Electroweak 2-loop corrections at high energies

The massive SU(2) form factor and an SU(2) × U(1) model with mass gap

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Electroweak 2-loop corrections at high energies

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   • how to treat the EW mass gaps $Z$ – $W$ – photon

V  Summary

I Why logarithmic 2-loop calculations in EW theory?

Electroweak (EW) precision physics

- experimentally measured by now at energy scales up to $\sim M_{W,Z}$
- future generation of accelerators (LHC, ILC) $\rightarrow$ TeV region
- new energy domain $\sqrt{s} \gg M_{W,Z}$ becomes accessible

Electroweak radiative corrections

at high energies $\sqrt{s} \sim$ TeV $\gg M_{W,Z}$

large negative corrections in exclusive cross sections

- EW corrections dominated by Sudakov logarithms $\alpha^n \ln^j(s/M_{W,Z}^2)$, $j = 2n$
  - large coefficients in front of subleading logarithms ($0 \leq j < 2n$)
- 1-loop corrections $\gtrsim 10\%$
- 2-loop corrections $\gtrsim 1\%$, need to be under control for LHC/ILC
- individual logarithmic contributions even larger, but strong cancellations
**Important class of processes: 4-fermion scattering**

\[
A = \frac{i g^2}{s} F^2 \tilde{A}
\]

Form factor \( F \) of vector current:

\[
q \sim \begin{array}{l}
\sum_{A} F \cdot \bar{u}(p_2) \gamma^\mu u(p_1) + F' \cdot \bar{u}(p_2) \sigma^{\mu\nu} u(p_1) q_\nu \\
\to 0, \ m_f \to 0
\end{array}
\]

**High energy behaviour** \( |s| \sim |t| \sim |u| \gg M_{W,Z}^2 \)

- all *collinear* logarithms of amplitude \( A \sim \) form factors \( F^2 \)
- *reduced amplitude* \( \tilde{A} \rightarrow \) only *soft* logarithms
- \( \tilde{A} \) satisfies an *evolution equation* (known from massless QCD calculations):
  \[
  \frac{\partial \tilde{A}}{\partial \ln s} = \chi(\alpha(s)) \tilde{A}, \quad \chi = \text{matrix of soft anomalous dimensions}
  \]

\( \Rightarrow \) still needed for 2-loop logarithms in \( A \): *form factor* \( F \)
### Simplified models

<table>
<thead>
<tr>
<th>Standard Model</th>
<th>SU(2)ₓ × U(1)ᵧ model</th>
<th>massive SU(2) model</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(2)ₓ × U(1)ᵧ</td>
<td>no mixing (sin θₓ ≈ 0)</td>
<td>only weak int. without γ</td>
</tr>
<tr>
<td>mixing (sin θₓ ≠ 0)</td>
<td>← M ≡ Mₓ ≈ Mₓ</td>
<td>1 mass M ≠ 0</td>
</tr>
<tr>
<td>Mₓ ≠ Mₓ, Mᵧ = 0</td>
<td>← M ≡ Mₓ ≈ Mₓ</td>
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<td></td>
<td>mass gap M ≫ λ → 0</td>
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### High energy behaviour of the form factor

← Sudakov limit:

\[ q = F(Q^2) \cdot \bar{u}(p_2) \gamma^\mu u(p_1) \]

- momentum transfer \(-q^2 ≡ Q^2 ≫ M^2 ≡ M^2_{W,Z}\)
- neglect fermion masses
- \textit{logarithmic approximation}: neglect terms \(\propto M^2/Q^2\)
- ← good approximation for 2-loop \(n_f\) contribution

B.J., Kühn, Moch '03
II Massive SU(2) form factor

Form factor in perturbation theory: \( F = 1 + \alpha F_1 + \alpha^2 F_2 + \ldots \)

sum up large logarithms to all orders in \( \alpha \):

\[
F = 1 + \alpha (\ln^2 + \ln + \text{const}) + \alpha^2 (\ln^4 + \ln^3 + \ln^2 + \ln + \text{const}) + \ldots \\
\leftrightarrow (1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots) \exp\left( \alpha (\ln^2 + \ln) + \alpha^2 (\ln^3 + \ln^2 + \ln) + \ldots \right)
\]

Evolution equation in logarithmic approximation: \( \text{Sen '81; Collins '89; Korchemsky '89; \ldots} \)

\[
\frac{\partial F(Q^2)}{\partial \ln Q^2} = \left[ \int_{M^2}^{Q^2} \frac{dx}{x} \gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2)) \right] F(Q^2)
\]

solution \( \rightarrow \) exponentiation:

\[
F(Q^2) = F_0(\alpha(M^2)) \exp \left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \left[ \int_{M^2}^{x} \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \right\}
\]
Exponentiated form factor from the evolution equation:

\[ F(Q^2) = F_0(\alpha(M^2)) \exp \left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \left[ \int_{M^2}^{x} \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \right\} \]

perturbative expansion of the functions \( \gamma, \zeta, \xi \) and \( F_0 \):

\[ \gamma(\alpha) = \alpha \gamma_1 + \alpha^2 \gamma_2 + \ldots \quad \text{etc.} \]

running of the coupling constant:

\[ \alpha(x) = \alpha(M^2) - \ln \left( \frac{x}{M^2} \right) \frac{\beta_0}{4\pi} \alpha(M^2)^2 + \ldots \]

\( \Rightarrow \) perform the integrals over \( x \) and \( x' \) in the exponent

\( \leftarrow \) expansion in \( \alpha \) and in powers of \( \ln(Q^2/M^2) \)

- compare to the perturbative result of a fixed order in \( \alpha \)
- determine the corresponding coefficients of \( \gamma, \zeta, \xi \) and \( F_0 \)
- obtain a leading logarithmic approximation to all orders in \( \alpha \)
Coefficients of $\gamma$, $\zeta$, $\xi$ and $F_0$ previously known for massive SU(N) and U(1) models:

- 1-loop result $\rightarrow \gamma$, $\zeta$, $\xi$ and $F_0$ up to $O(\alpha)$
- massless 2-loop result $\rightarrow \gamma$ up to $O(\alpha^2)$  

\[
\gamma(\alpha) = -2 C_F \frac{\alpha}{4\pi} \left\{ 1 + \frac{\alpha}{4\pi} \left[ \left( \frac{67}{9} - \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_F n_f - \frac{8}{9} T_F n_s \right] \right\} + O(\alpha^3)
\]

\[
\zeta(\alpha) = 3 C_F \frac{\alpha}{4\pi} + O(\alpha^2)
\]

\[
\xi(\alpha) = 0 + O(\alpha^2)
\]

\[
F_0(\alpha) = -C_F \left( \frac{7}{2} + \frac{2}{3} \pi^2 \right) \frac{\alpha}{4\pi} + O(\alpha^2)
\]

- 1-loop running of $\alpha \leftrightarrow$ 1-loop $\beta$-function:

\[
\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f - \frac{1}{3} T_F n_s
\]

$\Rightarrow$ NNLL approximation of 2-loop form factor $F_2$ known: $\alpha^2 \left( \ln^4 + \ln^3 + \ln^2 \right)$
Massive SU(2) form factor in 2-loop approximation: contributions & diagrams

2-loop vertex diagrams (massless fermions, massive bosons):

Abelian \((C_F^2)\):

non-Abelian \((C_FC_A)\): last 2 +

Higgs:

fermion \((C_FT_Fn_f)\):

+ 1-loop×1-loop corrections + renormalization
Size of the logarithmic contributions

2-loop form factor $F_2$ at $Q = 1\,\text{TeV}$ (in permill):

Abelian ($C_F^2$): $+ 0.3 \ln^4 - 1.7 \ln^3 + 8.2 \ln^2 - 11 \ln + 15$
\[+1.6 \quad -2.0 \quad +1.9 \quad -0.5 \quad +0.1\]

non-Abelian ($C_F C_A$): $+ 1.8 \ln^3 - 14 \ln^2 + 46 \ln - \ldots$
\[+2.1 \quad -3.3 \quad +2.1\]

Higgs: $- 0.04 \ln^3 + 0.5 \ln^2 - 2.3 \ln - \ldots$
\[-0.04 \quad +0.1 \quad -0.1\]

fermionic ($C_F T_F n_f$): $- 0.5 \ln^3 + 4.8 \ln^2 - 13 \ln + 21$
\[-0.6 \quad +1.1 \quad -0.6 \quad +0.2\]

$\ln^4,3,2$: Kühn, Moch, Penin, Smirnov '01
$\ln^{1,0}$: B.F., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04 '05

→ growing coefficients with alternating signs

⇒ cancellations between logarithmic terms

↔ NNLL approximation is not enough!

Abelian & fermionic contribution: $\ln^1$ small, $\ln^0$ negligible

⇒ $N^3LL$ approximation including $\ln^1$ is sufficient (non-Abelian $\ln^0$ more difficult)
Massive SU(2) form factor in 2-loop approximation: result

\[ \alpha^2 F_2 = \left( \frac{\alpha}{4\pi} \right)^2 \left[ \frac{9}{32} \ln^4 \left( \frac{Q^2}{M^2} \right) - \frac{19}{48} \ln^3 \left( \frac{Q^2}{M^2} \right) - \left( -\frac{7}{8} \pi^2 + \frac{463}{48} \right) \ln^2 \left( \frac{Q^2}{M^2} \right) + \left( \frac{39}{2} \frac{\text{Cl}_2}{\sqrt{3}} \right) + \frac{45}{4} \frac{\pi}{\sqrt{3}} - \frac{61}{2} \zeta_3 - \frac{11}{24} \pi^2 + 29 \right] \ln \left( \frac{Q^2}{M^2} \right) \]

\begin{align*}
\text{N}^3\text{LL approximation} \\
M_{\text{Higgs}} = M
\end{align*}

\[ M = 80 \text{ GeV}, \alpha/(4\pi) = 0.003 \]
Massive SU(2) form factor in 2-loop approximation: individual contributions
(N^3LL approximation, \( M_{\text{Higgs}} = M \), Feynman-'t Hooft gauge)
III Methods for loop calculations at high energies

Reduction to scalar diagrams

• **given** from Feynman rules: \( \mathcal{F}^\mu = \bar{u}(p_2) \Gamma^\mu(p_1, p_2) u(p_1) \)

• **wanted**: form factor \( F(Q^2) \) with \( \mathcal{F}^\mu = F(Q^2) \cdot \bar{u}(p_2) \gamma^\mu u(p_1) \)

• can be done using the properties of Dirac matrices and spinors,
  \( \{\gamma^\mu, \gamma^\nu\} = 2g^\mu\nu, \gamma_i^\dagger u(p_1) = 0, \bar{u}(p_2) \gamma^i = 0 \), combined with tensor reduction

• more elegantly with a **projector** on the form factor:

\[
F(Q^2) = \frac{\text{Tr} \left[ \gamma_\mu \gamma_i^\dagger \Gamma^\mu(p_1, p_2) \gamma_i \right]}{2(d-2)q^2}
\]

• **output**: form factor \( F(Q^2) \) in terms of **scalar Feynman integrals**

\[
\int d^d k_1 \int d^d k_2 \frac{\prod_{j=1}^N (\ell_j \cdot \ell_j')^{\nu_j}}{\prod_{i=1}^L (k_i^2 - M_i^2)^{n_i}}
\]

with \( L \) propagators and \( N \) irreducible scalar products in the numerator
Elimination of irreducible scalar products in the numerator

- Most scalar diagrams could directly be calculated with numerator.
- Diagrams with self-energy insertion:
  - tensor reduction for inner loop, e.g.
    \[ \int d^d k \, \frac{p \cdot k}{f(k, q)} = p_\nu \int d^d k \, \frac{k_\nu}{f(k, q)} = \frac{p \cdot q}{q^2} \int d^d k \, \frac{q \cdot k}{f(k, q)} \]
- Difficult diagrams where the absence of the numerator was desirable:
  - write propagators with Schwinger parameters (alpha parameters):
    \[ \frac{1}{(k^2 - M^2)^n} = \frac{1}{i^n \Gamma(n)} \int_0^\infty d\alpha \, \alpha^{n-1} \, e^{i\alpha(k^2 - M^2)} \]
    - diagonalize the argument of the exponential in the loop momenta
    - perform tensor reduction: numerator → factors of $g^{\mu\nu}$
    - rewrite as linear combinations of the original integral without numerator, but with higher powers of propagators ($n \rightarrow n + 1, n + 2, \ldots$) and higher dimension ($d \rightarrow d + 2, d + 4, \ldots$)

Anastasiou et al. '00
Expansion by regions

a powerful method for the asymptotic expansion of Feynman diagrams

- **given:** scalar Feynman integral & limit like $Q^2 \gg M^2$ (*Minkowskian limit!*)
- **wanted:** expansion of the integral in $M^2/Q^2$
- **problem:** direct expansion of the integrand leads to (new) IR/UV singularities

**Recipe for the method of expansion by regions:**

1. *divide* the integration domain into *regions* for the loop momenta
   (especially such regions where singularities are produced in the limit $M \to 0$)
2. in every region, *expand* the integrand in a *Taylor series* with respect to the parameters that are considered small *there*
3. *integrate* the expanded integrands over the *whole integration domain*
4. put to zero any *scaleless integral* (due to the properties of dimensional regularization)

- usually only a few regions give non-vanishing contributions
- for logarithmic approximation: only leading order of the expansion needed
  $\leftrightarrow$ in step 2. all small parameters in the integrand are simply set to zero
- sometimes additional regularization (apart from $\varepsilon$) needed for individual regions
Expansion by regions: example

Vertex form factor in the Sudakov limit $Q^2 \gg M^2$

- typical regions for each loop momentum $k$:
  - hard (h): all components of $k \sim Q$
  - soft (s): all components of $k \sim M$
  - ultrasoft (us): all components of $k \sim M^2/Q$
  - 1-collinear (1c): $k^2 \sim 2p_1 \cdot k \sim M^2$, $2p_2 \cdot k \sim Q^2$
  - 2-collinear (2c): $k^2 \sim 2p_2 \cdot k \sim M^2$, $2p_1 \cdot k \sim Q^2$

- 1-loop vertex correction: $f = \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{(k^2 - M^2)(k^2 - 2p_1 \cdot k)(k^2 - 2p_2 \cdot k)}$

\[
\begin{align*}
  f^{(h)} &= \frac{1}{Q^2} \left[ -\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \ln(Q^2) - \frac{1}{2} \ln^2(Q^2) + \frac{\pi^2}{12} + \mathcal{O}\left(\frac{M^2}{Q^2}\right) \right] \\
  f^{(1c)} + f^{(2c)} &= \frac{1}{Q^2} \left[ \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} \ln(Q^2) - \frac{1}{2} \ln^2(M^2) + \ln(M^2) \ln(Q^2) - \frac{5}{12} \pi^2 + \mathcal{O}\left(\frac{M^2}{Q^2}\right) \right] \\
  \Rightarrow f &= f^{(h)} + f^{(1c)} + f^{(2c)} = \frac{1}{Q^2} \left[ -\frac{1}{2} \ln^2\left(\frac{Q^2}{M^2}\right) - \frac{\pi^2}{3} + \mathcal{O}\left(\frac{M^2}{Q^2}\right) \right]
\end{align*}
\]
Expansion by regions: why it works

simple \( d = 1 \) example: \( f = \int_0^\infty \frac{dk}{(k + m)(k + q)} \), \( m \ll q \)

soft (s): \( k < \Lambda \)

hard (h): \( k > \Lambda \)

\[
f = \int_0^\Lambda \frac{dk}{(k + m)(k + q)} + \int_\Lambda^\infty \frac{dk}{(k + m)(k + q)} \\
= \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \int_0^\Lambda \frac{dk}{k + m} + \sum_{i=0}^\infty (-m)^i \int_\Lambda^\infty \frac{dk}{k + q} \\
= \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \left( \int_0^\infty \frac{dk}{k + m} - \int_\Lambda^\infty \frac{dk}{k + m} \right) + \sum_{i=0}^\infty (-m)^i \left( \int_0^\infty \frac{dk}{k + q} - \int_0^\Lambda \frac{dk}{k + q} \right) \\
= \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \int_0^\infty \frac{dk}{k + m} + \sum_{i=0}^\infty (-m)^i \int_0^\infty \frac{dk}{k + q} - \sum_{i=0}^\infty (-m)^i \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \int_0^\infty \frac{dk}{k + q} \\
\to 0, \text{ scaleless integral}
\]

\[
= f(s) + f(h) \quad \checkmark
\]

\[
= \frac{\Gamma(\varepsilon)\Gamma(1 - \varepsilon)}{(q - m) m^\varepsilon} + \frac{\Gamma(-\varepsilon)\Gamma(1 + \varepsilon)}{(q - m) q^\varepsilon} = \frac{\ln(q/m)}{q - m} + \mathcal{O}(\varepsilon) \quad \checkmark
\]
Parameterization of Feynman integrals

- Feynman parameters:
  \[ \prod_i \frac{1}{A_i^{n_i}} = \frac{\Gamma(\sum_i n_i)}{\prod_i \Gamma(n_i)} \left( \prod_i \int_0^1 dx_i x_i^{n_i-1} \right) \frac{\delta(\sum_i x_i - 1)}{\left(\sum_i x_i A_i\right)\sum_i n_i} \]

- Schwinger parameters → more general esp. with expansion by regions:
  \[ \frac{1}{A^n} = \frac{1}{i^n \Gamma(n)} \int_0^\infty d\alpha \alpha^{n-1} e^{i\alpha A}, \quad \text{numerator } A^n = \left( \frac{1}{i} \frac{\partial}{\partial \alpha} \right)^n e^{i\alpha A} \bigg|_{\alpha=0} \]

⇒ any number of propagators and numerators may be combined
⇒ can always be transformed to Feynman parameters
← evaluation:

\[ \int d^d k e^{i(\alpha k^2 + 2p \cdot k)} = i\pi^{d/2} (i\alpha)^{-d/2} e^{-ip^2/\alpha} \]

\[ \int_0^\infty d\alpha \alpha^{n-1} e^{i\alpha A} = \frac{i^n \Gamma(n)}{A^n} \]

\[ \int_0^\infty \frac{d\alpha \alpha^{n-1}}{(A + \alpha B)^r} = \frac{\Gamma(n) \Gamma(r - n)}{\Gamma(r) A^{r-n} B^n} \]
**Mellin-Barnes representation**

Feynman integrals with many scales / many massive propagators are hard to evaluate

$\rightarrow$ separate scales by Mellin-Barnes representation:

$$\frac{1}{(A + B)^n} = \frac{1}{\Gamma(n)} \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \Gamma(-z) \Gamma(n + z) \frac{B^z}{A^{n+z}}$$

- Mellin-Barnes integrals go along the imaginary axis, leaving poles of $\Gamma(-z + \ldots)$ to the right and poles of $\Gamma(z + \ldots)$ to the left of the integration contour

- applicable to massive propagators ($A = k^2$, $B = -M^2$) or to any complicated intermediate expression

- evaluation:
  close the integration contour to the right ($|B| \leq |A|$) or to the left ($|B| \geq |A|$) and pick up the residues within the contour using $\text{Res} \Gamma(z) \big|_{z=-i} = (-1)^i/i!$

  $\Rightarrow$ **sums over $\Gamma$-functions**

  $\Rightarrow$ **multiple $\zeta$-values / generalized (harmonic) polylogarithms** etc.

- close link to **expansion by regions**:
  Mellin-Barnes representation of the full integral

$\leftrightarrow$ contributions corresponding to the regions
Electroweak Standard Model: massive SU(2) and massless U(1) gauge bosons here: without mixing $M_W = M_Z$, neglect $\mathcal{O}\left(\sin^2 \theta_W \alpha^2 \ln^1\right) \approx 0.2$

- form factor $F_{SU(2)}(\alpha, Q, M) \to$ IR-finite
- form factor $F_{U(1)}(\alpha', Q, \lambda) \to$ IR-singularities regularized by $\lambda$ or $\varepsilon = \frac{4-d}{2}$

- $SU(2)_M \times U(1)_\lambda$: $\hat{F}(\alpha, \alpha', Q, M, \lambda)$ for $Q \gg M \gg \lambda \to 0$
  $\to$ factorization of IR-singularities:

$$\hat{F}(\alpha, \alpha', Q, M, \lambda) = F_{U(1)}(\alpha', Q, \lambda) \tilde{F}(\alpha, \alpha', Q, M) + \mathcal{O}\left(\alpha \alpha' \frac{\lambda^2}{M^2}\right)$$

$$\Rightarrow \tilde{F}(\alpha, \alpha', Q, M) = \lim_{\lambda \to 0} \frac{\hat{F}(\alpha, \alpha', Q, M, \lambda)}{F_{U(1)}(\alpha', Q, \lambda)} = \lim_{\varepsilon \to 0} \frac{\hat{F}(\alpha, \alpha', Q, M, 0; \varepsilon)}{F_{U(1)}(\alpha', Q, 0; \varepsilon)}$$

$\leftarrow$ set $\lambda = 0$ and calculate $\hat{F}(\alpha, \alpha', Q, M, 0; \varepsilon)$ in dimensional regularization
Factorization of IR-singularities:

\[
\hat{F}(\alpha, \alpha', Q, M, \lambda) = F_{U(1)}(\alpha', Q, \lambda) \hat{F}(\alpha, \alpha', Q, M) + \mathcal{O}\left(\alpha\alpha' \frac{\lambda^2}{M^2}\right)
\]

Calculation of \( \hat{F}(\alpha, \alpha', Q, M) \):

2-loop diagrams with 1 massive SU(2) and 1 massless U(1) gauge boson:

\[
\hat{F}(\alpha, \alpha', Q, M) = F_{SU(2)}(\alpha, Q, M) \times \\
\left\{ 1 + \frac{\alpha\alpha'}{(4\pi)^2} C_F \left[ (48\zeta_3 - 4\pi^2 + 3) \ln\left(\frac{Q^2}{M^2}\right) + \frac{7}{45} \pi^4 - 84\zeta_3 + \frac{20}{3} \pi^2 - 2 \right] \right\}
\]

\( \Rightarrow \) interference terms are finite \( \sim \) IR singularities factorize

\( \Rightarrow \) only single logarithm \( \ln^1 \) \( \sim \) evolution equation & NNLL prediction \( \checkmark \)
Factorization of the $\text{SU}(2) \times \text{U}(1)$ form factor for $\lambda = M$

Set $\lambda = M$ and parametrize:

$$\hat{F}(\alpha, \alpha', Q, M, M) = F_{\text{U}(1)}(\alpha', Q, M) \tilde{F}(\alpha, \alpha', Q, M) C(\alpha, \alpha', Q, M)$$

$\hat{F}(\alpha, \alpha', Q, M, M)$ known from $F_{\text{SU}(2)}(\alpha, Q, M)$ and $F_{\text{U}(1)}(\alpha', Q, M)$

$\Rightarrow$ calculate matching coefficient:

$$C(\alpha, \alpha', Q, M) = 1 + \frac{\alpha \alpha'}{(4\pi)^2} C_F \times$$

$$\left[ 512 \text{Li}_4(\frac{1}{2}) + \frac{64}{3} \ln^4 2 - \frac{64}{3} \pi^2 \ln^2 2 - \frac{113}{15} \pi^4 + 244 \zeta_3 + \frac{70}{3} \pi^2 + \frac{59}{4} \right]$$

no logarithm!

Applications:

- $\tilde{F}(\alpha, \alpha', Q, M) = \frac{\hat{F}(\alpha, \alpha', Q, M, M)}{F_{\text{U}(1)}(\alpha', Q, M)} + \mathcal{O}(\alpha \alpha' \ln^0)$

- $\hat{F}(\alpha, \alpha', Q, M, \lambda \approx M) = F_{\text{U}(1)}(\alpha', Q, \lambda \approx M) \tilde{F}(\alpha, \alpha', Q, M) + \mathcal{O}(\alpha \alpha' \ln^{0,1})$

$\leftarrow$ expansion in small mass difference, e.g. $M_W \approx M_Z$
IV Summary

Massive SU(2) form factor

- weak interaction with massive gauge bosons
- 2-loop result in $N^3LL$ approximation
- precise control of radiative corrections

SU(2)$\times$U(1) model with mass gap

- factorization of IR singularities shown explicitly
- calculation with mass gap reduced to the 1-mass case $M_W = M_Z = M_{\text{photon}}$
- $M_Z \neq M_W$ taken into account by expanding around the equal mass approximation
- prediction for electroweak 2-loop form factor

Combination with reduced amplitude

- scattering amplitude $f \bar{f} \rightarrow f' \bar{f'}$
- electroweak 2-loop corrections to cross sections, ...