SUDAKOV LOG'S
in
ELECTROWEAK PROCESSES

J. Kühn, Karlsruhe

NNLL in Four-Fermion Processes

J.H. Kühn, A.A. Penin; HEP-PH/9906545
J.H. Kühn, A.A. Penin, V.A. Smirnov; Eur. Phys. J. C17 (2000) 97
J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov; Nucl. Phys. B616 (2001) 286

and related work by

E. Accomando, A. Denner, S. Pozzorini,
M. Ciafaloni, P. Ciafaloni, D. Comelli,
M. Melles

1) Motivation

large one–loop corrections at high energies

2) NNLL for SU(2) Toy Model

A) Form Factor

B) Four-Fermion Scattering

3) Subleading n_f Terms

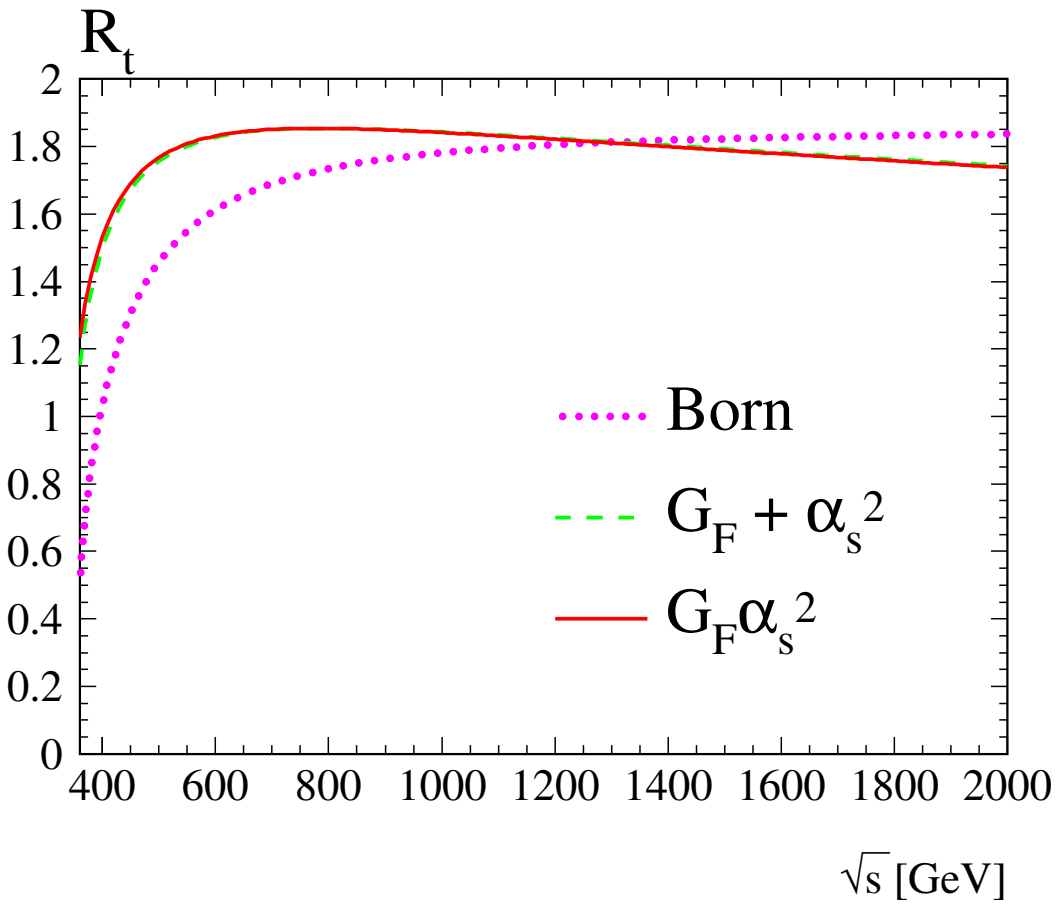
4) Standard Model: W, Z, γ

5) Questions about Experimental Setup

6) Related Work

1) MOTIVATION (One-Loop!)

example: $e^+ e^- \to t\bar{t}$

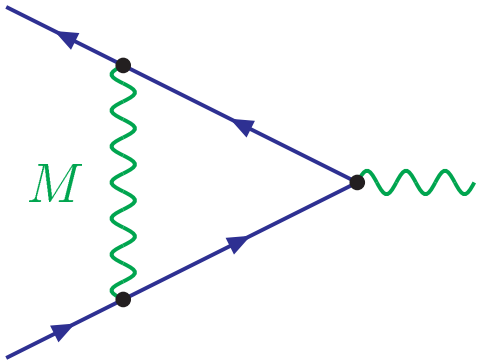


boxes + vertex corrections dominate

running coupling: $\sim \ln(s)$

boxes, vertices: $\sim \ln^2(s/M^2)$

example: massive U(1)



$$\Rightarrow \text{ Born} * \left(1 - 2\frac{\alpha}{\pi}\left(\frac{1}{4}\ln^2\frac{s}{M^2} + \ldots\right)\right)$$

magnitude ($\alpha_w/\pi = 10^{-2}$)

$\left(\frac{s}{M^2}\right)$	$-\frac{1}{4}\ln^2\frac{s}{M^2}$	$+\frac{3}{4}\ln\frac{s}{M^2}$	$-\frac{7}{8}+\frac{\pi^2}{12}$	Σ	$*2\frac{\alpha_w}{\pi}$
$\left(\frac{1000}{100}\right)^2$	- 5.30	+ 3.45	- 0.05	-1.9	-4%
$\left(\frac{2000}{100}\right)^2$	- 8.97	+ 4.49	- 0.05	-4.5	-9%

large negative correction, compensated by real radiation in inclusive rate!

massive boson: exclusive rate physically meaningfull!

higher orders, leading log: $\left(1 - \frac{1}{2}\underbrace{\frac{g^2}{16\pi^2}\ln^2\frac{s}{M^2}}_{\mathsf{L}}\right) \Rightarrow \mathrm{e}^{-\frac{1}{2}\mathsf{L}}$ (per leg) in amplitude

nonabelian theory: $\mathrm{e}^{-\frac{C_F}{2}\mathsf{L}}$, $\qquad C_F = \frac{3}{4}$ for SU(2)

numerically: $\mathsf{L}_{ew} = \begin{matrix}0.07\\0.11\end{matrix}$ for $\begin{matrix}1\ \text{TeV}\\2\ \text{TeV}\end{matrix}$

expected correction for $f_L\bar{f}_L \to f'_L\bar{f}'_L$: $\qquad \underset{\text{rate}}{\sim 2} \cdot \overset{\text{legs}}{4} \cdot \frac{3}{4} \cdot \frac{1}{2}\mathsf{L}_{ew} \sim 20 - 30\%$

$\Rightarrow$ large number of papers during past years:
NNLL; Yukawa coupling, external gauge bosons.

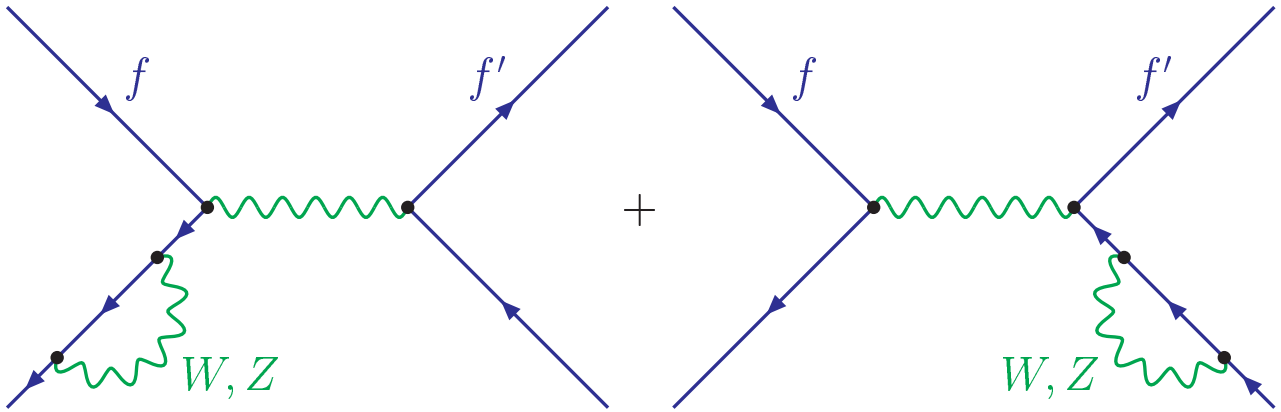
EW theory; One-Loop

$f_L \bar{f}_L \to f'_L \bar{f}'_L$: $\qquad \mathcal{M}_{\text{Born}} = \dfrac{ig^2}{s}\left(T^3_f T^3_{f'} + \tan^2\Theta_w \dfrac{Y_f Y_{f'}}{4}\right) \qquad (M_Z^2/s \to 0)$

one-loop: explicit calculation $\surd$

dominant correction: $*\left(1 - F^f_{\text{L,R}}\right)$ (per line) with

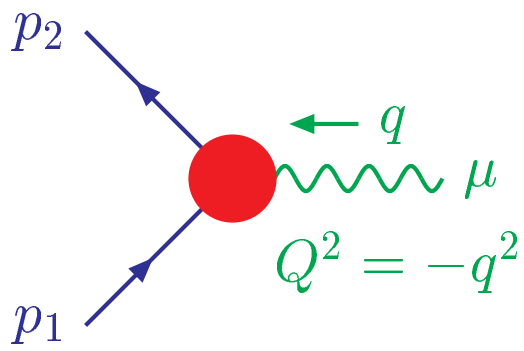
$$F^f_{\text{L}} = \Big(\underbrace{\frac{1}{2}}_{W} + \underbrace{\frac{1}{4c_{\text{w}}^2} + \tan^2_{\text{w}}\left(s^2_{\text{w}} Q_f^2 - 2T^3_f Q_f\right)}_{Z}\Big) L\,, \qquad F^f_{\text{R}} = \Big(\underbrace{\tan^2_{\text{w}}\, s^2_{\text{w}} Q_f^2}_{Z}\Big) L$$



axial gauge

2) NNLL for SU(2) Toy Model

A) Form Factor



Born:

$\mathcal{F}_B = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow\ \mathcal{F} = \mathcal{F}_B\, F_0(\alpha(M^2))\ \exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: NNLL $\Rightarrow$ corresponds to all terms of the form:

$$\alpha^n \ln^{2n}\left(\frac{Q^2}{M^2}\right) + \alpha^n \ln^{2n-1}\left(\frac{Q^2}{M^2}\right) + \alpha^n \ln^{2n-2}\left(\frac{Q^2}{M^2}\right)$$

LL NLL NNLL

requires running of α and:

$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$ up to $\mathcal{O}(\alpha)$

$\gamma(\alpha)$ up to $\mathcal{O}(\alpha^2)$!

expansion by regions:

$$\gamma^{(1)} = -2C_F\,,\ \zeta^{(1)} = 3C_F\,,\ \xi^{(1)} = 0\,,\ F_0^{(1)} = -C_F\left(\frac{7}{2} + \frac{2\pi^2}{3}\right)$$

$$\gamma^{(2)} = -2C_F\left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]$$

Matsuura +..., Kramer +...

Higgs doublet: $\gamma^{(2)} \to +\frac{16}{9}C_F\,T_F$

$$\mathcal{F} \;=\; \mathcal{F}_{\rm B} + \frac{\alpha}{4\pi}\,\mathcal{F}^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \mathcal{F}^{(2)} + \ldots$$

one-loop:

$$\mathcal{F}^{(1)} = \left(\frac{1}{2}\gamma^{(1)}\ln^2\left(\frac{Q^2}{M^2}\right) + \left(\xi^{(1)} + \zeta^{(1)}\right)\ln\left(\frac{Q^2}{M^2}\right) + F_0^{(1)}\right)\mathcal{F}_{\rm B}$$

two-loop:

$$\Rightarrow \quad \mathcal{F}^{(2)}_{LL} \;=\; \frac{1}{8}(\gamma^{(1)})^2\ln^4\left(\frac{Q^2}{M^2}\right)\mathcal{F}_{\rm B}$$

$$\mathcal{F}^{(2)}_{NLL} \;=\; \frac{1}{2}\left(\zeta^{(1)} - \frac{1}{3}\beta_0\right)\gamma^{(1)}\ln^3\left(\frac{Q^2}{M^2}\right)\mathcal{F}_{\rm B}$$

$$\mathcal{F}^{(2)}_{NNLL} \;=\; \frac{1}{2}\left(\gamma^{(2)} + \left(\zeta^{(1)} - \beta_0\right)\zeta^{(1)} + F_0^{(1)}\gamma^{(1)}\right)\ln^2\left(\frac{Q^2}{M^2}\right)\mathcal{F}_{\rm B}$$

concrete numbers:

SU(2)

$$\mathcal{F}^{(1)} = \left[-\frac{3}{4}\ln^2\left(\frac{Q^2}{M^2}\right)+\frac{9}{4}\ln\left(\frac{Q^2}{M^2}\right)-\left(\frac{21}{8}+\frac{\pi^2}{2}\right)\right]\mathcal{F}_{\rm B}$$

$$\mathcal{F}^{(2)} = \left[\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right)-\frac{19}{48}\ln^3\left(\frac{Q^2}{M^2}\right)-\left(\frac{463}{48}-\frac{7\pi^2}{8}\right)\ln^2\left(\frac{Q^2}{M^2}\right)\right]\mathcal{F}_{\rm B}$$

U(1)

$$\mathcal{F}^{(1)} = \left[-\ln^2\left(\frac{Q^2}{M^2}\right)+3\ln\left(\frac{Q^2}{M^2}\right)-\left(\frac{7}{2}+\frac{2\pi^2}{3}\right)\right]\mathcal{F}_{\rm B}$$

$$\mathcal{F}^{(2)} = \left[\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right)-\frac{52}{9}\ln^3\left(\frac{Q^2}{M^2}\right)+\left(\frac{625}{18}+\frac{2\pi^2}{3}\right)\ln^2\left(\frac{Q^2}{M^2}\right)\right]\mathcal{F}_{\rm B}$$

Large coefficients!

B) Four-Fermion Scattering $\qquad f + f' \to f + f'$

$$\frac{ig^2}{s}\left(\bar{\psi}_2 t^a \gamma^\mu \psi_1\right)\left(\bar{\psi}_4 t^a \gamma_\mu \psi_3\right) \equiv \frac{ig^2}{s}\mathcal{A}^\lambda$$

one-loop:

$$\frac{ig^2(Q^2)}{s}\frac{\alpha}{2\pi}\left[\left\{-C_F\left(\ln^2\left(\frac{-s}{M^2}\right) - 3\ln\left(\frac{-s}{M^2}\right)\right) + \left(-C_A \ln\left(\frac{u}{s}\right) + 2\left(C_F - \frac{T_F}{N}\right)\ln\left(\frac{u}{t}\right)\right)\ln\left(\frac{-s}{M^2}\right)\right\}\mathcal{A}^\lambda + \left\{2\frac{C_F T_F}{N}\ln\left(\frac{u}{t}\right)\ln\left(\frac{-s}{M^2}\right)\right\}\mathcal{A}^d\right] \quad + \text{ non-log-terms}$$

with $\mathcal{A}^d = \left(\bar{\psi}_2 \gamma^\mu \psi_1\right)\left(\bar{\psi}_4 \gamma_\mu \psi_3\right)$ diagonal!

$\Rightarrow$ evolution for $\tilde{\mathcal{A}} = \begin{pmatrix} \tilde{\mathcal{A}}^\lambda \\ \tilde{\mathcal{A}}^d \end{pmatrix}$ $\qquad (\tilde{\mathcal{A}} = \mathcal{A} * \text{collinear logs})$

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi\left(\alpha\left(Q^2\right)\right)\tilde{\mathcal{A}}$$

Sen + . . . , Sterman + . . .

$\chi_{\lambda\lambda}$, $\chi_{\lambda d}$ from the one-loop terms (single-log $\Rightarrow$ NLL, non-log $\Rightarrow$ NNLL)

$\chi_{d\lambda}$ analogous; $\chi_{dd} = 0$

$\tilde{\mathcal{A}}$ from complete one-loop result

diagonalize matrix χ

$$\tilde{\mathcal{A}} = \sum_i \tilde{\mathcal{A}}_{0i}(\alpha(M^2)) \exp\left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \chi_i(\alpha(x))\right]$$

starting values for $\tilde{\mathcal{A}}$ from one-loop result (including constant terms)

Angular dependence is important for NLL and NNLL.

result:
$$\sigma = \sigma_B + \frac{\alpha}{4\pi}\sigma^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \sigma^{(2)} + \ldots$$

$u\bar{u} \to u\bar{u}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{80}{3}\ln\left(\frac{s}{M^2}\right) - \left(\frac{22}{9} + 3\pi^2\right)\right]\sigma_B$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{449}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{2414}{9} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_B$$

$u\bar{u} \to d\bar{d}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{26}{3}\ln\left(\frac{s}{M^2}\right) + \left(\frac{221}{9} - 3\pi^2\right)\right]\sigma_B$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) - \left(\frac{1625}{18} - \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_B$$

U(1) theory

$$\sigma^{(1)} = \left[-4\ln^2\left(\frac{s}{M^2}\right) + 12\ln\left(\frac{s}{M^2}\right) - \left(\frac{376}{9} - \frac{4\pi^2}{3}\right)\right]\sigma_B$$

$$\sigma^{(2)} = \left[8\ln^4\left(\frac{s}{M^2}\right) - \frac{532}{9}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{1130}{3} + \frac{16\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_B$$

3) Subleading n_f Terms

Stability, subleading terms?

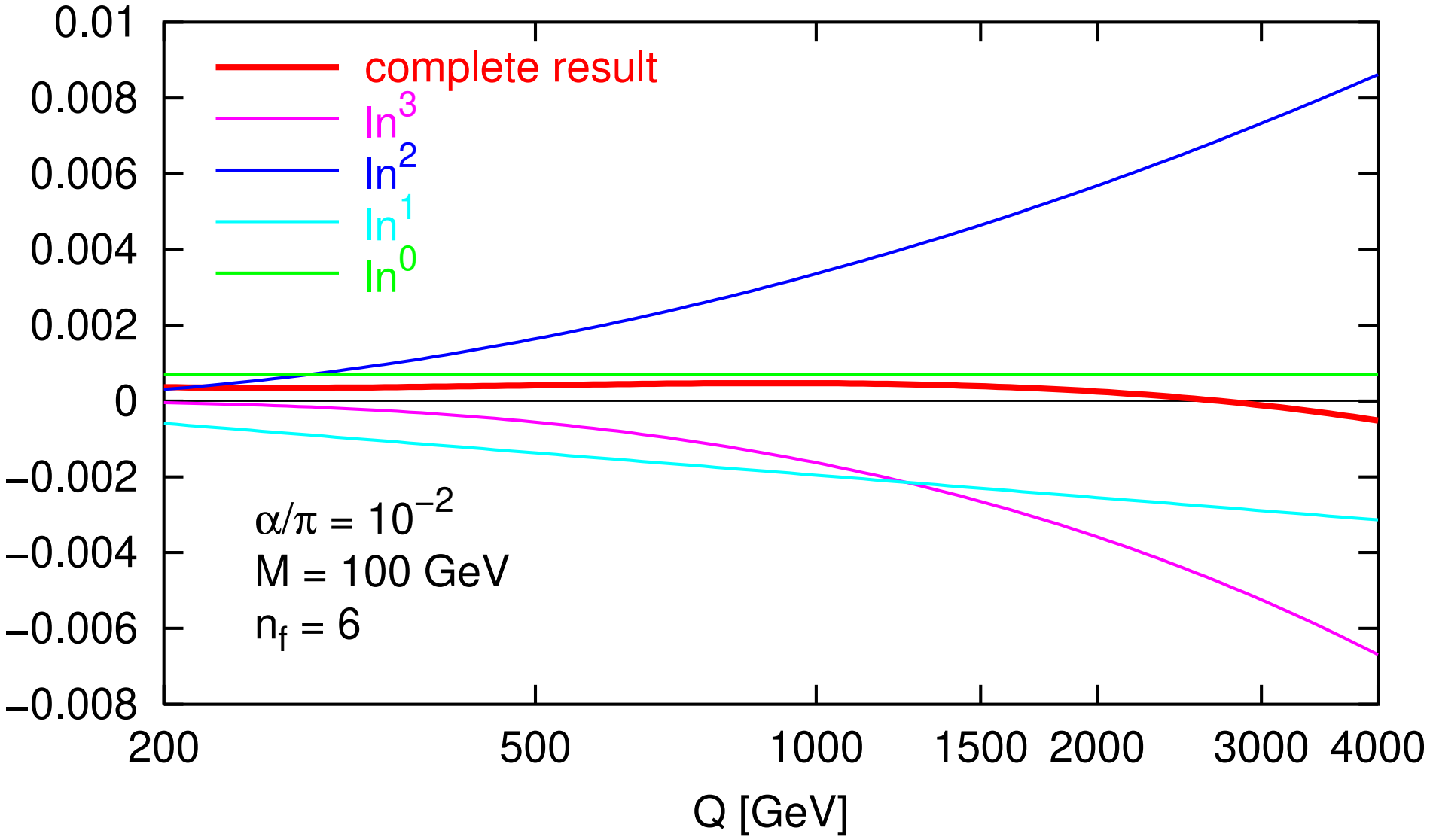
consider the full series of the n_f terms for the form factor

$$\mathcal{F} = \left\{1 + \left(\frac{\alpha}{4\pi}\right)^2 n_f \left[-\frac{4}{9}\ln^3\left(\frac{Q^2}{M^2}\right) + \frac{38}{9}\ln^2\left(\frac{Q^2}{M^2}\right) - \frac{34}{3}\ln\left(\frac{Q^2}{M^2}\right) + \frac{16\pi^2}{27} + \frac{115}{9}\right]\right\}\mathcal{F}_B$$

Feucht, Kühn, Moch

$\ln^3$ and $\ln^2$ consistent with NNLL,
$\ln^1$ and $\ln^0$ new;
alternating sign!
increasing coefficients!

energy dependence of correction:



4) Standard Model: W, Z, γ

examine $f'\bar{f}' \to f\bar{f}$

$$A_B = \frac{ig^2}{s} \sum_{I,J=L,R} \left(T^3_{f'} T^3_f + t^2_W \frac{Y_{f'} Y_f}{4} \right) A^{f'f}_{IJ} \quad \text{with} \quad A^{f'f}_{IJ} = \left(\bar{f}'_I \gamma^\mu f'_I \right) \left(\bar{f}_J \gamma_\mu f_J \right)$$

corrections from photon radiation up to cutoff $\omega \ll M_{W,Z}$ must be taken into account separately (prescription of Fadin et al.).

define

$$L(s) = \frac{g^2}{16\pi^2} \ln^2 \left(\frac{s}{M^2} \right) = 0.07 \quad (0.11)$$

$$l(s) = \frac{g^2}{16\pi^2} \ln \left(\frac{s}{M^2} \right) = 0.014 \, (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

for $\sqrt{s} = 1$ TeV (2 TeV)

result:

$$\begin{aligned}
\sigma/\sigma_{\mathrm{B}}(e^+e^- \to Q\bar{Q}) &= 1 - 1.66\,L(s) + 5.31\,l(s) - 15.86\,a \\
&\quad +1.93\,L^2(s) - 9.43\,L(s)l(s) + 28.73\,l^2(s) \\
\sigma/\sigma_{\mathrm{B}}(e^+e^- \to q\bar{q}) &= 1 - 2.18\,L(s) + 20.58\,l(s) - 36.34\,a \\
&\quad +2.79\,L^2(s) - 50.06\,L(s)l(s) + 295.12\,l^2(s) \\
\sigma/\sigma_{\mathrm{B}}(e^+e^- \to \mu^+\mu^-) &= 1 - 1.39\,L(s) + 10.12\,l(s) - 31.33\,a \\
&\quad +1.42\,L^2(s) - 18.43\,L(s)l(s) + 99.89\,l^2(s)
\end{aligned}$$

similar and even larger corrections for A_{LR}
smaller corrections for A_{FB}

5) Questions about Experimental Setup

- How exclusive are "exclusive" four-fermion processes?
- Can collinear ISR of W, Z be detected at TESLA or at LHC?
- Can collinear FSR of W, Z be detected in quark jets?
  If we constrain the jet mass, will we encounter QCD Sudakov logs?
- Interplay?

6) Related Work

Accomando, Denner, Pozzorini (ABS 924, hep-ph/0110114)

one-loop logarithmic corrections in $p + p \to W Z + X$
$\to W \gamma + X$

large negative corrections (5–20%)

difference between transverse and longitudinal gauge bosons

$$\text{transverse} \sim C_A \ln^2 \frac{s}{M^2}$$
$$\text{longitudinal} \sim C_F \ln^2 \frac{s}{M^2}$$

important differences between double resonant and
narrow width approximations!

M. Ciafaloni, P. Ciafaloni, Comelli (ABS 15, hep-ph/0111109)

hard inclusive (!) processes in spontaneously broken gauge theory
(fermion and boson initiated)
e.g. $f\bar{f} \to f'\bar{f}' + X$

difference with respect to QCD: initial beam charge $\neq 0$
$\Rightarrow$ different evolution equation
$\Rightarrow$ Bloch-Nordsieck theorem violated!
(infrared cutoff provided by SSB)

Melles (ABS 26, hep-ph/0108221)
Beccaria, Melles, Renard, Verzegnassi (hep-ph/0112273)

Melles
angular dependent (one-loop) terms important for NLL analysis
(cf. KPS for fermions)
application to gauge bosons

Beccaria, Melles, Renard, Verzegnassi
SUSY-scalar production at lepton colliders: assume "light" SUSY-scalars
leading and subleading terms in one-loop LL and NLL

claim: Sudakov region starts around 1 TeV

(role of subleading logarithmic terms?)

SUMMARY

- large double logarithmic corrections
  e.g. $e^+e^- \to q\bar{q}$ (2 TeV), $M = M_W$ for IR cutoff

  $\sim -24\%$ (one-loop) LL $\sim +3.4\%$ (two-loop) LL
- large compensations from subleading terms

  $\sim +35\%$ (one-loop) NLL $\sim -9.3\%$ (two-loop) NLL
- again alternating sign in NNLL order

  $\sim -10\%$ (one-loop) NNLL $\sim +8.5\%$ (two-loop) NNLL
- corresponding pattern in subleading n_f terms
- large differences between different flavours
- results for σ, A_{FB}, A_{LR} available
- rapidly evolving field