SUDAKOV LOG'S
in
ELECTROWEAK PROCESSES

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The Two-Loop Form Factor in a Massive U(1) Theory

J.H. Kühn, A.A. Penin; hep-ph/9906545
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1) Motivation

large corrections at high energies

2) U(1)×U(1) Model

A) Form Factor and Evolution Equations

B) Two-Loop Results

C) Exponentiation, Factorization and Matching

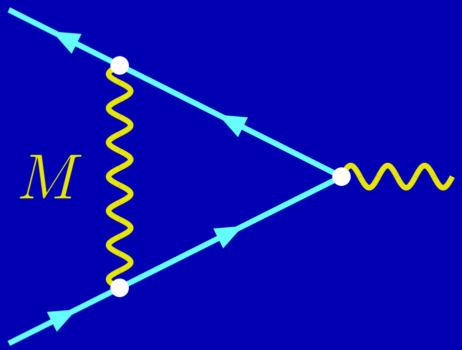
3) n_f and n_s Terms

4) Summary

1) MOTIVATION

One-Loop

example: massive U(1)



$$\Rightarrow \quad \text{Born} * \left[1 + \frac{\alpha}{4\pi}\left(-\ln^2\frac{s}{M^2} + 3\ln\frac{s}{M^2} - \frac{7}{2} + \frac{\pi^2}{3}\right)\right]$$

magnitude ($\frac{\alpha_w}{4\pi} = 3 \cdot 10^{-3}$)

$\frac{s}{M^2}$	$-\ln^2\frac{s}{M^2}$	$+3\ln\frac{s}{M^2}$	$-\frac{7}{2}+\frac{\pi^2}{3}$	Σ	$* 4\,\frac{\alpha_w}{4\pi}$
$\left(\frac{1000}{80}\right)^2$	-25.52	$+15.15$	-0.21	-10.6	-13%
$\left(\frac{2000}{80}\right)^2$	-41.44	$+19.31$	-0.21	-22.3	-27%

(four-fermion cross section $\Rightarrow$ factor 4)

Two-Loop

Four-fermion processes, status:

LL:	Fadin et al. (2000)
NLL:	J.H.K., Penin, Smirnov (2000) Large (!) subleading corrections important angular dependent terms
NNLL:	J.H.K., Moch, Penin, Smirnov (2001) Large (!) NNLL terms, oscillating signs of LL, NLL, NNLL $\Rightarrow$ compensations

$\Rightarrow$ N^3LL and constant terms desirable

additional complication in SM: massless photon

$$|Q^2| \gg M^2_{W,Z} \gg m^2_\gamma$$

Toy Model: $\mathbf{U(1)} \times \mathbf{U(1)}$

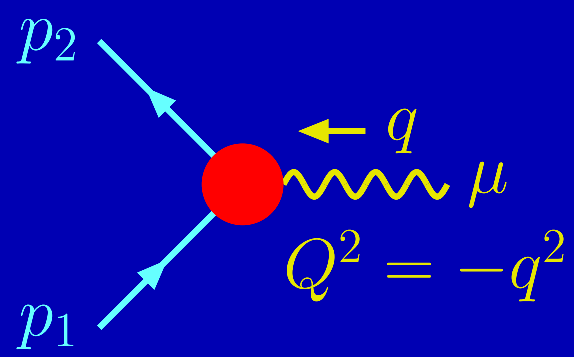
with couplings α and α',
masses $M \gg \lambda$

- calculate form factor in two loops, including linear log and constant terms,
- verify NNLL results,
- provide input for four-fermion N^3LL calculation,
- investigate matching between massive and massless theory,
- phenomenological relevance for gluon-quark form factor.

techniques: $Q^2 \gg M^2 \Rightarrow$ expansion by regions

2) U(1)×U(1) MODEL

A) Form Factor and Evolution Equations



Born:

$$\mathcal{F}_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow \mathcal{F} = \mathcal{F}_{\text{Born}}\, F_0(\alpha(M^2)) \exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: N^4LL $\Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \Bigg[\underbrace{\ln^{2n}\left(\frac{Q^2}{M^2}\right)}_{\text{LL}} + \underbrace{\ln^{2n-1}\left(\frac{Q^2}{M^2}\right)}_{\text{NLL}} + \underbrace{\ln^{2n-2}\left(\frac{Q^2}{M^2}\right)}_{\text{NNLL}} + \underbrace{\ln^{2n-3}\left(\frac{Q^2}{M^2}\right)}_{\text{N}^3\text{LL}} + \underbrace{\ln^{2n-4}\left(\frac{Q^2}{M^2}\right)}_{\text{N}^4\text{LL}} \Bigg]$$

NNLL (previous result) requires running of α (i.e. β_0 and β_1) and:

$$\begin{array}{ll} \zeta(\alpha),\ \xi(\alpha),\ F_0(\alpha) & \text{up to } \mathcal{O}(\alpha) \\ \gamma(\alpha) & \text{up to } \mathcal{O}(\alpha^2) \end{array}$$

N^4LL requires complete two-loop calculation in high-energy limit (new!)

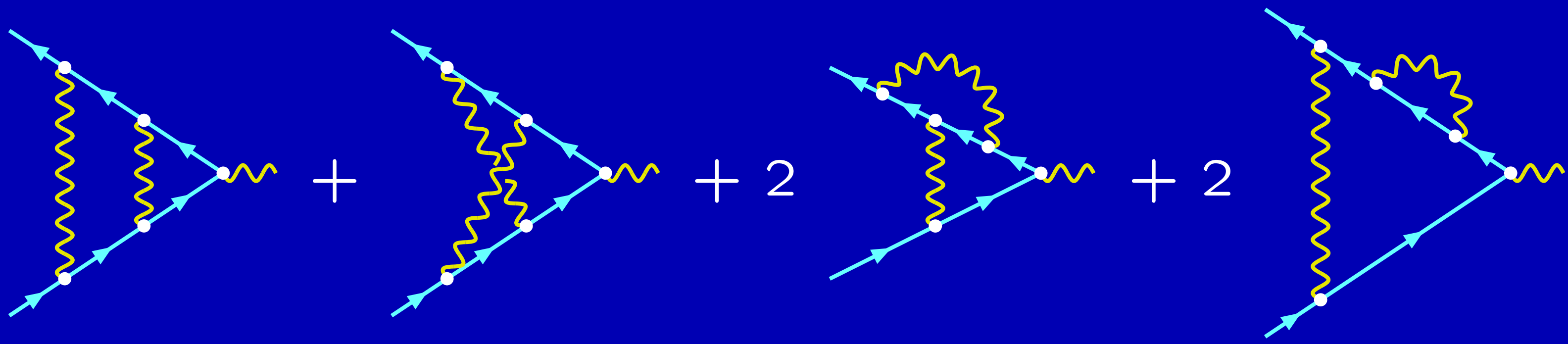
B) Two-Loop Results

Massive U(1) Model

$$\mathcal{F}_\alpha(M,Q) = \mathcal{F}_{\text{Born}}\left[1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \ldots\right]$$

$$\begin{aligned}
f^{(1)} &= -\mathcal{L}^2 + 3\,\mathcal{L} - \frac{7}{2} - \frac{2}{3}\pi^2 \\
f^{(2)} &= \frac{1}{2}\mathcal{L}^4 - 3\,\mathcal{L}^3 + \left(8 + \frac{2}{3}\pi^2\right)\mathcal{L}^2 - \left(9 + 4\pi^2 - 24\zeta_3\right)\mathcal{L} \\
&\quad + \frac{25}{2} + \frac{52}{3}\pi^2 + 80\zeta_3 - \frac{52}{15}\pi^4 - \frac{32}{3}\pi^2 \ln^2 2 + \frac{32}{3}\ln^4 2 + 256\,\mathrm{Li}_4\left(\frac{1}{2}\right) \\
\mathcal{L} &\equiv \ln(Q^2/M^2)
\end{aligned}$$

NNLL in agreement with previous results!



numerically:

$$f^{(1)} = -\mathcal{L}^2 + 3\,\mathcal{L} - 10.1$$

$$f^{(2)} = +0.5\,\mathcal{L}^4 - 3\,\mathcal{L}^3 + 14.6\,\mathcal{L}^2 - 19.6\,\mathcal{L} + 26.4$$

rescaling of argument of $\mathcal{L}$: $M \to e^{3/4}M \Rightarrow$ NLL $\to$ 0

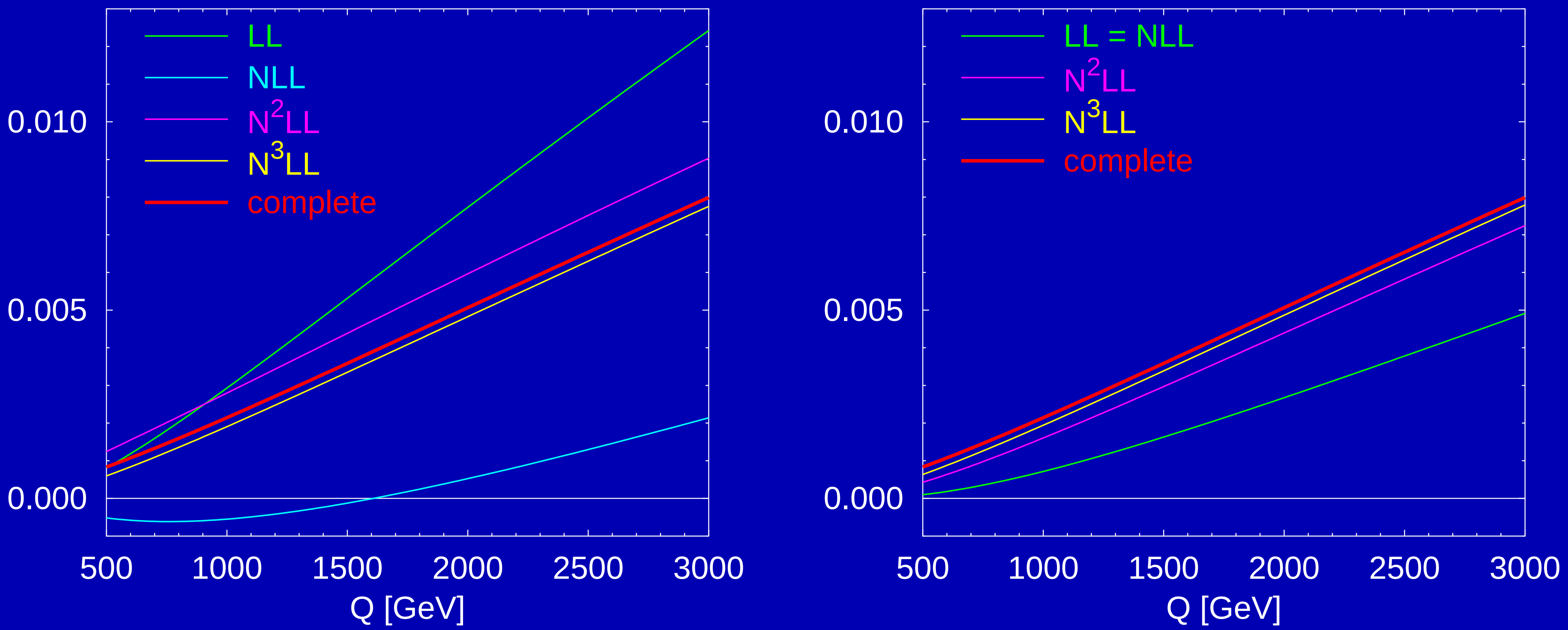
$$f^{(1)} = -\widehat{\mathcal{L}}^2 - 7.8$$

$$f^{(2)} = +0.5\,\widehat{\mathcal{L}}^4 + 7.8\,\widehat{\mathcal{L}}^2 + 10.6\,\widehat{\mathcal{L}} + 22.2$$

$$\widehat{\mathcal{L}} \equiv \ln\left(\frac{Q^2}{(e^{3/4}M)^2}\right)$$

Two-loop result $f^{(2)}$:

logs $\ln\left(\frac{Q^2}{M^2}\right)$ rescaled logs $\ln\left(\frac{Q^2}{(e^{3/4}M)^2}\right)$



with $M = 80\,\text{GeV}$, $\frac{\alpha}{4\pi} = 3 \cdot 10^{-3}$

U(1)×U(1) Model

$(\alpha, M) \times (\alpha', \lambda)$

factorization for $Q^2 \gg M^2 \gg \lambda^2$:

$$\mathcal{F}_{\alpha',\alpha}(\lambda, M, Q) = \tilde{F}_{\alpha',\alpha}(M, Q)\, \underbrace{\mathcal{F}_{\alpha'}(\lambda, Q)}_{\text{as before}} + \mathcal{O}(\lambda/M)$$

$$\Rightarrow \quad \tilde{F}_{\alpha',\alpha}(M, Q) = \left[\frac{\mathcal{F}_{\alpha',\alpha}(\lambda, M, Q)}{\mathcal{F}_{\alpha'}(\lambda, Q)}\right]_{\lambda \to 0}$$

evaluated with dimensional regularization for IR singularities

$$\tilde{F}_{\alpha',\alpha}(M, Q) = 1 + \frac{\alpha}{4\pi} f^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 f^{(2)} + \frac{\alpha'\alpha}{(4\pi)^2} \tilde{f}^{(1,1)} + \ldots$$

$$\tilde{f}^{(1,1)} = \left(3 - 4\pi^2 + 48\zeta_3\right) \mathcal{L} - 2 + \frac{20}{3}\pi^2 - 84\zeta_3 + \frac{7}{45}\pi^4$$

important observation: no $\mathcal{L}^2$ terms $\Rightarrow$ consistent with evolution equations

Kühn, Penin, Smirnov (2000)

C) Exponentiation, Factorization and Matching

Massive U(1) Theory

5 terms in the two-loop result $\Rightarrow$ N^{4}LL approximation in all orders:

$$\mathcal{F}_\alpha(M,Q) = \exp\left\{\frac{\alpha}{4\pi}\left[-\mathcal{L}^2 + \left(3 + \frac{\alpha}{4\pi}\left(\frac{3}{2} - 2\pi^2 + 24\zeta_3\right) + \mathcal{O}(\alpha^2)\right)\mathcal{L}\right]\right\}\mathcal{F}_\alpha(M,M)$$

U(1)×U(1) Theory

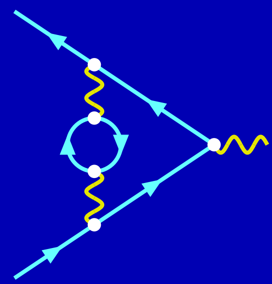
matching relation: $\mathcal{F}_{\alpha',\alpha}(M,M,Q) = C_{\alpha',\alpha}(M,Q)\,\tilde{F}_{\alpha',\alpha}(M,Q)\,\mathcal{F}_{\alpha'}(M,Q)$

$$\Rightarrow\ C_{\alpha',\alpha}(M,Q) = 1 + \frac{\alpha'\alpha}{(4\pi)^2}\left[\frac{59}{4} + \frac{70}{3}\pi^2 + 244\zeta_3 - \frac{113}{15}\pi^4 - \frac{64}{3}\pi^2\ln^2 2 + \frac{64}{3}\ln^4 2 + 512\,\mathrm{Li}_4\left(\frac{1}{2}\right)\right]$$

- no logarithmic terms!
- $\tilde{F}_{\alpha',\alpha}(M,Q)\,\mathcal{F}_{\alpha'}(\lambda,Q)$ approaches $\mathcal{F}_{\alpha',\alpha}(\lambda,M,Q)$ for $\lambda \to M$ in N^{3}LL accuracy!
- *all* logs in theory with mass gap are obtained from symmetric phase

3) n_f AND n_s TERMS

loops with (massless) fermions and scalars affect the beta-function and the anomalous dimension (γ !)



explicit result: $$\mathcal{F} = 1 + \left(\frac{\alpha}{4\pi}\right)^2 n_f\, f_f^{(2)} + \left(\frac{\alpha}{4\pi}\right)^2 n_s\, f_s^{(2)} \qquad \left[z = \tfrac{M^2}{Q^2}\right]$$

$$\begin{aligned}
f_f^{(2)} &= (1-4z+3z^2)\left[\tfrac{8}{3}\,\mathrm{Li}_3(z) + \tfrac{8}{3}\ln(z)\,\mathrm{Li}_2(1-z) + \tfrac{4}{9}\ln^3(z) + \tfrac{4}{3}\ln^2(z)\ln(1-z) - \tfrac{4}{9}\pi^2\ln(z)\right] \\
&\quad +(1-z)^2\left[\tfrac{16}{9}\,\mathrm{Li}_2(1-z) + \tfrac{8}{27}\pi^2\right] + \left(\tfrac{38}{9} - \tfrac{52}{9}z + \tfrac{8}{9}z^2\right)\ln^2(z) + \left(\tfrac{34}{3} - \tfrac{88}{9}z\right)\ln(z) + \tfrac{115}{9} - \tfrac{88}{9}z \\
&\to -\frac{4}{9}\mathcal{L}^3 + \frac{38}{9}\mathcal{L}^2 - \frac{34}{3}\mathcal{L} + \frac{115}{9} + \frac{16}{27}\pi^2\,, \qquad Q^2 \gg M^2 \\
&= -0.44\,\mathcal{L}^3 + 4.22\,\mathcal{L}^2 - 11.3\,\mathcal{L} + 18.6 \\
f_s^{(2)} &= \ldots
\end{aligned}$$

- large Q^2 approximation works well (M^2/Q^2 terms small for $M/Q \lesssim 5$)
- large compensations between subleading terms

4) SUMMARY

- subleading Sudakov logarithms are important
- NNLL approximation for four-fermion processes is available since long
- results from evolution equation verified by explicit calculation for the two-loop form factor
- fermion (and scalar) loops under full control
- next step: four-fermion processes in N^3LL