Charm and bottom masses at NNLO from electron-positron annihilation at low energies

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+ updates

I. Experimental Results for R below $B\bar{B}$-Threshold ➪ α_s

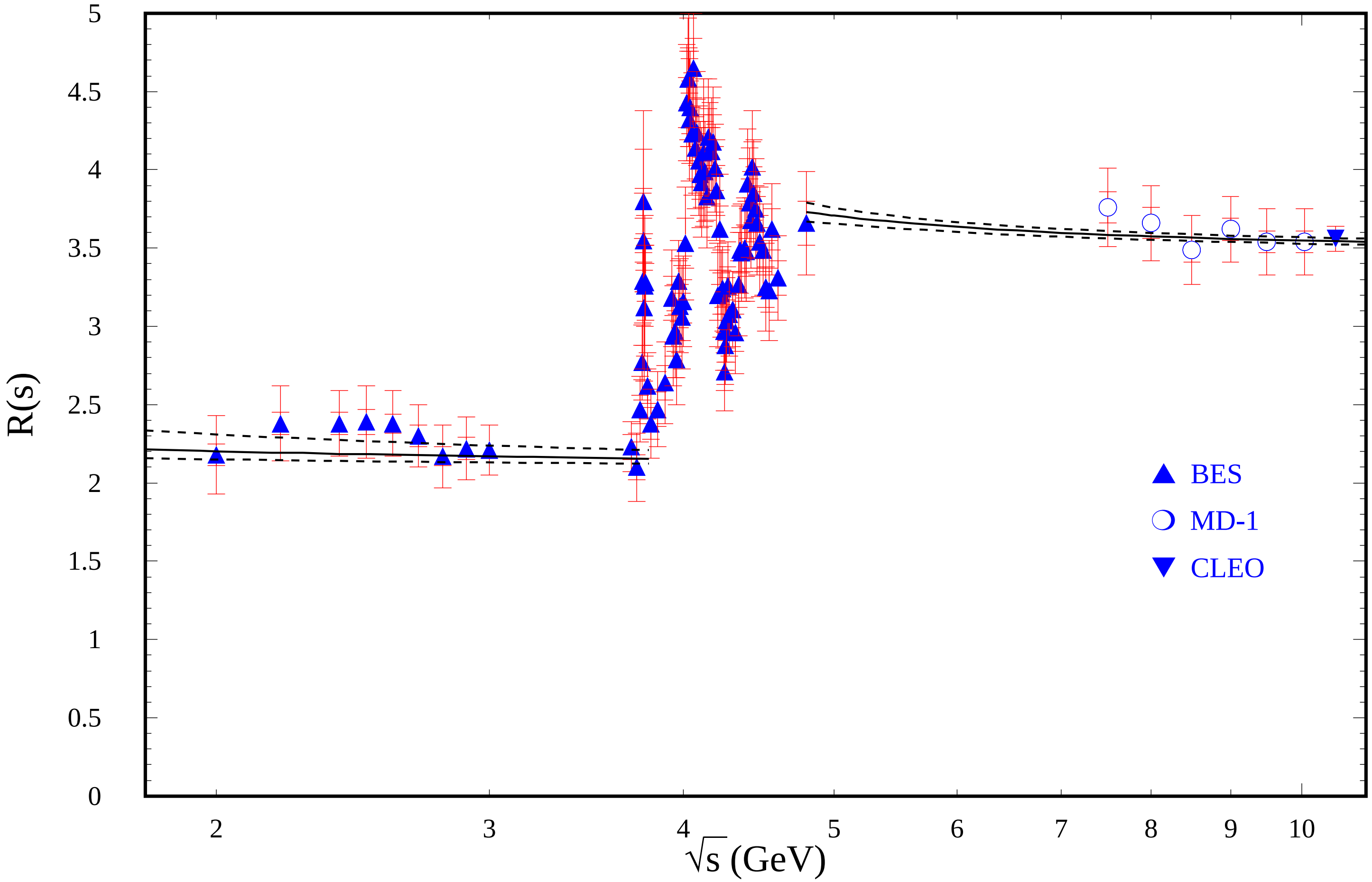
II. Sum Rules to NNLO with Massive Quarks ➪ $m_Q(m_Q)$
updates based on recent data

III. Summary

I. Experimental Results for R below $B\bar{B}$-Threshold ➪ α_s

- data
- α_s

Data vs. Theory



experiment	energy [GeV]	date	systematic error
BES	2 — 5	2001	4%
MD-1	7.2 — 10.34	1996	4%
CLEO	10.52	1998	2%
PDG	J/ψ		(7%) 3%
PDG	ψ'		(9%) 5.7%
PDG	ψ''		15%

pQCD and data agree well in the regions

2 — 3.73 GeV and 5 — 10.52 GeV

α_s

pQCD includes full m_Q-dependence up to $\mathcal{O}(\alpha_s^2)$ and terms of $\mathcal{O}(\alpha_s^3(m^2/s)^n)$ with $n = 0, 1, 2$

can we deduce α_s from the low energy data?

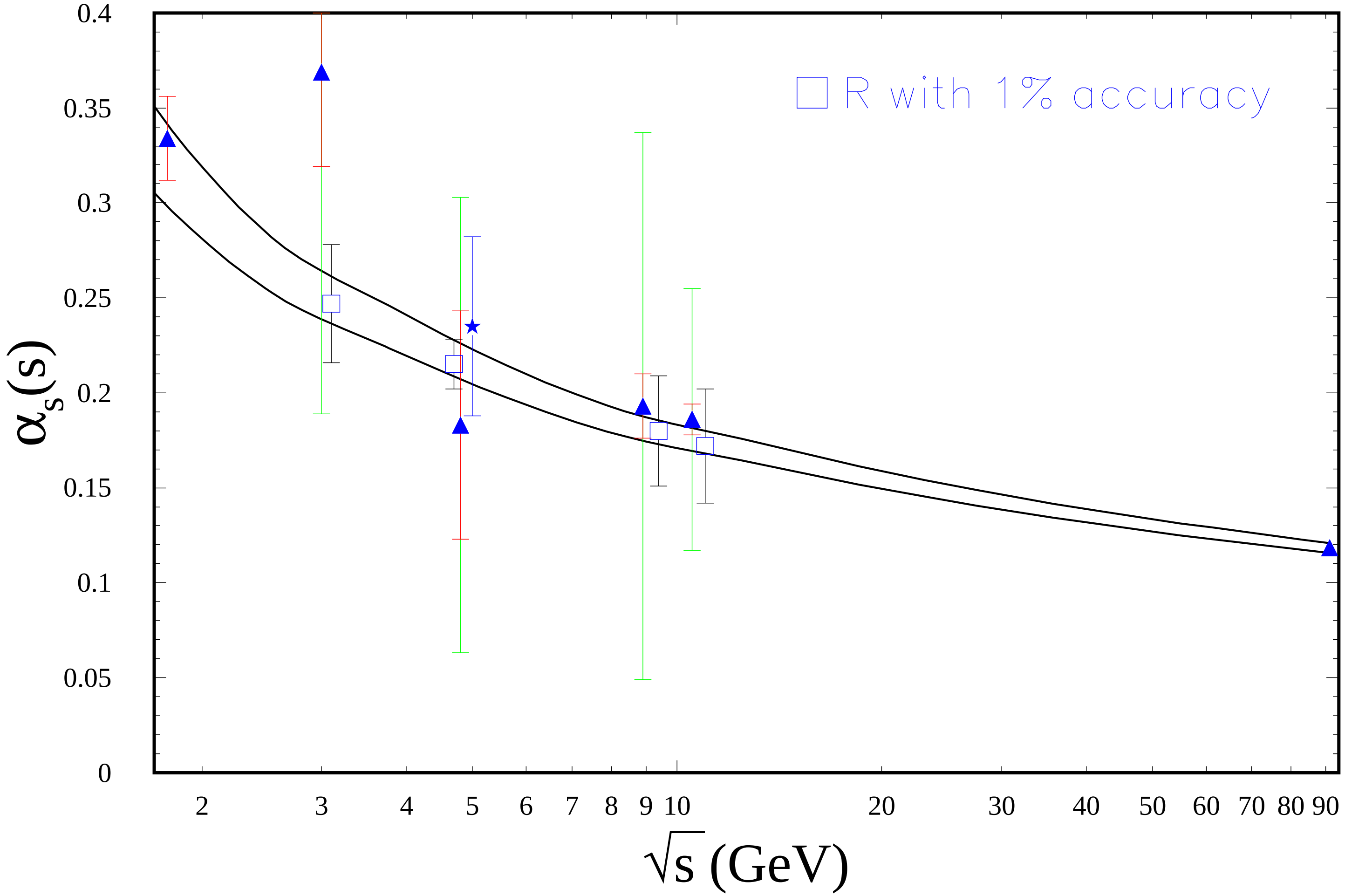
Result:

BES below 3.73 GeV: $\alpha_s^{(3)}(3\text{ GeV}) = 0.369^{+0.047+0.123}_{-0.046-0.130}$

BES at 4.8 GeV: $\alpha_s^{(4)}(4.8\text{ GeV}) = 0.183^{+0.059+0.053}_{-0.064-0.057}$

MD-1: $\alpha_s^{(4)}(8.9\text{ GeV}) = 0.193^{+0.017+0.127}_{-0.017-0.107}$

CLEO: $\alpha_s^{(4)}(10.52\text{ GeV}) = 0.186^{+0.008+0.061}_{-0.008-0.057}$



R with 1% accuracy
$\alpha_s(s)$
$\sqrt{s}$ (GeV)
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90

combined, assuming uncorrelated errors:

$$\alpha_s^{(4)}(5\text{ GeV}) = 0.235 \pm 0.047$$

Evolve up to M_Z : $\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$

confirmation of running !

Result consistent with LEP, but not competitive

(precision of 0.4% at 3.7 GeV (0.7% at 2 GeV) would be required)

The evaluation of $R(s)$ in order α_s^4 is within reach

(Baikov,Chetyrkin, JK)

II. Sum Rules to NNLO with Massive Quarks

- m_Q from SVZ Sum Rules, Moments and Tadpoles
- Tadpoles at Three Loop
- Results for Charm and Bottom Masses

m_Q from
SVZ Sum Rules, Moments and Tadpoles

Some definitions

$$R(s) \;=\; 12\pi \, \mathrm{Im} \left[\Pi(q^2 = s + i\epsilon) \right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu \, q_\nu \right) \; \Pi(q^2) \;\equiv\; i \int \mathrm{d}x \, e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ

Taylor expansion: $\Pi_c(q^2) \;=\; Q_c^2 \dfrac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n \, z^n$

with $z = q^2/(4m_c^2)$ and $m_c = m_c(\mu)$ the $\overline{\mathrm{MS}}$ mass.

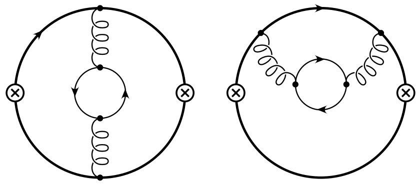
Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2
(Chetyrkin, JK, Steinhauser)

recently also $\bar{C}_0$ in order α_s^3 (four loops!)
(Chetyrkin, JK, Sturm)

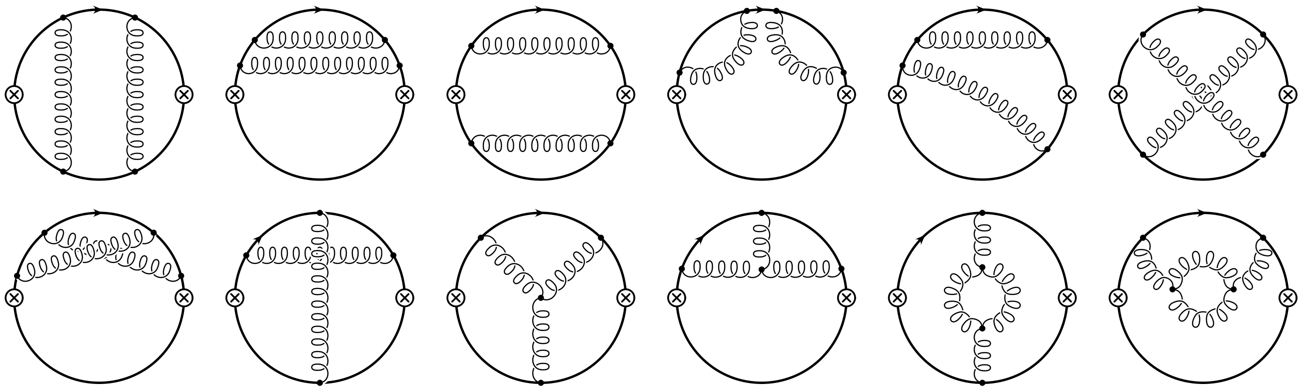
$\bar{C}_1$ to order α_s^3 is within reach

Tadpoles in NNLO

all three-loop – one-scale tadpole amplitudes can be calculated with "arbitrary" power of propagators (Broadhurst; Chetyrkin, JK, Steinhauser); FORM-program MATAD (Steinhauser)



Three-loop diagrams contributing to $\Pi_l^{(2)}$ (inner quark massless) and $\Pi_F^{(2)}$ (both quarks with mass m).



Purely gluonic contribution to $\mathcal{O}(\alpha_s^2)$

$\bar{C}_n$ depend on the charm quark mass through

$$
\begin{aligned}
l_{m_c} &\equiv \ln(m_c^2(\mu)/\mu^2) \\
\bar{C}_n &= \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right) \\
&\quad + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right)
\end{aligned}
$$

n	1	2	3	4
$\bar{C}_n^{(0)}$	1.0667	0.4571	0.2709	0.1847
$\bar{C}_n^{(10)}$	2.5547	1.1096	0.5194	0.2031
$\bar{C}_n^{(11)}$	2.1333	1.8286	1.6254	1.4776
$\bar{C}_n^{(20)}$	2.4967	2.7770	1.6388	0.7956
$\bar{C}_n^{(21)}$	3.3130	5.1489	4.7207	3.6440
$\bar{C}_n^{(22)}$	−0.0889	1.7524	3.1831	4.3713

Define the moments

$$\mathcal{M}_n^{\rm th} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Bigg|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\,\frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

➪ $\mathcal{M}_n^{\rm exp} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$

constraint: $\mathcal{M}_n^{\rm exp} = \mathcal{M}_n^{\rm th}$

➪ m_c

SVZ:

$\mathcal{M}_n^{\mathrm{th}}$ can be reliably calculated in pQCD: low n:

- fixed order in α_s is sufficient, in particular no resummation of $1/v$ - terms from higher orders required

- condensates are unimportant

- pQCD in terms of short distance mass : $m_c(3\ \mathrm{GeV}) \Rightarrow m_c(m_c)$ stable expansion : no pole mass or closely related definition (1S-mass, potential-subtracted mass) involved

- moments available in NNLO

- and soon $\bar{C}_0$, $\bar{C}_1$, $\bar{C}_2(?)$ in $\mathrm{N}^3\mathrm{LO}$

Results from Nucl. Phys. B 619 (2001)

input for $R(s)$

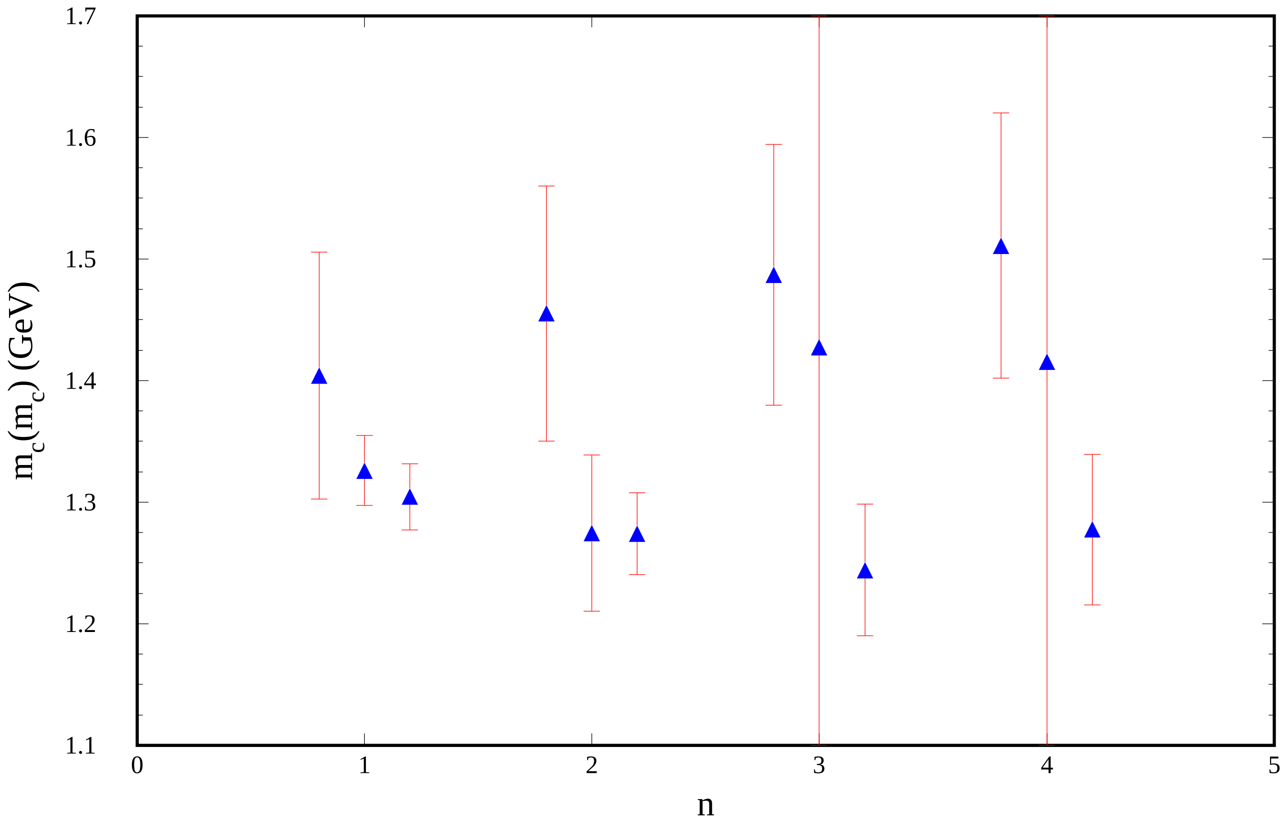
- resonances (J/ψ, ψ')
- continuum below 4.8 GeV (BES)
- continuum above 4.8 GeV (theory)

experimental error of the moments dominated by resonances

n	1	2	3	4
$m_c(3\ \text{GeV})$	1.027(30)	0.994(37)	0.961(59)	0.997(67)
$m_c(m_c)$	1.304(27)	1.274(34)	1.244(54)	1.277(62)

error in m_c dominated by experiment for n=1,
by theory (variation of μ, α_s) for $n = 3, 4, \ldots$

stability: compare LO, NLO, NNLO ➪ clear improvement



$m_c(m_c)$ for $n = 1, 2, 3, 4$ in LO, NLO, NNLO.

anatomy of errors and update: m_c

old results and

new results (update on $\Gamma_e(J/\psi, \psi')$ and $\alpha_s = 0.1187 \pm 0.0020$)

	$J/\psi, \psi'$	charm threshold region	continuum	sum
n	$\mathcal{M}_n^{\text{exp,res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp,cc}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$
1	0.1114(82)	0.0313(15)	0.0638(10)	0.2065(84)
1	0.1138(40)	0.0313(15)	0.0639(10)	0.2090(44)
2	0.1096(79)	0.0174(8)	0.0142(3)	0.1412(80)
2	0.1121(38)	0.0174(8)	0.0142(3)	0.1437(39)

old:

$$m_c(m_c) = \begin{cases} 1.304(27)\ \text{GeV (from } n{=}1) \\ 1.274(34)\ \text{GeV (from } n{=}2) \end{cases}$$

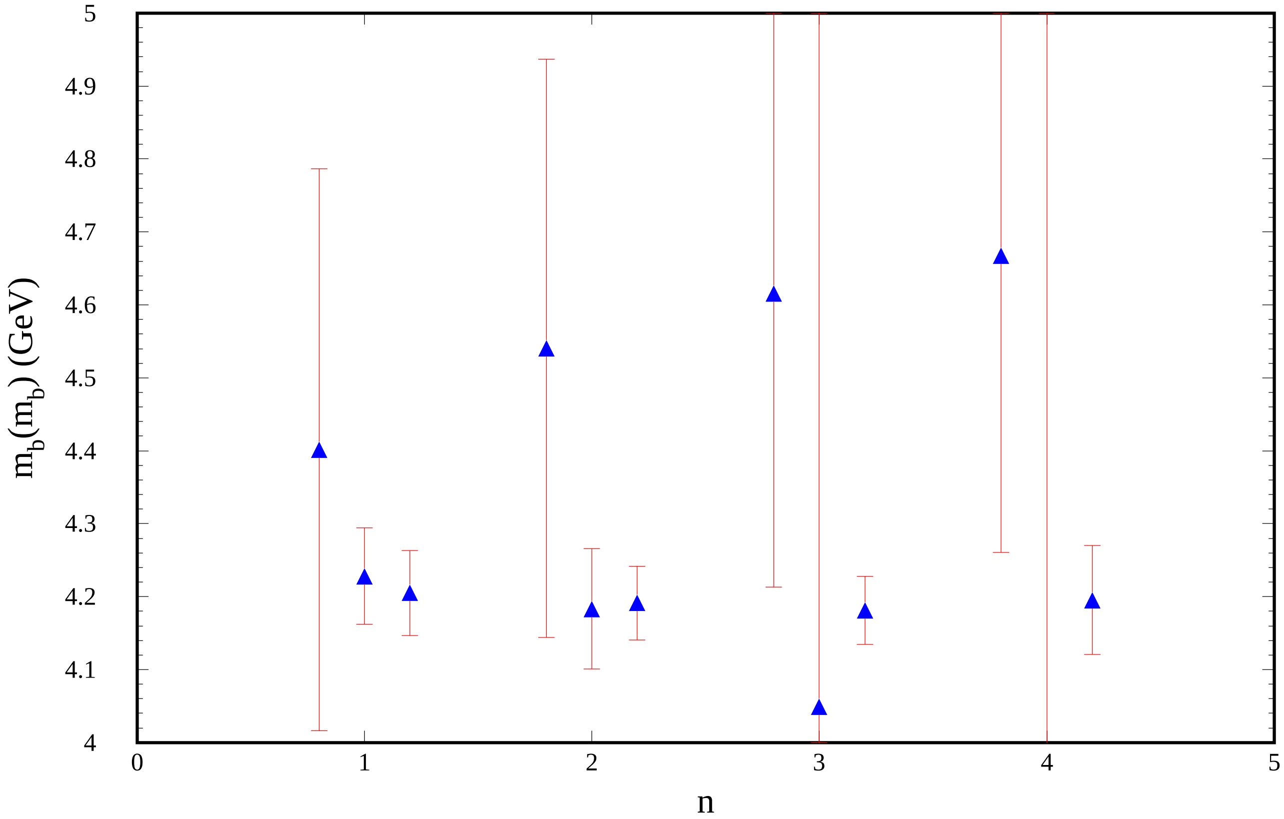
new:

$$m_c(m_c) = \begin{cases} 1.300(15)\ \text{GeV (from } n{=}1\text{), error dominated by exp.} \\ 1.269(25)\ \text{GeV (from } n{=}2\text{), error dominated by th.} \end{cases}$$

new results consistent with old results; smaller error

Similar analysis for the **bottom quark :**

resonances include $\Upsilon(1)$ up to $\Upsilon(3)$, "continuum" starts at 11.2 GeV



$m_b(m_b)$ for $n = 1, 2, 3$ and 4 in LO, NLO and NNLO

Results from Nucl. Phys. B 619 (2001)

n	1	2	3	4
$m_b(10\ \text{GeV})$	3.665(60)	3.651(52)	3.641(48)	3.655(77)
$m_b(m_b)$	4.205(58)	4.191(51)	4.181(47)	4.195(75)

anatomy of errors and update: m_b

old results and update (Γ_e from CLEO; α_s)

n	$\mathcal{M}_n^{\text{exp,res}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{exp,thr}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(2n+1)}$
1	1.237(63)	0.306(62)	2.913(21)	4.456(121)
1	1.271(24)	0.306(62)	2.918(16)	4.494(84)
2	1.312(65)	0.261(54)	1.182(12)	2.756(113)
2	1.348(25)	0.261(52)	1.185(9)	2.795(75)
3	1.399(68)	0.223(44)	0.634(8)	2.256(108)
3	1.437(26)	0.223(44)	0.636(6)	2.296(68)

old:

$$m_b(m_b) = \begin{cases} 4.205(58) \text{ GeV (from } n{=}1\text{) error dominated by exp.} \\ 4.191(51) \text{ GeV (from } n{=}2\text{)} \\ 4.181(47) \text{ GeV (from } n{=}3\text{) error dominated by exp.} \end{cases}$$

new:

$$m_b(m_b) = \begin{cases} 4.191(40) \text{ GeV (from } n{=}1\text{) error dominated by exp.} \\ 4.179(35) \text{ GeV (from } n{=}2\text{)} \\ 4.170(33) \text{ GeV (from } n{=}3\text{) equal distr. of exp.,}\alpha_s\text{,th.} \end{cases}$$

III. Summary

$$\alpha_s^{(4)}(5\ \text{GeV}) = 0.235 \pm 0.047$$

➪ $\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$

➪ drastic improvement in δm_c, δm_b from moments with low n in N^2LO

➪ direct determination of short-distance mass

old results:

$$m_c(m_c) = 1.304(27)\ \text{GeV}$$
$$m_b(m_b) = 4.19(5)\ \text{GeV}$$

improved measurements of $\Gamma_e(J/\psi, \psi')$ and $\Gamma_e(\Upsilon, \Upsilon', \Upsilon'')$ lead to significant improvements

preliminary results:

$$m_c(m_c) = 1.300(15)\ \text{GeV}$$
$$M_c = 1.696(19)\ \text{GeV}$$

$$m_b(m_b) = 4.179(35)\ \text{GeV}$$
$$M_b = 4.815(40)\ \text{GeV}$$