Charm and bottom masses at NNLO from electron-positron annihilation at low energies

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I Experimental Results for R below $B\bar{B}$-Threshold

➪ α_s

II Sum Rules to NNLO with Massive Quarks

➪ $m_Q(m_Q)$

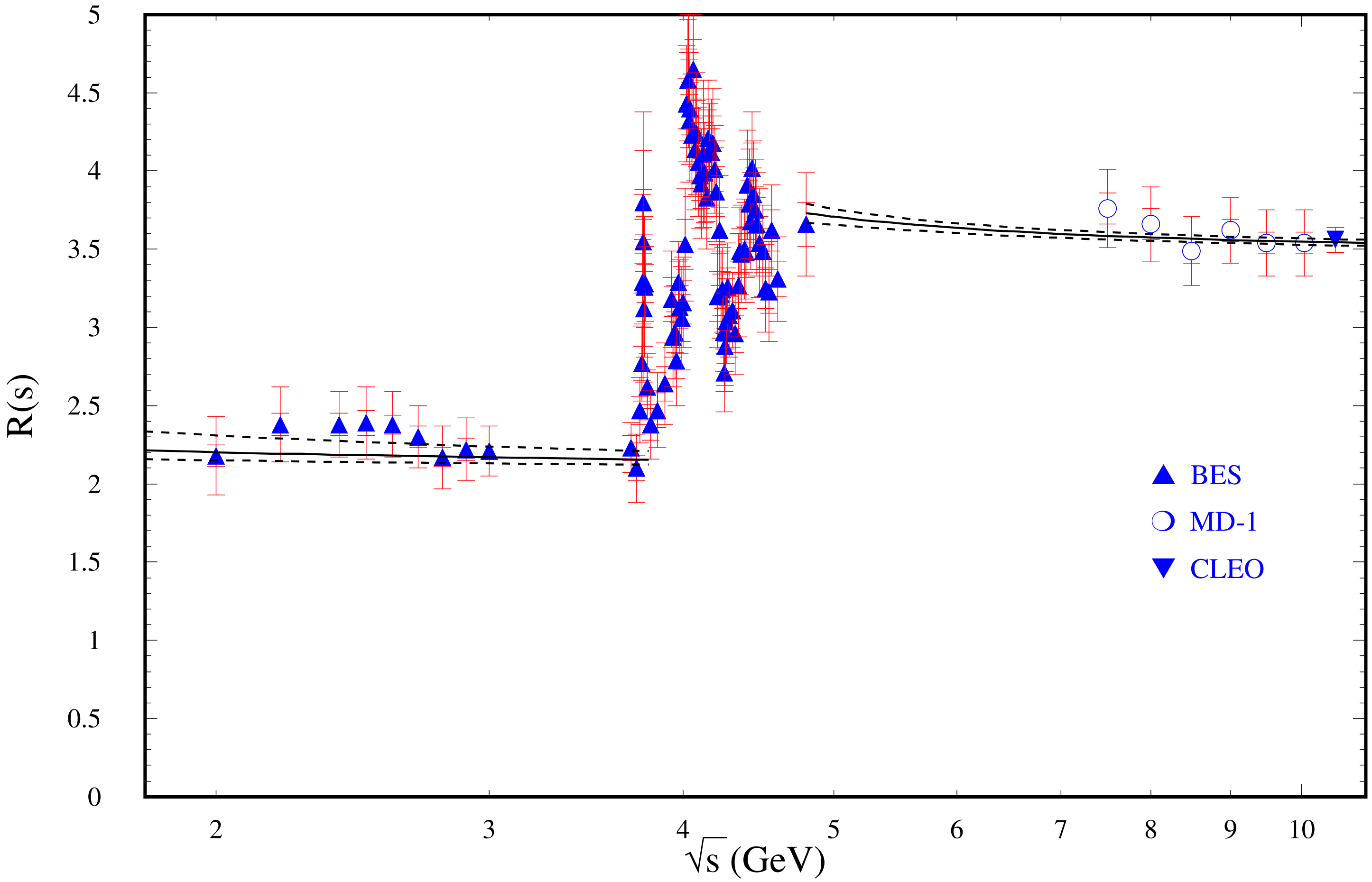
III Summary

I Experimental Results for R below $B\bar{B}$-Threshold

➪ α_s

- data
- α_s

Recent "precision" data on $R(s)$



experiment	energy [GeV]	date	systematic error
BES	2 — 5	2001	4%
MD-1	7.2 — 10.34	1996	4%
CLEO	10.52	1998	2%
PDG	J/ψ		7%
PDG	ψ'		9%
PDG	ψ''		15%

pQCD and data agree well in the regions

2 — 3.73 GeV and 5 — 10.52 GeV

α_s

pQCD includes full m_Q-dependence up to $\mathcal{O}(\alpha_s^2)$ and terms of $\mathcal{O}(\alpha_s^3(m^2/s)^n)$ with $n = 0, 1, 2$

can we deduce α_s from the low energy data?

Result:

BES below 3.73 GeV: $\alpha_s^{(3)}(3\ \text{GeV}) = 0.369^{+0.047+0.123}_{-0.046-0.130}$

BES at 4.8 GeV: $\alpha_s^{(4)}(4.8\ \text{GeV}) = 0.183^{+0.059+0.053}_{-0.064-0.057}$

MD-1: $\alpha_s^{(4)}(8.9\ \text{GeV}) = 0.193^{+0.017+0.127}_{-0.017-0.107}$

CLEO: $\alpha_s^{(4)}(10.52\ \text{GeV}) = 0.186^{+0.008+0.061}_{-0.008-0.057}$

combined, assuming uncorrelated errors: $\boxed{\alpha_s^{(4)}(5 \text{ GeV}) = 0.235 \pm 0.047}$

Evolve up to M_Z : $\quad \alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$

confirmation of running !

result consistent with LEP, but not competitive

(precision of 0.4% at 3.7 GeV (0.7% at 2 GeV) would be required)

II. Sum Rules to NNLO with Massive Quarks

– SVZ Sum Rules, Moments and Tadpoles

– Tadpoles at Three Loop

– Results for Charm and Bottom Masses

SVZ Sum Rules, Moments and Tadpoles

Some definitions

$$R(s) \;=\; 12\pi \,\mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right)\, \Pi(q^2) \;\equiv\; i \int \mathrm{d}x\, e^{iqx} \langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion: $$\Pi_c(q^2) \;=\; Q_c^2 \frac{3}{16\pi^2} \sum_{n \geq 0} \bar{C}_n\, z^n$$

with $z = q^2/(4m_c^2)$ and $m_c = m_c(\mu)$ the $\overline{\mathrm{MS}}$ mass.

Coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2

(Chetyrkin, JK, Steinhauser)

$\bar{C}_n$ depend on the charm quark mass through

$$
\begin{aligned}
l_{m_c} &\equiv \ln(m_c^2(\mu)/\mu^2) \\
\bar{C}_n &= \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right) \\
&\quad + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right)
\end{aligned}
$$

n	1	2	3	4
$\bar{C}_n^{(0)}$	1.0667	0.4571	0.2709	0.1847
$\bar{C}_n^{(10)}$	2.5547	1.1096	0.5194	0.2031
$\bar{C}_n^{(11)}$	2.1333	1.8286	1.6254	1.4776
$\bar{C}_n^{(20)}$	2.4967	2.7770	1.6388	0.7956
$\bar{C}_n^{(21)}$	3.3130	5.1489	4.7207	3.6440
$\bar{C}_n^{(22)}$	−0.0889	1.7524	3.1831	4.3713

Define the moments

$$\mathcal{M}_n^{\mathrm{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\bigg|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\,\frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

$$\Rightarrow \qquad \mathcal{M}_n^{\mathrm{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

$$\mathcal{M}_n^{\mathrm{exp}} = \mathcal{M}_n^{\mathrm{th}}$$

$\Rightarrow m_c$

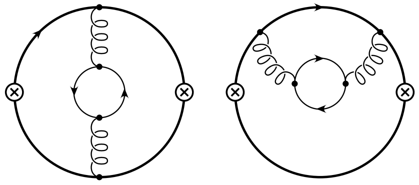
SVZ:

$\Pi^n(0)$ can be reliably calculated in pQCD: low n:

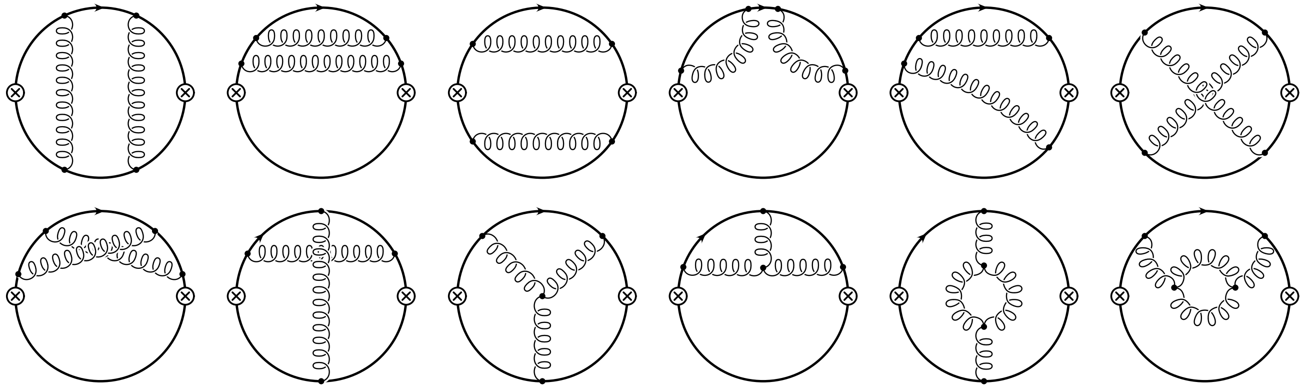
- fixed order in α_s is sufficient, in particular no resummation of $1/v$ - terms from higher orders required
- condensates are unimportant
- pQCD in terms of short distance mass : $m_c(3\text{ GeV}) \Rightarrow m_c(m_c)$ stable expansion : no pole mass or closely related definition (1S-mass, potential-subtracted mass) involved
- moments available in NNLO

Tadpoles in NNLO

all three-loop – one-scale tadpole amplitudes can be calculated with "arbitrary" power of propagators (Broadhurst; Chetyrkin, JK, Steinhauser); FORM-program MATAD (Steinhauser)



Three-loop diagrams contributing to $\Pi_l^{(2)}$ (inner quark massless) and $\Pi_F^{(2)}$ (both quarks with mass m).



Purely gluonic contribution to $\mathcal{O}(\alpha_s^2)$

Results

input for $R(s)$

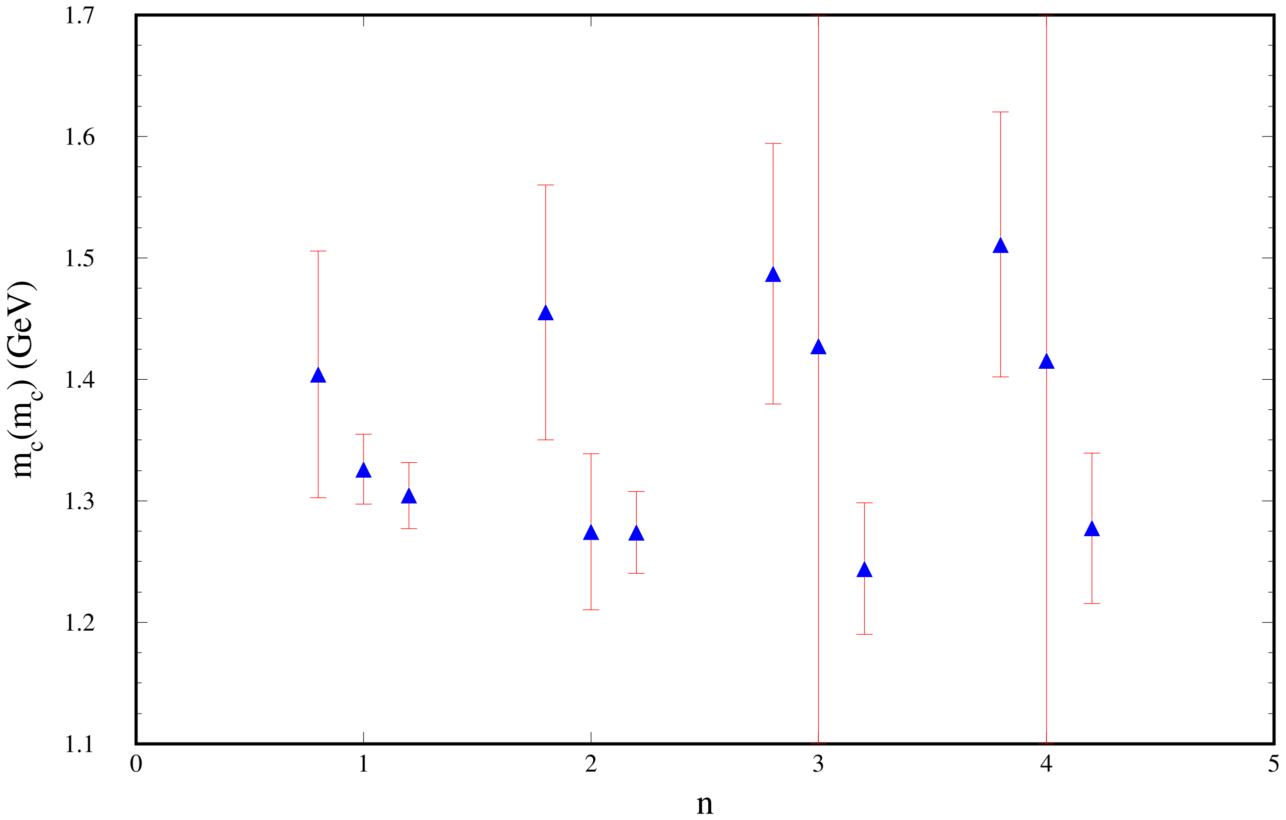
- resonances (J/ψ, ψ', ψ'')
- continuum below 4.8 GeV (BES)
- continuum above 4.8 GeV (theory)

experimental error of the moments dominated by resonances

n	1	2	3	4
$m_c(3\text{ GeV})$	1.027(30)	0.994(37)	0.961(59)	0.997(67)
$m_c(m_c)$	1.304(27)	1.274(34)	1.244(54)	1.277(62)

error in m_c dominated by experiment for n=1,
by theory (variation of μ, α_s) for $n = 3, 4, \ldots$

stability: compare LO, NLO, NNLO ➪ clear improvement



$m_c(m_c)$ for $n = 1, 2, 3, 4$ in LO, NLO, NNLO.

FINAL RESULT: $m_c(m_c) = 1.304(27)$ GeV convertion to pole mass (drastic shift from NLO to NNLO!):

$$M_c^{(2-\text{loop})} = 1.514(34)\ \text{GeV}$$
$$M_c^{(3-\text{loop})} = 1.691(35)\ \text{GeV}$$

$\overline{\text{MS}}$-mass more stable

other results from sum rules:

1.23 ± 0.09 GeV Eidemüller, Jamin

1.37 ± 0.09 GeV Penarrocha, Schilcher

1.10 ± 0.04 GeV Narison

1.275 ± 0.015 GeV Ioffe,Zyablyuk

results from lattice simulations:

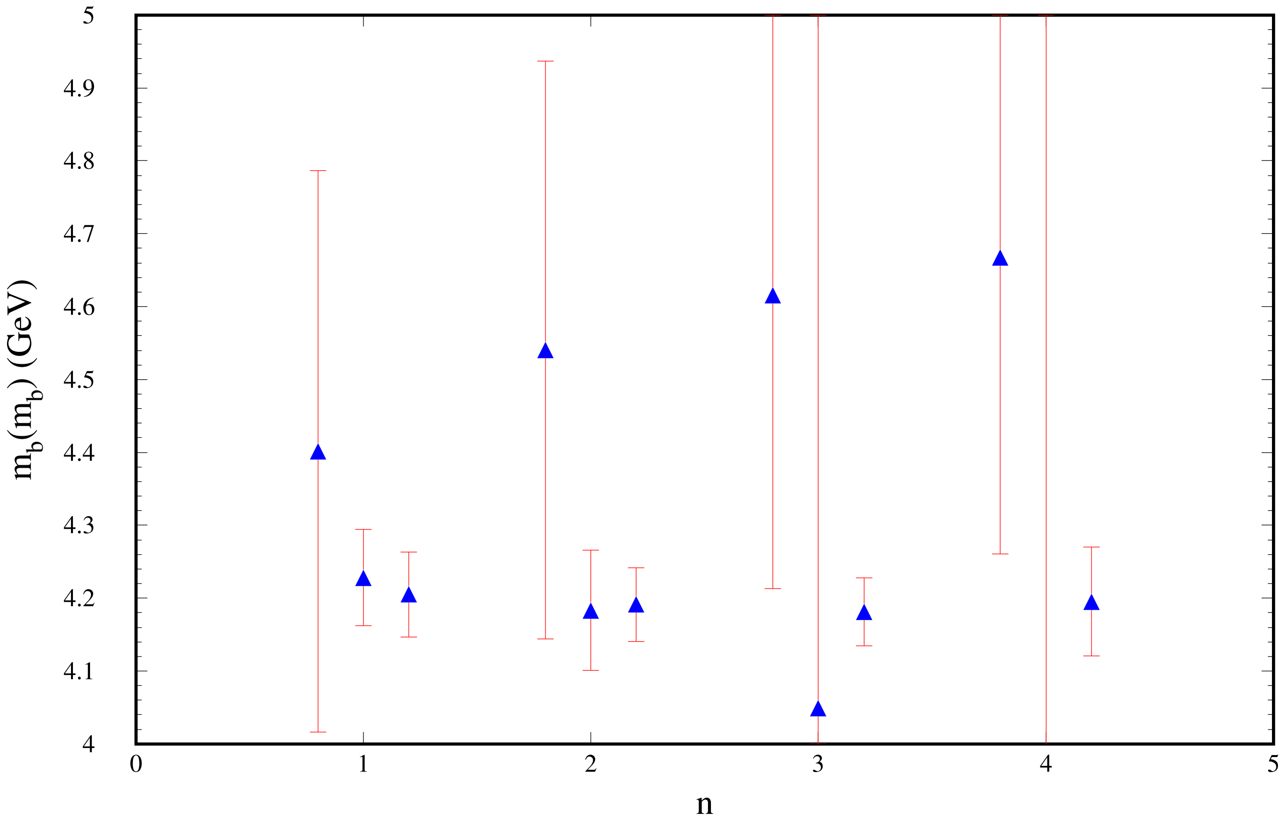
1.26(4)(12) GeV Becirevic, Lubicz, Martinelli

1.314(40)(20)(7) GeV Rolf, Sint

effect of "perturbative quenching": ± 30 MeV !

Similar analysis for the **bottom quark :**

resonances include $\Upsilon(1)$ up to $\Upsilon(6)$, "continuum" starts at 11.2 GeV



$m_b(m_b)$ for $n = 1, 2, 3$ and 4 in LO, NLO and NNLO

RESULT

n	1	2	3	4
$m_b(10\ \text{GeV})$	3.665(60)	3.651(52)	3.641(48)	3.655(77)
$m_b(m_b)$	4.205(58)	4.191(51)	4.181(47)	4.195(75)

$$m_b(m_b) = 4.19(5)\ \text{GeV}$$

corresponds to a pole mass of

$$M_b^{(2-\text{loop})} = 4.63(6)\ \text{GeV}$$
$$M_b^{(3-\text{loop})} = 4.80(6)\ \text{GeV}$$

consistent with analysis of bottonium through moments with large n

- direct determination of m_{pole} less stable !

III SUMMARY

$$\alpha_s^{(4)}(5\ \mathrm{GeV}) = 0.235 \pm 0.047$$

➪ $\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$

➪ drastic improvement in δm_c and δm_b from moments with low n in NNLO

➪ direct determination of short-distance mass

$$\boxed{\begin{aligned} m_c(m_c) &= 1.304(27)\ \mathrm{GeV} \\ m_b(m_b) &= 4.19(5)\ \mathrm{GeV} \end{aligned}}$$

improved measurements of the cross section in the charm region (J/ψ !) would help