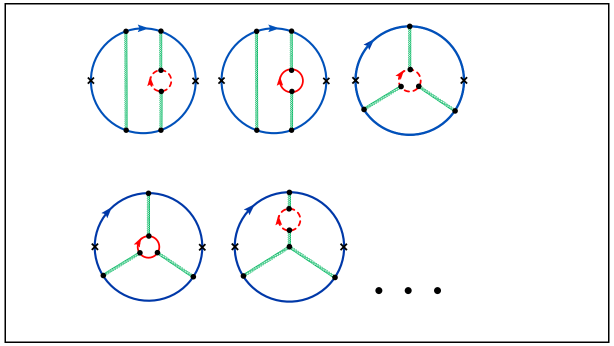
Analysis in N^{3}LO

Algebraic reduction to 13 master integrals (Laporta algorithm);
numerical evaluation of master integrals

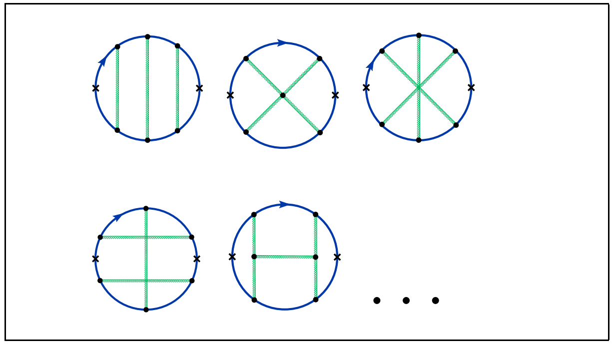
n_f^2-contributions



n_f^1-contributions



n_f^0-contributions



 : heavy quarks,  : light quarks,

n_f: number of active quarks

$\Longrightarrow$ About 700 Feynman-diagrams

$\bar{C}_n$ depend on the charm quark mass through $l_{m_c} \equiv \ln(m_c^2(\mu)/\mu^2)$

$$\begin{aligned}\bar{C}_n &= \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right)\\ &+ \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right)\\ &+ \left(\frac{\alpha_s(\mu)}{\pi}\right)^3 \left(\dots\right)\end{aligned}$$

n	1	2	3	4
$\bar{C}_n^{(0)}$	1.0667	0.4571	0.2709	0.1847
$\bar{C}_n^{(10)}$	2.5547	1.1096	0.5194	0.2031
$\bar{C}_n^{(11)}$	2.1333	1.8286	1.6254	1.4776
$\bar{C}_n^{(20)}$	2.4967	2.7770	1.6388	0.7956
$\bar{C}_n^{(21)}$	3.3130	5.1489	4.7207	3.6440
$\bar{C}_n^{(22)}$	−0.0889	1.7524	3.1831	4.3713

Define the moments

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Big|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\,\frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

➪ $\mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$

constraint: $\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$

➪ m_c

SVZ:

$\mathcal{M}_n^{\mathrm{th}}$ can be reliably calculated in pQCD: low n:

- fixed order in α_s is sufficient, in particular no resummation of $1/v$ - terms from higher orders required
- condensates are unimportant
- pQCD in terms of short distance mass : $m_c(3\ \mathrm{GeV}) \Rightarrow m_c(m_c)$ stable expansion : no pole mass or closely related definition (1S-mass, potential-subtracted mass) involved
- moments available in NNLO
- and $\bar{C}_0$, $\bar{C}_1$ in N^{3}LO

anatomy of errors and update: m_c

old results (NPB 619(2001) and

new results (update on $\Gamma_e(J/\psi, \psi')$ and $\alpha_s = 0.1187 \pm 0.0020$)

n	$J/\psi, \psi'$ $\mathcal{M}_n^{\text{exp,res}}$ $\times 10^{(n-1)}$	charm threshold region $\mathcal{M}_n^{\text{exp,cc}}$ $\times 10^{(n-1)}$	continuum $\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	sum $\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$
1	0.1114(82)	0.0313(15)	0.0638(10)	0.2065(84)
1	0.1138(40)	0.0313(15)	0.0639(10)	0.2090(44)
2	0.1096(79)	0.0174(8)	0.0142(3)	0.1412(80)
2	0.1121(38)	0.0174(8)	0.0142(3)	0.1437(39)

new analysis

$\boxed{n=1}$ N^2LO

$m_c(3\text{ GeV}) = 1017 \pm 14(\text{exp}) \pm 7(\alpha_s) \pm 1.3(\text{scale})$

➪ $m_c(m_c) = 1301 \pm 15$ (3-loop-running)

$\boxed{n=1}$ N^3LO

$m_c(3\text{ GeV}) = 1012 \pm 14(\text{exp}) \pm 8(\alpha_s) \pm 0.6(\text{scale})$

➪ $m_c(m_c) = 1297 \pm 15$ (4-loop-running)

$\boxed{n=2}$ N^2LO

$m_c(3\text{ GeV}) = 982 \pm 10(\text{exp}) \pm 17\ (\alpha_s) \pm 19\ (\text{scale})$

➪ $m_c(m_c) = 1269 \pm 25$ (3-loop-running)

combine ➪ $m_c(3\text{ GeV}) = 1007 \pm 16$

$m_c(m_c) = 1290 \pm 15,$ old result 1304 ± 27

anatomy of errors and update: m_b

old results (NPP 619 (2001)) and update (Γ_e from CLEO);

$$\alpha_s = 0.1187 \pm 0.0020$$

n	$\mathcal{M}_n^{\rm exp,res} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\rm exp,thr} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\rm cont} \times 10^{(2n+1)}$	$\mathcal{M}_n^{\rm exp} \times 10^{(2n+1)}$
1	1.237(63)	0.306(62)	2.913(21)	4.456(121)
1	1.271(24)	0.306(62)	2.918(16)	4.494(84)
2	1.312(65)	0.261(54)	1.182(12)	2.756(113)
2	1.348(25)	0.261(52)	1.185(9)	2.795(75)
3	1.399(68)	0.223(44)	0.634(8)	2.256(108)
3	1.437(26)	0.223(44)	0.636(6)	2.296(68)

new analysis

$\boxed{n=1}$ N^2LO

$m_b(10\ \mathrm{GeV}) = 3644 \pm 40(\mathrm{exp})\ \pm 7(\alpha_s)\ \pm 4(\mathrm{scale})$

➪ $m_b(m_b) = 4191 \pm 40$ (3-loop-running)

$\boxed{n=1}$ N^3LO

$m_b(10\ \mathrm{GeV}) = 3644 \pm 40(\mathrm{exp})\ \pm 7(\alpha_s)\ \pm 1(\mathrm{scale})$

➪ $m_b(m_b) = 4191 \pm 40$ (4-loop-running)

$\boxed{n=2}$ N^2LO

$m_b(10\ \mathrm{GeV}) = 3631 \pm 29(\mathrm{exp}) \pm 13\ (\alpha_s) \pm 17\ (\mathrm{scale})$

➪ $m_b(m_b) = 4179 \pm 35$ (3-loop-running)

$\boxed{n=3}$ N^2LO

$m_b(10\ \mathrm{GeV}) = 3622 \pm 22(\mathrm{exp}) \pm 17\ (\alpha_s) \pm 20\ (\mathrm{scale})$

➪ $m_b(m_b) = 4170 \pm 33$ (3-loop-running)

combine ➪ $m_b(10\ \mathrm{GeV}) = 3630 \pm 35$

$m_b(m_b) = 4180 \pm 35,$ old result 4210 ± 50

Summary on m_c and m_b

- drastic improvement in δm_c, δm_b from moments with low n in N^{2}LO
- direct determination of short-distance mass

improved measurements of $\Gamma_e(J/\psi, \psi')$ and $\Gamma_e(\Upsilon, \Upsilon', \Upsilon'')$
and new N^3LO results lead to significant improvements

preliminary results:

$$m_c(3\text{ GeV}) = 1.007(16)\text{ GeV}$$
$$m_c(m_c) = 1.300(15)\text{ GeV}$$

$$m_b(10\text{ GeV}) = 3.630(35)\text{ GeV}$$
$$m_b(m_b) = 4.179(35)\text{ GeV}$$

II m_c and m_b: other characteristic results

no review

charm

- moments of B-decay distributions (hadron mass, lepton energy)
  HQE up to $\mathcal{O}(1/m_b^3)$, pQCD up to $\mathcal{O}(\alpha_s^2\beta_0)$
  1240 ± 70 Buchmüller, Flächer
  $1224 \pm 17 \pm 54$ Hoang, Manohar
- Lattice, from D_s (quenched ➪ $\pm(40-60)$)
  $1260 \pm 40 \pm 120$ Becirevic, Lubicz, Martinelli
  1301 ± 34 Rolf, Sint

bottom

- moments of B-decay distributions (hadron mass, lepton energy)
  HQE up to $\mathcal{O}(1/m_b^3)$, pQCD up to $\mathcal{O}(\alpha_s^2\beta_0)$
  4200 ± 40 Buchmüller, Flächer
  4170 ± 30 Bauer et al.
- Υ-spectroscopy (1S-state), pNRQCD + nonperturbative effects
  4346 ± 70 Penin, Steinhauser (N^3LO)
  $4210 \pm 90 \pm 25$ Pineda (N^2LO)
- Lattice (HQET + $1/m_b$ terms, quenched)
  4301 ± 70 ALPHA-Coll. Dellta Morte et al.

III m_s from τ-decays and sum rules

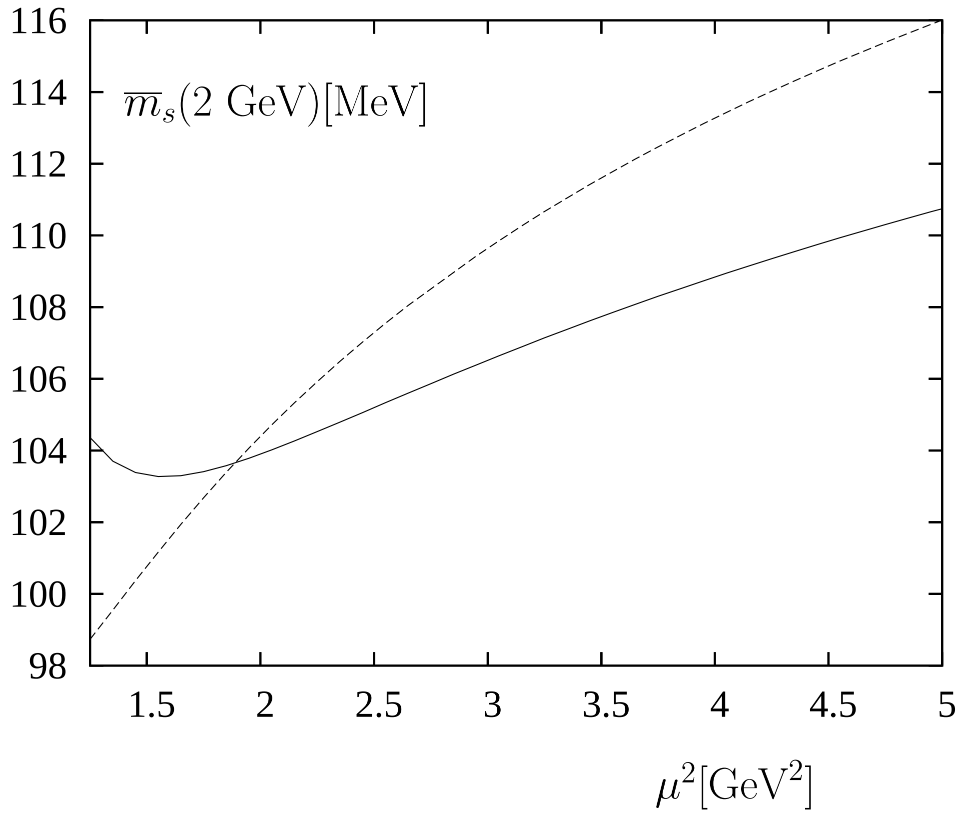
$\boxed{\tau \longrightarrow \nu s \bar{d}}$

input:	moments of $m(s\bar{d})$	(ALEPH, OPAL)
	V_{us}	(Czarnecki, Marciano, Sirlin)
	phenomenology	(Gamiz et al)
	pQCD in $\mathcal{O}(\alpha^3)$	(Baikov, Chetyrkin, JK)
	(finite part of massless four-loop correlator)	

➪ $ms(M_\tau) = 100 \pm \binom{+5}{-3}_{\text{theo}} \pm \binom{+17}{-19}_{\text{rest}}$

pseudoscalar sum rules in $\mathcal{O}(\alpha_s^4)$

$\bar{m}_s(2\,\mathrm{GeV}) = 105 \pm 6(\mathrm{param}) \pm 7(\mathrm{hadr})$ Chetyrkin, Khodjamirian



Method	$\overline{m}_s(2\text{ GeV})$ [MeV]	Ref.
Pseudoscalar Borel sum rule	$105 \pm 6 \pm 7$	Chetyrkin
Pseudoscalar FESR	100 ± 12	Maltman
Scalar Borel sum rule	99 ± 16	Jamin
Vector FESR	139 ± 31	Eidemüller
Spectral function	> 77	Baikov
Hadronic τ decays	81 ± 22	Gamiz
	96^{+5+16}_{-3-18}	Baikov
	104 ± 28	Narison
τ decays $\oplus$ sum rules	99 ± 28	Narison
Lattice QCD ($n_f = 2$)	97 ± 22	Della Morte
	100 -130	Gockeler
	$101 \pm 8^{+25}_{-0}$	Becirevic
Lattice QCD ($n_f = 3$)	$76 \pm 3 \pm 7$	Aubin
	86.7 ± 5.9	Ishikawa
	$87 \pm 4 \pm 4$	Mason
PDG04 average	80 -130	Eidelman

Summary

new multiloop results from pQCD + improved data

$$m_c(3\text{ GeV}) = 1007 \pm 16\text{ MeV} \qquad m_c(m_c) = 1290 \pm 15\text{ MeV}$$
$$m_b(10\text{ GeV}) = 3630 \pm 35\text{ MeV} \qquad m_b(m_b) = 4180 \pm 35\text{ MeV}$$

significantly reduced errors, consistent with other determinations, but more precise

$$m_s(2\text{ GeV}) = 105 \pm 10\text{ MeV}$$

on the basis of N^3LO pseudoscalar sumrules