SUDAKOV LOGARITHMS
in
N^3LL APPROXIMATION

J.H. Kühn, Karlsruhe

with B. Feucht, A. Penin, V. Smirnov

J.H. Kühn, A.A. Penin; hep-ph/9906545
J.H. Kühn, A.A. Penin, V.A. Smirnov; Eur. Phys. J. C17 (2000) 97
J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov; Nucl. Phys. B616 (2001) 286
B. Feucht, J.H. Kühn, S. Moch; Phys. Lett. B561 (2003) 111
B. Feucht, J.H. Kühn, A.A. Penin, V.A. Smirnov; Phys. Rev. Letts. 93 (2004) 101802

- **Introduction**

- **Four fermion scattering**

- **Form factors at two loop**

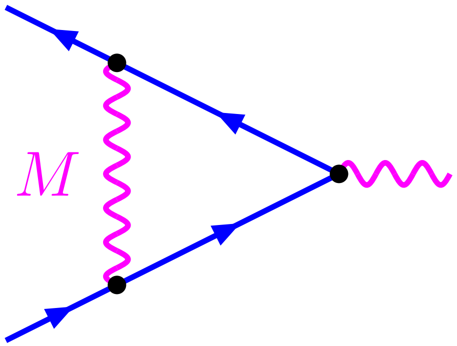
- **Z-boson production**

- **Summary**

Introduction

One-Loop

example: massive U(1)



$$\Rightarrow \quad \text{Born} * \left[1 + \frac{\alpha}{4\pi}\left(-\ln^2\frac{s}{M^2} + 3\ln\frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3}\right)\right]$$

magnitude ($\frac{\alpha_w}{4\pi} = 3 \cdot 10^{-3}$)

$\frac{s}{M^2}$	$-\ln^2\frac{s}{M^2}$	$+3\ln\frac{s}{M^2}$	$-\frac{7}{2}+\frac{\pi^2}{3}$	Σ	$* 4\,\frac{\alpha_w}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	−25.52	+15.15	−0.21	−10.6	−13%
$\left(\frac{2000}{80}\right)^2$	−41.44	+19.31	−0.21	−22.3	−27%

(four-fermion cross section ⇒ factor 4)

Two-Loop

Four-fermion processes, status:

LL: Fadin et al. (2000)

NLL: J.H.K., Penin, Smirnov (2000)
Large (!) subleading corrections
important angular dependent terms

NNLL: J.H.K., Moch, Penin, Smirnov (2001)
Large (!) NNLL terms,
oscillating signs of LL, NLL, NNLL
$\Rightarrow$ compensations

$\Rightarrow$ N^3LL and constant terms desirable

additional complication in SM: massless photon

$$|Q^2| \gg M_{W,Z}^2 \gg m_\gamma^2$$

Four fermion scattering

examine $f'\bar{f}' \to f\bar{f}$

$$A_B = \frac{ig^2}{s} \sum_{I,J=L,R} \left(T^3_{f'} T^3_f + \tan^2 \Theta_W \frac{Y_{f'} Y_f}{4} \right) A^{f'f}_{IJ} \quad \text{with} \quad A^{f'f}_{IJ} = \left(\bar{f}'_I \gamma^\mu f'_I \right) \left(\bar{f}_J \gamma_\mu f_J \right)$$

corrections from photon radiation up to cutoff $\omega \ll M_{W,Z}$ must be taken into account separately (prescription of Fadin et al.).

define

$$L(s) = \frac{g^2}{16\pi^2} \ln^2 \left(\frac{s}{M^2} \right) = 0.07 \quad (0.11)$$

$$l(s) = \frac{g^2}{16\pi^2} \ln \left(\frac{s}{M^2} \right) = 0.014 \quad (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

for $\sqrt{s} = 1$ TeV (2 TeV)

result (based on evolution equations):

$$\sigma/\sigma_{\rm B}(e^+e^- \to Q\bar{Q}) = 1 - 1.66\,L(s) + 5.31\,l(s) - 15.86\,a + 1.93\,L^2(s) - 9.43\,L(s)l(s) + 29.73\,l^2(s)$$

$$\sigma/\sigma_{\rm B}(e^+e^- \to q\bar{q}) = 1 - 2.18\,L(s) + 20.58\,l(s) - 36.34\,a + 2.79\,L^2(s) - 50.06\,L(s)l(s) + 295.12\,l^2(s)$$

$$\sigma/\sigma_{\rm B}(e^+e^- \to \mu^+\mu^-) = 1 - 1.39\,L(s) + 10.12\,l(s) - 31.33\,a + 1.42\,L^2(s) - 18.43\,L(s)l(s) + 99.89\,l^2(s)$$

(result very close to J.K., Penin, hep-ph/9906545!)

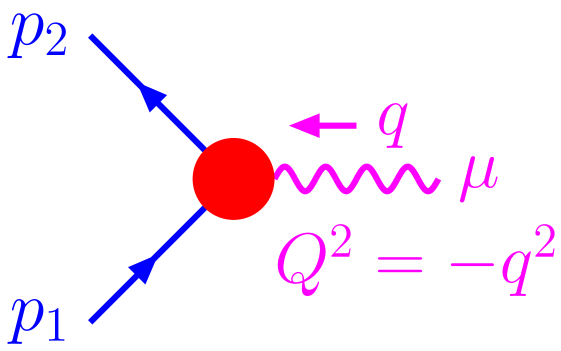
NLL terms confirmed by diagrammatic calculation — Pozzorini

subleading terms important
$\to$ evaluate N^{3}LL and N^{4}LL

first step: form factor

Form factors at two loop

A) Form Factor and Evolution Equations



Born:

$$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu \psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow \; \mathcal{F} = \mathcal{F}_{\text{Born}}\, F_0(\alpha(M^2)) \exp\left\{\int\limits_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\left[\int\limits_{M^2}^{x} \frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: N^{4}LL $\Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \left[\underbrace{\ln^{2n}\left(\frac{Q^2}{M^2}\right)}_{\text{LL}} + \underbrace{\ln^{2n-1}\left(\frac{Q^2}{M^2}\right)}_{\text{NLL}} + \underbrace{\ln^{2n-2}\left(\frac{Q^2}{M^2}\right)}_{\text{NNLL}} + \underbrace{\ln^{2n-3}\left(\frac{Q^2}{M^2}\right)}_{\text{N}^3\text{LL}} + \underbrace{\ln^{2n-4}\left(\frac{Q^2}{M^2}\right)}_{\text{N}^4\text{LL}} \right]$$

NNLL (previous result) requires running of α (i.e. β_0 and β_1) and:

$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$ up to $\mathcal{O}(\alpha)$

$\gamma(\alpha)$ up to $\mathcal{O}(\alpha^2)$

N^{3}LL requires two-loop calculation in high-energy limit including linear logarithms (available for non-abelian theory)

N^{4}LL requires complete two-loop calculation in high-energy limit (available for abelian theory)

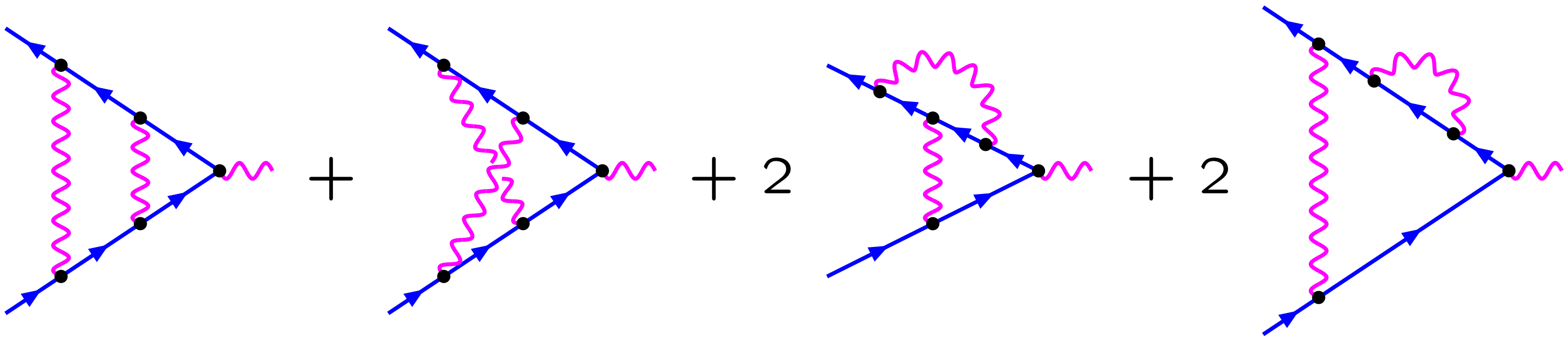
B) Two-Loop Results

Massive U(1) Model

$$\mathcal{F}_\alpha(M,Q) = \mathcal{F}_{\text{Born}}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \dots\right]$$

$$\begin{aligned}
f^{(1)} &= -\mathcal{L}^2 + 3\,\mathcal{L} - \frac{7}{2} - \frac{2}{3}\pi^2 \\
f^{(2)} &= \frac{1}{2}\mathcal{L}^4 - 3\,\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right)\mathcal{L} \\
&\quad + \frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3}\ln^4 2 + 256\,\mathrm{Li}_4\left(\frac{1}{2}\right) \\
\mathcal{L} &\equiv \ln(Q^2/M^2)
\end{aligned}$$

NNLL in agreement with previous results!



numerically:

$$f^{(1)} = -\mathcal{L}^2 + 3\,\mathcal{L} - 10.1$$

$$f^{(2)} = +0.5\,\mathcal{L}^4 - 3\,\mathcal{L}^3 + 14.6\,\mathcal{L}^2 - 19.6\,\mathcal{L} + 26.4$$

rescaling of argument of $\mathcal{L}$: $M \rightarrow e^{3/4}M \Rightarrow$ NLL $\rightarrow$ 0

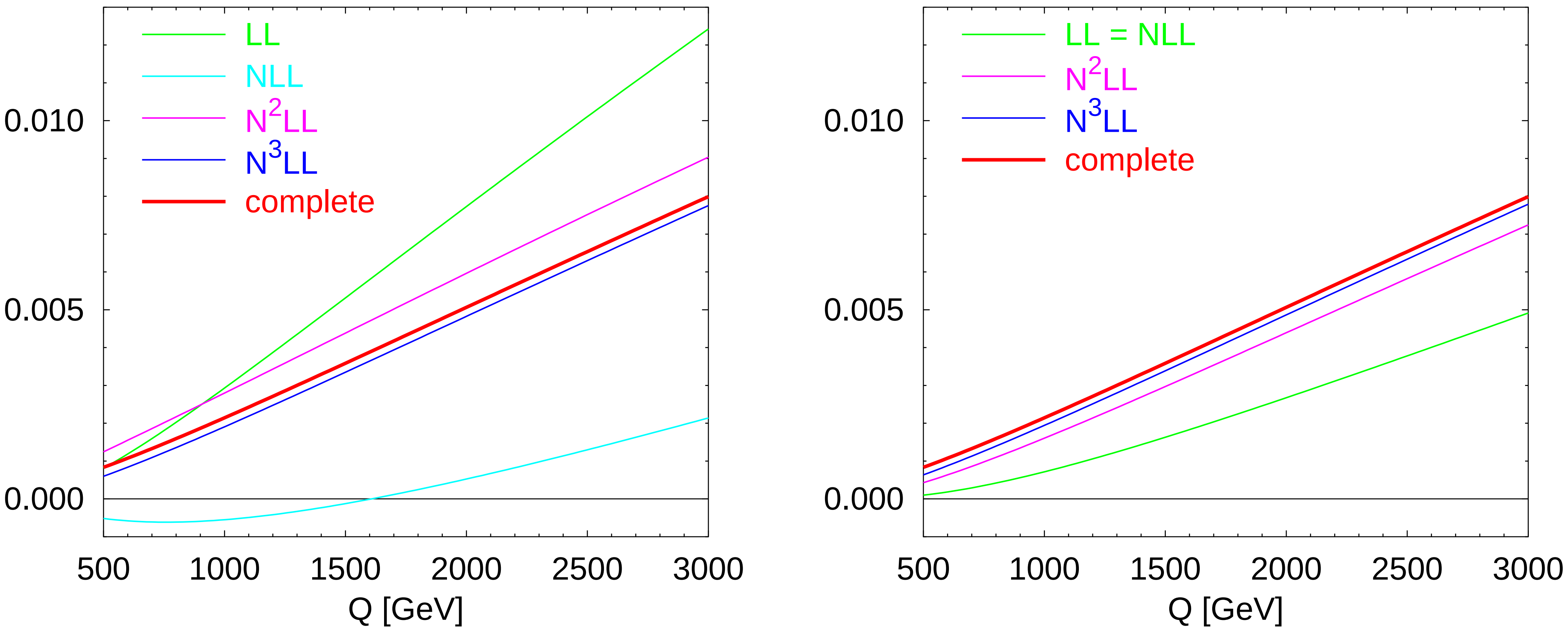
$$f^{(1)} = -\widehat{\mathcal{L}}^2 - 7.8$$

$$f^{(2)} = +0.5\,\widehat{\mathcal{L}}^4 + 7.8\,\widehat{\mathcal{L}}^2 + 10.6\,\widehat{\mathcal{L}} + 22.2$$

$$\widehat{\mathcal{L}} \equiv \ln\left(\frac{Q^2}{(e^{3/4}M)^2}\right)$$

Two-loop result $f^{(2)}$:

logs $\ln\left(\frac{Q^2}{M^2}\right)$ — rescaled logs $\ln\left(\frac{Q^2}{(e^{3/4}M)^2}\right)$

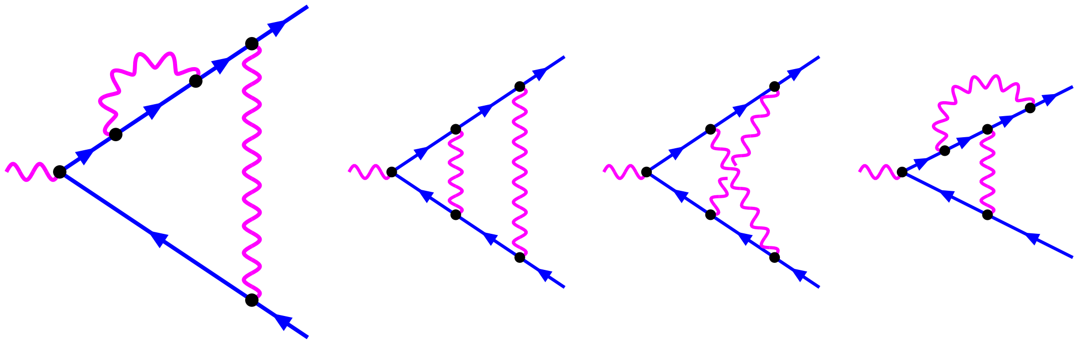


with $M = 80\,\text{GeV}$, $\frac{\alpha}{4\pi} = 3 \cdot 10^{-3}$

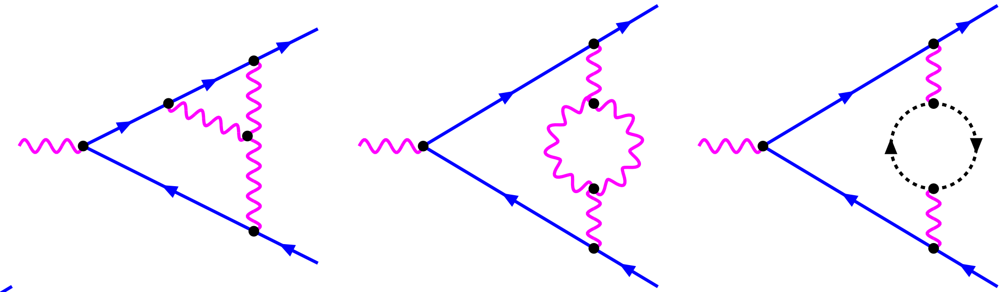
C) Massive SU(2) form factor in 2-loop approximation

2-loop vertex diagrams (massless fermions, massive bosons):

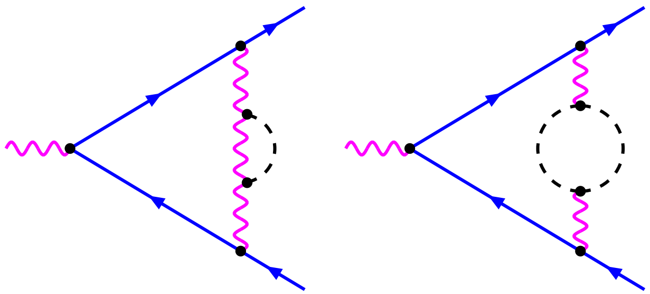
Abelian (C_F^2):



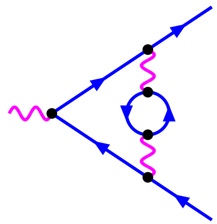
non-Abelian ($C_F C_A$): last 2 +



Higgs:



fermion ($C_F T_F n_f$):



+ 1-loop×1-loop corrections + renormalization

Size of the logarithmic contributions

2-loop form factor F_2 at $Q = 1\,\text{TeV}$ (in 1/1000):

Abelian (C_F^2):	$+0.3\ln^4$	$-1.7\ln^3$	$+8.2\ln^2$	$-11\ln$	$+15$
	$+1.6$	-2.0	$+1.9$	-0.5	$+0.1$
non-Abelian ($C_F C_A$):		$+1.8\ln^3$	$-14\ln^2$	$+46\ln$	$-\ldots$
		$+2.1$	-3.3	$+2.1$	
Higgs:		$-0.04\ln^3$	$+0.5\ln^2$	$-2.3\ln$	$+\ldots$
		-0.04	$+0.1$	-0.1	
fermionic ($C_F T_F n_f$):		$-0.5\ln^3$	$+4.8\ln^2$	$-13\ln$	$+21$
		-0.6	$+1.1$	-0.6	$+0.2$

$\ln^{4,3,2}$: J.H.K, Moch, Penin, Smirnov

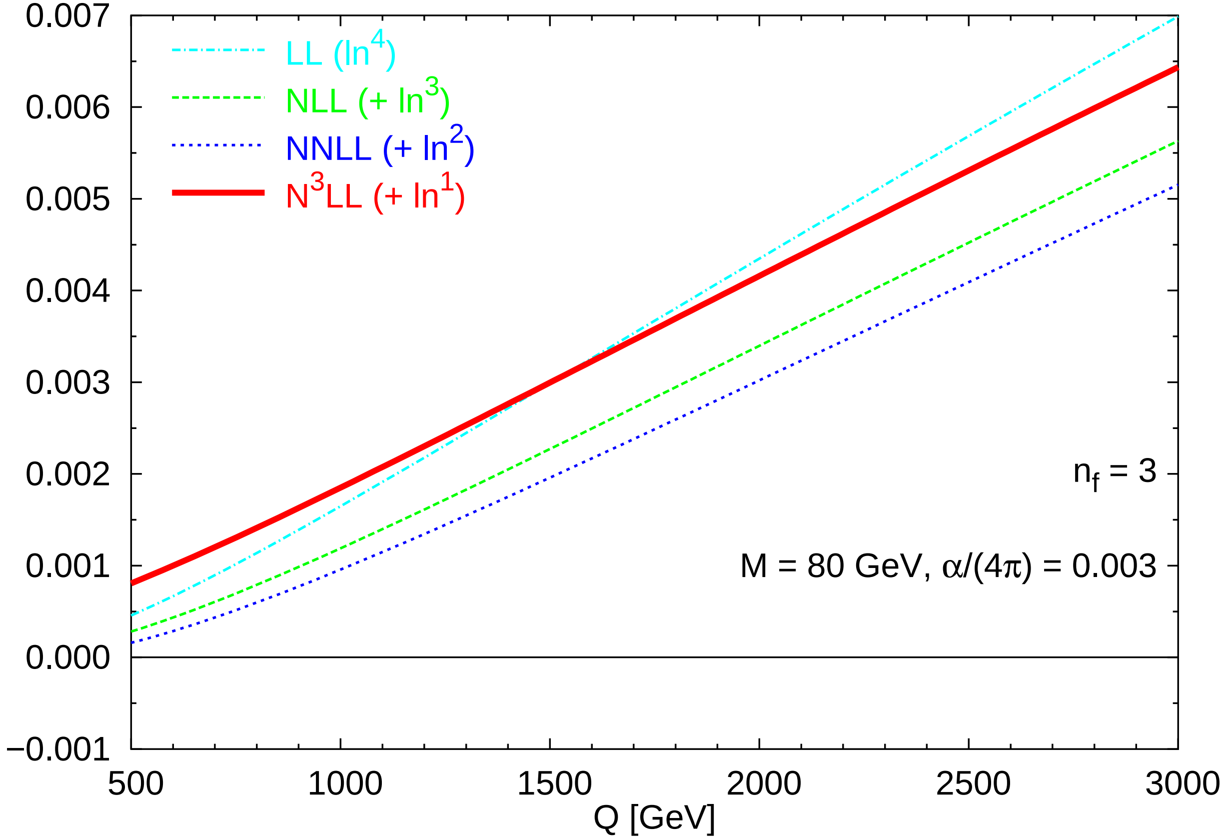
$\ln^{1,0}$: Feucht, J.H.K, Moch; Feucht, J.H.K, Penin, Smirnov

$\longrightarrow$ growing coefficients with alternating signs
$\Rightarrow$ cancellations between logarithmic terms
$\hookrightarrow$ **NNLL approximation is not enough!**

Abelian & fermionic contribution: $\ln^1$ small, $\ln^0$ negligible
$\Rightarrow$ **N^3LL approximation** including $\ln^1$ is sufficient (non-Abelian $\ln^0$ more difficult)

Massive SU(2) form factor in 2-loop approximation: result

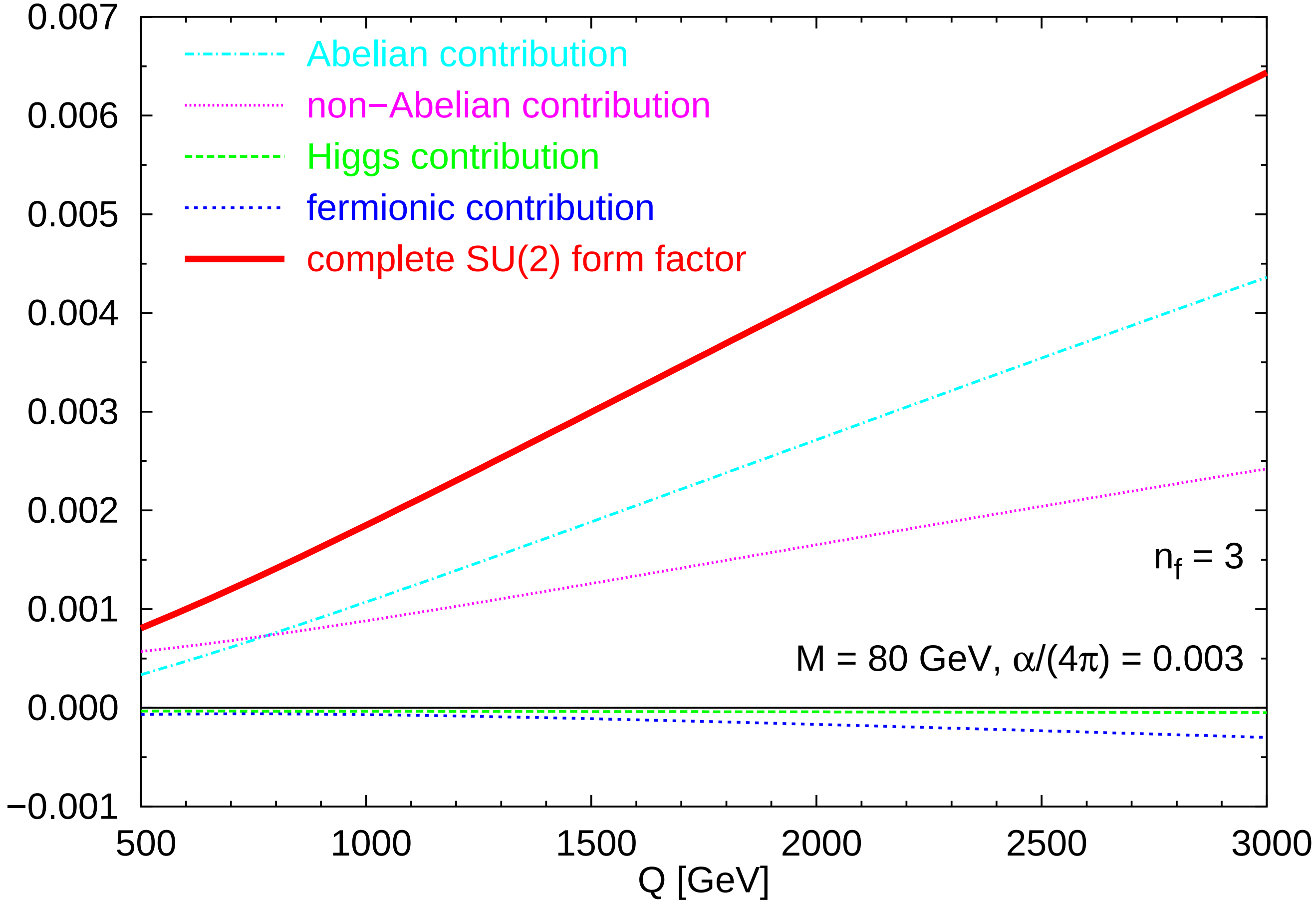
$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \Bigg[+\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{19}{48}\ln^3\left(\frac{Q^2}{M^2}\right) - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\ln^2\left(\frac{Q^2}{M^2}\right)$$
$$+ \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\ln\left(\frac{Q^2}{M^2}\right)\Bigg]$$



N^{3}LL approximation
$M_{\mathrm{Higgs}} = M$
$n_f = 3$

Massive SU(2) form factor in 2-loop approximation: individual contributions

(N^{3}LL approximation, $M_{\text{Higgs}} = M$, $n_f = 3$, Feynman-'t Hooft gauge)



U(1)×U(1) Model useful for QED×Weak and QCD×EW

$(\alpha, M) \times (\alpha', \lambda)$

factorization for $Q^2 \gg M^2 \gg \lambda^2$:

$$\mathcal{F}_{\alpha',\alpha}(\lambda, M, Q) = \tilde{F}_{\alpha',\alpha}(M, Q) \underbrace{\mathcal{F}_{\alpha'}(\lambda, Q)}_{\text{as before}} + \mathcal{O}(\lambda/M)$$

$$\Rightarrow \tilde{F}_{\alpha',\alpha}(M, Q) = \left[\frac{\mathcal{F}_{\alpha',\alpha}(\lambda, M, Q)}{\mathcal{F}_{\alpha'}(\lambda, Q)}\right]_{\lambda \to 0}$$

evaluated with dimensional regularization for IR singularities

$$\tilde{F}_{\alpha',\alpha}(M, Q) = 1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \frac{\alpha'\alpha}{(4\pi)^2} \tilde{f}^{(1,1)} + \ldots$$

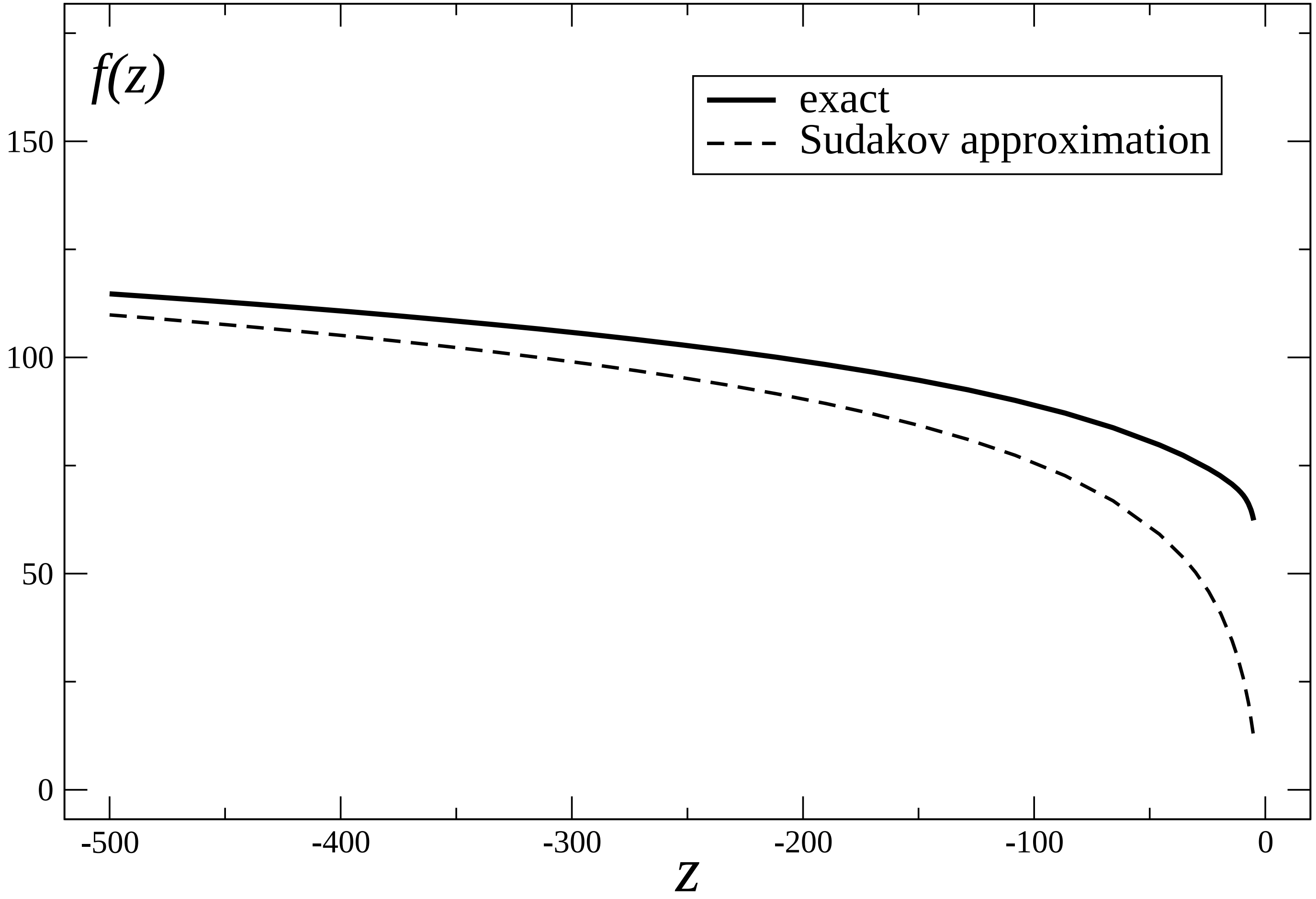
$$\tilde{f}^{(1,1)} = \left(3 - 4\pi^2 + 48\zeta_3\right) \mathcal{L} - 2 + \frac{20}{3}\pi^2 - 84\zeta_3 + \frac{7}{45}\pi^4$$

important observation: no $\mathcal{L}^2$ terms $\Rightarrow$ consistent with evolution equations

J.H.K., Penin, Smirnov (2000)

Complete result for $\tilde{f}^{(1,1)}(z)$ available in analytical form ($z = \frac{Q^2}{M^2}$)

Kotikov, J.H.K., Veretin



Exponentiation, Factorization and Matching

Massive U(1) Theory

5 terms in the two-loop result $\Rightarrow$ N^{4}LL approximation in all orders:

$$\mathcal{F}_\alpha(M,Q) = \exp\left\{\frac{\alpha}{4\pi}\left[-\mathcal{L}^2 + \left(3 + \frac{\alpha}{4\pi}\left(\frac{3}{2} - 2\pi^2 + 24\zeta_3\right) + \mathcal{O}(\alpha^2)\right)\mathcal{L}\right]\right\}\mathcal{F}_\alpha(M,M)$$

U(1)×U(1) Theory

matching relation: $\mathcal{F}_{\alpha',\alpha}(M,M,Q) = C_{\alpha',\alpha}(M,Q)\,\tilde{F}_{\alpha',\alpha}(M,Q)\,\mathcal{F}_{\alpha'}(M,Q)$

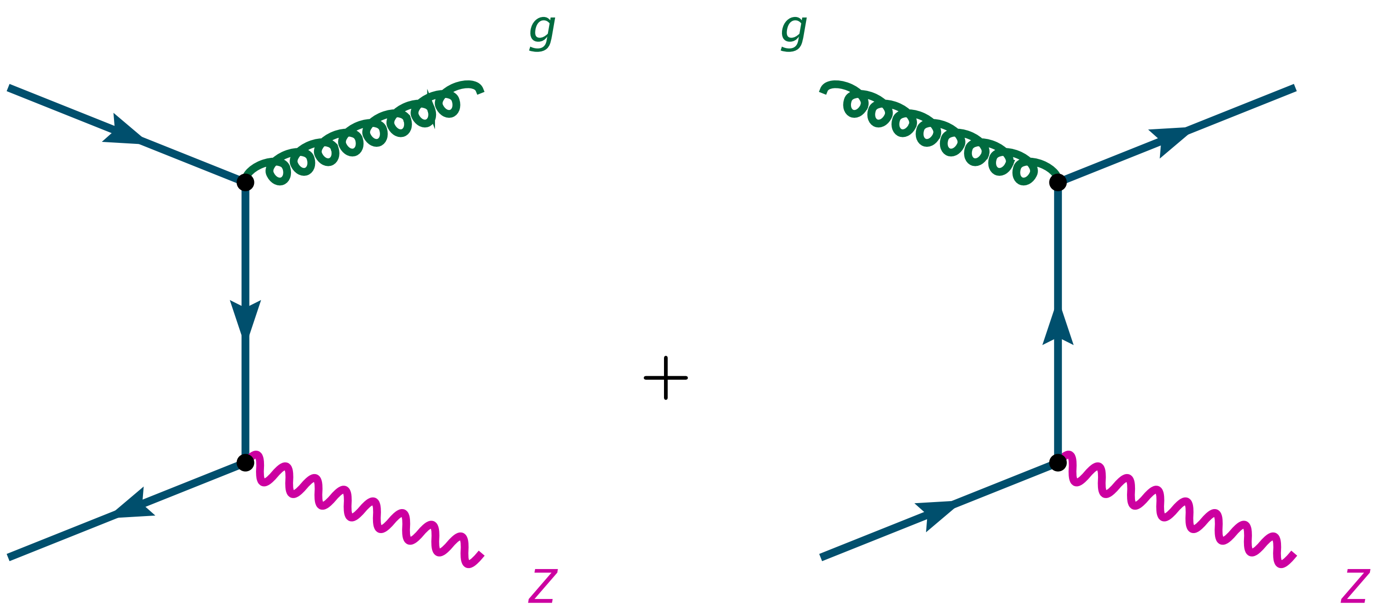
$$\Rightarrow\ C_{\alpha',\alpha}(M,Q) = 1 + \frac{\alpha'\alpha}{(4\pi)^2}\left[\frac{59}{4} + \frac{70}{3}\pi^2 + 244\zeta_3 - \frac{113}{15}\pi^4 - \frac{64}{3}\pi^2\ln^2 2 + \frac{64}{3}\ln^4 2 + 512\,\mathrm{Li}_4\left(\frac{1}{2}\right)\right]$$

- no logarithmic terms!
- $\tilde{F}_{\alpha',\alpha}(M,Q)\,\mathcal{F}_{\alpha'}(\lambda,Q)$ approaches $\mathcal{F}_{\alpha',\alpha}(\lambda,M,Q)$ for $\lambda \to M$ in N^{3}LL accuracy!
- *all* logs in theory with mass gap are obtained from symmetric phase

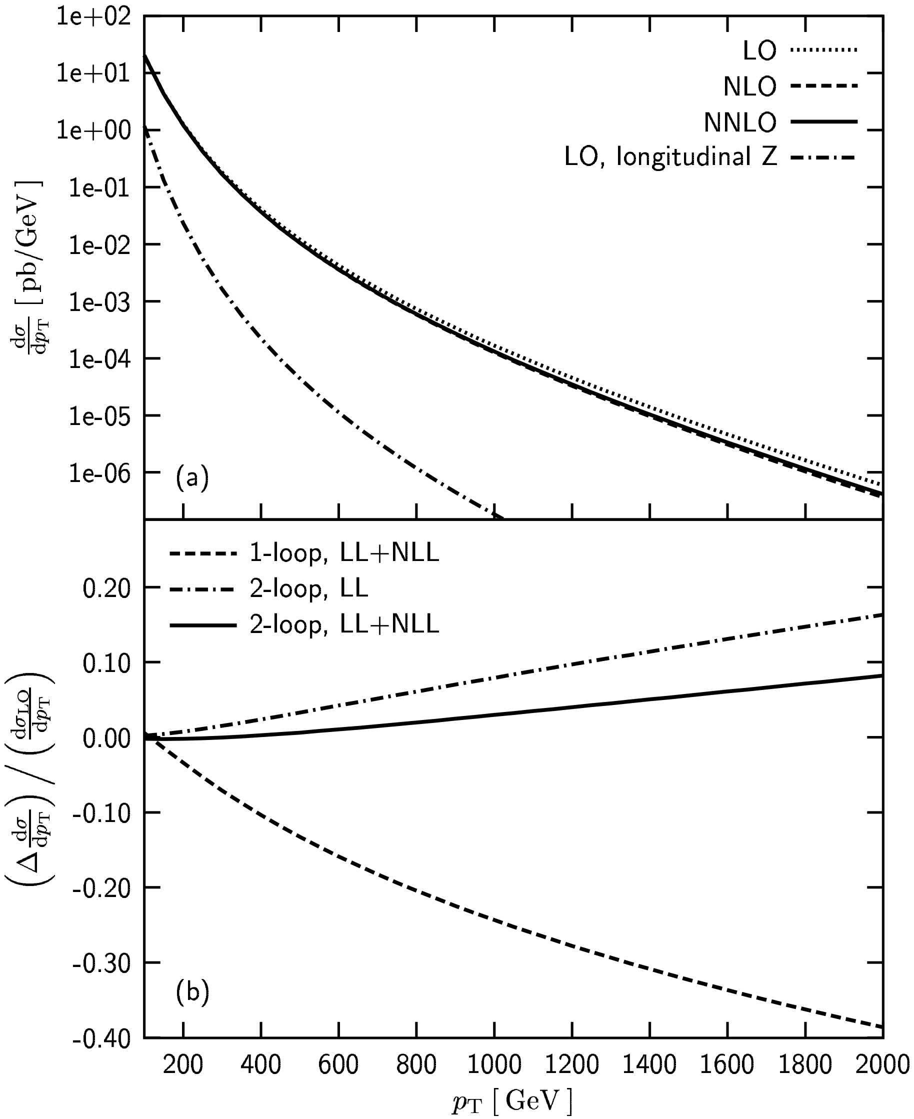
Z-boson production

J.H.K., Kulesza, Pozzorini, Schulze

Z-boson production at LHC with large p_T
$\rightarrow$ large electroweak corrections



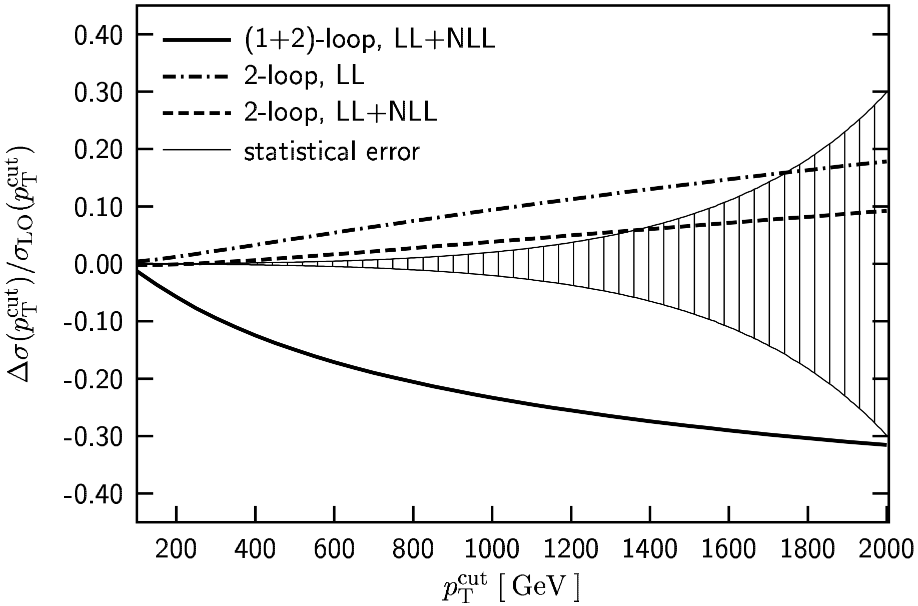
results:



(a) Transverse momentum distribution for $pp \to Zj$ at $\sqrt{s} = 14$ TeV: LO (dotted), NLO (dashed) and NNLO (solid) result for unpolarised Z and LO contribution from longitudinally polarised Z (dash-dotted).

(b) Relative electroweak correction to the lowest order unpolarised p_T distribution for $pp \to Zj$ at $\sqrt{s} = 14$ TeV: 1-loop LLs+NLLs (dashed), 2-loop LLs (dash-dotted) and 2-loop LLs+NLLs (solid).

results:



Relative electroweak correction and statistical error for the unpolarised integrated cross section for $pp \to Zj$ at $\sqrt{s} =$ 14 TeV as a function of p_T^{cut}: (1+2)-loop LL+NLL (solid), 2-loop LL (dash-dotted) and 2-loop LL+NLL (dashed) correction and statistical error (shaded region) with respect to the lowest order cross section.

Summary

- Large logarithmic corrections at large energies
- NLL, N^2LL, N^3LL terms are important
- N^3LL and N^4LL (partly) available for form factor
- special role of massless bosons (γ and g)
  $\rightarrow$ factorization of IR singularities
- first applications: LHC
  important issue for LC