SUDAKOV LOG'S
in
ELECTROWEAK PROCESSES

J.H. Kühn, Karlsruhe

NNLL in Four-Fermion Processes

J.H. Kühn, A.A. Penin; hep-ph/9906545
J.H. Kühn, A.A. Penin, V.A. Smirnov; Eur. Phys. J. C17 (2000) 97
J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov; Nucl. Phys. B616 (2001) 286
B. Feucht, J.H. Kühn, S. Moch; Phys. Lett. B561 (2003) 111

1) Motivation
   large one–loop corrections at high energies

2) NNLL for SU(2) Toy Model
   A) Form Factor
   B) Four-Fermion Scattering
      Technical aspects
      Angular dependent logarithms

3) Subleading n_f and n_s Terms

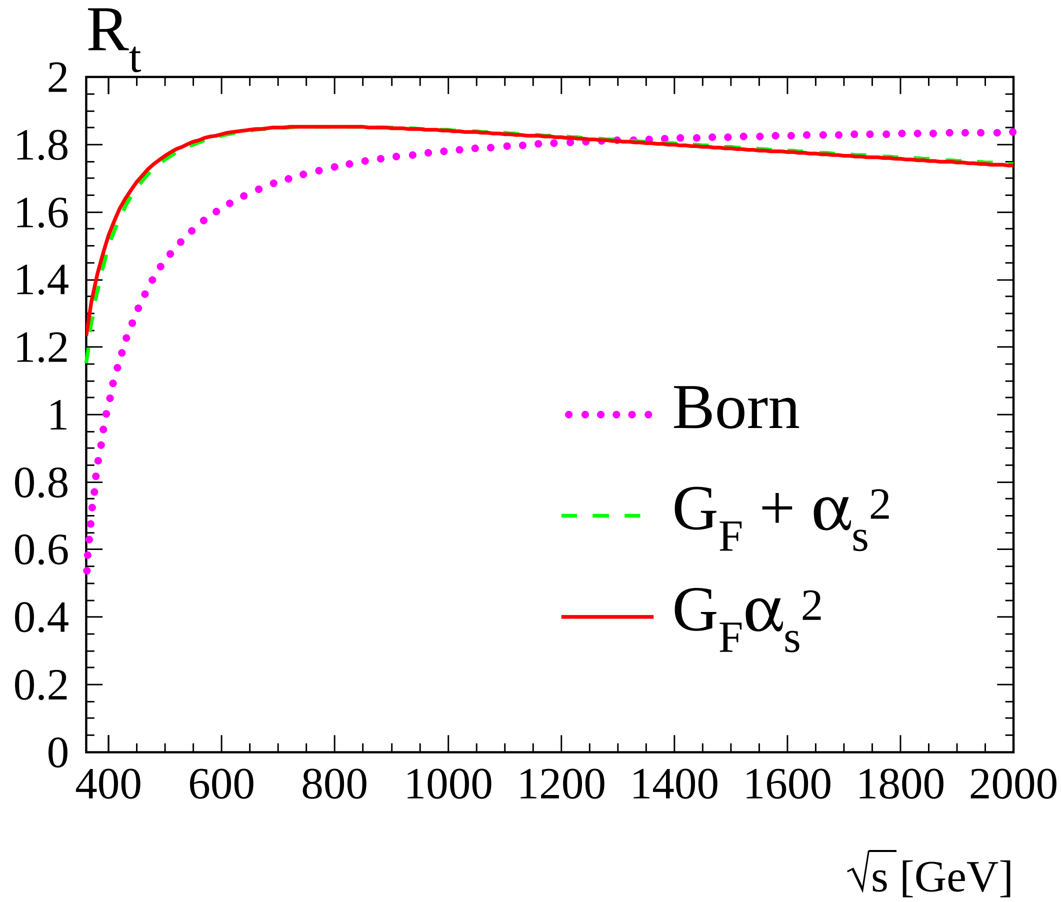
4) Standard Model: W, Z, γ

5) Questions about Experimental Setup

6) Summary

1) MOTIVATION (One-Loop!)

example: $e^+ e^- \to t\bar{t}$

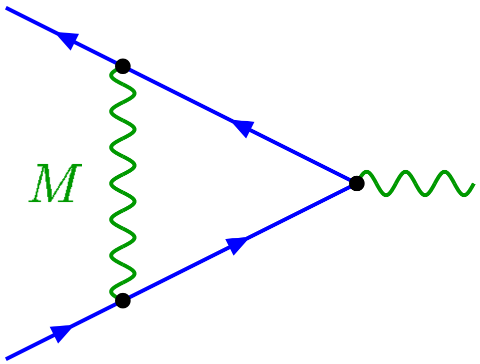


boxes + vertex corrections dominate

running coupling: $\sim \ln(s)$

boxes, vertices: $\sim \ln^2(s/M^2)$

example: massive U(1)



$$\Rightarrow \text{ Born} * \left(1 - 2\frac{\alpha}{\pi}\left(\frac{1}{4}\ln^2\frac{s}{M^2} + \ldots\right)\right)$$

magnitude ($\alpha_w/\pi = 10^{-2}$)

$\left(\frac{s}{M^2}\right)$	$-\frac{1}{4}\ln^2\frac{s}{M^2}$	$+\frac{3}{4}\ln\frac{s}{M^2}$	$-\frac{7}{8}+\frac{\pi^2}{12}$	Σ	$*2\frac{\alpha_w}{\pi}$
$\left(\frac{1000}{100}\right)^2$	- 5.30	+ 3.45	- 0.05	-1.9	-4%
$\left(\frac{2000}{100}\right)^2$	- 8.97	+ 4.49	- 0.05	-4.5	-9%

large negative correction, compensated by real radiation in inclusive rate!

massive boson: exclusive rate physically meaningful!

higher orders, leading log: $\Big(1 - \frac{1}{2}\underbrace{\frac{g^2}{16\pi^2}\ln^2\frac{s}{M^2}}_{\mathsf{L}}\Big) \Rightarrow e^{-\frac{1}{2}\mathsf{L}}$ (per leg) in amplitude

nonabelian theory: $e^{-\frac{C_F}{2}\mathsf{L}}$, $\qquad C_F = \frac{3}{4}$ for SU(2)

numerically: $\mathsf{L}_{ew} = \begin{matrix} 0.07 \\ 0.11 \end{matrix}$ for $\begin{matrix} 1 \text{ TeV} \\ 2 \text{ TeV} \end{matrix}$

expected correction for $f_L\bar{f}_L \to f'_L\bar{f}'_L$: $\qquad \sim \underset{\text{rate}}{2} \cdot \overset{\text{legs}}{4} \cdot \frac{3}{4} \cdot \frac{1}{2}\mathsf{L}_{ew} \sim 20 - 30\%$

$\Rightarrow$ large number of papers during past years:
NNLL; Yukawa coupling, external gauge bosons.

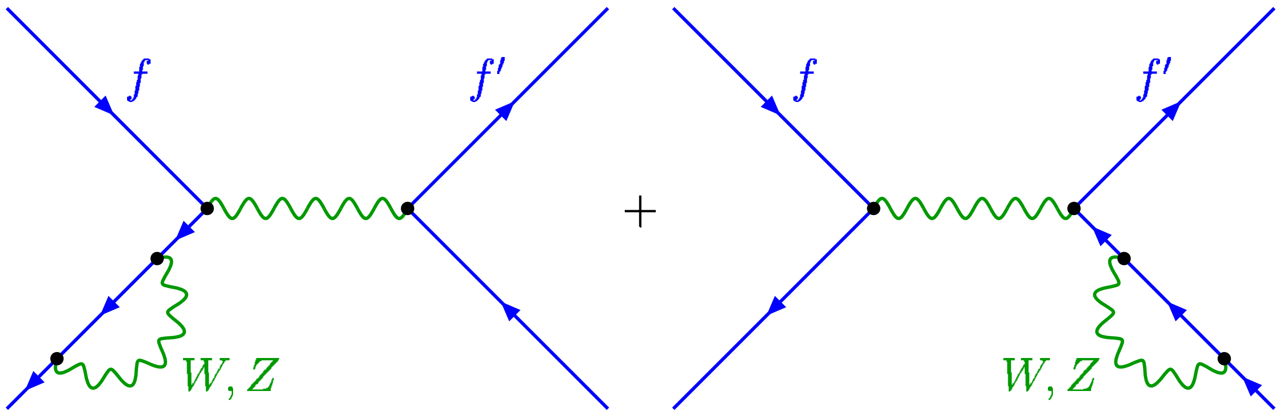
EW theory; One-Loop

$$f_L \bar{f}_L \to f'_L \bar{f}'_L : \qquad \mathcal{M}_{\text{Born}} = \frac{ig^2}{s}\left(T^3_f T^3_{f'} + \tan^2\Theta_w \frac{Y_f Y_{f'}}{4}\right) \qquad (M_Z^2/s \to 0)$$

one-loop: explicit calculation $\surd$

dominant correction: $*\left(1 - F^f_{\text{L,R}}\right)$ (per line) with

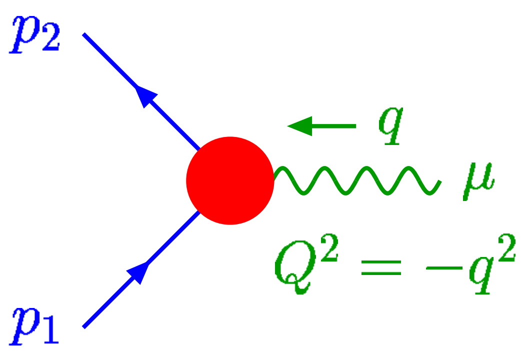
$$F^f_{\text{L}} = \Big(\underbrace{\frac{1}{2}}_{W} + \underbrace{\frac{1}{4c^2_{\text{w}}} + \tan^2_{\text{w}}\left(s^2_{\text{w}} Q^2_f - 2T^3_f Q_f\right)}_{Z}\Big) L\,, \qquad F^f_{\text{R}} = \Big(\underbrace{\tan^2_{\text{w}} s^2_{\text{w}} Q^2_f}_{Z}\Big) L$$



axial gauge

2) NNLL for SU(2) Toy Model

A) Form Factor



Born:

$\mathcal{F}_{\mathrm{B}} = \bar{\psi}(p_2)\gamma_\mu \psi(p_1)$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow \; \mathcal{F} = \mathcal{F}_{\mathrm{B}}\, F_0(\alpha(M^2))\, \exp\left\{\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x} \frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: NNLL $\Rightarrow$ corresponds to all terms of the form:

$$\underbrace{\alpha^n \ln^{2n}\left(\frac{Q^2}{M^2}\right)}_{\text{LL}} + \underbrace{\alpha^n \ln^{2n-1}\left(\frac{Q^2}{M^2}\right)}_{\text{NLL}} + \underbrace{\alpha^n \ln^{2n-2}\left(\frac{Q^2}{M^2}\right)}_{\text{NNLL}}$$

requires running of α (i.e. β_0 and β_1) and:

$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$ up to $\mathcal{O}(\alpha)$
$\gamma(\alpha)$ up to $\mathcal{O}(\alpha^2)$!

Expansion by regions:

relevant regions:

hard (h):	$k \sim Q$		
1-collinear (1c):	$k_+ \sim Q,$	$k_- \sim M^2/Q,$	$\underline{k} \sim M$
2-collinear (1c):	$k_- \sim Q,$	$k_+ \sim M^2/Q,$	$\underline{k} \sim M$
soft (s):	$k \sim M$		

where $k_\pm = k_0 \pm k_3$, $\underline{k} = (k_1, k_2)$

$\gamma(\alpha)$ from double pole of hard contribution

$\zeta(\alpha)$ from single pole of hard contribution

$\xi(\alpha)$: collinear region

$F_0(\alpha)$: complete answer

$$\mathcal{F}^{(1)} = \left(\Delta_h^{(1)} + \Delta_c^{(1)} + \Delta_s^{(1)}\right) \mathcal{F}_{\mathrm{B}}$$

$$\Delta_h^{(1)} = C_F \left(-\frac{2}{\epsilon^2} + \frac{1}{\epsilon}\left(2\ln(Q^2) - 3\right) - \ln^2(Q^2) + 3\ln(Q^2) + \frac{\pi^2}{6} - 8 \right)$$

and

$$\mathcal{F}^{(1)} = -C_F \left(\ln^2\left(\frac{Q^2}{M^2}\right) - 3\ln\left(\frac{Q^2}{M^2}\right) + \frac{7}{2} + \frac{2\pi^2}{3} \right) \mathcal{F}_{\mathrm{B}}$$

$$\Rightarrow \quad \gamma^{(1)} = -2C_F\,, \ \zeta^{(1)} = 3C_F\,, \ \xi^{(1)} = 0\,, \ F_0^{(1)} = -C_F\left(\frac{7}{2} + \frac{2\pi^2}{3}\right)$$

Two-loop result for $\gamma(\alpha)$:

$$\gamma^{(2)} = -2C_F\left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]$$

from Matsuura + ..., Kramer + ...

Higgs doublet (scalar particle $\rightarrow n_s = 1$):

$$\gamma^{(2)} \rightarrow +\frac{16}{9}C_F T_F$$

Final result for form factor

$$\mathcal{F} \;=\; \mathcal{F}_{\mathrm{B}} + \frac{\alpha}{4\pi}\mathcal{F}^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \mathcal{F}^{(2)} + \ldots$$

one-loop:

$$\mathcal{F}^{(1)} = \left(\frac{1}{2}\gamma^{(1)} \ln^2\left(\frac{Q^2}{M^2}\right) + \left(\xi^{(1)} + \zeta^{(1)}\right) \ln\left(\frac{Q^2}{M^2}\right) + F_0^{(1)}\right)\mathcal{F}_{\mathrm{B}}$$

two-loop:

$$\Rightarrow \quad \mathcal{F}^{(2)}_{LL} \;=\; \frac{1}{8}(\gamma^{(1)})^2 \ln^4\left(\frac{Q^2}{M^2}\right)\mathcal{F}_{\mathrm{B}}$$

$$\mathcal{F}^{(2)}_{NLL} \;=\; \frac{1}{2}\left(\zeta^{(1)} - \frac{1}{3}\beta_0\right)\gamma^{(1)} \ln^3\left(\frac{Q^2}{M^2}\right)\mathcal{F}_{\mathrm{B}}$$

$$\mathcal{F}^{(2)}_{NNLL} \;=\; \frac{1}{2}\left(\gamma^{(2)} + \left(\zeta^{(1)} - \beta_0\right)\zeta^{(1)} + F_0^{(1)}\gamma^{(1)}\right) \ln^2\left(\frac{Q^2}{M^2}\right)\mathcal{F}_{\mathrm{B}}$$

concrete numbers:

SU(2)

$$\mathcal{F}^{(1)} = \left[-\frac{3}{4}\ln^2\left(\frac{Q^2}{M^2}\right) + \frac{9}{4}\ln\left(\frac{Q^2}{M^2}\right) - \left(\frac{21}{8} + \frac{\pi^2}{2}\right)\right]\mathcal{F}_B$$

$$\mathcal{F}^{(2)} = \left[\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{19}{48}\ln^3\left(\frac{Q^2}{M^2}\right) - \left(\frac{463}{48} - \frac{7\pi^2}{8}\right)\ln^2\left(\frac{Q^2}{M^2}\right)\right]\mathcal{F}_B$$

U(1)

$$\mathcal{F}^{(1)} = \left[-\ln^2\left(\frac{Q^2}{M^2}\right) + 3\ln\left(\frac{Q^2}{M^2}\right) - \left(\frac{7}{2} + \frac{2\pi^2}{3}\right)\right]\mathcal{F}_B$$

$$\mathcal{F}^{(2)} = \left[\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{52}{9}\ln^3\left(\frac{Q^2}{M^2}\right) + \left(\frac{625}{18} + \frac{2\pi^2}{3}\right)\ln^2\left(\frac{Q^2}{M^2}\right)\right]\mathcal{F}_B$$

large coefficients of subleading terms in $\mathcal{F}^{(2)}$!
alternating signs!

B) Four-Fermion Scattering $\quad f + f' \to f + f'$

$$\frac{ig^2}{s}\left(\bar\psi_2 t^a \gamma^\mu \psi_1\right)\left(\bar\psi_4 t^a \gamma_\mu \psi_3\right) \equiv \frac{ig^2}{s}\mathcal{A}^\lambda$$

one-loop:

$$\frac{ig^2(Q^2)}{s}\frac{\alpha}{2\pi}\left[\left\{-C_F\left(\ln^2\left(\frac{-s}{M^2}\right) - 3\ln\left(\frac{-s}{M^2}\right)\right) + \left(-C_A\ln\left(\frac{u}{s}\right) + 2\left(C_F - \frac{T_F}{N}\right)\ln\left(\frac{u}{t}\right)\right)\ln\left(\frac{-s}{M^2}\right)\right\}\mathcal{A}^\lambda + \left\{2\frac{C_F T_F}{N}\ln\left(\frac{u}{t}\right)\ln\left(\frac{-s}{M^2}\right)\right\}\mathcal{A}^d\right] \quad + \text{ non-log}(-s/M^2) \text{ terms}$$

with $\mathcal{A}^d = \left(\bar\psi_2 \gamma^\mu \psi_1\right)\left(\bar\psi_4 \gamma_\mu \psi_3\right)$ diagonal!

angular dependent NLL terms! J.K., Penin, Smirnov, 1999

non-log$(-s/M^2)$ terms required for NNLL result J.K., Moch, Penin, Smirnov, 2001

$\Rightarrow$ evolution for $\tilde{\mathcal{A}} = \begin{pmatrix} \tilde{\mathcal{A}}^\lambda \\ \tilde{\mathcal{A}}^d \end{pmatrix}$ $\qquad (\tilde{\mathcal{A}} = \mathcal{A} * \text{collinear logs})$

$$\frac{\partial}{\partial \ln Q^2} \tilde{\mathcal{A}} = \chi\left(\alpha\left(Q^2\right)\right)\tilde{\mathcal{A}}$$

Sen $+\ldots$, Sterman $+\ldots$

$\chi_{\lambda\lambda}$, $\chi_{\lambda d}$ from the one-loop terms (single-log $\Rightarrow$ NLL, non-log $\Rightarrow$ NNLL)

$\chi_{d\lambda}$ analogous; $\chi_{dd} = 0$

χ is angular dependent, e.g.:

$$\chi^{(1)}_{\lambda\lambda} = -2C_A\left(\ln\left(\frac{1+\cos\theta}{2}\right) + i\pi\right) + 4\left(C_F - \frac{T_F}{N}\right)\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right)$$

$\tilde{\mathcal{A}}$ from complete one-loop result

(involves also angular dependent terms $\Rightarrow$ NNLL) diagonalize matrix χ

$$\tilde{\mathcal{A}} = \sum_i \tilde{\mathcal{A}}_{0i}(\alpha(M^2)) \, \mathrm{exp}\left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \chi_i(\alpha(x))\right]$$

starting values for $\tilde{\mathcal{A}}$ from one-loop result (including constant terms)

angular dependence is important for NLL and NNLL!

result strictly valid for fixed non-vanishing scattering angle
(or total cross section)

result for SU(2)-theory with SSB: $\sigma = \sigma_{\mathrm{B}} + \frac{\alpha}{4\pi}\sigma^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \sigma^{(2)} + \ldots$

explicit result for differential cross section $\rightarrow$ J.K., Penin, Smirnov, 1999; J.K., Moch, Penin, Smirnov, 2001

integrated cross section:

$u\bar{u} \longrightarrow u\bar{u}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{80}{3}\ln\left(\frac{s}{M^2}\right) - \left(\frac{25}{9} + 3\pi^2\right)\right]\sigma_{\mathrm{B}}$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{449}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{4855}{18} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_{\mathrm{B}}$$

$u\bar{u} \longrightarrow d\bar{d}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{26}{3}\ln\left(\frac{s}{M^2}\right) + \left(\frac{218}{9} - 3\pi^2\right)\right]\sigma_{\mathrm{B}}$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) - \left(\frac{799}{9} - \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_{\mathrm{B}}$$

U(1) theory

$$\sigma^{(1)} = \left[-4\ln^2\left(\frac{s}{M^2}\right) + 12\ln\left(\frac{s}{M^2}\right) - \left(\frac{382}{9} - \frac{4\pi^2}{3}\right)\right]\sigma_{\mathrm{B}}$$

$$\sigma^{(2)} = \left[8\ln^4\left(\frac{s}{M^2}\right) - \frac{532}{9}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{1142}{3} + \frac{16\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_{\mathrm{B}}$$

3) Subleading n_f and n_s Terms

Stability, subleading terms?

consider the full series of the n_f (fermionic) and n_s (scalar) terms for the form factor

$$\mathcal{F} = \Bigg\{ 1$$

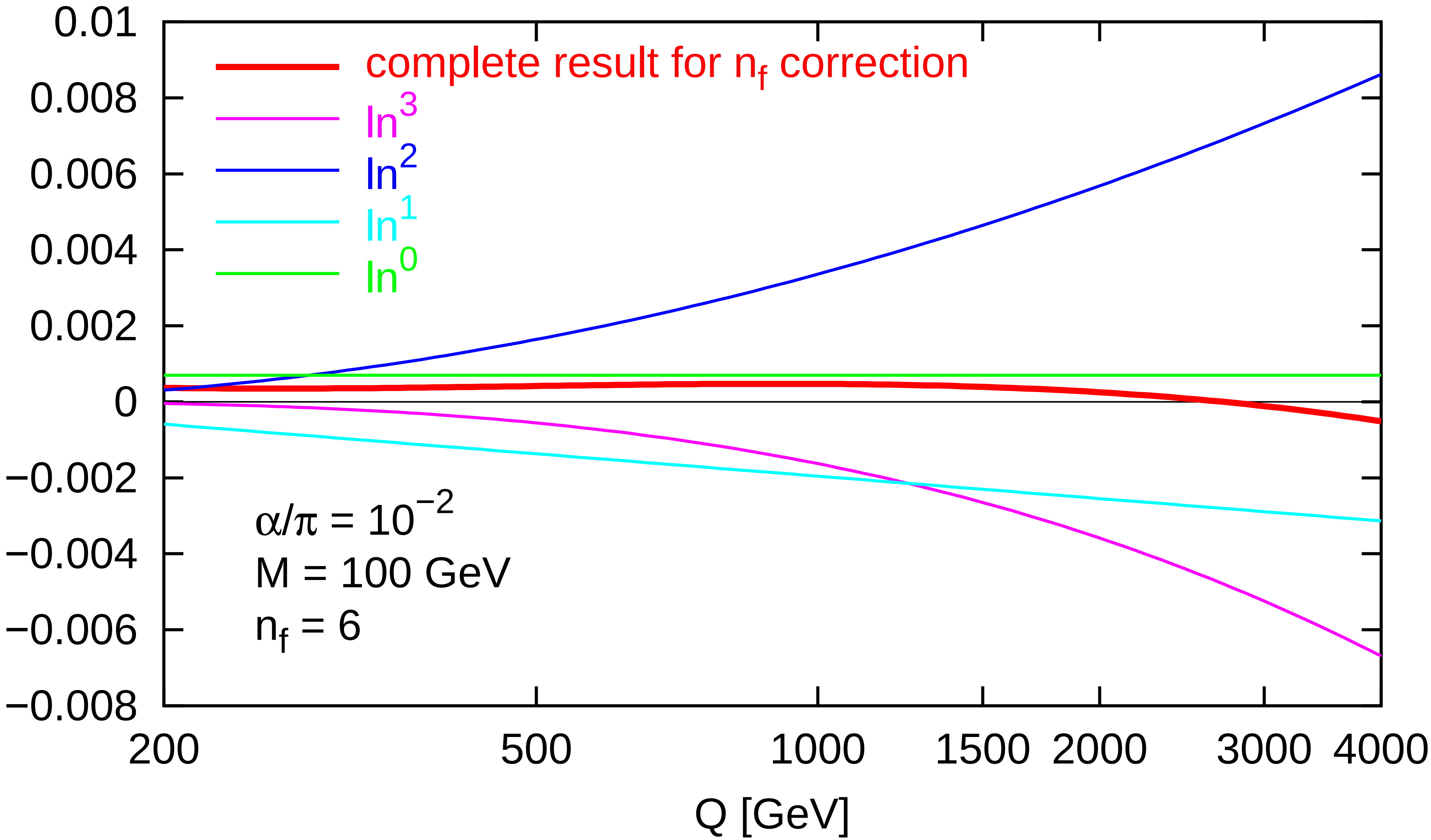
$$+ \left(\frac{\alpha}{4\pi}\right)^2 n_f \left[-\frac{4}{9} \ln^3\left(\frac{Q^2}{M^2}\right) + \frac{38}{9} \ln^2\left(\frac{Q^2}{M^2}\right) - \frac{34}{3} \ln\left(\frac{Q^2}{M^2}\right) + \frac{16\pi^2}{27} + \frac{115}{9} \right]$$

$$+ \left(\frac{\alpha}{4\pi}\right)^2 n_s \left[-\frac{1}{9} \ln^3\left(\frac{Q^2}{M^2}\right) + \frac{25}{18} \ln^2\left(\frac{Q^2}{M^2}\right) - \frac{23}{6} \ln\left(\frac{Q^2}{M^2}\right) + \frac{10\pi^2}{27} + \frac{157}{36} \right] \Bigg\} \mathcal{F}_\text{B}$$

Feucht, J.K., Moch

$\ln^3$ and $\ln^2$ consistent with NNLL,
$\ln^1$ and $\ln^0$ new;
alternating sign!
increasing coefficients!

energy dependence of correction for n_f terms:



4) Standard Model: W, Z, γ

examine $f'\bar{f}' \to f\bar{f}$

$$A_B = \frac{ig^2}{s} \sum_{I,J=L,R} \left(T^3_{f'} T^3_f + t^2_W \frac{Y_{f'} Y_f}{4} \right) A^{f'f}_{IJ} \quad \text{with} \quad A^{f'f}_{IJ} = \left(\bar{f}'_I \gamma^\mu f'_I \right) \left(\bar{f}_J \gamma_\mu f_J \right)$$

corrections from photon radiation up to cutoff $\omega \ll M_{W,Z}$ must be taken into account separately (prescription of Fadin et al.).

define

$$L(s) = \frac{g^2}{16\pi^2} \ln^2 \left(\frac{s}{M^2} \right) = 0.07 \quad (0.11)$$

$$l(s) = \frac{g^2}{16\pi^2} \ln \left(\frac{s}{M^2} \right) = 0.014 \; (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

for $\sqrt{s} = 1$ TeV (2 TeV)

result:

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to Q\bar{Q}) \;=\; 1 - 1.66\,L(s) + 5.31\,l(s) - 15.86\,a$$
$$+1.93\,L^2(s) - 9.43\,L(s)l(s) + 29.73\,l^2(s)$$

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to q\bar{q}) \;=\; 1 - 2.18\,L(s) + 20.58\,l(s) - 36.34\,a$$
$$+2.79\,L^2(s) - 50.06\,L(s)l(s) + 295.12\,l^2(s)$$

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to \mu^+\mu^-) \;=\; 1 - 1.39\,L(s) + 10.12\,l(s) - 31.33\,a$$
$$+1.42\,L^2(s) - 18.43\,L(s)l(s) + 99.89\,l^2(s)$$

(result very close to J.K., Penin, hep-ph/9906545!)

similar and even larger corrections for A^{LR}, e.g.

$$A^{LR}/A^{LR}_{\mathrm{B}}(e^+e^- \to \mu^+\mu^-) \;=\; 1 - 13.24\,L(s) + 113.77\,l(s) - 60.36\,a$$
$$-0.79\,L^2(s) + 32.40\,L(s)l(s) - 291.25\,l^2(s)$$

smaller corrections for A^{FB}, e.g.

$$A^{FB}/A^{FB}_{\mathrm{B}}(e^+e^- \to \mu^+\mu^-) \;=\; 1 - 0.04\,L(s) + 5.49\,l(s) - 10.77\,a$$
$$+0.27\,L^2(s) - 6.29\,L(s)l(s) + 6.65\,l^2(s)$$

5) Questions about Experimental Setup

- How exclusive are "exclusive" four-fermion processes?
- Can collinear ISR of W, Z be detected at TESLA or at LHC?
- Can collinear FSR of W, Z be detected in quark jets?
  If we constrain the jet mass, will we encounter QCD Sudakov logs?
- Interplay?

6) SUMMARY

- large double logarithmic corrections
  e.g. $e^+e^- \to q\bar{q}$ (2 TeV), $M = M_W$ for IR cutoff

  $\sim -24\%$ (one-loop) LL $\qquad \sim +3.4\%$ (two-loop) LL

- large compensations from subleading terms

  $\sim +35\%$ (one-loop) NLL $\qquad \sim -9.3\%$ (two-loop) NLL

- again alternating sign in NNLL order

  $\sim -10\%$ (one-loop) NNLL $\qquad \sim +8.5\%$ (two-loop) NNLL

- corresponding pattern in subleading n_f and n_s terms
- important angular dependent terms in NLL and NNLL
- large differences between different flavours
- results for σ, A_{FB}, A_{LR} available