ELECTROWEAK SUDAKOV LOGARITHMS

J.H. Kühn, Karlsruhe

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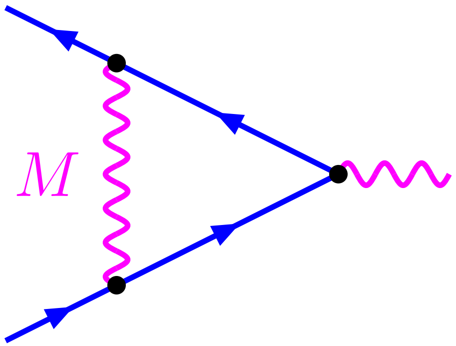
- **Introduction**
- **Form factors at two loop in N^{3}LL approximation**
- **Four fermion scattering in N^{3}LL approximation**
- **Z-boson and photon production at hadron colliders**
- **Summary**

Introduction

One-Loop

example: massive U(1)



$$\Rightarrow \quad \text{Born} * \left[1 + \frac{\alpha}{4\pi}\left(-\ln^2\frac{s}{M^2} + 3\ln\frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3}\right)\right]$$

magnitude ($\frac{\alpha_w}{4\pi} = 3 \cdot 10^{-3}$)

$\frac{s}{M^2}$	$-\ln^2\frac{s}{M^2}$	$+3\ln\frac{s}{M^2}$	$-\frac{7}{2}+\frac{\pi^2}{3}$	Σ	$* 4\,\frac{\alpha_w}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	$+15.15$	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	$+19.31$	-0.21	-22.3	-27%

(four-fermion cross section $\Rightarrow$ factor 4)

Two-Loop

Four-fermion processes, status:

LL: Fadin et al. (2000)

NLL: J.H.K., Penin, Smirnov (2000)
Large (!) subleading corrections
important angular dependent terms

NNLL: J.H.K., Moch, Penin, Smirnov (2001)
Large (!) NNLL terms,
oscillating signs of LL, NLL, NNLL
$\Rightarrow$ compensations

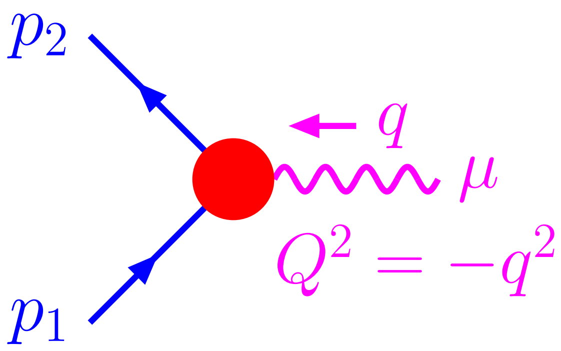
$\Rightarrow$ N^3LL and constant terms desirable
$\Rightarrow$ N^3LL now available (Jantzen, J.H.K., Penin, Smirnov)

Additional complication in SM: massless photon

$$|Q^2| \gg M_{W,Z}^2 \gg m_\gamma^2$$

Form factors at two loop

A) Form Factor and Evolution Equations



Born:

$$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow\ \mathcal{F} = \mathcal{F}_{\text{Born}}\, F_0(\alpha(M^2))\,\exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: N^{4}LL $\Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \left[\underbrace{\ln^{2n}\left(\frac{Q^2}{M^2}\right)}_{\text{LL}} + \underbrace{\ln^{2n-1}\left(\frac{Q^2}{M^2}\right)}_{\text{NLL}} + \underbrace{\ln^{2n-2}\left(\frac{Q^2}{M^2}\right)}_{\text{NNLL}} + \underbrace{\ln^{2n-3}\left(\frac{Q^2}{M^2}\right)}_{\text{N}^3\text{LL}} + \underbrace{\ln^{2n-4}\left(\frac{Q^2}{M^2}\right)}_{\text{N}^4\text{LL}} \right]$$

NNLL (previous result) requires running of α (i.e. β_0 and β_1) and:

$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$ up to $\mathcal{O}(\alpha)$

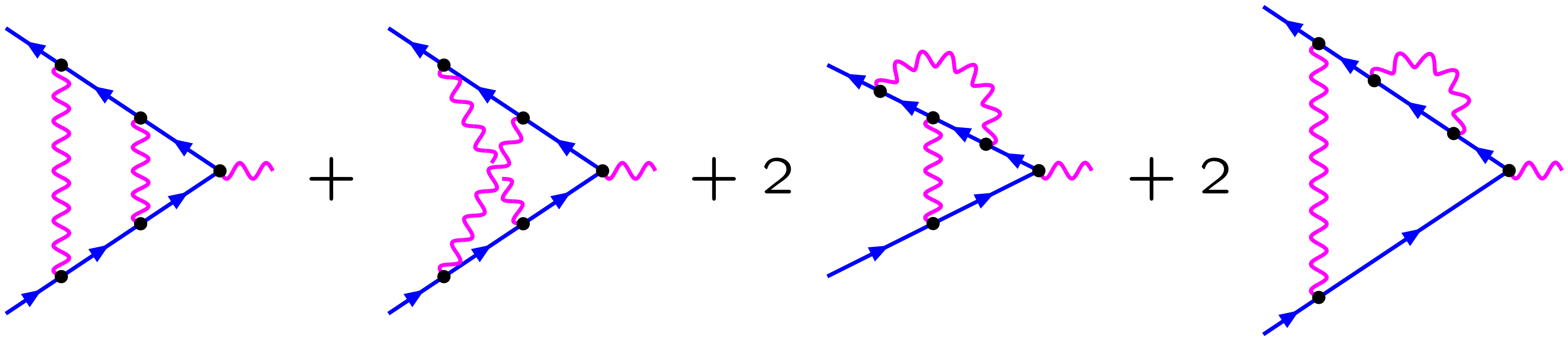
$\gamma(\alpha)$ up to $\mathcal{O}(\alpha^2)$

N^{3}LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

N^{4}LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

B) Two-Loop Results: Massive U(1) Model

$$\mathcal{F}_\alpha(M,Q) = \mathcal{F}_{\text{Born}}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \dots\right]$$



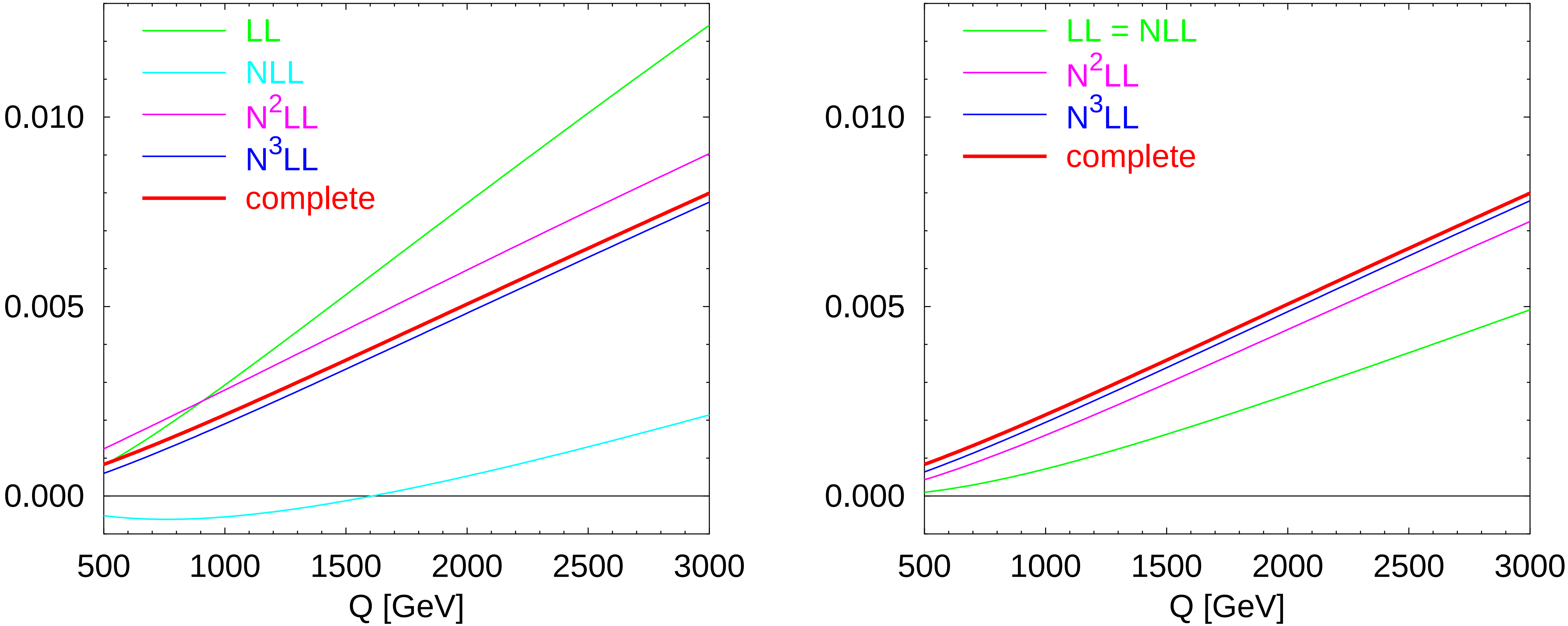
$$f^{(1)} = -\mathcal{L}^2 + 3\,\mathcal{L} - \frac{7}{2} - \frac{2}{3}\pi^2 \approx -\mathcal{L}^2 + 3\,\mathcal{L} - 10.1, \qquad \mathcal{L} \equiv \ln(Q^2/M^2)$$

$$\begin{aligned} f^{(2)} &= \frac{1}{2}\mathcal{L}^4 - 3\,\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right)\mathcal{L} + \frac{25}{2} \\ &\quad + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3}\ln^4 2 + 256\,\text{Li}_4\left(\frac{1}{2}\right) \\ &\approx +0.5\,\mathcal{L}^4 - 3\,\mathcal{L}^3 + 14.6\,\mathcal{L}^2 - 19.6\,\mathcal{L} + 26.4 \end{aligned}$$

NNLL in agreement with previous results!

Two-loop result $f^{(2)}$:

logs $\ln\left(\frac{Q^2}{M^2}\right)$ rescaled logs $\ln\left(\frac{Q^2}{(e^{3/4}M)^2}\right)$

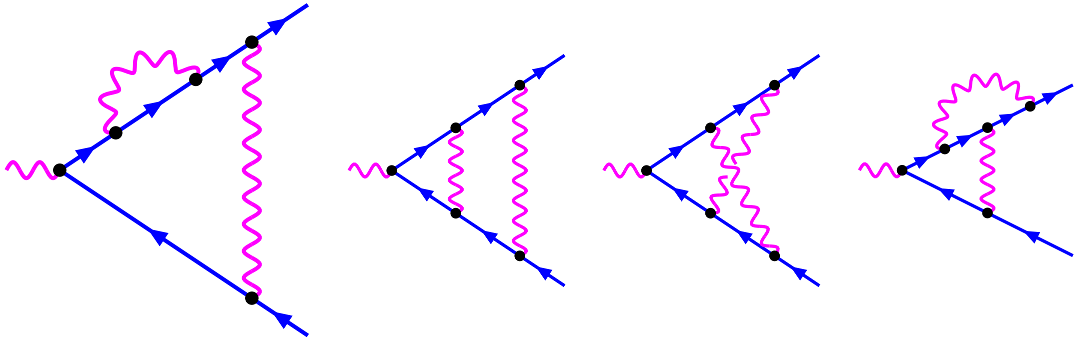


with $M = 80\,\mathrm{GeV}$, $\frac{\alpha}{4\pi} = 3 \cdot 10^{-3}$

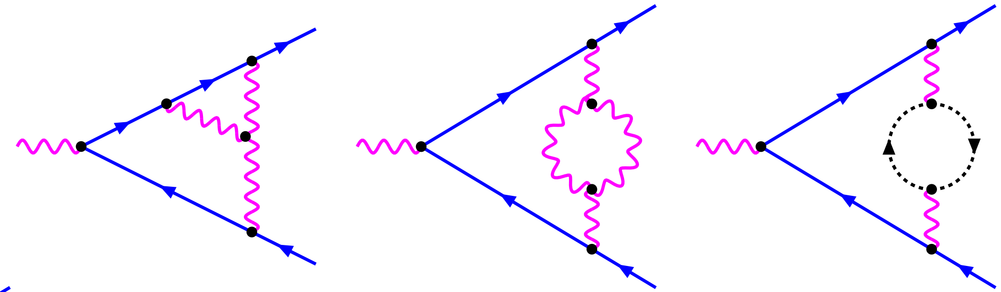
C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

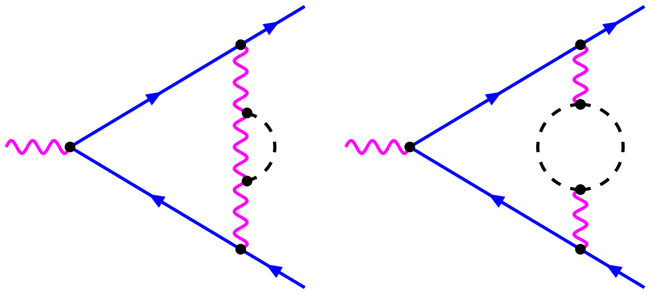
Abelian (C_F^2):



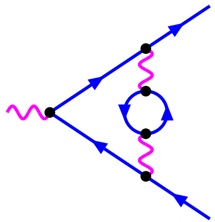
non-Abelian ($C_F C_A$): last 2 +



Higgs:



fermion ($C_F T_F n_f$):



+ 1-loop×1-loop corrections + renormalization

Size of the logarithmic contributions

2-loop form factor F_2 at $Q = 1$ TeV (in 1/1000):

	$\ln^4$	$\ln^3$	$\ln^2$	$\ln$	$\ln^0$
Abelian (C_F^2):	$+0.3\ln^4$ ($+1.6$)	$-1.7\ln^3$ (-2.0)	$+8.2\ln^2$ ($+1.9$)	$-11\ln$ (-0.5)	$+15$ ($+0.1$)
non-Abelian ($C_F C_A$):		$+1.8\ln^3$ ($+2.1$)	$-14\ln^2$ (-3.3)	$+46\ln$ ($+2.1$)	$-\ldots$
Higgs:		$-0.04\ln^3$ (-0.04)	$+0.5\ln^2$ ($+0.1$)	$-2.3\ln$ (-0.1)	$+\ldots$
fermionic ($C_F T_F n_f$):		$-0.5\ln^3$ (-0.6)	$+4.8\ln^2$ ($+1.1$)	$-13\ln$ (-0.6)	$+21$ ($+0.2$)

$\ln^{4,3,2}$: J.H.K., Moch, Penin, Smirnov

$\ln^{1,0}$: Jantzen, J.H.K., Moch; Jantzen, J.H.K., Penin, Smirnov

$\longrightarrow$ growing coefficients with alternating signs

$\Rightarrow$ cancellations between logarithmic terms

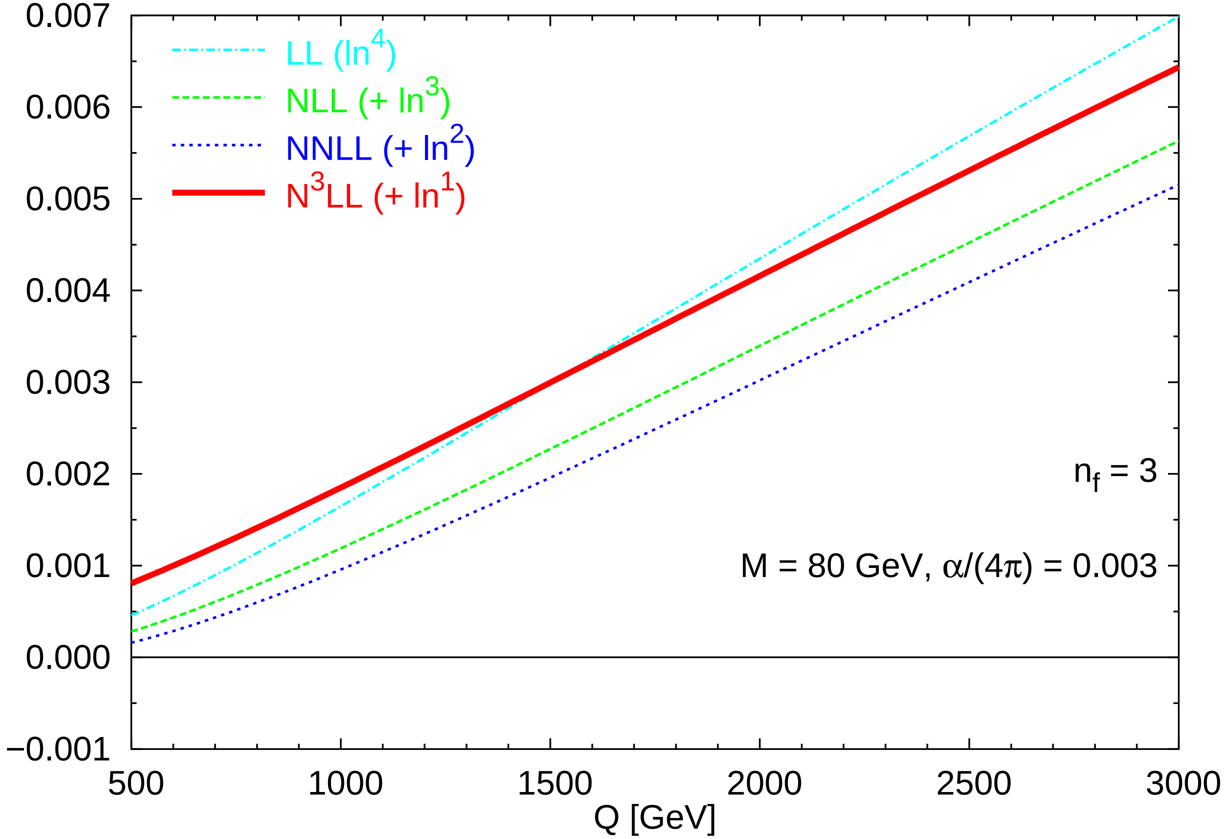
$\hookrightarrow$ **NNLL approximation is not enough!**

Abelian & fermionic contribution: $\ln^1$ small, $\ln^0$ negligible

$\Rightarrow$ **N^{3}LL approximation** including $\ln^1$ is sufficient (non-Abelian $\ln^0$ more difficult)

Massive SU(2) form factor in 2-loop approximation: result

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \Bigg[+\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{19}{48}\ln^3\left(\frac{Q^2}{M^2}\right) - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\ln^2\left(\frac{Q^2}{M^2}\right) + \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\ln\left(\frac{Q^2}{M^2}\right)\Bigg]$$



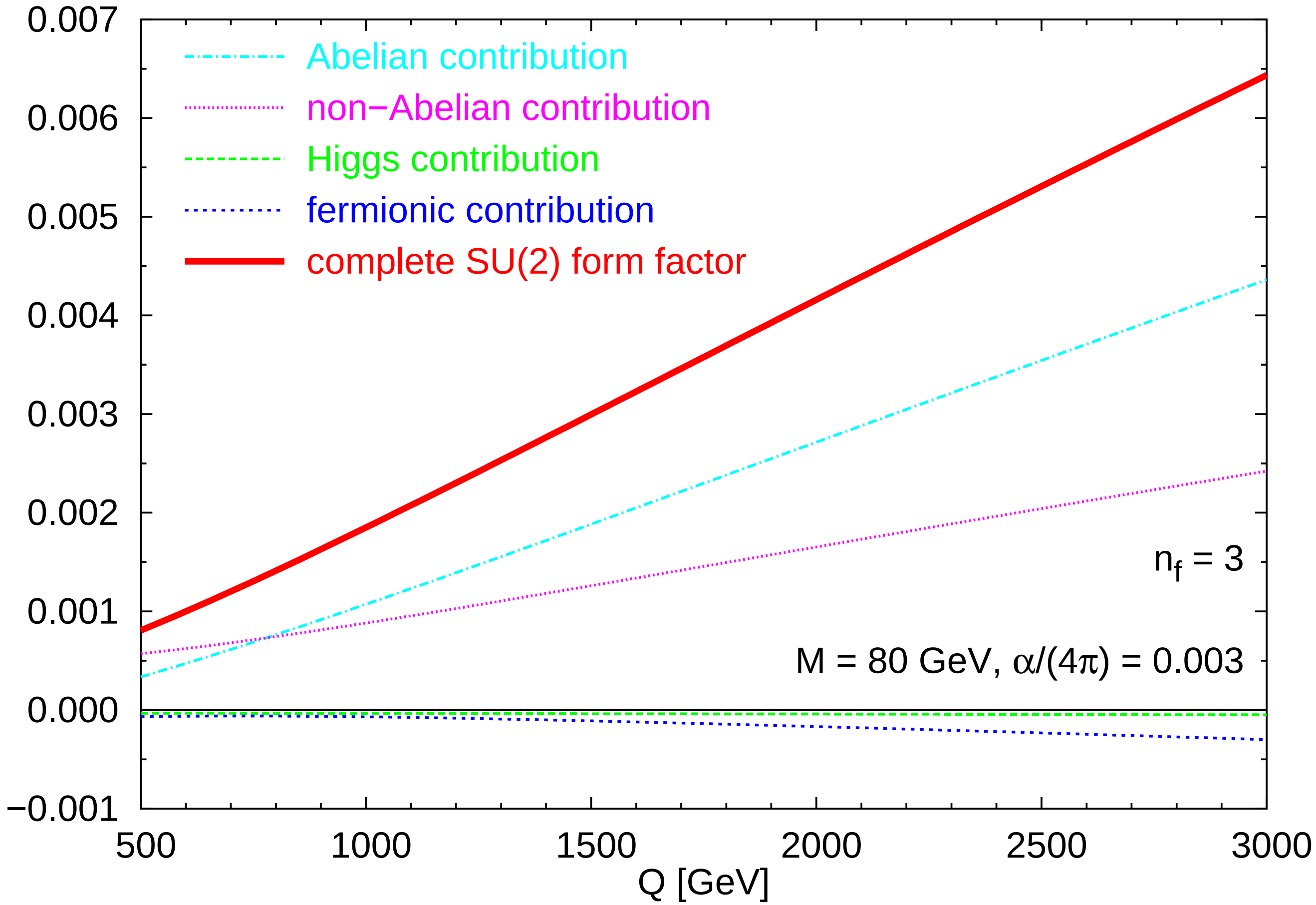
N^3LL approximation

$M_{\text{Higgs}} = M$

$n_f = 3$

Massive SU(2) form factor in 2-loop approximation: individual contributions

(N^{3}LL approximation, $M_{\text{Higgs}} = M$, $n_f = 3$, Feynman-'t Hooft gauge)



U(1)×U(1) Model useful for QED×Weak and QCD×EW

$(\alpha, M) \times (\alpha', \lambda)$

factorization for $Q^2 \gg M^2 \gg \lambda^2$:

$$\mathcal{F}_{\alpha',\alpha}(\lambda, M, Q) = \tilde{F}_{\alpha',\alpha}(M, Q)\ \underbrace{\mathcal{F}_{\alpha'}(\lambda, Q)}_{\text{as before}} + \mathcal{O}(\lambda/M)$$

$$\Rightarrow\ \tilde{F}_{\alpha',\alpha}(M, Q) = \left[\frac{\mathcal{F}_{\alpha',\alpha}(\lambda, M, Q)}{\mathcal{F}_{\alpha'}(\lambda, Q)}\right]_{\lambda\to 0}$$

evaluated with dimensional regularization for IR singularities

$$\tilde{F}_{\alpha',\alpha}(M, Q) = 1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \frac{\alpha'\alpha}{(4\pi)^2}\tilde{f}^{(1,1)} + \dots$$

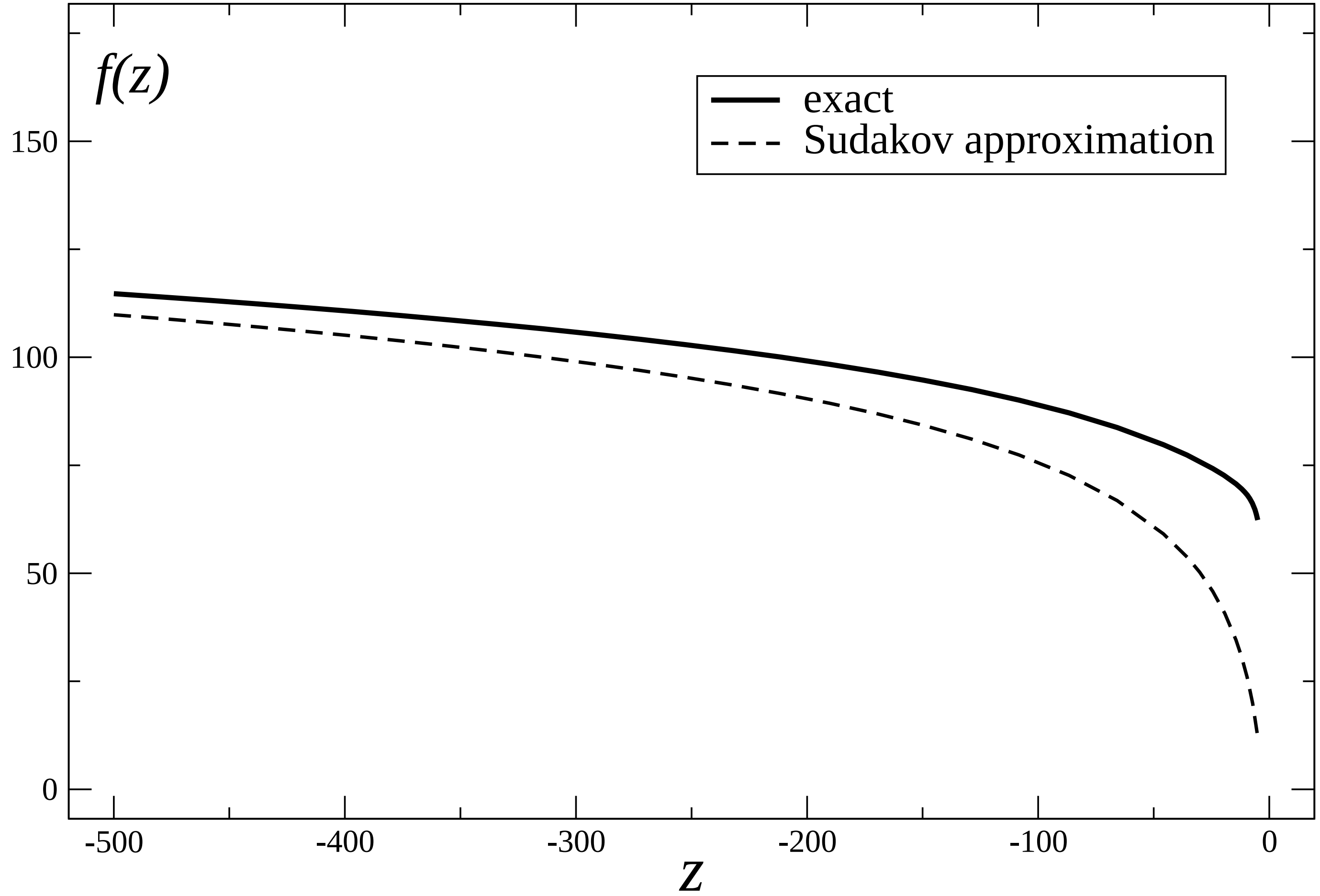
$$\tilde{f}^{(1,1)} = \left(3 - 4\pi^2 + 48\zeta_3\right)\mathcal{L} - 2 + \frac{20}{3}\pi^2 - 84\zeta_3 + \frac{7}{45}\pi^4$$

important observation: no $\mathcal{L}^2$ terms $\Rightarrow$ consistent with evolution equations

J.H.K., Penin, Smirnov (2000)

Complete result for $\tilde{f}^{(1,1)}(z)$ available in analytical form ($z = \frac{Q^2}{M^2}$)

Kotikov, J.H.K., Veretin



Exponentiation, Factorization and Matching

Massive U(1) Theory

5 terms in the two-loop result $\Rightarrow$ N^{4}LL approximation in all orders:

$$\mathcal{F}_\alpha(M,Q) = \exp\left\{\frac{\alpha}{4\pi}\left[-\mathcal{L}^2 + \left(3 + \frac{\alpha}{4\pi}\left(\frac{3}{2} - 2\pi^2 + 24\zeta_3\right) + \mathcal{O}(\alpha^2)\right)\mathcal{L}\right]\right\}\mathcal{F}_\alpha(M,M)$$

U(1)×U(1) Theory

matching relation: $\quad \mathcal{F}_{\alpha',\alpha}(M,M,Q) = C_{\alpha',\alpha}(M,Q)\,\tilde{F}_{\alpha',\alpha}(M,Q)\,\mathcal{F}_{\alpha'}(M,Q)$

$$\Rightarrow\ C_{\alpha',\alpha}(M,Q) \ = \ 1 + \frac{\alpha'\alpha}{(4\pi)^2}\left[\frac{59}{4} + \frac{70}{3}\pi^2 + 244\zeta_3 - \frac{113}{15}\pi^4 - \frac{64}{3}\pi^2 \ln^2 2 + \frac{64}{3}\ln^4 2 + 512\,\mathrm{Li}_4\left(\frac{1}{2}\right)\right]$$

- no logarithmic terms!
- $\tilde{F}_{\alpha',\alpha}(M,Q)\,\mathcal{F}_{\alpha'}(\lambda,Q)$ approaches $\mathcal{F}_{\alpha',\alpha}(\lambda,M,Q)$ for $\lambda \to M$ in N^{3}LL accuracy!
- *all* logs in theory with mass gap are obtained from symmetric phase

Four fermion scattering

Evaluation in the high energy limit

define

$$\begin{aligned}\mathcal{A}^{\lambda} &= \bar{\psi}_2 t^a \gamma_\mu \psi_1 \bar{\psi}_4 t^a \gamma_\mu \psi_3 \\ \mathcal{A}^{\lambda}_{LL} &= \bar{\psi}_{2L} t^a \gamma_\mu \psi_{1L} \bar{\psi}_{4L} t^a \gamma_\mu \psi_{3L} \\ \mathcal{A}^{d}_{LR} &= \bar{\psi}_{2L} \gamma_\mu \psi_{1L} \bar{\psi}_{4R} \gamma_\mu \psi_{3R}\end{aligned}$$

define "reduced" amplitude $\tilde{\mathcal{A}}$

$$\mathcal{A} = \frac{ig^2}{s}\mathcal{F}^2\tilde{\mathcal{A}}$$

evolution equation

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi(\alpha(Q^2))\tilde{\mathcal{A}}$$

$\tilde{\mathcal{A}}$: vector in isospin/chiral basis
χ: matrix

N^3LL **requires:**

- form factor up to N^3LL
- χ up to two loop, as obtained from hard contribution to single pole part of 4-fermion scattering amplitude

e.g. pure massive SU(2) theory with SSB:

$$\sigma^{(2)} = \left[\frac{9}{2}\mathcal{L}^4 - \frac{449}{6}\mathcal{L}^3 + \left(\frac{4855}{18} + \frac{37}{3}\pi^2\right)\mathcal{L}^2 + \left(\frac{34441}{216} - \frac{1247}{18}\pi^2 - 122\zeta(3) + 15\sqrt{3}\pi + 26\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right)\mathcal{L}\right]\sigma_B$$

for identical isospin in initial and final state

Electroweak theory

- infrared logs must be separated

- NNLL: complete
  - result insensitive to form of gauge-boson mass generation
  - term of order $1 - M_W^2/M_Z^2 = \sin^2\theta$ included

- N^3LL
  - sensitive to details of mass generation, gauge boson mixing
  - Approximation: terms of $\mathcal{O}(\sin^2\theta)$ neglected

Result for the correction factor

$$R(e^+e^- \to Q\bar{Q}) = 1 - 1.66\,L(s) + 5.60\,l(s) - 8.39\,a + 1.93\,L^2(s) - 11.28\,L(s)\,l(s) + 33.79\,l^2(s) - 150.95\,l(s)\,a$$

$$R(e^+e^- \to q\bar{q}) = 1 - 2.18\,L(s) + 20.94\,l(s) - 35.07\,a + 2.79\,L^2(s) - 51.98\,L(s)\,l(s) + 321.34\,l^2(s) - 603.43\,l(s)\,a$$

$$R(e^+e^- \to \mu^+\mu^-) = 1 - 1.39\,L(s) + 10.12\,l(s) - 21.26\,a + 1.42\,L^2(s) - 20.33\,L(s)\,l(s) + 112.57\,l^2(s) - 260.15\,l(s)\,a$$

with

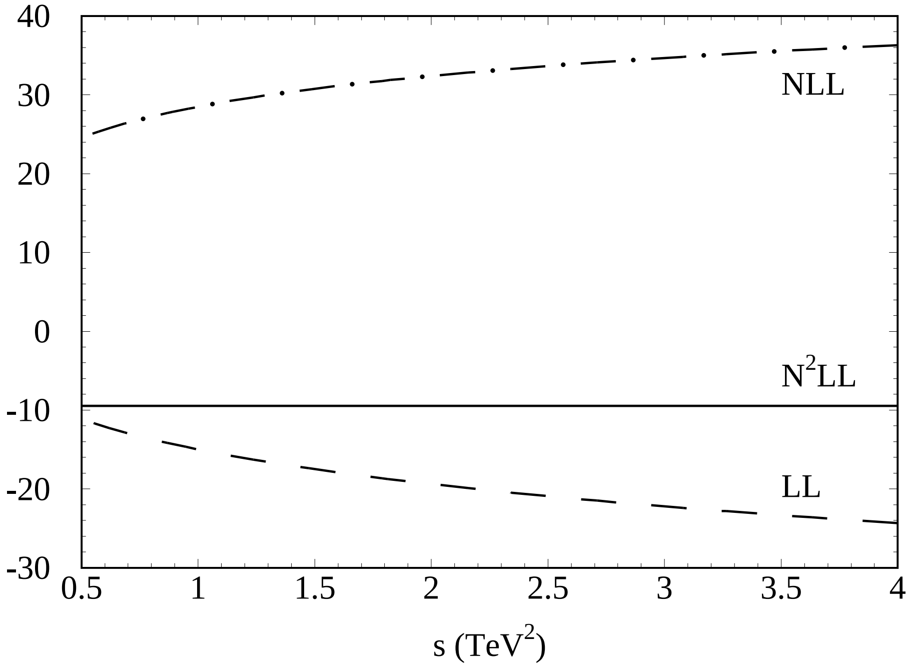
$$L(s) = \frac{g^2}{16\pi^2}\ln^2\left(\frac{s}{M^2}\right) = 0.07 \quad (0.11)$$

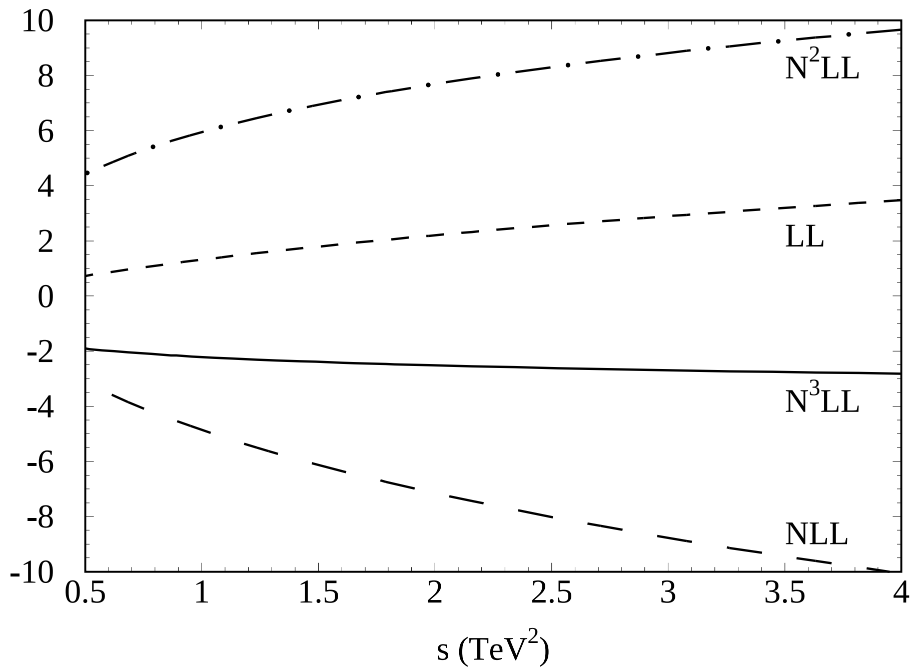
$$l(s) = \frac{g^2}{16\pi^2}\ln\left(\frac{s}{M^2}\right) = 0.014 \; (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

for $\sqrt{s} = 1$ TeV (2 TeV)

Separate logarithmic contributions to $R(e^+e^- \to q\bar{q})$ in % to the Born approximation

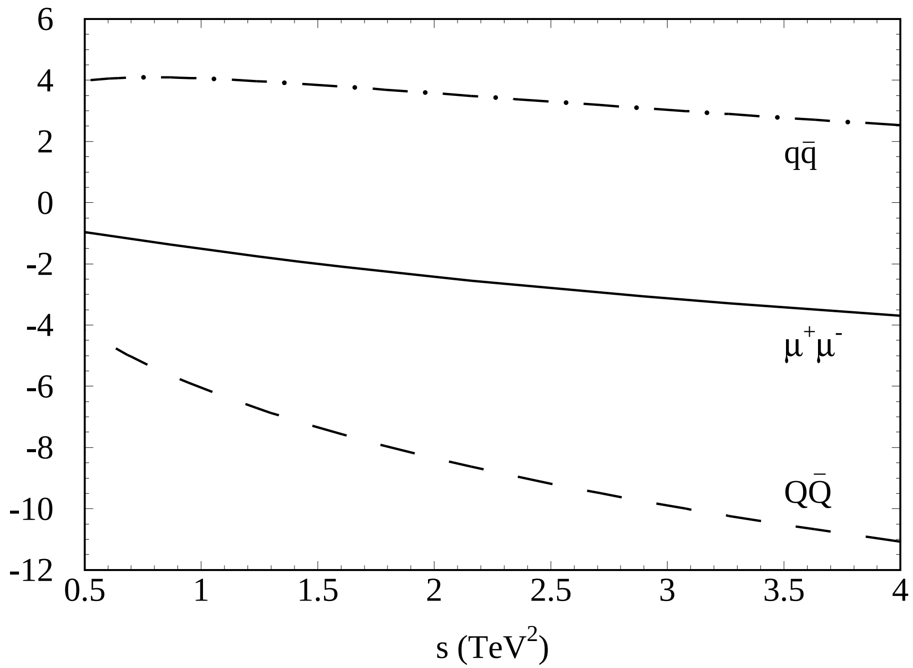


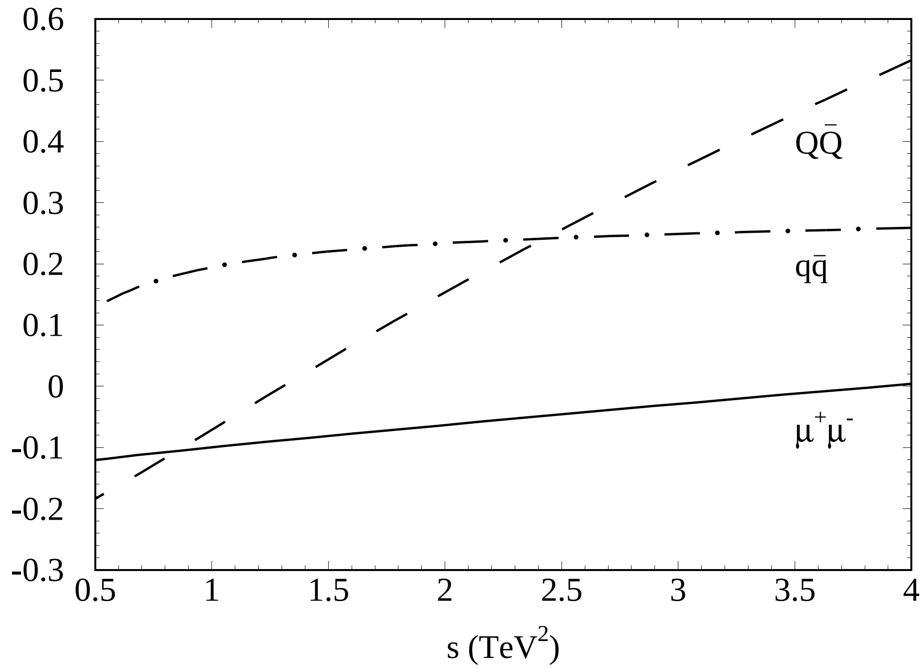


one-loop LL $\left(\ln^2(s/M^2)\right)$, NLL $\left(\ln^1(s/M^2)\right)$ and N^{2}LL $\left(\ln^0(s/M^2)\right)$

two-loop LL $\left(\ln^4(s/M^2)\right)$, NLL $\left(\ln^3(s/M^2)\right)$, NNLL $\left(\ln^2(s/M^2)\right)$ and N^{3}LL $\left(\ln^1(s/M^2)\right)$

Total logarithmic corrections in % to the Born approximation: $R(e^+e^- \to Q\bar{Q})$, $R(e^+e^- \to q\bar{q})$ and $R(e^+e^- \to \mu^+\mu^-)$





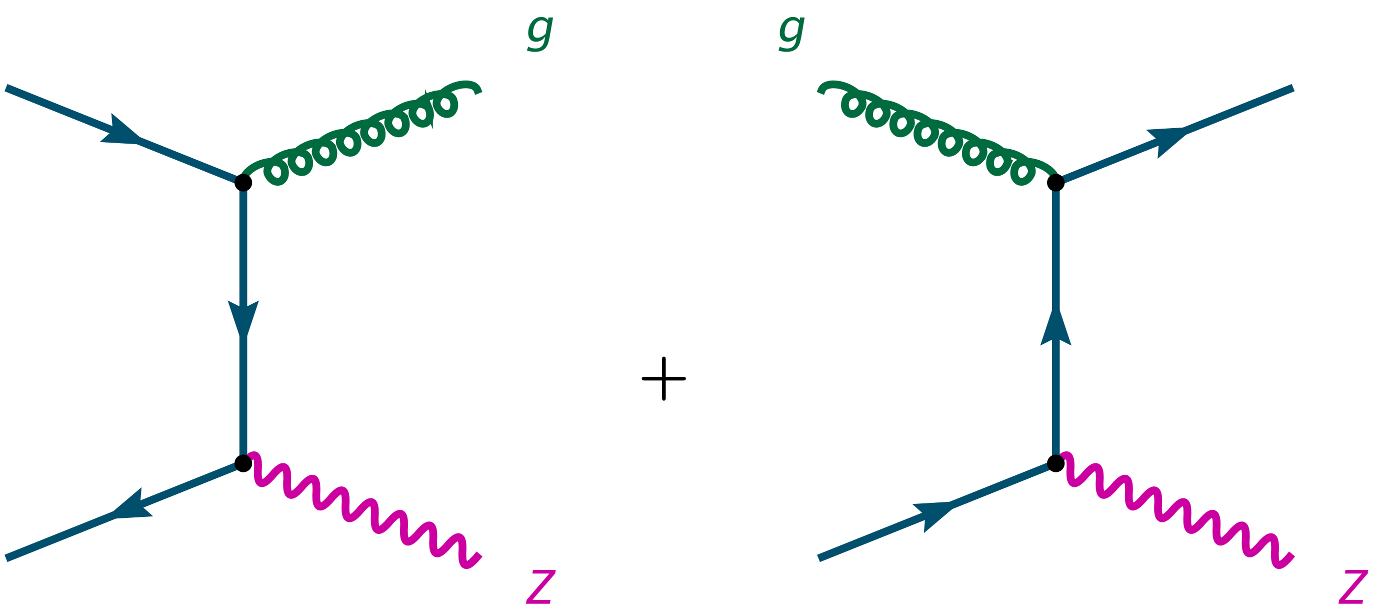
one-loop correction up to N^2LL term

two-loop correction up to N^3LL term

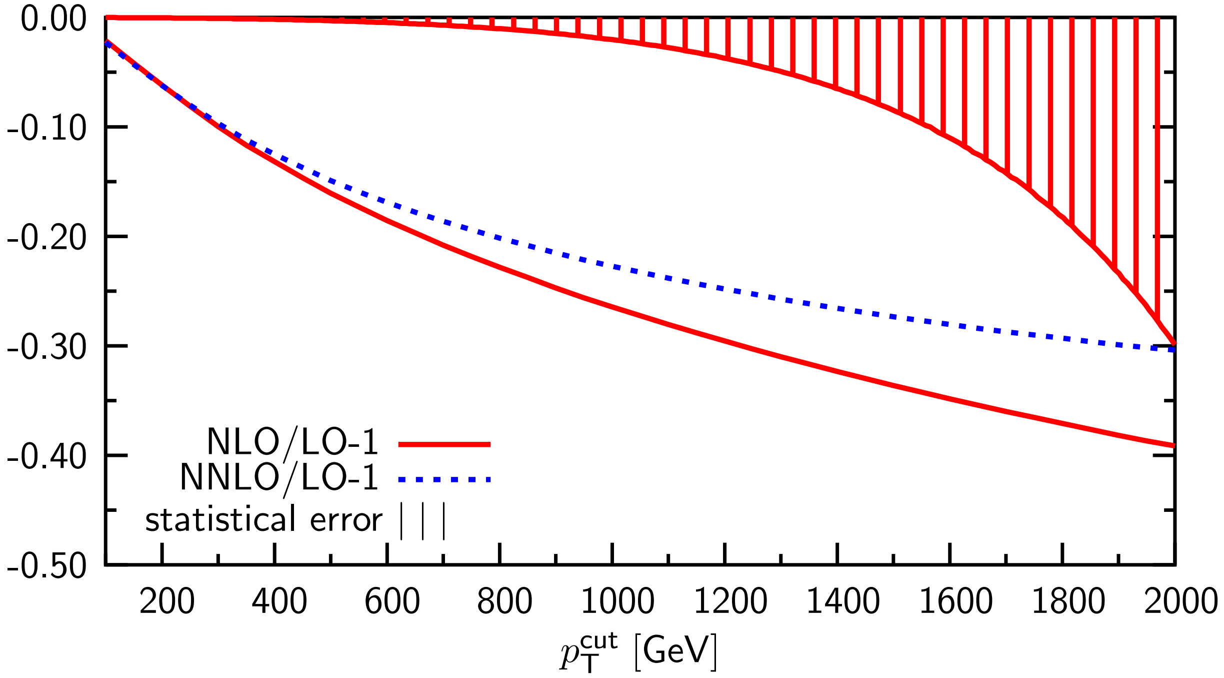
Z/photon production at large transverse momenta

J.H.K., Kulesza, Pozzorini, Schulze

Large rate for Z-boson and photon production at LHC at large p_T (1-2 TeV)
Large electroweak corrections ($\hat{s} \gg M_{W,Z}^2$)



Complete one loop calculation NLL approximation at two loops

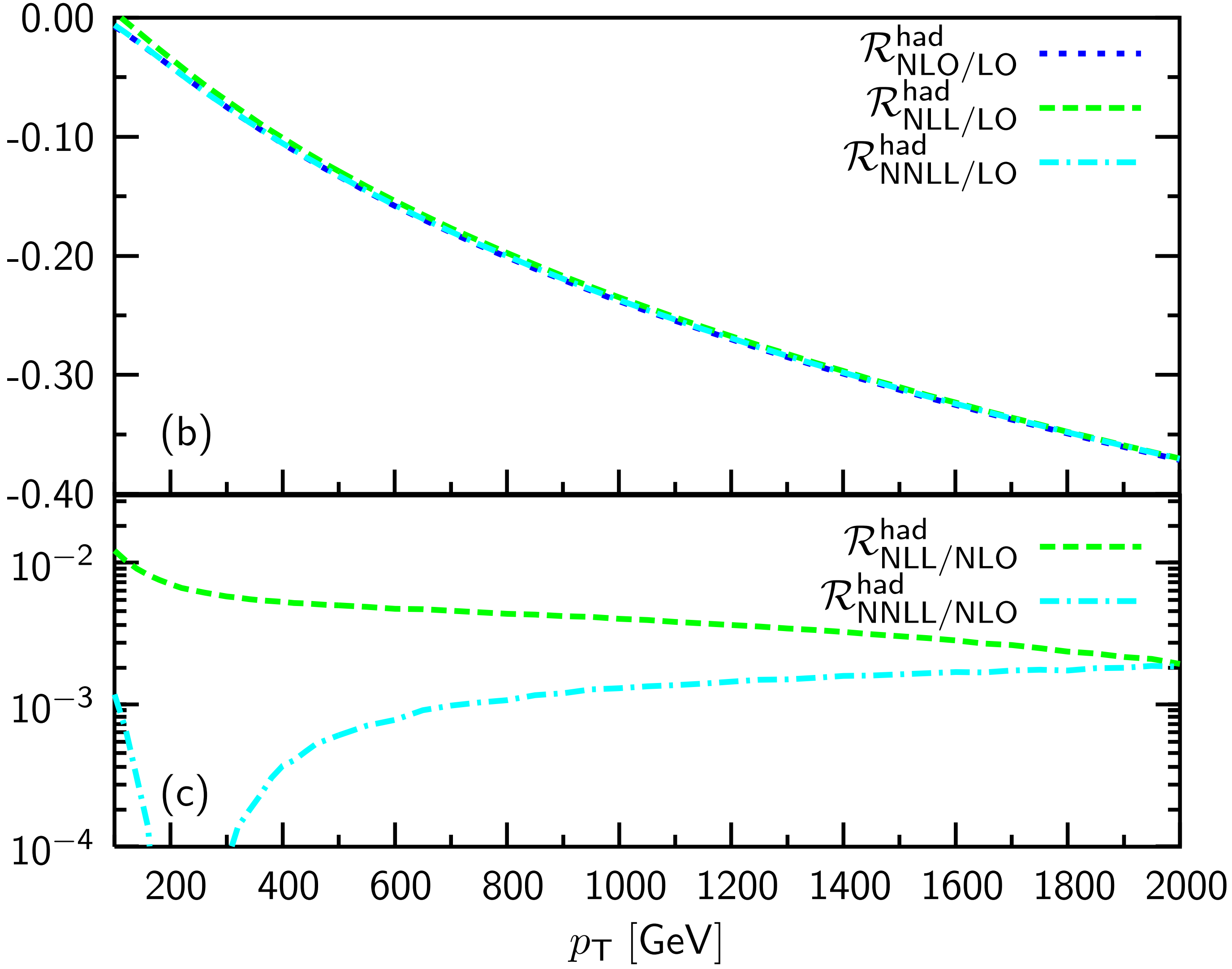


- one loop effects are large (∼ 30% at $p_T \sim 1$Tev)
- two loop effects (based on Denner, Melles, Pozzorini; Melles)
- become relevant above 1 TeV
- important angular-dependent logarithmic terms
- experiment will explore p_T up to 2 TeV

Relative NLO and NNLO corrections w.r.t. the LO and statistical error for the unpolarized integrated cross section for $pp \to Zj$ at $\sqrt{s} = 14$ TeV.

Compact analytical formulae for one loop results

in NNLL approximation ($\ln^2 + \ln +$const.) provide an excellent description (better than 2×10^{-3}) of complete result



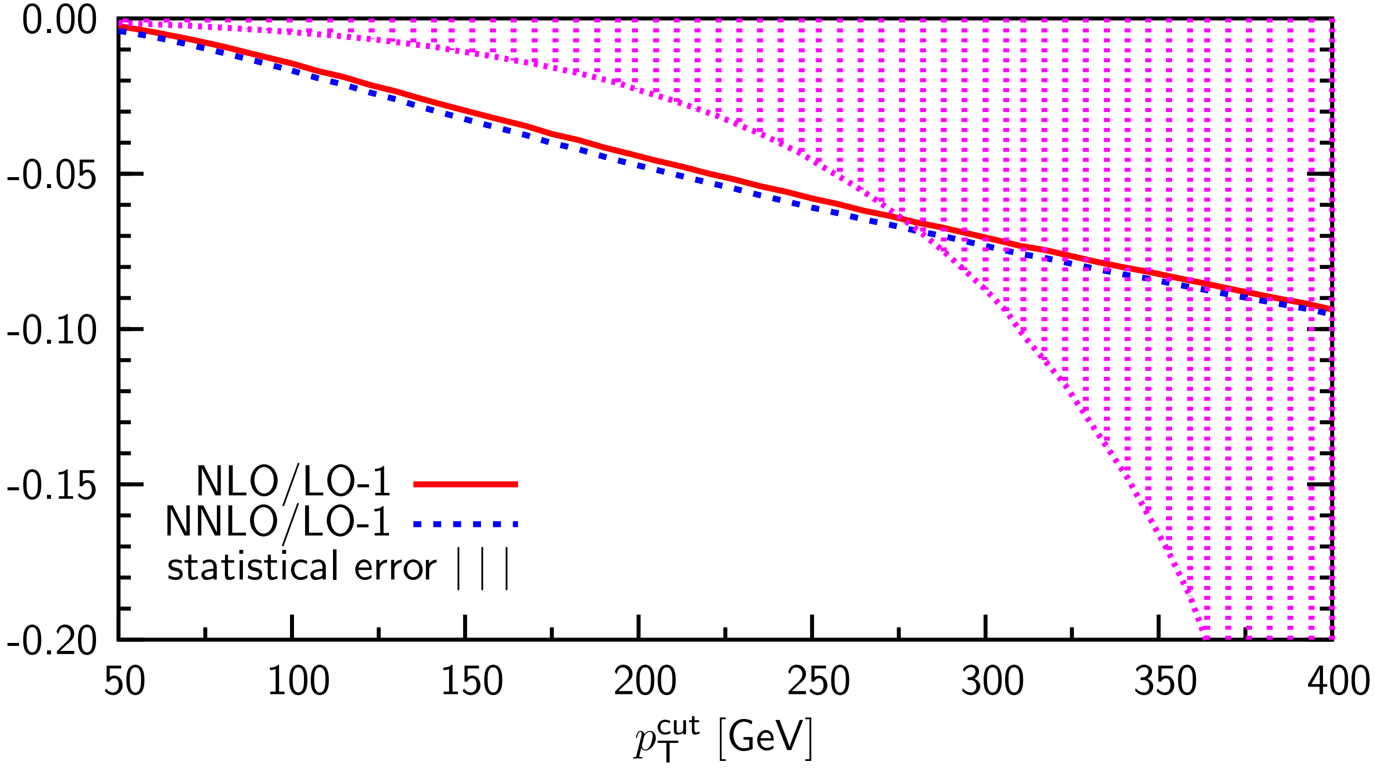
p_T **distribution for** $pp \to Zj$ **at** $\sqrt{s} = 14$ **TeV:**

(b) Relative NLO, NLL and NNLL weak correction w.r.t. the LO distribution.

(c) NLL and NNLL approximations compared to the full NLO result

Corrections at the Tevatron ($\sqrt{s} = 2$ TeV)

amount up to 5%



Relative NLO and NNLO corrections w.r.t. the LO and statistical error (shaded area) for the unpolarized integrated cross section for $p\bar{p} \to Zj$ at $\sqrt{s} = 2$ TeV as a function of p_T^{cut}.

Similar results for $pp \to \gamma + X$

Summary

- Large logarithmic corrections at large energies: NLL, N^2LL, N^3LL important
- N^3LL and N^4LL (partly) available for form factor
- N^3LL available for 4-fermion scattering
- special role of massless bosons (γ and g) $\rightarrow$ factorization of IR singularities
- Z-boson and γ production at large p_T accessible at LHC
- Full one loop and NLL-terms at two loop are under control
- **first applications: LHC; important issue for LC**