The Radiative Return
at $\Phi$- and $B$-Meson Factories

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III Charge Asymmetry and Radiative $\Phi$-Decays ($\Rightarrow$ H. Czyż)
IV Nucleon Form Factor at $B$-Factories
V Conclusions

(with H. Czyż, A. Grzelinska, E. Nowak, G. Rodrigo)
1 BASIC IDEA

photon radiated off the initial $e^+e^-$ (ISR) reduces the effective energy of the collision

$$d\sigma(e^+e^- \rightarrow \text{hadrons} + \gamma) = H(Q^2, \theta_\gamma) \ d\sigma(e^+e^- \rightarrow \text{hadrons})$$

- measurement of $R(s)$ over the full range of energies, from threshold up to $\sqrt{s}$
- large luminosities of factories compensate $\alpha/\pi$ from photon radiation
- radiative corrections essential (NLO)
- advantage over energy scan (BES, CMD2, SND): systematics (e.g. normalization) only once

High precision measurement of the hadronic cross-section at DAΦNE, CLEO-C, B-factories
DAΦNE versus B-factories:

configurations in the cms - frame

10 GeV

very hard photon: clear kinematic separation between photon and hadrons

1 GeV

no natural kinematic separation

⇒ cuts to control FSR versus ISR

( two step process: $e^+e^- \rightarrow \gamma \rho(\rightarrow \gamma\pi\pi) \Rightarrow$ see below )
Rough estimates for rates:

\[ \pi^+ \pi^- \gamma : E_\gamma \geq 100\,MeV \]

<table>
<thead>
<tr>
<th>(\sqrt{s}) [GeV]</th>
<th>(\int L) [fb(^{-1})]</th>
<th>#events, (\theta_{min} = 7^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.02</td>
<td>1.35</td>
<td>16 (\cdot 10^6)</td>
</tr>
<tr>
<td>10.6</td>
<td>100</td>
<td>3.5 (\cdot 10^6)</td>
</tr>
</tbody>
</table>

multi-hadron-events (R \(\equiv 2\)) \(\sqrt{s} = 10.6\,GeV\)

<table>
<thead>
<tr>
<th>(Q^2)-interval [GeV]</th>
<th>#events, (\theta_{min} = 7^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>([1.5, 2.0])</td>
<td>9.9 (\cdot 10^5)</td>
</tr>
<tr>
<td>([2.0, 2.5])</td>
<td>7.9 (\cdot 10^5)</td>
</tr>
<tr>
<td>([2.5, 3.0])</td>
<td>6.6 (\cdot 10^5)</td>
</tr>
<tr>
<td>([3.0, 3.5])</td>
<td>5.8 (\cdot 10^5)</td>
</tr>
</tbody>
</table>
Lowest order

\[
\frac{d\sigma}{dQ^2} (e^+e^- \rightarrow \gamma + \text{had}(Q^2)) = \sigma (e^+e^- \rightarrow \text{had}(Q^2)) \\
\times \frac{\alpha}{\pi s} \left\{ \frac{s^2+Q^4}{s(s-Q^2)} \left( \log \left( \frac{s}{m^2_e} \right) - 1 \right) , \text{no angular cut} \right. \\
\left. \frac{s^2+Q^4}{s(s-Q^2)} \log \left( \frac{1+\cos \theta_{\text{min}}}{1-\cos \theta_{\text{min}}} \right) - \frac{s-Q^2}{s} \cos \theta_{\text{min}} \right\}
\]

⇒ differential luminosity: \[
\frac{dL}{dQ^2} (Q^2, s) = \frac{\alpha}{\pi s} \left\{ \cdots \right\} L(\text{at } s)
\]
Basic Ingredients for Pion Formfactor

- **ISR**

  ![ISR Diagram]
  
  pion form factor $\leftrightarrow$ to be tested

- **FSR**

  ![FSR Diagram]
  
  radiation from point-like pions (probably overestimated)

- **additional radiation:**
  
  - collinear (EVA MC) (Binner, JK, Melnikov)
  - or NLO calculation (PHOKHARA MC)
References etc. → http://cern.ch/german.rodrigo/phokhara
http://cern.ch/german.rodrigo/phokhara

| **PHOKHARA**  
<table>
<thead>
<tr>
<th>radiative return at meson factories</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Physics</strong></td>
</tr>
<tr>
<td><strong>Content</strong></td>
</tr>
<tr>
<td><strong>Downloads</strong></td>
</tr>
</tbody>
</table>
  - manual [Postscript, PDF], source [unencoded] |
Forthcoming features
- Simulation of other exclusive hadronic channels
  (see also hep-ph/0312217)

Previous versions
版本3.0 (August 2003): simulates ISR at NLO for two charged pions or muons, and four-pion channels, and FSR at leading order for two charged pions or muons. FSR at NLO for two charged pions added.
- manual [Postscript, PDF], source [unencoded]

版本2.0 (December 2002): includes small angle photon generation, four-pion channels and FSR at leading order for two charged pions or muons.
- manual [Postscript, PDF], source [unencoded]

版本1.0 (December 2001): includes ISR only and is limited to two charged pions or muons together with one or two hard photons as final states.
- manual [Postscript, PDF], source [unencoded]

EVA: simulates two pion events and includes ISR, FSR, and their interference at the leading order (LO), and the dominant radiative corrections from additional collinear radiation through structure function techniques.
- manual [Postscript, PDF], source [unencoded]
The Radiative Return at $\Phi$- and $B$-Meson Factories

References

- PHOKHARA 1.0:
  G. Rodríguez, H. Czyz, J.H. Kühn, M. Szopa, 
  G. Rodríguez, A. Gehrmann-De Ridder, M. Guilleaume, J.H. Kühn,

- PHOKHARA 2.0:
  H. Czyz, A. Grzelinska, J.H. Kühn, G. Rodríguez,
  J.H. Kühn, G. Rodríguez,

- PHOKHARA 3.0:
  H. Czyz, A. Grzelinska, J.H. Kühn, G. Rodríguez,

- PHOKHARA 4.0:
  H. Czyz, J.H. Kühn, E. Nowak, G. Rodríguez,
  H. Czyz, A. Grzelinska, J.H. Kühn, G. Rodríguez,

- EVA:
  S. Binnew, J.H. Kühn and K. Melnikov,

- EVA-4pi:
  H. Czyz and J.H. Kühn,

- Further reading:
  H. Czyz, A. Grzelinska, J.H. Kühn, G. Rodríguez,
  H. Czyz and E. Nowak,
  G. Rodríguez, hep-ph/0311588;
PHOKHARA 2.0:
\[ \pi^+ \pi^-, \, \mu^+ \mu^- , \, 4\pi \]

- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level

- **FSR at LO:** \( \pi^+ \pi^- , \, \mu^+ \mu^- \)
- tagged or untagged photons
- modular structure
QED CORRECTIONS AT LEPTONIC SIDE

⇒ BASIC BUILDING BLOCK
FOR ALL HADRONIC FINAL STATES
specifically developed for $\pi^+\pi^-$ (plus photons)
allows for simultaneous emission of photons from initial and final state, including virtual corrections (interference neglected).

$\Rightarrow$ dominated by “two step process”: $e^+e^- \rightarrow \gamma \rho (\rightarrow \gamma \pi\pi)$
$\Rightarrow$ importance of $\pi\pi\gamma$ as input for $\alpha_\mu$
Large effect for $Q^2 < m_\rho^2$ eliminated by suitable cuts on $\pi^+\pi^-$ configuration (suppress 2$\gamma$ events)

or measure photon
Experimental Perspectives

KLOE

- \( \pi \pi \)
- pion form factor

BABAR, BELLE

- higher \( Q^2 \) available

\( \Rightarrow \) measurement of \( R(Q^2) \) from threshold up to at least 5 GeV.

Examples:

- \( 3\pi \)
- \( 4\pi^\pm \)
- \( K K \pi \pi \)
- \( K K K K \)
- \( J/\Psi \)
The background-subtracted $3\pi$ mass spectrum for masses between 0.70 and 1.05 MeV/$c^2$ (plot on the left) and for masses from 1.05 to 1.80 MeV/$c^2$ (plot on the right).  

BaBar
The $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section measured by BaBar (filled circles), by SND (open circles), and DM2 (open triangles).
FIGURE 2. On the left: The $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ cross section obtained from ISR at BABAR in comparison with all $e^+e^-$ data. On the right: The signals from $J/\psi$ and $\psi(2S)$ in $4\pi$ invariant mass. The shaded region at the latter corresponds to $\psi(2S) \rightarrow J/\psi \pi^+\pi^-$, with $J/\psi \rightarrow \mu^+\mu^-$.  

BaBar
The Radiative Return at $\Phi$- and $B$-Meson Factories
$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \gamma(\gamma)$

$M_{3\pi} = (1.4, 1.8)$ GeV

$\sqrt{s} = 10.52$ GeV

$\frac{d\sigma}{dM_{\pi^+\pi^-}} (\text{nb/(GeV)}^2)$

$\text{events}/(25 \text{ MeV/c}^2)$

$M(\pi^+\pi^-) (\text{GeV/c}^2)$

BaBar
• $\mu^+\mu^-\gamma$ with FSR at NLO

• vacuum polarisation can be switched on

• nucleon pair production included
III Charge Asymmetries and Radiative $\Phi$-Decays


talk by Henryk Czyż
IV NUCLEON FORM FACTORS

(with Czyż, Nowak, Rodrigo, hep-ph/0403062)

\[ Q^2 \gtrsim 4m_N^2 \] accessible at B-factories
\[ \Rightarrow \text{study } e^+e^- \rightarrow \gamma N\bar{N} \text{ (with } N = p \text{ or } n) \]

hadronic current:

\[ J_\mu = -ie \cdot \bar{u}(q_2) \left( F_1^N(Q^2) \gamma_\mu - \frac{F_2^N(Q^2)}{4m_N} [\gamma_\mu, Q] \right) v(q_1), \]

\[ Q = q_1 + q_2, \quad q = (q_1 - q_2)/2 \]

or

\[ G_M = F_1 + F_2, \quad G_E = F_1 + \frac{Q^2}{4m^2} F_2 \]
Separation of $|G_M|^2$ and $|G_E|^2$ through angular distribution:

$$L_{\mu \nu} H^{\mu \nu} = \frac{(4\pi \alpha)^3}{Q^2} \frac{(1 + \cos^2 \theta_\gamma)}{(1 - \cos^2 \theta_\gamma)}$$

$$\times 4 \left( |G_M^N|^2 (1 + \cos^2 \hat{\theta}) + \frac{1}{\tau} |G_E^N|^2 \sin^2 \hat{\theta} \right)$$

$\hat{\theta}$ = angle of nucleon with respect to $\gamma$-direction in hadronic rest frame

(valid for $s/Q^2 \ll 1$, corrections and “optimal frame” → hep-ph/0403062)

Similarity to $e^+e^- \rightarrow N\bar{N}$:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta_N}{4Q^2} \left( |G_M^N|^2 (1 + \cos^2 \theta) + \frac{1}{\tau} |G_E^N|^2 \sin^2 \theta \right)$$
At least one photon satisfies:

\( 0^\circ < \theta_{p\bar{p}} < 180^\circ \)
\( 5^\circ < \theta_\gamma < 175^\circ \)
\( 30^\circ < \theta_{p\bar{p}} < 150^\circ \)
\( 25^\circ < \theta_\gamma < 155^\circ \)
\( E_\gamma > 4 \text{GeV} \)

\( e^+ e^- \rightarrow p\bar{p} \gamma \)

implementation in PHOKHARA

(large rates \( \sim 400 \text{ evt/10MeV around } 4 \text{GeV}^2 \))
Angular distributions of nucleon

$e^+e^- \rightarrow p\bar{p}\gamma$

$G_M = \mu_p G_E$

$4.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$

$30^\circ < \theta_{p,\phi} < 150^\circ$

$25^\circ < \theta_\gamma < 155^\circ$

$\sqrt{s} = 10.52 \text{ GeV}$

hadronic rest frame

(two choices for $G_M/G_E$)

J.H. Kühn

The Radiative Return at $\Phi$- and $B$-Meson Factories
• similar results for neutron pair production

• NLO corrections from ISR included (corrections $\sim 1\text{-}2\%$)

• no FSR

thousands of events around 4–5 GeV$^2$
several events up to 7–8 GeV$^2$
V Conclusions

- continuous development of PHOKHARA
  - radiative corrections (completed for ISR)
  - more channels
  - cooperation between theory and experiment crucial

- charge asymmetry as analysis tool

- nucleon form factors:
  - $G_E$ and $G_M$ can be measured for a wide range of $Q^2$

central issue: hadronic form factors!