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Electroweak 2-loop corrections at high energies

The massive SU(2) form factor and an SU(2)$\times$U(1) model with mass gap

Bernd Jantzen

Institut für Theoretische Teilchenphysik, Universität Karlsruhe

In collaboration with

Johann H. Kühn, **Alexander A. Penin**

and **Vladimir A. Smirnov**

Electroweak 2-loop corrections at high energies

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- how to treat the EW mass gaps Z – W – photon

V Summary

J.H. Kühn, A.A. Penin, *hep-ph/9906545*
J.H. Kühn, A.A. Penin, V.A. Smirnov, *Eur. Phys. J. C17 (2000) 97*
J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov, *Nucl. Phys. B616 (2001) 286*
B. Feucht, J.H. Kühn, S. Moch, *Phys. Lett. B561 (2003) 111*
B. Feucht, J.H. Kühn, A.A. Penin, V.A. Smirnov, *Phys. Rev. Letts. 93 (2004) 101802*
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I Why logarithmic 2-loop calculations in EW theory?

Electroweak (EW) precision physics

- experimentally measured by now at energy scales up to $\sim M_{W,Z}$
- future generation of accelerators (LHC, ILC) $\rightarrow$ TeV region
- new energy domain $\sqrt{s} \gg M_{W,Z}$ becomes accessible

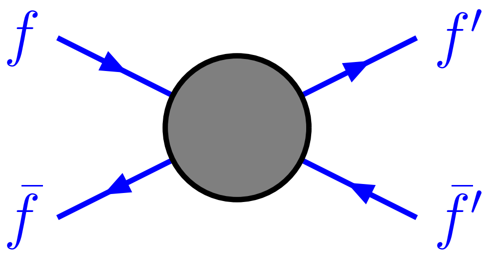
Electroweak radiative corrections at high energies $\sqrt{s} \sim$ TeV $\gg M_{W,Z}$

Fadin et al. '00; Kühn et al. '00, '01; Denner et al. '01, '03, '04; Pozzorini '04; B.J. et al. '03, '04, '05; ...

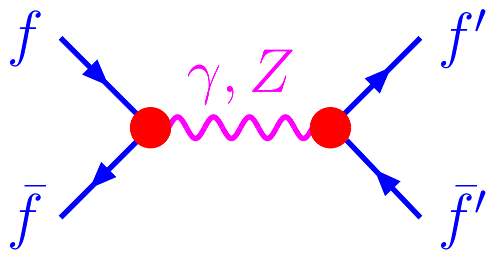
large negative corrections in *exclusive* cross sections

- EW corrections dominated by Sudakov logarithms $\alpha^n \ln^j(s/M_{W,Z}^2)$, $j = 2n$, large coefficients in front of subleading logarithms $(0 \leq j < 2n)$
- 1-loop corrections $\gtrsim 10\%$
- 2-loop corrections $\gtrsim 1\%$, need to be under control for LHC/ILC
- individual logarithmic contributions even larger, but strong cancellations

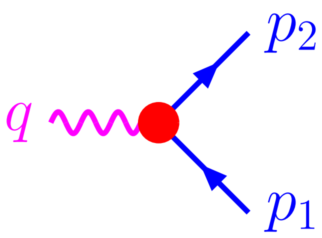
Important class of processes: 4-fermion scattering



$$A = \frac{ig^2}{s} F^2 \tilde{A}$$



Form factor F of vector current:



$$= F \cdot \bar{u}(p_2)\, \gamma^\mu\, u(p_1) + \underbrace{F' \cdot \bar{u}(p_2)\, \sigma^{\mu\nu}\, u(p_1)\, q_\nu}_{\to 0,\ m_f \to 0}$$

High energy behaviour $|s| \sim |t| \sim |u| \gg M^2_{W,Z}$

references: see Kühn et al. '01

- all *collinear* logarithms of amplitude $A \rightsquigarrow$ form factors F^2
- *reduced amplitude* $\tilde{A} \to$ only *soft* logarithms
- $\tilde{A}$ satisfies an *evolution equation* (known from massless QCD calculations):

$$\frac{\partial \tilde{A}}{\partial \ln s} = \chi\big(\alpha(s)\big)\, \tilde{A}\,, \quad \chi = \text{matrix of soft anomalous dimensions}$$

$\Rightarrow$ still needed for 2-loop logarithms in A: form factor F

Simplified models

Standard Model
SU(2)$_\mathrm{L}$ $\times$ U(1)$_\mathrm{Y}$
mixing $(\sin\theta_W \neq 0)$
$M_W \neq M_Z$, $M_\gamma = 0$

$\longrightarrow$

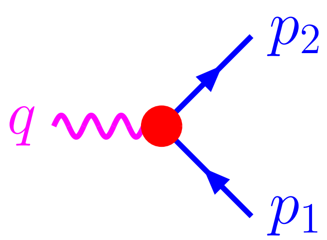
SU(2)$_M$ $\times$ U(1)$_\lambda$ model
no mixing $(\sin\theta_W \approx 0)$
$\hookrightarrow M \equiv M_W \approx M_Z$
mass gap $M \gg \lambda \to 0$

$\longrightarrow$

massive SU(2) model
only weak int. without γ
1 mass $M \neq 0$

High energy behaviour of the form factor

$\hookrightarrow$ Sudakov limit:



$$= F(Q^2) \cdot \bar{u}(p_2)\,\gamma^\mu\,u(p_1)$$

- momentum transfer $-q^2 \equiv Q^2 \gg M^2 \equiv M^2_{W,Z}$
- neglect fermion masses
- *logarithmic approximation*: neglect terms $\propto M^2/Q^2$

  $\hookrightarrow$ good approximation for 2-loop n_f contribution

B.J., Kühn, Moch '03

II Massive SU(2) form factor

Form factor in perturbation theory: $F = 1 + \alpha\, F_1 + \alpha^2\, F_2 + \ldots$

sum up large logarithms to all orders in α:

$$F = 1 + \alpha\,(\ln^2 + \ln + \text{const}) + \alpha^2\,(\ln^4 + \ln^3 + \ln^2 + \ln + \text{const}) + \ldots$$
$$\leftrightarrow \big(1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots\big) \exp\Big(\alpha\,(\ln^2 + \ln) + \alpha^2\,(\ln^3 + \ln^2 + \ln) + \ldots\Big)$$

Evolution equation in logarithmic approximation: Sen '81; Collins '89; Korchemsky '89; ...

$$\frac{\partial F(Q^2)}{\partial \ln Q^2} = \left[\int_{M^2}^{Q^2} \frac{dx}{x} \gamma\big(\alpha(x)\big) + \zeta\big(\alpha(Q^2)\big) + \xi\big(\alpha(M^2)\big)\right] F(Q^2)$$

solution → exponentiation:

$$F(Q^2) = F_0\big(\alpha(M^2)\big) \exp\left\{\int_{M^2}^{Q^2} \frac{dx}{x} \left[\int_{M^2}^{x} \frac{dx'}{x'} \gamma\big(\alpha(x')\big) + \zeta\big(\alpha(x)\big) + \xi\big(\alpha(M^2)\big)\right]\right\}$$

Exponentiated form factor from the evolution equation:

$$F(Q^2) = F_0\big(\alpha(M^2)\big)\exp\left\{\int_{M^2}^{Q^2}\frac{dx}{x}\left[\int_{M^2}^{x}\frac{dx'}{x'}\gamma\big(\alpha(x')\big)+\zeta\big(\alpha(x)\big)+\xi\big(\alpha(M^2)\big)\right]\right\}$$

perturbative expansion of the functions γ, ζ, ξ and F_0:

$$\gamma(\alpha) = \alpha\,\gamma_1 + \alpha^2\,\gamma_2 + \ldots \quad \text{etc.}$$

running of the coupling constant:

$$\alpha(x) = \alpha(M^2) - \ln\left(\frac{x}{M^2}\right)\frac{\beta_0}{4\pi}\,\alpha(M^2)^2 + \ldots$$

$\Rightarrow$ perform the integrals over x and x' in the exponent

$\hookrightarrow$ expansion in α and in powers of $\ln(Q^2/M^2)$

- compare to the perturbative result of a fixed order in α
- determine the corresponding coefficients of γ, ζ, ξ and F_0
- obtain a *leading logarithmic approximation* to all orders in α

Coefficients of γ, ζ, ξ and F_0 previously known for massive SU(N) and U(1) models:

- 1-loop result $\longrightarrow$ γ, ζ, ξ and F_0 up to $\mathcal{O}(\alpha)$
- massless 2-loop result $\longrightarrow$ γ up to $\mathcal{O}(\alpha^2)$ Kodaira, Trentadue '81

$$\gamma(\alpha) = -2C_F \frac{\alpha}{4\pi} \left\{ 1 + \frac{\alpha}{4\pi} \left[\left(\frac{67}{9} - \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_F n_f - \frac{8}{9} T_F n_s \right] \right\} + \mathcal{O}(\alpha^3)$$

$$\zeta(\alpha) = 3C_F \frac{\alpha}{4\pi} + \mathcal{O}(\alpha^2)$$

$$\xi(\alpha) = 0 + \mathcal{O}(\alpha^2)$$

$$F_0(\alpha) = -C_F \left(\frac{7}{2} + \frac{2}{3}\pi^2 \right) \frac{\alpha}{4\pi} + \mathcal{O}(\alpha^2)$$

- 1-loop running of α $\leftrightarrow$ 1-loop β-function:

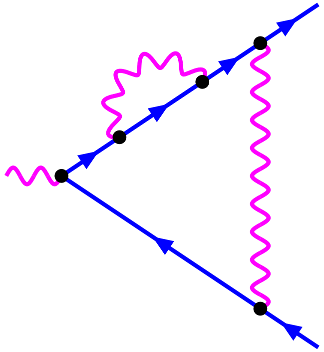
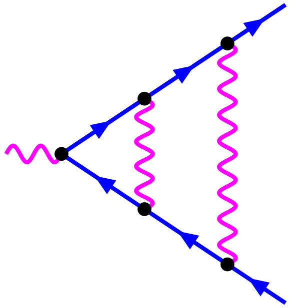
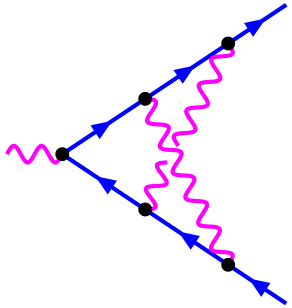
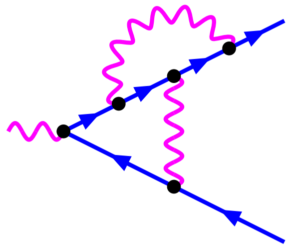
$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f - \frac{1}{3} T_F n_s$$

$\Rightarrow$ NNLL approximation of 2-loop form factor F_2 known: $\alpha^2 \left(\ln^4 + \ln^3 + \ln^2 \right)$

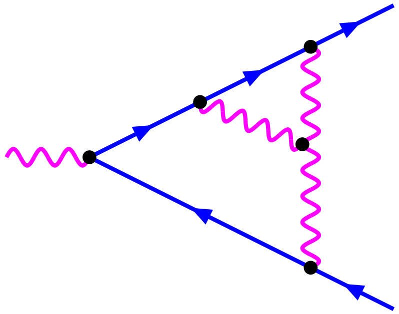
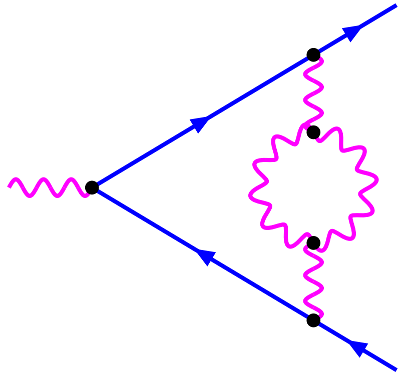
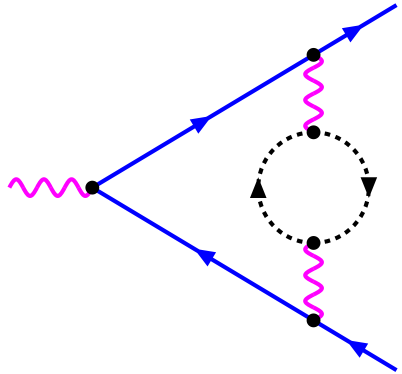
Massive SU(2) form factor in 2-loop approximation: contributions & diagrams

2-loop vertex diagrams (massless fermions, massive bosons):

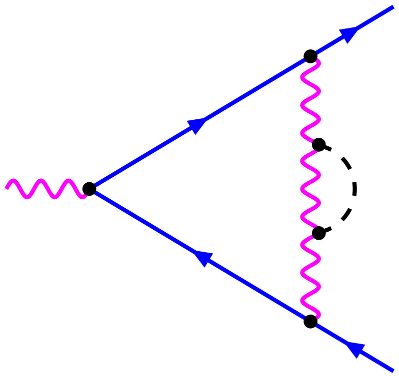
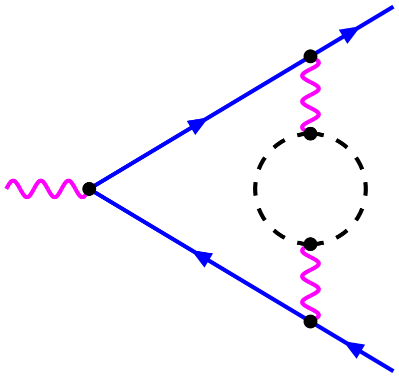
Abelian (C_F^2):

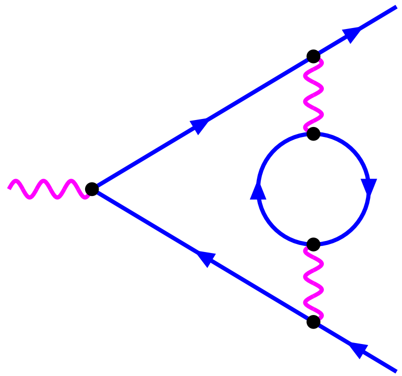
non-Abelian ($C_F C_A$): last 2 +

Higgs:

fermion ($C_F T_F n_f$):



+ 1-loop×1-loop corrections + renormalization

Size of the logarithmic contributions

2-loop form factor F_2 at $Q = 1$ TeV (in permill):

Abelian (C_F^2):	$+\,0.3\ln^4$	$-\,1.7\ln^3$	$+\,8.2\ln^2$	$-\,11\ln$	$+\,15$
	$+1.6$	-2.0	$+1.9$	-0.5	$+0.1$
non-Abelian ($C_F C_A$):		$+\,1.8\ln^3$	$-\,14\ln^2$	$+\,46\ln$	$-\,\ldots$
		$+2.1$	-3.3	$+2.1$	
Higgs:		$-\,0.04\ln^3$	$+\,0.5\ln^2$	$-\,2.3\ln$	$+\,\ldots$
		-0.04	$+0.1$	-0.1	
fermionic ($C_F T_F n_f$):		$-\,0.5\ln^3$	$+\,4.8\ln^2$	$-\,13\ln$	$+\,21$
		-0.6	$+1.1$	-0.6	$+0.2$

$\ln^{4,3,2}$: Kühn, Moch, Penin, Smirnov '01

$\ln^{1,0}$: B.F., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04 '05

$\longrightarrow$ growing coefficients with alternating signs

$\Rightarrow$ cancellations between logarithmic terms

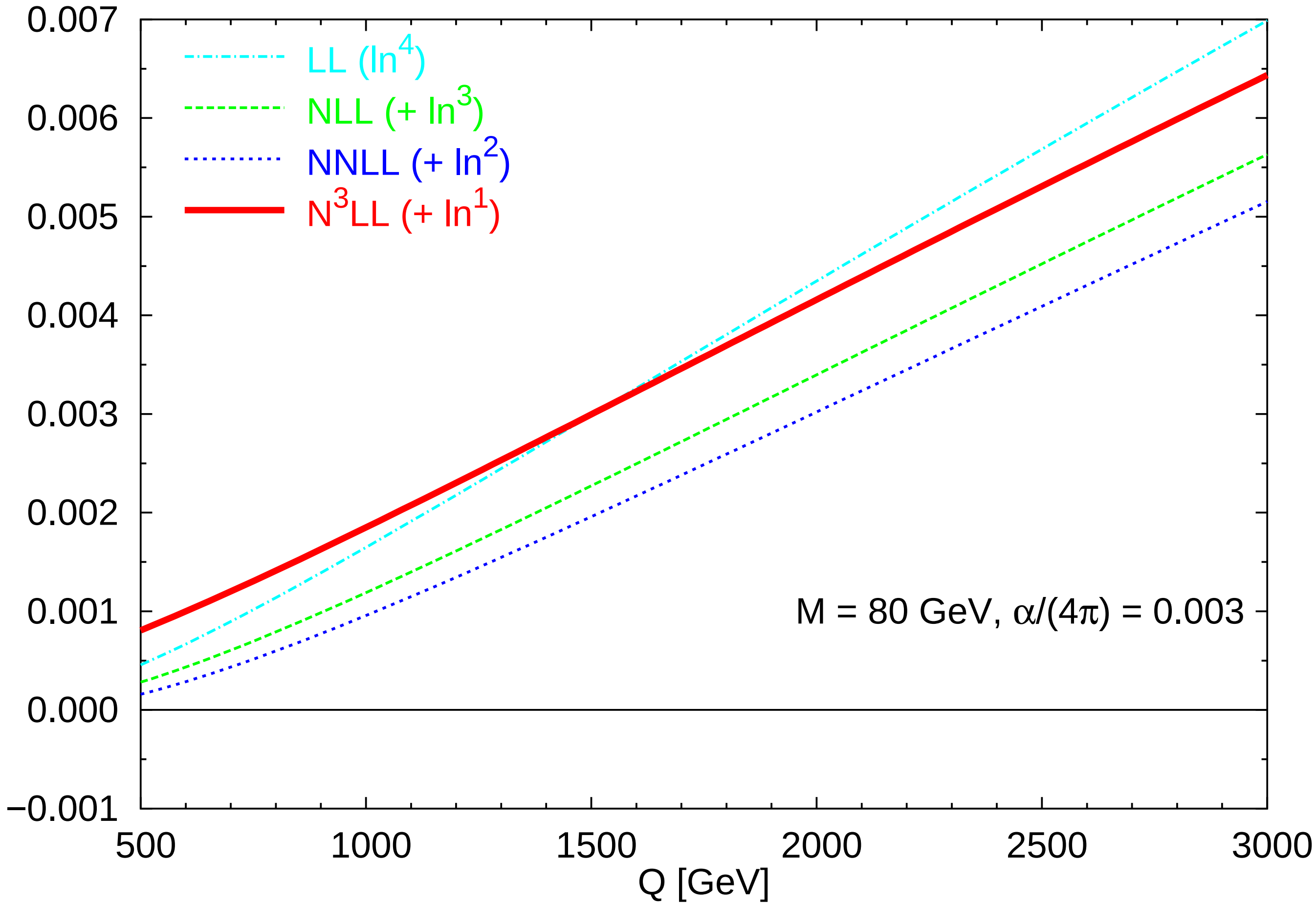
$\hookrightarrow$ **NNLL approximation is not enough!**

Abelian & fermionic contribution: $\ln^1$ small, $\ln^0$ negligible

$\Rightarrow$ **N^{3}LL approximation** including $\ln^1$ is sufficient (non-Abelian $\ln^0$ more difficult)

Massive SU(2) form factor in 2-loop approximation: result

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \Bigg[+\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{19}{48}\ln^3\left(\frac{Q^2}{M^2}\right) - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\ln^2\left(\frac{Q^2}{M^2}\right)$$
$$+ \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\ln\left(\frac{Q^2}{M^2}\right) \Bigg]$$

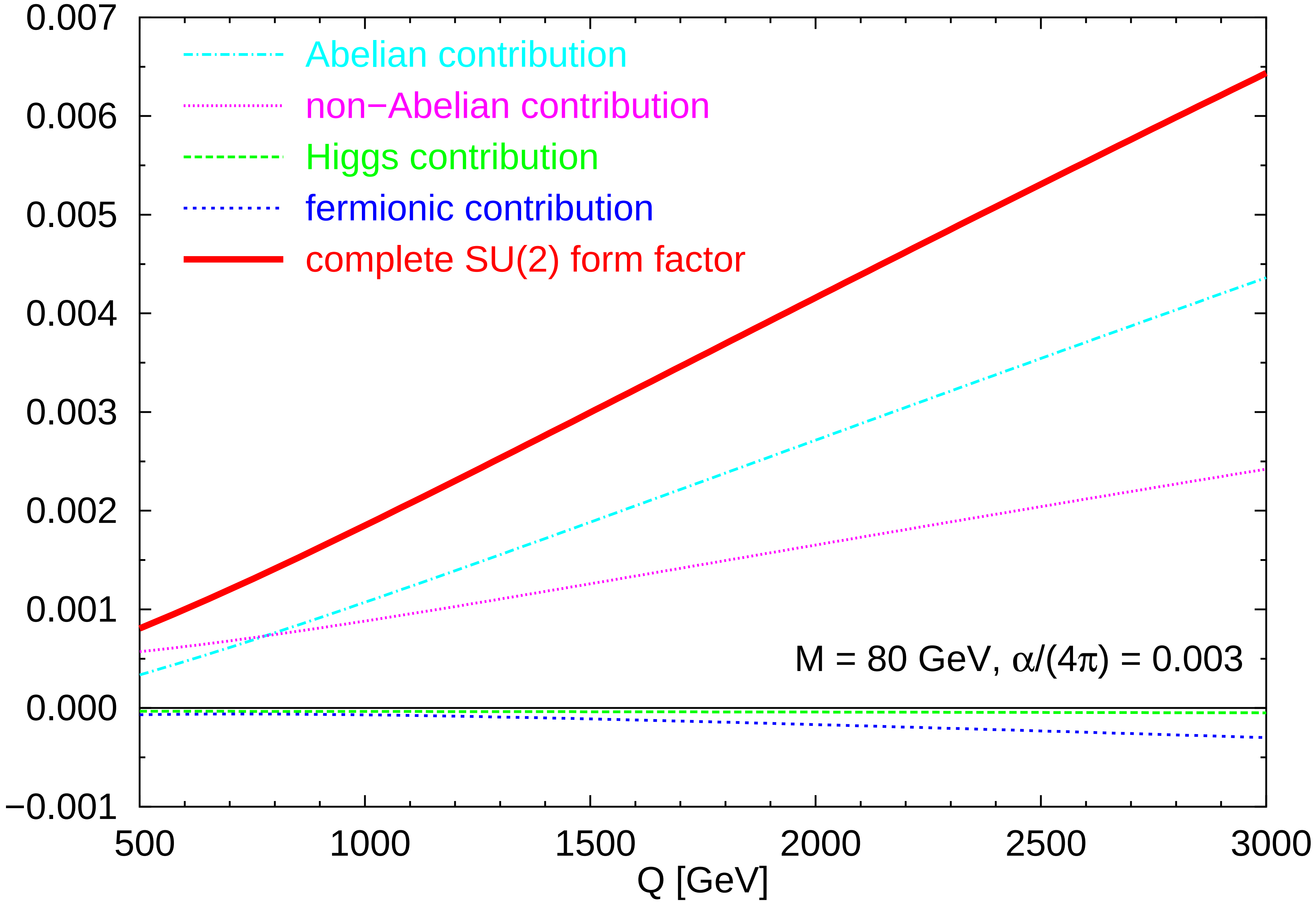


N^3LL approximation

$M_{\text{Higgs}} = M$

Massive SU(2) form factor in 2-loop approximation: individual contributions

(N^{3}LL approximation, $M_{\text{Higgs}} = M$, Feynman-'t Hooft gauge)



III Methods for loop calculations at high energies

Reduction to scalar diagrams

- **given** from Feynman rules: $\mathcal{F}^\mu = \bar{u}(p_2)\,\Gamma^\mu(p_1,p_2)\,u(p_1)$
- **wanted:** form factor $F(Q^2)$ with $\mathcal{F}^\mu = F(Q^2)\cdot\bar{u}(p_2)\,\gamma^\mu\,u(p_1)$
- can be done using the properties of Dirac matrices and spinors, $\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}$, $\not{p}_1\,u(p_1) = 0$, $\bar{u}(p_2)\,\not{p}_2 = 0$, combined with tensor reduction
- more elegantly with a *projector* on the form factor:

$$F(Q^2) = \frac{\mathrm{Tr}\left[\gamma_\mu\,\not{p}_2\,\Gamma^\mu(p_1,p_2)\,\not{p}_1\right]}{2(d-2)\,q^2}$$

- **output:** form factor $F(Q^2)$ in terms of *scalar Feynman integrals*

$$\int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2\,\frac{\prod_{j=1}^{N}(\ell_j\cdot\ell_j')^{\nu_j}}{\prod_{i=1}^{L}({k_i'}^2 - M_i^2)^{n_i}}$$

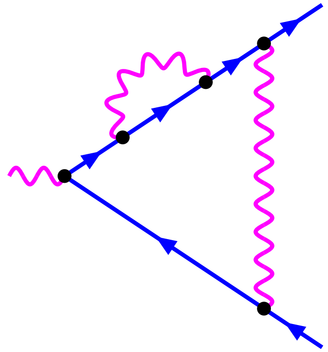
with L propagators and N irreducible scalar products in the numerator

Elimination of irreducible scalar products in the numerator

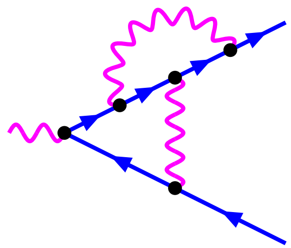
- Most scalar diagrams could directly be calculated *with numerator*.

- Diagrams with self-energy insertion:
  tensor reduction for inner loop, e.g.

$$\int \mathrm{d}^d k \, \frac{p \cdot k}{f(k,q)} = p_\nu \int \mathrm{d}^d k \, \frac{k^\nu}{f(k,q)} = \frac{p \cdot q}{q^2} \int \mathrm{d}^d k \, \frac{q \cdot k}{f(k,q)}$$



- Difficult diagrams where the absence of the numerator was desirable:



  - ⋆ write propagators with Schwinger parameters (alpha parameters):

$$\frac{1}{(k^2 - M^2)^n} = \frac{1}{i^n \, \Gamma(n)} \int_0^\infty \mathrm{d}\alpha \, \alpha^{n-1} \, e^{i\alpha(k^2 - M^2)}$$

  - ⋆ diagonalize the argument of the exponential in the loop momenta
  - ⋆ perform tensor reduction: numerator $\to$ factors of $g^{\mu\nu}$
  - ⋆ rewrite as linear combinations of the original integral *without numerator*, but with *higher powers of propagators* $(n \to n+1, \, n+2, \, \ldots)$ and *higher dimension* $(d \to d+2, \, d+4, \, \ldots)$

Anastasiou et al. '00

Expansion by regions

a powerful method for the asymptotic expansion of Feynman diagrams Beneke, Smirnov '98

- **given:** scalar Feynman integral & limit like $Q^2 \gg M^2$ *(Minkowskian limit!)*
- **wanted:** expansion of the *integral* in M^2/Q^2
- **problem:** direct expansion of the *integrand* leads to (new) IR/UV singularities

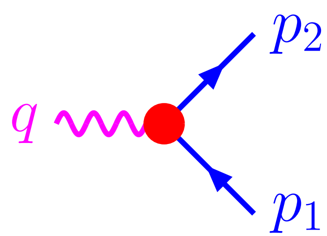
Recipe for the method of expansion by regions:

1. *divide* the integration domain into *regions* for the loop momenta
   (especially such regions where singularities are produced in the limit $M \to 0$)
2. in every region, *expand* the integrand in a *Taylor series* with respect to the parameters that are considered small *there*
3. *integrate* the expanded integrands over the *whole integration domain*
4. put to zero any *scaleless integral* (due to the properties of dimensional regularization)

- usually only a few regions give non-vanishing contributions
- for logarithmic approximation: only leading order of the expansion needed
  $\hookrightarrow$ in step 2. all small parameters in the integrand are simply set to zero
- sometimes additional regularization (apart from ε) needed for individual regions

Expansion by regions: example

Vertex form factor in the Sudakov limit $Q^2 \gg M^2$



- typical regions for each loop momentum k:

hard	(h):	all components of $k \sim Q$
soft	(s):	all components of $k \sim M$
ultrasoft	(us):	all components of $k \sim M^2/Q$
1-collinear	(1c):	$k^2 \sim 2p_1 \cdot k \sim M^2, \quad 2p_2 \cdot k \sim Q^2$
2-collinear	(2c):	$k^2 \sim 2p_2 \cdot k \sim M^2, \quad 2p_1 \cdot k \sim Q^2$

- 1-loop vertex correction: $f = \int \frac{\mathrm{d}^d k}{i\pi^{d/2}} \frac{1}{(k^2 - M^2)(k^2 - 2p_1 \cdot k)(k^2 - 2p_2 \cdot k)}$

$$f^{(h)} = \frac{1}{Q^2} \left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} \ln(Q^2) - \frac{1}{2} \ln^2(Q^2) + \frac{\pi^2}{12} + \mathcal{O}\left(\frac{M^2}{Q^2} \right) \right]$$

$$f^{(1c)} + f^{(2c)} = \frac{1}{Q^2} \left[\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} \ln(Q^2) - \frac{1}{2} \ln^2(M^2) + \ln(M^2) \ln(Q^2) - \frac{5}{12}\pi^2 + \mathcal{O}\left(\frac{M^2}{Q^2} \right) \right]$$

$$\Rightarrow f = f^{(h)} + f^{(1c)} + f^{(2c)} = \frac{1}{Q^2} \left[-\frac{1}{2} \ln^2\left(\frac{Q^2}{M^2} \right) - \frac{\pi^2}{3} + \mathcal{O}\left(\frac{M^2}{Q^2} \right) \right]$$

Expansion by regions: why it works

simple $d = 1$ example: $f = \displaystyle\int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon}}{(k+m)(k+q)}\,, \quad m \ll q$

$$\left.\begin{array}{ll} \text{soft (s):} & k < \Lambda \\ \text{hard (h):} & k > \Lambda \end{array}\right\} \text{where } m \ll \Lambda \ll q$$

$$\begin{aligned}
f &= \int_0^\Lambda \frac{\mathrm{d}k\, k^{-\varepsilon}}{(k+m)(k+q)} + \int_\Lambda^\infty \frac{\mathrm{d}k\, k^{-\varepsilon}}{(k+m)(k+q)} \\
&= \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \int_0^\Lambda \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m} + \sum_{i=0}^\infty (-m)^i \int_\Lambda^\infty \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q} \\
&= \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \left(\int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m} - \int_\Lambda^\infty \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m} \right) + \sum_{i=0}^\infty (-m)^i \left(\int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q} - \int_0^\Lambda \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q} \right) \\
&= \underbrace{\sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m}}_{f^{(s)}} + \underbrace{\sum_{i=0}^\infty (-m)^i \int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q}}_{f^{(h)}} - \sum_{i=0}^\infty (-m)^i \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \underbrace{\int_0^\infty \mathrm{d}k\, k^{-\varepsilon-i+j-1}}_{\to 0,\ \text{scaleless integral}} \\
&= f^{(s)} + f^{(h)} \quad \checkmark \\
&= \frac{\Gamma(\varepsilon)\Gamma(1-\varepsilon)}{(q-m)\, m^\varepsilon} + \frac{\Gamma(-\varepsilon)\Gamma(1+\varepsilon)}{(q-m)\, q^\varepsilon} = \frac{\ln(q/m)}{q-m} + \mathcal{O}(\varepsilon) \quad \checkmark
\end{aligned}$$

Parameterization of Feynman integrals

- Feynman parameters:

$$\prod_i \frac{1}{A_i^{n_i}} = \frac{\Gamma(\sum_i n_i)}{\prod_i \Gamma(n_i)} \left(\prod_i \int_0^1 \mathrm{d}x_i \, x_i^{n_i - 1} \right) \frac{\delta(\sum_i x_i - 1)}{(\sum_i x_i A_i)^{\sum_i n_i}}$$

- Schwinger parameters $\longrightarrow$ more general esp. with expansion by regions:

$$\frac{1}{A^n} = \frac{1}{i^n \, \Gamma(n)} \int_0^\infty \mathrm{d}\alpha \, \alpha^{n-1} \, e^{i\alpha A} \,, \quad \text{numerator } A^n = \left(\frac{1}{i} \frac{\partial}{\partial \alpha} \right)^n e^{i\alpha A} \Big|_{\alpha=0}$$

$\Rightarrow$ any number of propagators and numerators may be combined

$\Rightarrow$ can always be transformed to Feynman parameters

$\hookrightarrow$ evaluation:

$$\int \mathrm{d}^d k \, e^{i(\alpha k^2 + 2p\cdot k)} = i\pi^{d/2} \, (i\alpha)^{-d/2} \, e^{-ip^2/\alpha}$$

$$\int_0^\infty \mathrm{d}\alpha \, \alpha^{n-1} \, e^{i\alpha A} = \frac{i^n \, \Gamma(n)}{A^n}$$

$$\int_0^\infty \frac{\mathrm{d}\alpha \, \alpha^{n-1}}{(A + \alpha B)^r} = \frac{\Gamma(n) \, \Gamma(r-n)}{\Gamma(r) \, A^{r-n} \, B^n}$$

Mellin-Barnes representation

Feynman integrals with many scales / many massive propagators are hard to evaluate
$\hookrightarrow$ separate scales by Mellin-Barnes representation:

$$\frac{1}{(A+B)^n} = \frac{1}{\Gamma(n)} \int_{-i\infty}^{i\infty} \frac{\mathrm{d}z}{2\pi i}\, \Gamma(-z)\, \Gamma(n+z)\, \frac{B^z}{A^{n+z}}$$

- Mellin-Barnes integrals go along the imaginary axis, leaving poles of $\Gamma(-z+\dots)$ to the right and poles of $\Gamma(z+\dots)$ to the left of the integration contour
- applicable to massive propagators ($A=k^2$, $B=-M^2$) or to any complicated intermediate expression
- evaluation:
  close the integration contour to the right ($|B| \leq |A|$) or to the left ($|B| \geq |A|$) and pick up the residues within the contour using $\operatorname{Res}\Gamma(z)\big|_{z=-i} = (-1)^i/i!$
  $\Rightarrow$ *sums over Γ-functions*
  $\Rightarrow$ *multiple ζ-values* / *generalized (harmonic) polylogarithms* etc.
- close link to *expansion by regions*:
  Mellin-Barnes representation of the full integral
  $\hookrightarrow$ contributions corresponding to the regions

III SU(2)×U(1) model with mass gap

Electroweak Standard Model: massive SU(2) and massless U(1) gauge bosons

here: without mixing → $M_W = M_Z$, neglect $\mathcal{O}(\underbrace{\sin^2\theta_W}_{\approx 0.2} \alpha^2 \ln^1)$

- form factor $F_{\text{SU(2)}}(\alpha, Q, M)$ → IR-finite
- form factor $F_{\text{U(1)}}(\alpha', Q, \lambda)$ → IR-singularities regularized by λ or $\varepsilon = \frac{4-d}{2}$
- $\text{SU(2)}_M \times \text{U(1)}_\lambda$: $\hat{F}(\alpha, \alpha', Q, M, \lambda)$ for $Q \gg M \gg \lambda \to 0$
  → **factorization of IR-singularities:**

$$\hat{F}(\alpha, \alpha', Q, M, \lambda) = \underbrace{F_{\text{U(1)}}(\alpha', Q, \lambda)}_{\text{IR-singular}} \underbrace{\tilde{F}(\alpha, \alpha', Q, M)}_{\text{IR-finite}} + \mathcal{O}\left(\alpha\alpha' \frac{\lambda^2}{M^2}\right)$$

$$\Rightarrow \tilde{F}(\alpha, \alpha', Q, M) = \lim_{\lambda \to 0} \frac{\hat{F}(\alpha, \alpha', Q, M, \lambda)}{F_{\text{U(1)}}(\alpha', Q, \lambda)} = \lim_{\varepsilon \to 0} \frac{\hat{F}(\alpha, \alpha', Q, M, 0; \varepsilon)}{F_{\text{U(1)}}(\alpha', Q, 0; \varepsilon)}$$

↪ set $\lambda = 0$ and calculate $\hat{F}(\alpha, \alpha', Q, M, 0; \varepsilon)$ in dimensional regularization

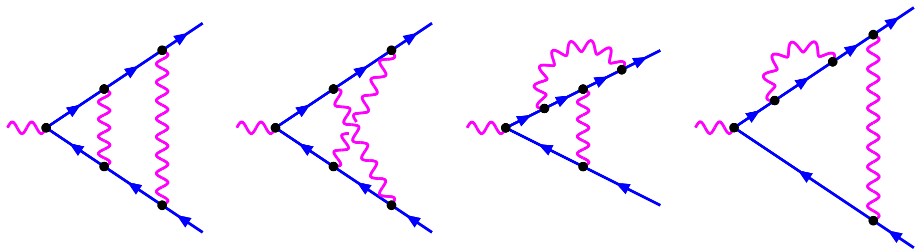
Factorization of IR-singularities:

$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = \underbrace{F_{\mathsf{U(1)}}(\alpha',Q,\lambda)}_{\text{IR-singular}}\,\underbrace{\tilde{F}(\alpha,\alpha',Q,M)}_{\text{IR-finite}} + \mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$$

Calculation of $\tilde{F}(\alpha,\alpha',Q,M)$:

2-loop diagrams with 1 massive SU(2)

and 1 massless U(1) gauge boson:



$$\tilde{F}(\alpha,\alpha',Q,M) = F_{\mathsf{SU(2)}}(\alpha,Q,M)\times \left\{1+\frac{\alpha\alpha'}{(4\pi)^2}C_F\left[\left(48\zeta_3-4\pi^2+3\right)\ln\left(\frac{Q^2}{M^2}\right)+\frac{7}{45}\pi^4-84\zeta_3+\frac{20}{3}\pi^2-2\right]\right\}$$

⇒ interference terms are finite ⇝ IR singularities factorize

⇒ only single logarithm $\ln^1$ ⇝ evolution equation & NNLL prediction ✓

Factorization of the SU(2)×U(1) form factor for $\lambda = M$

Set $\lambda = M$ and parametrize:

$$\hat{F}(\alpha, \alpha', Q, M, M) = F_{\mathsf{U(1)}}(\alpha', Q, M)\, \tilde{F}(\alpha, \alpha', Q, M)\, C(\alpha, \alpha', Q, M)$$

$\hat{F}(\alpha, \alpha', Q, M, M)$ known from $F_{\mathsf{SU(2)}}(\alpha, Q, M)$ and $F_{\mathsf{U(1)}}(\alpha', Q, M)$
$\Rightarrow$ calculate matching coefficient:

$$C(\alpha, \alpha', Q, M) = 1 + \frac{\alpha\alpha'}{(4\pi)^2} C_F \times \left[512\,\mathrm{Li}_4\left(\tfrac{1}{2}\right) + \tfrac{64}{3}\ln^4 2 - \tfrac{64}{3}\pi^2 \ln^2 2 - \tfrac{113}{15}\pi^4 + 244\zeta_3 + \tfrac{70}{3}\pi^2 + \tfrac{59}{4}\right]$$

no logarithm!

Applications:

- $\tilde{F}(\alpha, \alpha', Q, M) = \dfrac{\hat{F}(\alpha, \alpha', Q, M, M)}{F_{\mathsf{U(1)}}(\alpha', Q, M)} + \mathcal{O}\big(\alpha\alpha' \ln^0\big)$
- $\hat{F}(\alpha, \alpha', Q, M, \lambda \approx M) = F_{\mathsf{U(1)}}(\alpha', Q, \lambda \approx M)\tilde{F}(\alpha, \alpha', Q, M) + \mathcal{O}\big(\alpha\alpha' \ln^{0,1}\big)$
  $\hookrightarrow$ expansion in small mass difference, e.g. $M_W \approx M_Z$

IV Summary

Massive SU(2) form factor

- weak interaction with massive gauge bosons
- 2-loop result in N^{3}LL approximation ✓

⇒ precise control of radiative corrections

SU(2)×U(1) model with mass gap

- factorization of IR singularities shown explicitly ✓
- calculation with mass gap reduced to the 1-mass case $M_W = M_Z = M_{\text{photon}}$
- $M_Z \neq M_W$ taken into account by expanding around the equal mass approximation

⇒ prediction for electroweak 2-loop form factor

Combination with reduced amplitude

- scattering amplitude $f\bar{f} \to f'\bar{f}'$
- electroweak 2-loop corrections to cross sections, ...
- B. Jantzen, J.H. Kühn, A.A. Penin, V.A. Smirnov, *hep-ph/0504111*