Charm and bottom masses at NNLO from electron-positron annihilation at low energies

J.H. Kühn

I Experimental Results for R
below $B\bar{B}$-Threshold ➪ α_s

II Sum Rules to NNLO
with Massive Quarks ➪ $m_Q(m_Q)$

III Perspectives for $R(s)$ below ≈ 3 GeV
➪ the Radiative Return

IV Summary

I Experimental Results for R below $B\bar{B}$-Threshold ⇨ α_s

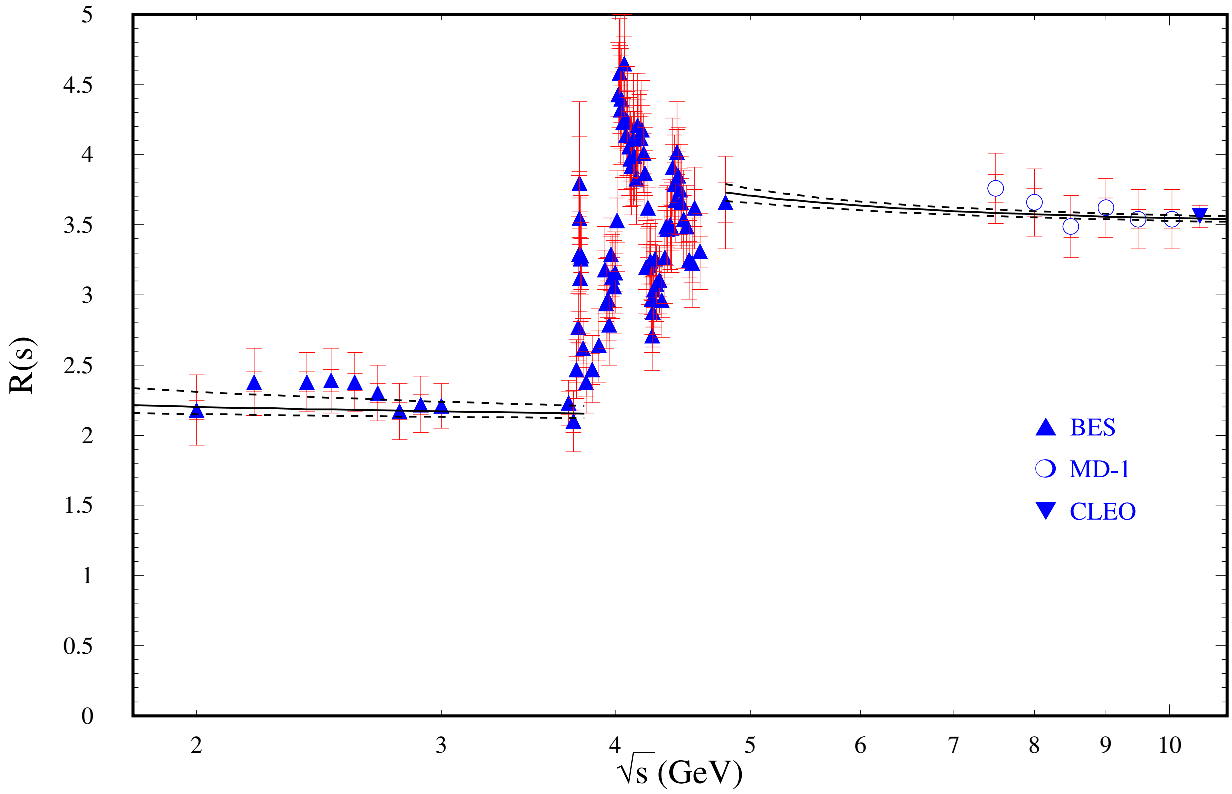
– data

– status of theory

– α_s

Steinhauser, JK

1.) Recent "precision" data on $R(s)$

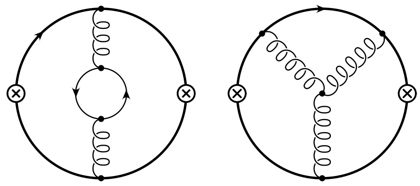


experiment	energy [GeV]	date	systematic error
BES	2 — 5	2001	4%
MD-1	7.2 — 10.34	1996	4%
CLEO	10.52	1998	2%
PDG	J/ψ		7%
PDG	ψ'		9%
PDG	ψ''		15%

pQCD and data agree well in the regions
2 — 3.73 GeV and 5 — 10.52 GeV

2.) Status of Theory

pQCD includes full m_Q-dependence up to $\mathcal{O}(\alpha_s^2)$, for example in



$Q = u, d, s, c, b$

Chetyrkin, Hoang, JK, Steinhauser, Teubner

terms of $\mathcal{O}(\alpha_s^3)$ with

$(m^2/s)^0$ Gorishny, Kataev, Larin

Surguladze, Samuel

$(m^2/s)^1$ Chetyrkin, JK

$(m^2/s)^2$ Chetyrkin, Harlander, JK

n_f^3 and n_f^2 terms of $\mathcal{O}(\alpha_s^4)$ are available (Baikov, Chetyrkin, JK) but not included.

3.) α_s

input in the plot: $\alpha_s(M_Z) = 0.118 \pm 0.003$

can we deduce α_s from the low energy data?

Result:

BES below 3.73 GeV:

$$\alpha_s^{(3)}(3 \text{ GeV}) = 0.369^{+0.047+0.123}_{-0.046-0.130}$$

BES at 4.8 GeV:

$$\alpha_s^{(4)}(4.8 \text{ GeV}) = 0.183^{+0.059+0.053}_{-0.064-0.057}$$

MD-1:

$$\alpha_s^{(4)}(8.9 \text{ GeV}) = 0.193^{+0.017+0.127}_{-0.017-0.107}$$

CLEO:

$$\alpha_s^{(4)}(10.52 \text{ GeV}) = 0.186^{+0.008+0.061}_{-0.008-0.057}$$

combined assuming uncorrelated (?) errors

$$\alpha_s^{(4)}(5 \text{ GeV}) = 0.235^{+0.047}_{-0.047}$$

Evolve up to M_Z

$$\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$$

confirmation of running

result consistent with LEP, but not competitive

(precision of 0.4% at 3.7 Gev (0.7% at 2 GeV) would be required)

II. Sum Rules to NNLO with Massive Quarks

– SVZ Sum Rules, Moments and Tadpoles

– Tadpoles at Three Loop

– Results for Charm and Bottom Masses

Steinhauser, JK

1.) SVZ Sum Rules, Moments and Tadpoles

Some definitions

$$R(s) \;=\; 12\pi \,\mathrm{Im}\left[\Pi(q^2 = s + i\epsilon)\right]$$

where $\Pi(q^2)$

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)\Pi(q^2) \;=\; i\int \mathrm{d}x\, e^{iqx}\langle T j_\mu(x) j_\nu(0)\rangle$$

with the electromagnetic current j_μ

Taylor expansion:

$$\Pi_c(q^2) \;=\; Q_c^2 \frac{3}{16\pi^2}\sum_{n\geq 0}\bar{C}_n z^n$$

with $z = q^2/(4m_c^2)$
where $m_c = m_c(\mu)$ is the $\overline{\mathrm{MS}}$ charm quark mass.

coefficients $\bar{C}_n$ up to $n = 8$ known analytically in order α_s^2 (Chetyrkin, JK, Steinhauser)

$\bar{C}_n$ depend on the charm quark mass through $l_{m_c} \equiv \ln(m_c^2(\mu)/\mu^2)$

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 \left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right)$$

n	1	2	3	4
$\bar{C}_n^{(0)}$	1.0667	0.4571	0.2709	0.1847
$\bar{C}_n^{(10)}$	2.5547	1.1096	0.5194	0.2031
$\bar{C}_n^{(11)}$	2.1333	1.8286	1.6254	1.4776
$\bar{C}_n^{(20)}$	2.4967	2.7770	1.6388	0.7956
$\bar{C}_n^{(21)}$	3.3130	5.1489	4.7207	3.6440
$\bar{C}_n^{(22)}$	−0.0889	1.7524	3.1831	4.3713

Define the moments

$$\mathcal{M}_n^{\rm th} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\bigg|_{q^2=0}$$
$$= \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\, \frac{R_c(s)}{s(s-q^2)} + subtraction$$

➪

$$\mathcal{M}_n^{\rm exp} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

$$\mathcal{M}_n^{\rm exp} = \mathcal{M}_n^{\rm th}$$

➪

m_c

Alternative: consider ratios of moments

less sensitivity, larger theoretical uncertainties.

SVZ:

$\Pi^n(0)$ can be reliably calculated in pQCD:

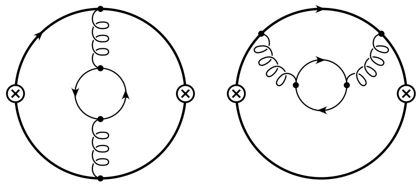
low n:

- fixed order in α_s is sufficient, in particular no resummation of $1/v$ - terms from higher orders required
- condensates are unimportant
- pQCD in terms of short distance mass
  $m_c(3GeV) \Rightarrow m_c(m_c)$
  stable expansion
  no pole mass or closely related definition
  (1S-mass, potential-subtracted mass) involved
- moments available in NNLO

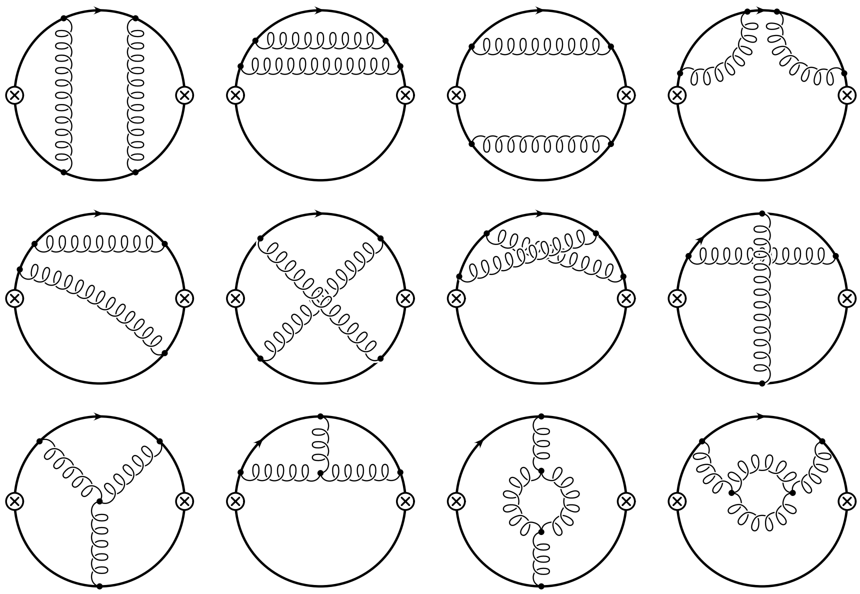
2.) Tadpoles in NNLO

all three-loop – one-scale tadpole amplitudes can be calculated with "arbitrary" power of propagators (Broadhurst; Chetyrkin, JK, Steinhauser)

FORM-program MATAD (Steinhauser)



Three-loop diagrams contributing to $\Pi_l^{(2)}$ (inner quark massless) and $\Pi_F^{(2)}$ (both quarks with mass m).



Purely gluonic contribution to $\mathcal{O}(\alpha_s^2)$

other applications

- mixed QCD and electroweak corrections
  top contribution to the rho-parameter

$$\delta\rho_{OS} = 3\frac{G_F m_t^2}{8\sqrt{2}\pi^2}\left(1 - 2.8599\,\frac{\alpha_s}{\pi} - 14.594\,\left(\frac{\alpha_s}{\pi}\right)^2\right)$$

similarly Δr

- electroweak corrections in order $(G_F m_t^2)^3$
  (assuming $m_W^2 \ll m_t^2$)

- $H \to gg$ in NNLO

- etc

3. Results

input for $R(s)$

- resonances (J/ψ, ψ', ψ'')
- continuum below 4.8GeV (BES)
- continuum above 4.8 GeV (theory)

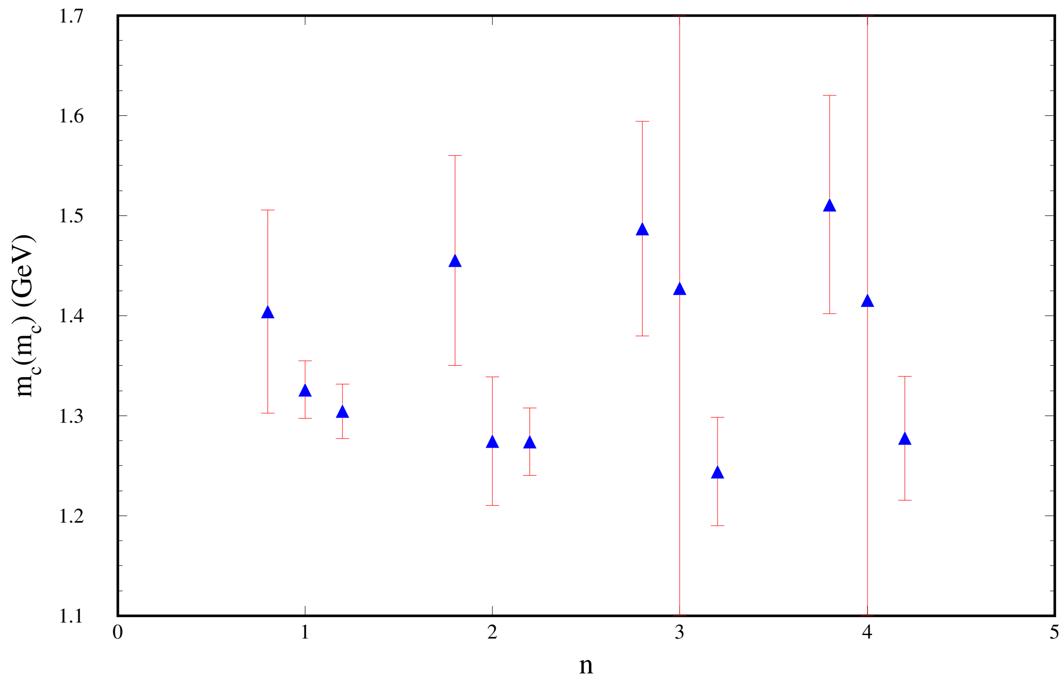
n	$\mathcal{M}_n^{\mathrm{exp,res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{exp,cc}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{exp}}$ $\times 10^{(n-1)}$
1	0.1114(82)	0.0313(15)	0.0638(10)	0.2065(84)
2	0.1096(79)	0.0174(8)	0.0142(3)	0.1412(80)
3	0.1094(79)	0.0099(5)	0.0042(1)	0.1234(79)
4	0.1105(79)	0.0057(3)	0.0014(0)	0.1175(79)

error of the moments dominated by resonances

n	1	2	3	4
$m_c(3\text{ GeV})$	1.027(30)	0.994(37)	0.961(59)	0.997(67)
$m_c(m_c)$	1.304(27)	1.274(34)	1.244(54)	1.277(62)

error in m_c dominated by experiment for n=1,
by theory (variation of μ, α_s) for n=3

stability: compare LO, NLO, NNLO
clear improvement



$m_c(m_c)$ for $n = 1, 2, 3$ and 4 in LO, NLO, NNLO.

FINAL RESULT: $m_c(m_c) = 1.304(27)$ GeV

convertion to pole mass:

$$
\begin{aligned}
M_c^{(2-\text{loop})} &= 1.514(34)\ \text{GeV} \\
M_c^{(3-\text{loop})} &= 1.691(35)\ \text{GeV}
\end{aligned}
$$

drastic shift from NLO to NNLO!

$\overline{MS}$-mass more stable

other results from sum rules:

1.23 ± 0.09 GeV Eidemüller, Jamin

1.37 ± 0.09 GeV Penarrocha, Schilcher

1.10 ± 0.04 GeV Narison

results from lattice simulations:

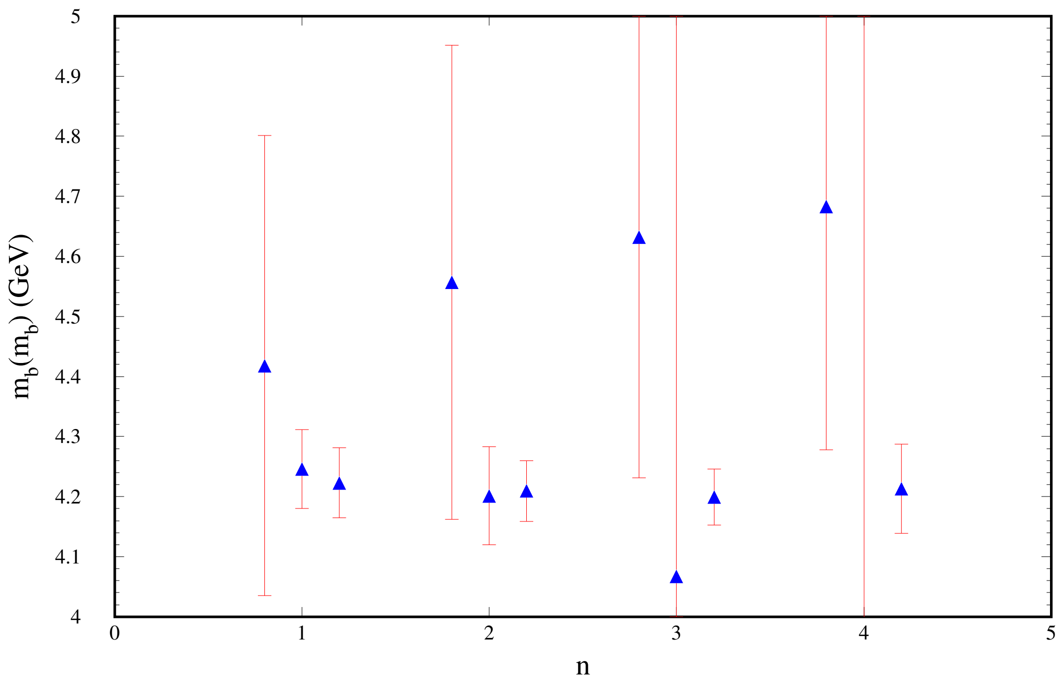
1.26(4)(12) GeV Becirevic, Lubicz, Martinelli

1.314(40)(20)(7) GeV Rolf, Sint

Similar analysis for the **bottom quark**
resonances include $\Upsilon(1)$ up to $\Upsilon(6)$,
"continuum" starts at 11.2 GeV

n	$\mathcal{M}_n^{\mathrm{exp,res}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{exp,lin}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{cont}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\mathrm{exp}}$ $\times 10^{(2n+1)}$
1	1.508(114)	0.035(35)	2.913(21)	4.456(121)
2	1.546(109)	0.028(28)	1.182(12)	2.756(113)
3	1.600(106)	0.022(22)	0.634(8)	2.256(108)
4	1.671(104)	0.018(18)	0.381(5)	2.070(105)

Moments for the bottom quark system.



$m_b(m_b)$ for $n = 1, 2, 3$ and 4 in LO, NLO and NNLO

RESULT

n	1	2	3	4
$m_b(10\ \mathrm{GeV})$	3.665(60)	3.651(52)	3.641(48)	3.655(77)
$m_b(m_b)$	4.223(58)	4.209(50)	4.199(46)	4.213(74)

the value

$$m_b(m_b) \;=\; 4.209(50)\ \mathrm{GeV}$$

is consistent with evaluations based on the analysis of bottonium and moments with large n

and corresponds to a pole mass of

$$M_b^{(2-\mathrm{loop})} \;=\; 4.651(57)\ \mathrm{GeV}$$
$$M_b^{(3-\mathrm{loop})} \;=\; 4.819(57)\ \mathrm{GeV}$$

III Radiative Return to low s at Φ and B-factories

– basic idea

– MC programs

– some preliminary experimental results

Binner, Kühn, Melnikov, *PLB*

Czyż, Kühn, *Eur. Phys. J.C*

Rodrigo, Gehrmann-De Ridder, Guilleaume, Kühn, *Eur. Phys.J.C*

Rodrigo, Czyż, Kühn, Szopa, *Eur. Phys.J.C*

Basic Idea

Large luminosities of factories compensate α/π from photon radiation

$\pi^+ \pi^- \gamma : E_\gamma > 100 MeV$

$\sqrt{s}$ $[GeV]$	$\int \mathcal{L}$ $[fb^{-1}]$	#events, $\theta_{min} = 7°$
1.02	1.35	$16 \cdot 10^6$
10.6	100	$3.5 \cdot 10^6$

multi-hadron-events (R $\equiv$ 2) $\sqrt{s} = 10.6\, GeV$

Q^2-interval $[GeV]$	#events, $\theta_{min} = 7°$
[1.5 , 2.0]	$9.9 \cdot 10^5$
[2.0 , 2.5]	$7.9 \cdot 10^5$
[2.5 , 3.0]	$6.6 \cdot 10^5$
[3.0 , 3.5]	$5.8 \cdot 10^5$

DAΦNE vs. B-factory: important difference

10 GeV

clear kinematic separation between photon and hadrons

1 GeV

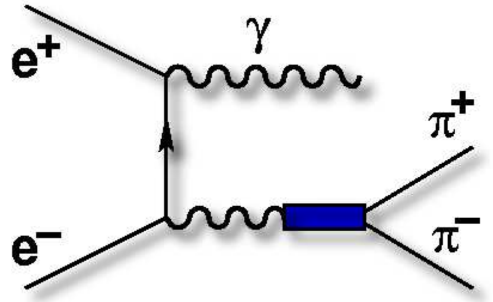
no natural kinematic separation

➪ cuts to control FSR vs. ISR

The Generator (EVA MC)

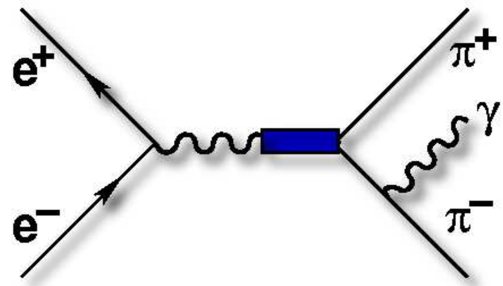
[Binner, Kühn, Melnikov, 99]

- ISR



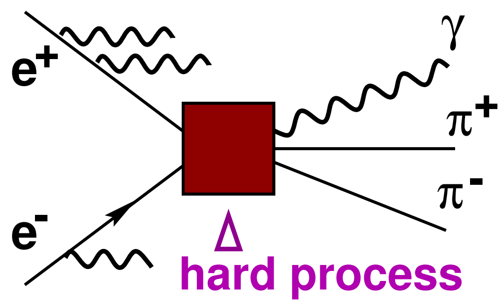
pion form factor
➪ to be tested

- FSR



radiation from point-like pions (probably overestimated)

- additional collinear radiation



Structure Function

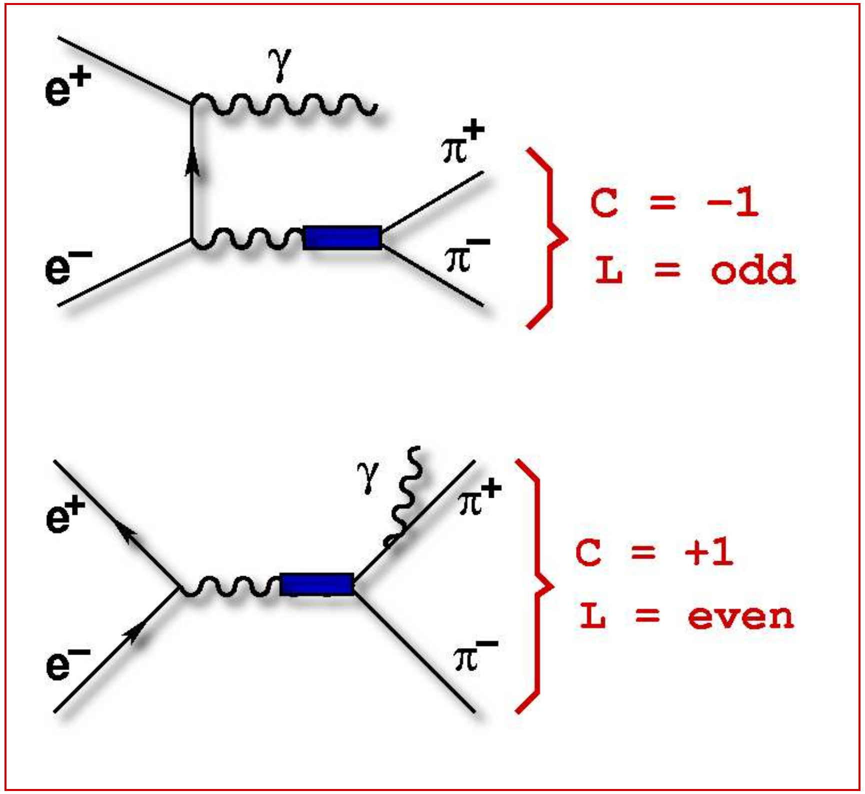
$\sigma = \int \int dx_1 \, dx_2$

$\times D(x_1) \, D(x_2) \, \sigma(x_1 x_2 s)$

[Caffo *et al.*, 94]

Test of FSR model

interference:

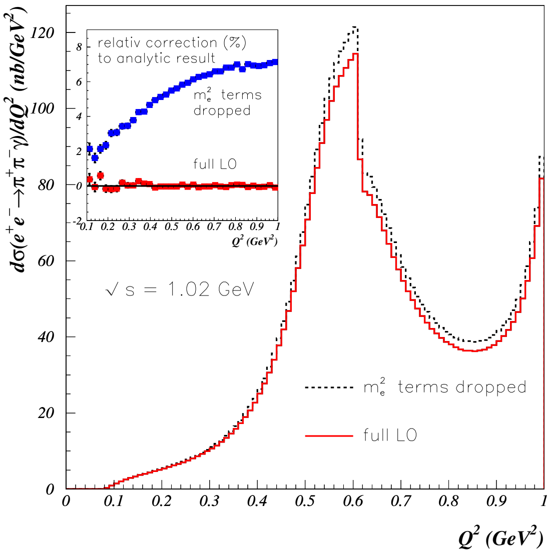


- interference odd under $\pi^+ \leftrightarrow \pi^-$
- asymmetric differential distribution: $\int$ interf. $= 0$

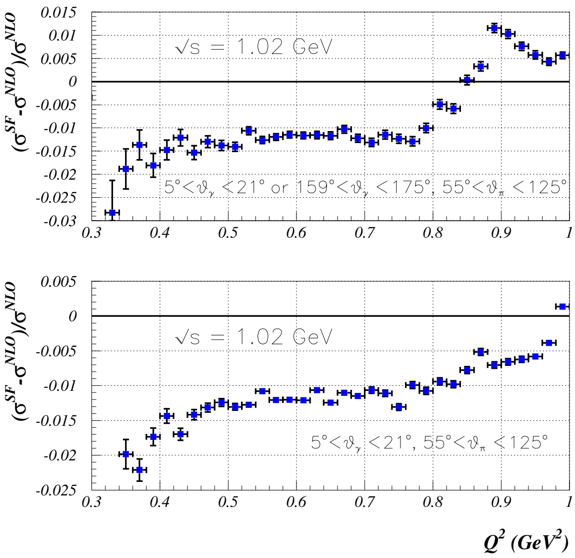
$$A(\theta) = \frac{N^{\pi^+}(\theta) - N^{\pi^-}(\theta)}{N^{\pi^+}(\theta) + N^{\pi^-}(\theta)}$$

Recent results

A) LO generator extended to small angles (important m_e terms)

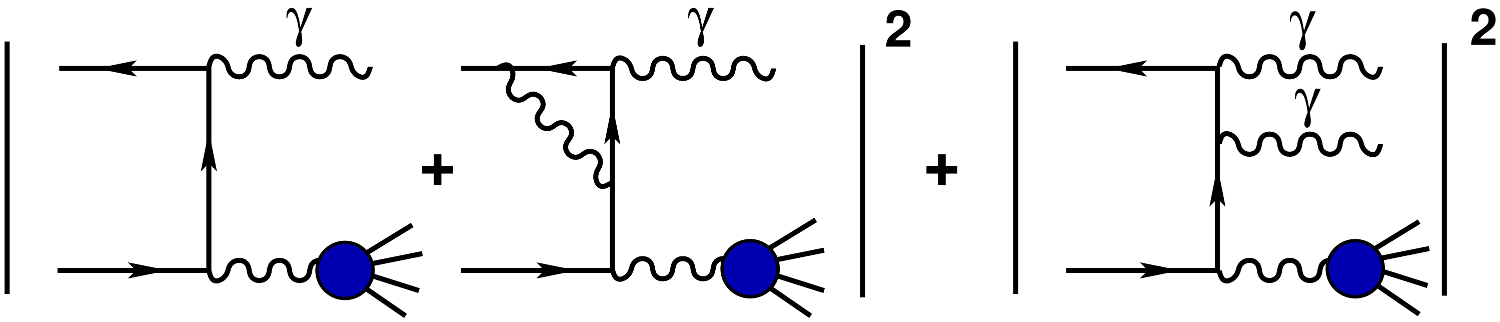


B) NLO generator ($\theta_\gamma > \theta_{cut}$) reasonable good agreement with EVA (LO + structure function)



C) NLO generator ($\theta_\gamma \geq 0$) under construction

NLO corrections to ISR



Dimensional Regularization: $D = 4 - 2\epsilon$

- **one-loop**
- **two photons**

UV: on-shell renormalization

IR divergences

soft: *analytically* ⇨ $E_\gamma < w\sqrt{s}$

finite but w - dependent

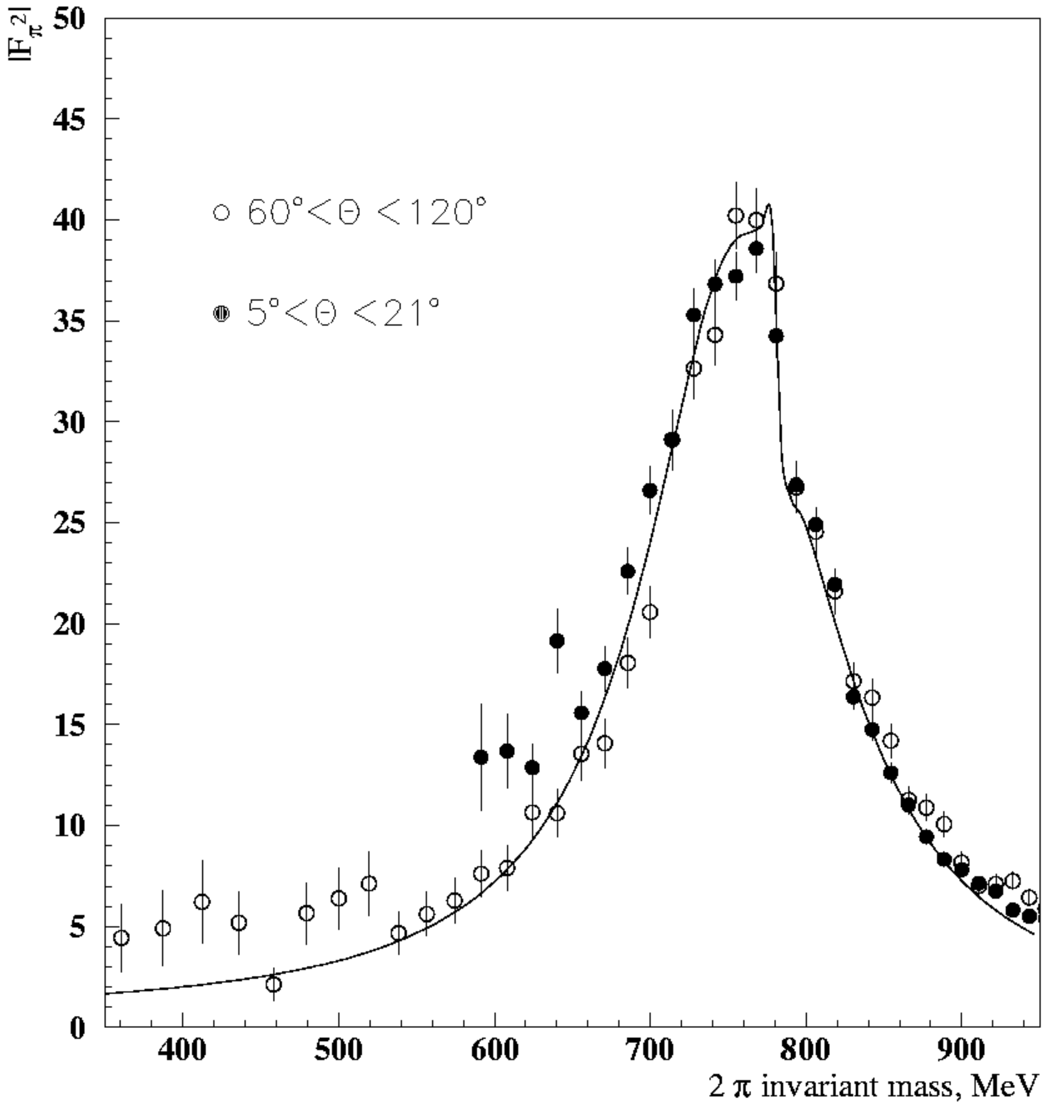
hard: *numerically* ⇨ helicity amplitude method

[Kołodziej, Zrałek, Jegerlehner]

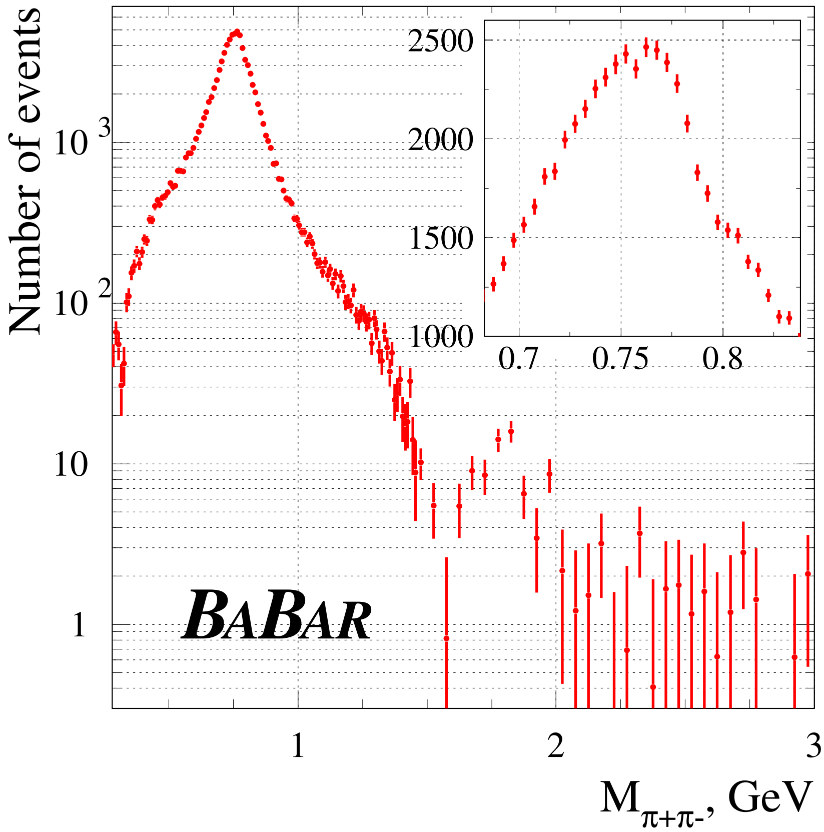
Generators - the next steps

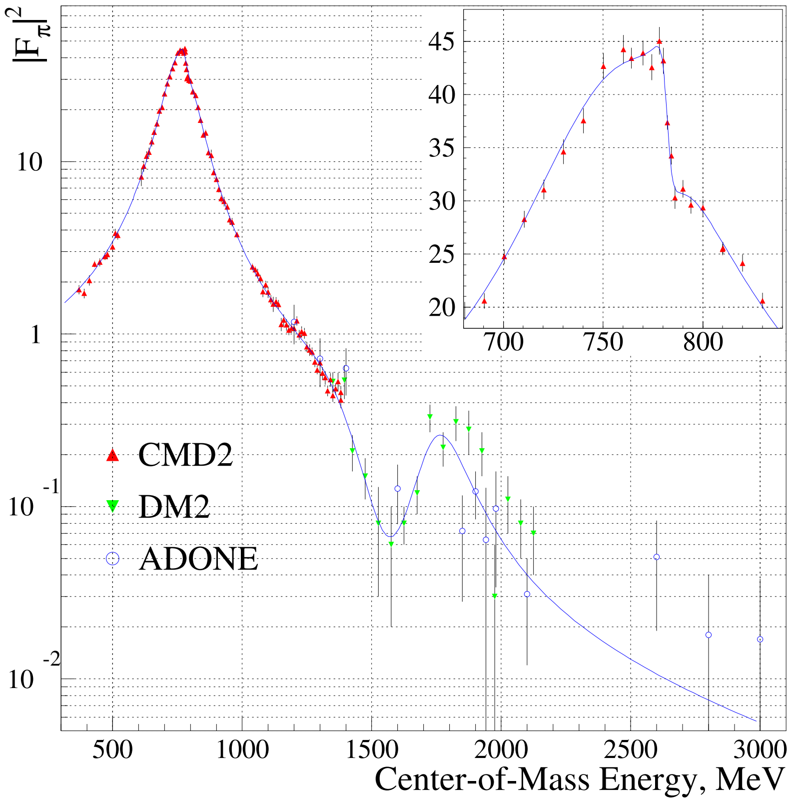
- provide a **fully tested** NLO generator for tagged events (➪ Publication) ($\pi^+\pi^-$, $\mu^+\mu^-$)
- same for $\theta_\gamma \to 0$
- more final states (3π, $KK\pi$, $\cdots$)
  - improved matrix elements, m_{had} = 1-2 GeV
  - B-factories
- final state radiation and NLO

KLOE

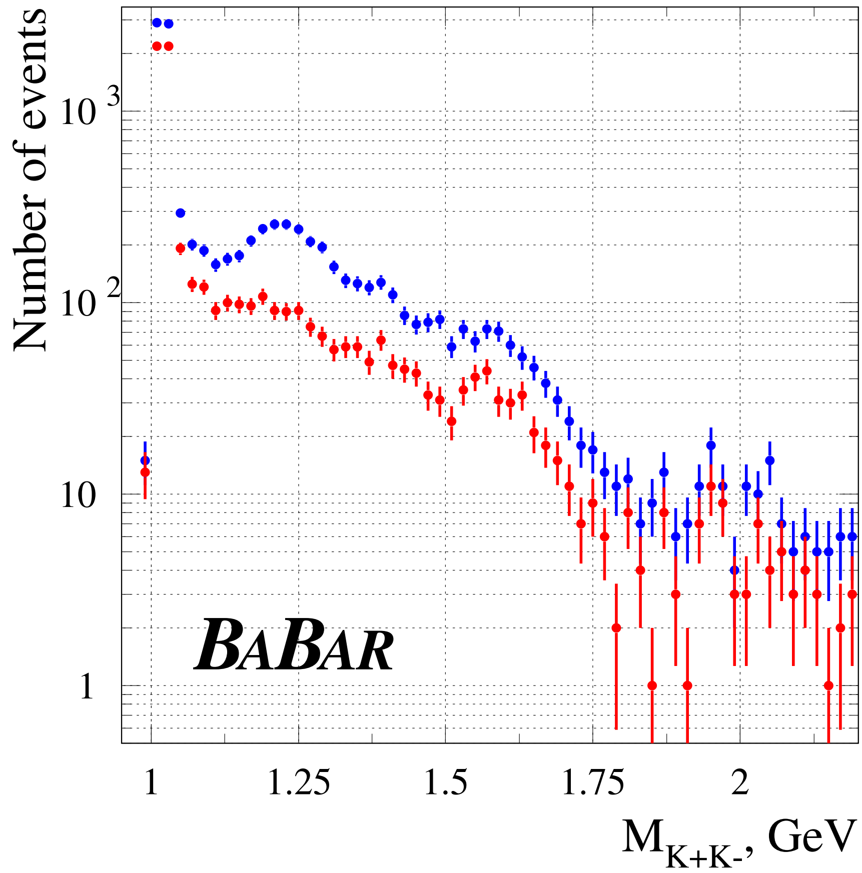


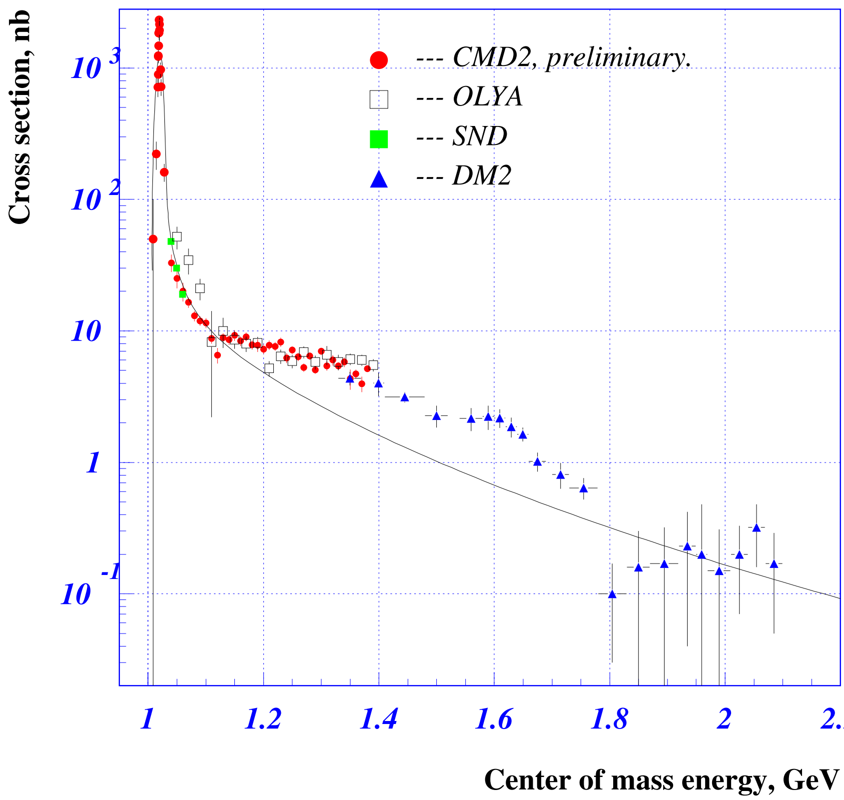
Pion form factor for small and large angles. The line is a Gounaris-Sakurai parametrization with fixed parameters. No fit attempted.



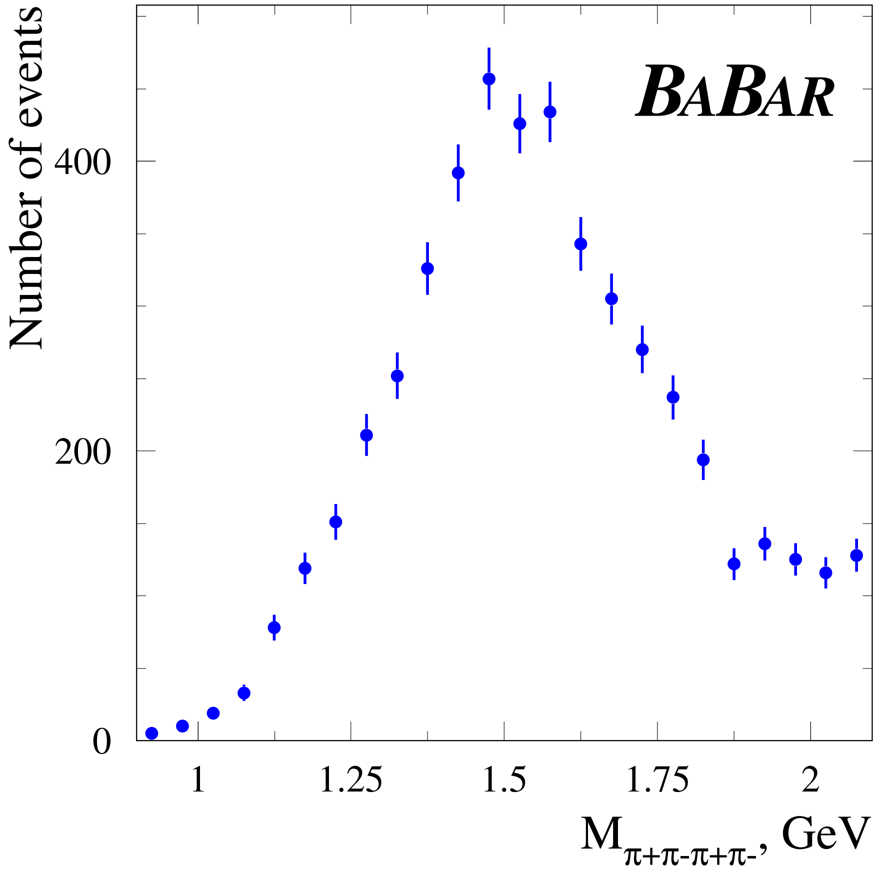


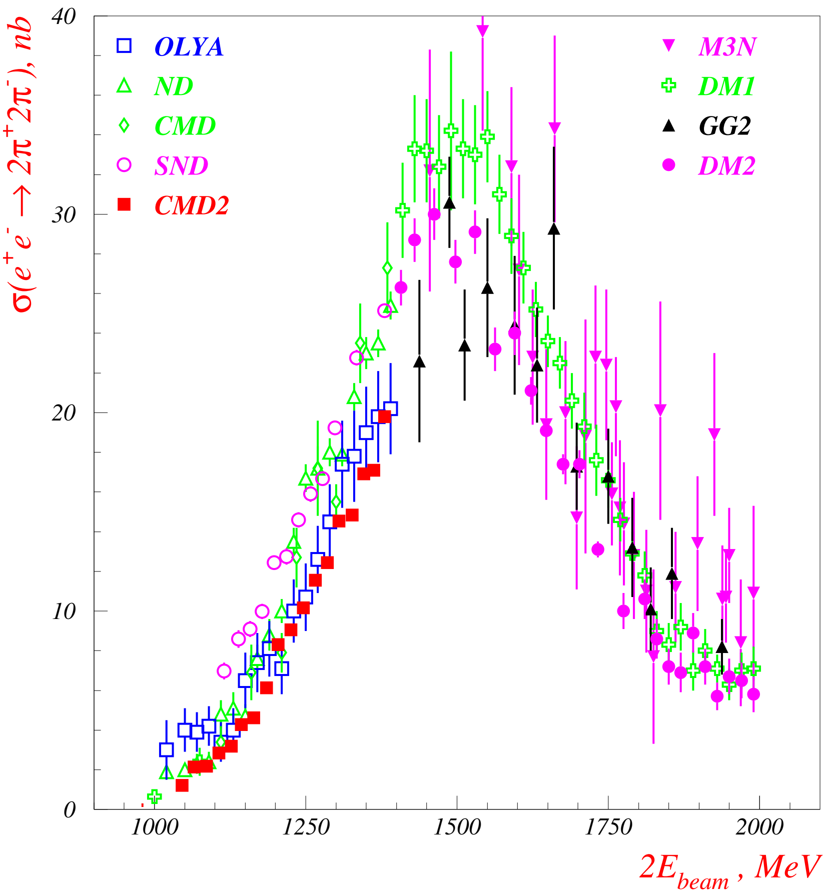
Uncorrected $\pi^+\pi^-$ invariant mass distribution for $\pi^+\pi^-\gamma$ events (top), compared to pion form factor measurements.



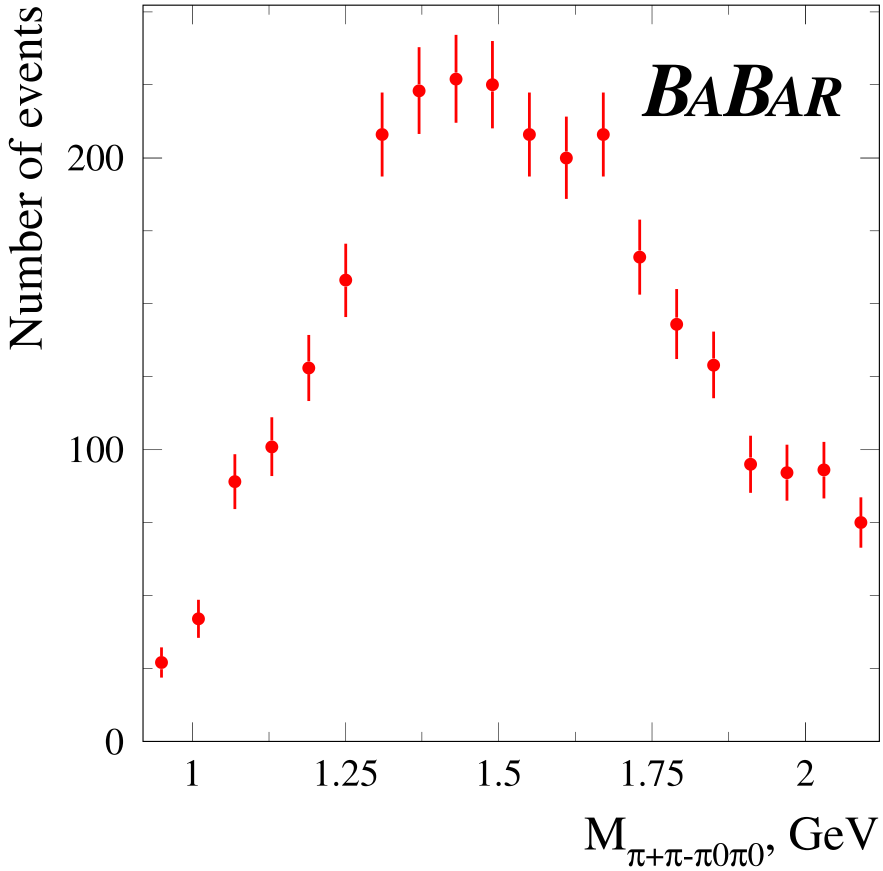


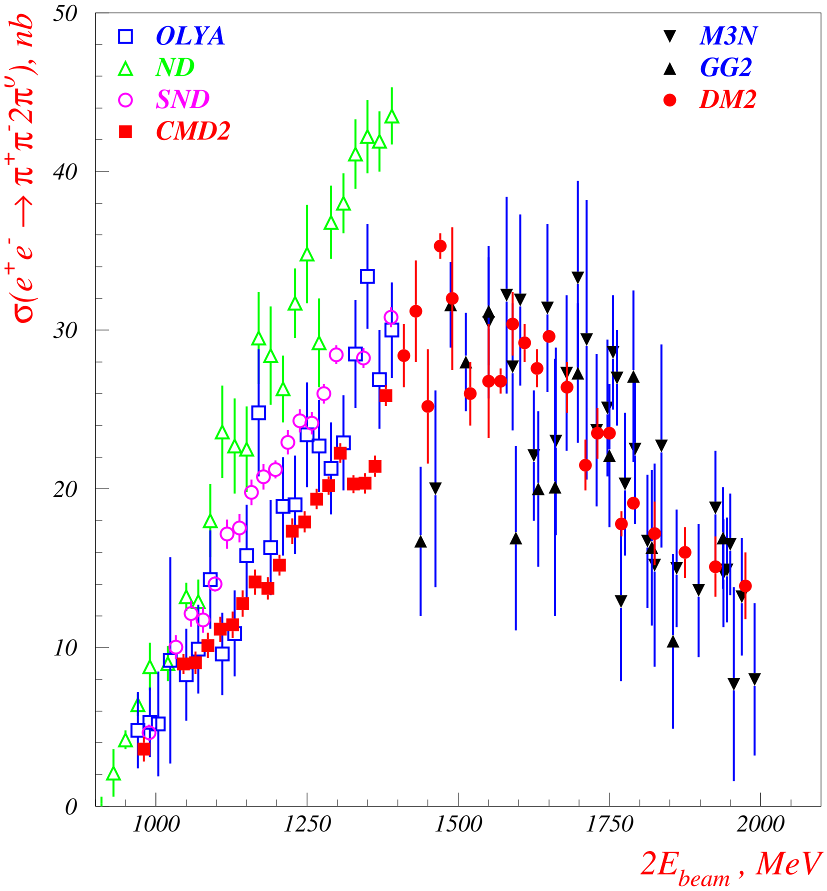
Uncorrected K^+K^- invariant mass distribution for $K^+K^-\gamma$ events (top), compared to K^+K^- cross section measurements.





Uncorrected $2\pi^+2\pi^-$ mass distribution from ISR data (top) compared to the c.m. energy dependence of the $2\pi^+2\pi^-$ cross section in e^+e^- experiments





Uncorrected $\pi^+\pi^-2\pi^0$ mass distribution from ISR data (top) compared to the c.m. energy dependence of the $\pi^+\pi^-2\pi^0$ cross section in e^+e^- experiments

IV SUMMARY

moments in NNLO:

➪ drastic improvement in δm_c and δm_b

➪ direct determination of short-distance mass from moments with low n

improved measurements of the cross section in the charm region (J/ψ !) would help

$R(s)$ at very low s remains an important issue for many applications

➪ radiative return