SUDAKOV LOG'S

in

ELECTROWEAK PROCESSES

NNLL in Four-Fermion Processes

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Nucl. Phys. B616 (2001) 286

1) Motivation

large one–loop corrections at high energies

2) NNLL for SU(2) Toy Model

A) form factor

B) scattering

3) Subleading n_f Terms

4) Standard Model: W,Z,γ

5) Questions about
Experimental Setup

1) MOTIVATION (One-Loop!)

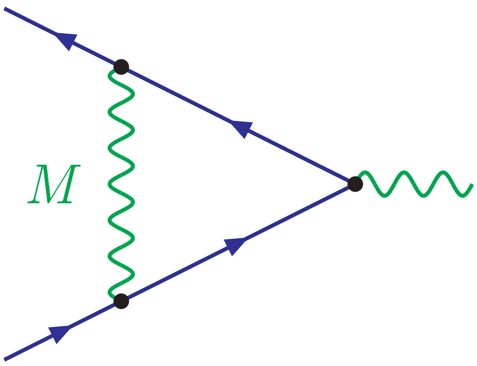
example: $e^+e^- \to t\bar{t}$ FIG

boxes + vertex corrections dominate

running coupling: $\sim \ln(s)$

boxes, vertices: $\sim \ln^2(s/M^2)$

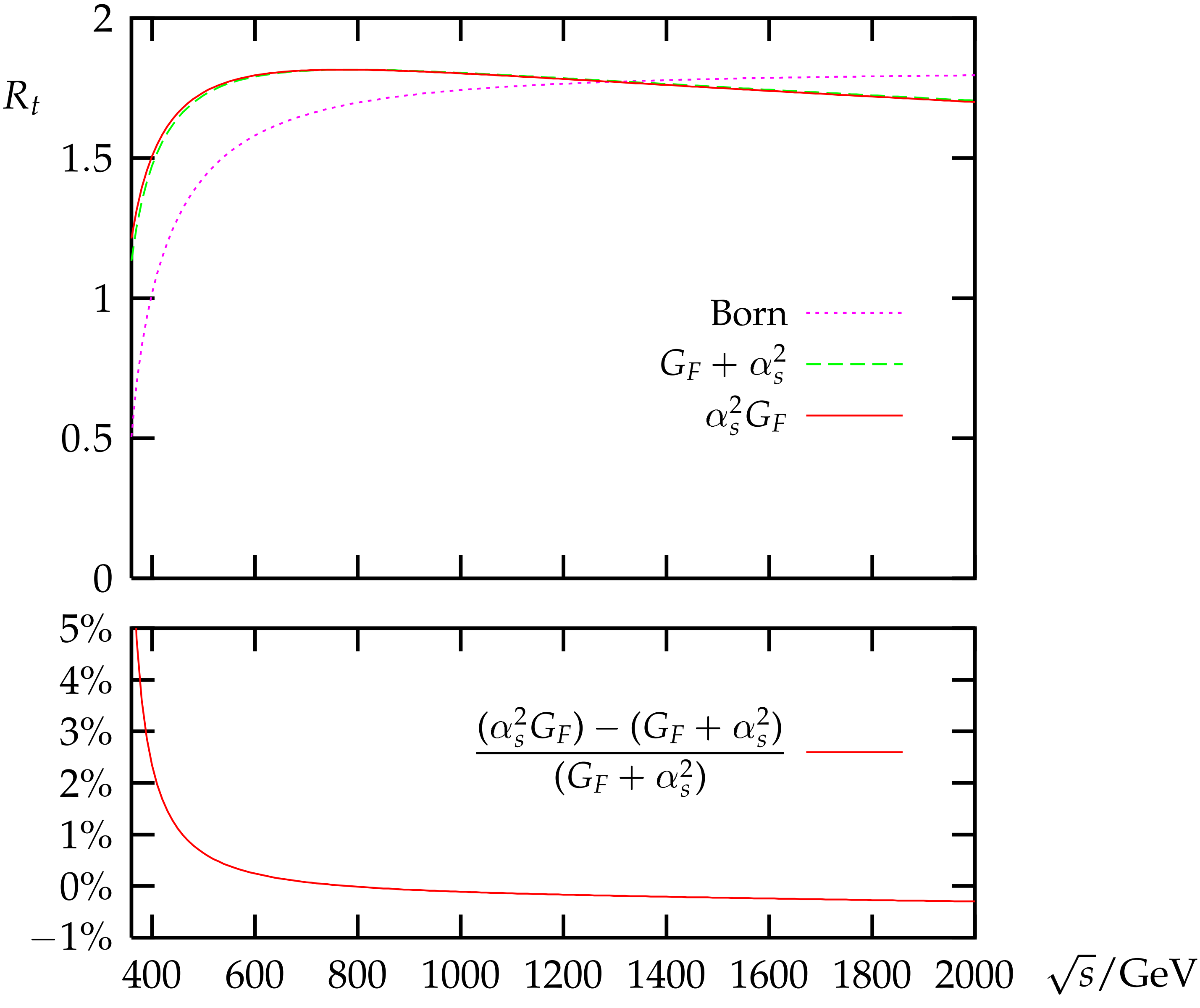
example: massive U(1)

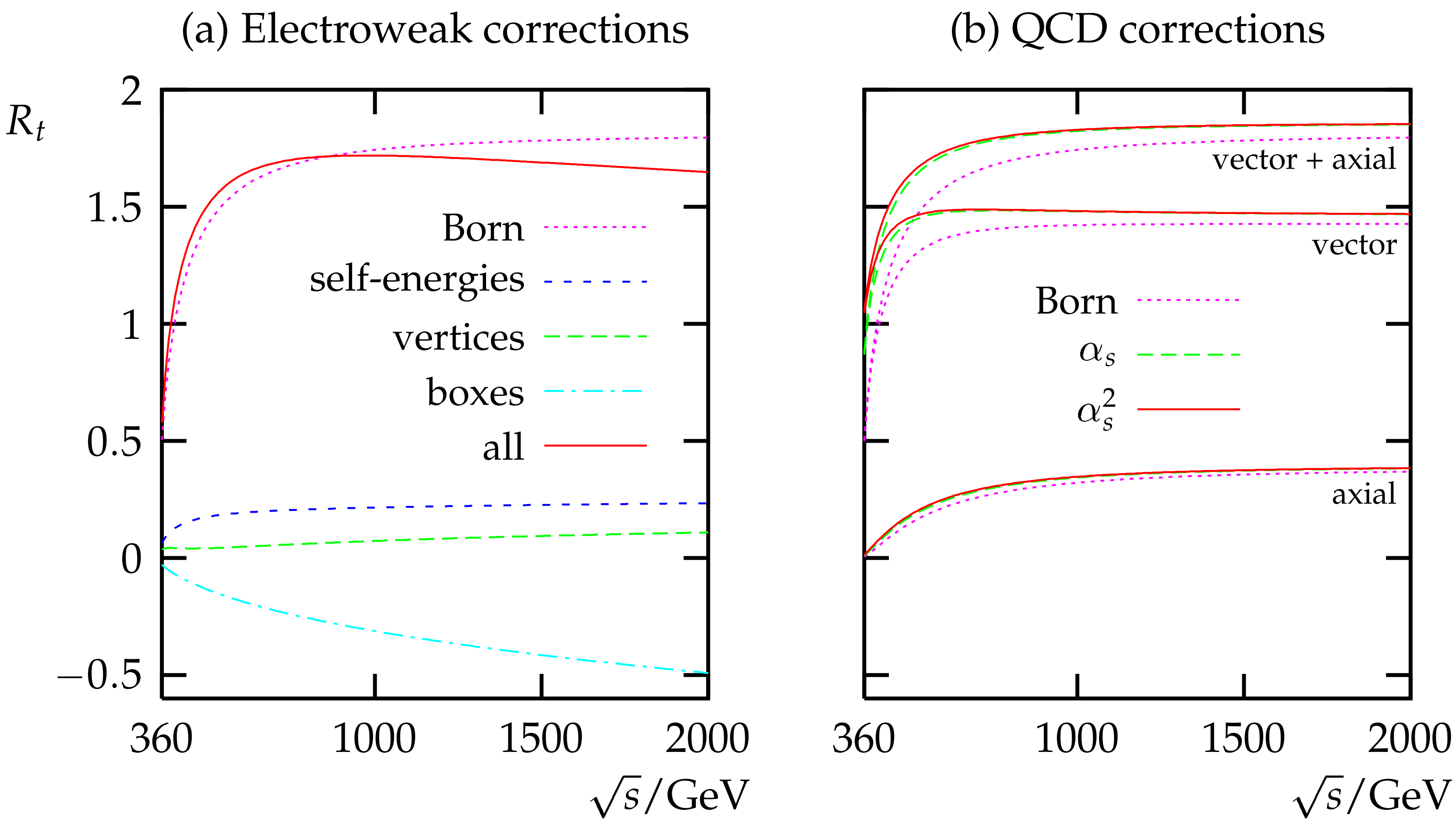


$$\Rightarrow \quad \text{Born} * \left(1 - 2\frac{\alpha}{\pi}\left(\frac{1}{4}\ln^2\frac{s}{M^2} + \ldots\right)\right)$$

magnitude $(\alpha_w/\pi = 10^{-2})$

$\left(\frac{s}{M^2}\right)$	$-\frac{1}{4}\ln^2\frac{s}{M^2}$	$+\frac{3}{4}\ln\frac{s}{M^2}$	$-\frac{7}{8}+\frac{\pi^2}{12}$	Σ	$*2\frac{\alpha_w}{\pi}$
$\left(\frac{1000}{100}\right)^2$	- 5.30	+ 3.45	- 0.05	-1.9	-4%
$\left(\frac{2000}{100}\right)^2$	- 8.97	+ 4.49	- 0.05	-4.5	-9%


R_t
2
1.5
1
0.5
0
Born
$G_F + \alpha_s^2$
$\alpha_s^2 G_F$
5%
4%
3%
2%
1%
0%
−1%
$\frac{(\alpha_s^2 G_F) - (G_F + \alpha_s^2)}{(G_F + \alpha_s^2)}$
400
600
800
1000
1200
1400
1600
1800
2000
$\sqrt{s}$/GeV



(a) Electroweak corrections
(b) QCD corrections
R_t
2
1.5
1
0.5
0
−0.5
Born
self-energies
vertices
boxes
all
vector + axial
vector
axial
Born
α_s
α_s^2
360
1000
1500
2000
$\sqrt{s}$/GeV

large negative correction,
compensated by real radiation in inclusive rate!

massive boson:
exclusive rate physically meaningfull!

higher orders, leading log:

$$\Big(1 - \frac{1}{2}\underbrace{\frac{g^2}{16\pi^2}\ln^2\frac{s}{M^2}}_{\mathsf{L}}\Big) \Rightarrow e^{-\frac{1}{2}\mathsf{L}}$$

(per leg) in amplitude

nonabelian theory: $e^{-\frac{C_F}{2}\mathsf{L}}$

numerically: $\mathsf{L}_{ew} = \begin{matrix} 0.07 \\ 0.11 \end{matrix}$ for $\begin{matrix} 1\ \text{TeV} \\ 2\ \text{TeV} \end{matrix}$

$C_F = \frac{3}{4}$ for SU(2)

expected correction for $f_L \bar{f}_L \to f'_L \bar{f}'_L$

$$\sim \underset{\text{rate}}{2} \cdot \overset{\text{legs}}{4} \cdot \frac{3}{4} \cdot \frac{1}{2}\mathsf{L}_{ew} \sim 20 - 30\%$$

EW theory; One-Loop

$f_L \bar{f}_L \to f'_L \bar{f}'_L$

$$\mathcal{M}_{\text{Born}} = \frac{ig^2}{s}\left(T^3_f T^3_{f'} + \tan^2\Theta_w \frac{Y_f Y_{f'}}{4}\right)$$

$$\left(M_Z^2/s \to 0\right)$$

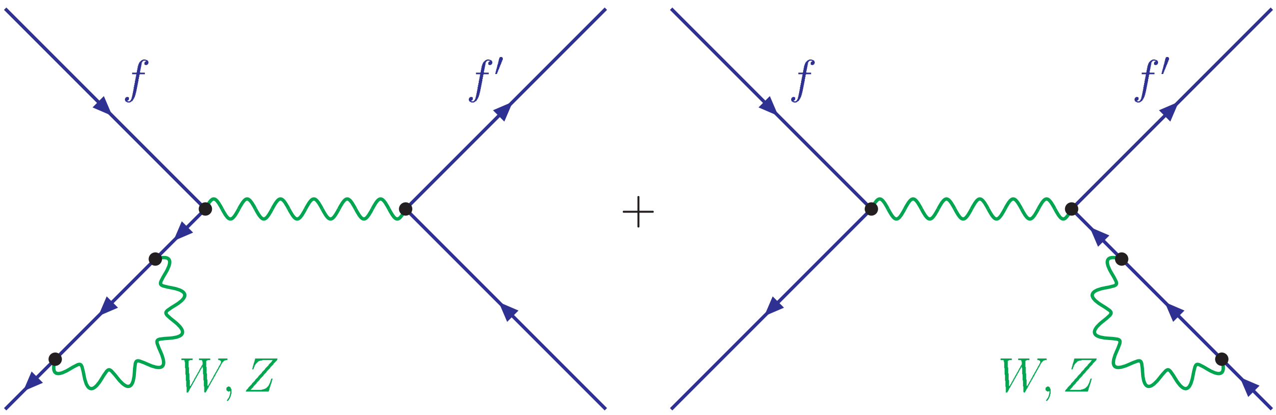
one-loop: explicit calculation $\surd$

dominant correction: $*\left(1 - F^f_{\text{L,R}}\right)$ with

$$F^f_{\text{L}} = \Big(\underbrace{\frac{1}{2}}_{W} + \underbrace{\frac{1}{4c^2_{\text{w}}} + \tan^2_{\text{w}}\,(s^2_{\text{w}}Q^2_f - 2T^3_f Q_f)}_{Z}\Big)L$$

$$F^f_{\text{R}} = \Big(\underbrace{\tan^2_{\text{w}}\, s^2_{\text{w}} Q^2_f}_{Z}\Big)L$$

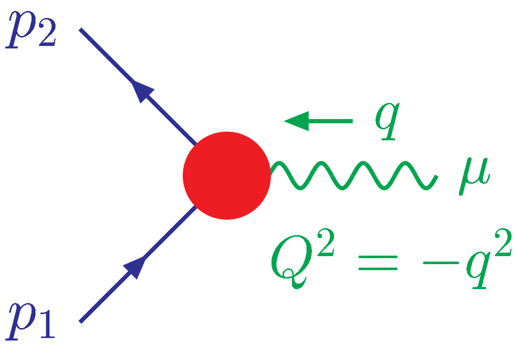
(per line)



axial gauge

2) NNLL for SU(2) Toy Model

A) Form Factor



Born:

$$\mathcal{F}_{\mathsf{B}} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

Collins, Sen

$$\Rightarrow \mathcal{F} = \mathcal{F}_{\mathsf{B}}\, F_0(\alpha(M^2)) * \exp\left\{\int_{M^2}^{Q^2}\frac{\mathrm{d}x}{x}\left[\int_{M^2}^{x}\frac{\mathrm{d}x'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

aim: NNLL

corresponds to all terms of the form:

$$\alpha^n \ln^{2n}\left(\frac{Q^2}{M^2}\right) + \alpha^n \ln^{2n-1}\left(\frac{Q^2}{M^2}\right) + \alpha^n \ln^{2n-2}\left(\frac{Q^2}{M^2}\right)$$

LL NLL NNLL

requires:

$\zeta(\alpha)$, $\xi(\alpha)$, $F_0(\alpha)$ up to $\mathcal{O}(\alpha)$

$\gamma(\alpha)$ up to $\mathcal{O}(\alpha^2)$!

and running of α

expansion by regions:

$$\gamma^{(1)} = -2C_F\,,\ \zeta^{(1)} = 3C_F\,,\ \xi^{(1)} = 0\,,\ F_0^{(1)} = -C_F\left(\frac{7}{2} + \frac{2\pi^2}{3}\right)$$

$$\gamma^{(2)} = -2C_F\left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]$$

Matsuura +..., Kramer +...

$$\mathcal{F} \;=\; \mathcal{F}_{\mathrm{B}} + \frac{\alpha}{4\pi}\mathcal{F}^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2\mathcal{F}^{(2)} + \ldots$$

one-loop:

$$\mathcal{F}^{(1)} = \left(\frac{1}{2}\gamma^{(1)} \ln^2\left(\frac{Q^2}{M^2}\right) + \left(\xi^{(1)} + \zeta^{(1)}\right) \ln\left(\frac{Q^2}{M^2}\right) + F_0^{(1)} \right) \mathcal{F}_{\mathrm{B}}$$

two-loop:

$$\begin{aligned}
\Rightarrow \mathcal{F}_{LL}^{(2)} &= \frac{1}{8}(\gamma^{(1)})^2 \ln^4\left(\frac{Q^2}{M^2}\right) \mathcal{F}_{\mathrm{B}} \\
\mathcal{F}_{NLL}^{(2)} &= \frac{1}{2}\left(\zeta^{(1)} - \frac{1}{3}\beta_0\right) \gamma^{(1)} \ln^3\left(\frac{Q^2}{M^2}\right) \mathcal{F}_{\mathrm{B}} \\
\mathcal{F}_{NNLL}^{(2)} &= \frac{1}{2}\left(\gamma^{(2)} + \left(\zeta^{(1)} - \beta_0\right)\zeta^{(1)} + F_0^{(1)}\gamma^{(1)}\right) \ln^2\left(\frac{Q^2}{M^2}\right) \mathcal{F}_{\mathrm{B}}
\end{aligned}$$

concrete numbers:

SU(2)

$$\mathcal{F}^{(1)} = \left[-\frac{3}{4}\ln^2\left(\frac{Q^2}{M^2}\right) + \frac{9}{4}\ln\left(\frac{Q^2}{M^2}\right) - \left(\frac{21}{8} + \frac{\pi^2}{2}\right) \right] \mathcal{F}_{\mathrm{B}}$$

$$\mathcal{F}^{(2)} = \left[\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{19}{48}\ln^3\left(\frac{Q^2}{M^2}\right) - \left(\frac{469}{48} - \frac{7\pi^2}{8}\right)\ln^2\left(\frac{Q^2}{M^2}\right) \right] \mathcal{F}_{\mathrm{B}}$$

U(1)

$$\mathcal{F}^{(1)} = \left[-\ln^2\left(\frac{Q^2}{M^2}\right) + 3\ln\left(\frac{Q^2}{M^2}\right) - \left(\frac{7}{2} + \frac{2\pi^2}{3}\right) \right] \mathcal{F}_{\mathrm{B}}$$

$$\mathcal{F}^{(2)} = \left[\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{52}{9}\ln^3\left(\frac{Q^2}{M^2}\right) + \left(\frac{619}{18} + \frac{2\pi^2}{3}\right)\ln^2\left(\frac{Q^2}{M^2}\right) \right] \mathcal{F}_{\mathrm{B}}$$

Large coefficients!

B) Four-Fermion Scattering $\quad f + f' \to f + f'$

$$\frac{ig^2}{s}\left(\bar\psi_2 t^a \gamma^\mu \psi_1\right)\left(\bar\psi_4 t^a \gamma_\mu \psi_3\right) \equiv \frac{ig^2}{s}\mathcal{A}^\lambda$$

one-loop:

$$\frac{ig^2(Q^2)}{s}\frac{\alpha}{2\pi}\left[\left\{-C_F\left(\ln^2\left(\frac{-s}{M^2}\right) - 3\ln\left(\frac{-s}{M^2}\right)\right) + \left(-C_A \ln\left(\frac{u}{s}\right) + 2\left(C_F - \frac{T_F}{N}\right)\ln\left(\frac{u}{t}\right)\right)\ln\left(\frac{-s}{M^2}\right)\right\}\mathcal{A}^\lambda + \left\{2\frac{C_F T_F}{N}\ln\left(\frac{u}{t}\right)\ln\left(\frac{-s}{M^2}\right)\right\}\mathcal{A}^d\right]$$

$$+ \text{ non-log-terms}$$

with $\mathcal{A}^d = \left(\bar\psi_2 \gamma^\mu \psi_1\right)\left(\bar\psi_4 \gamma_\mu \psi_3\right)$ diagonal!

$\Rightarrow$ evolution for $\tilde{\mathcal{A}} = \begin{pmatrix}\tilde{\mathcal{A}}^\lambda \\ \tilde{\mathcal{A}}^d\end{pmatrix}$ $\qquad$ ($\tilde{\mathcal{A}} = \mathcal{A}*$collinear logs)

$$\frac{\partial}{\partial \ln Q^2}\tilde{\mathcal{A}} = \chi\left(\alpha\left(Q^2\right)\right)\tilde{\mathcal{A}}$$

Sen $+ \ldots$, Sterman $+ \ldots$

$\chi_{\lambda\lambda}$, $\chi_{\lambda d}$ from above
$\chi_{d\lambda}$ analogous; $\chi_{dd} = 0$

$\tilde{\mathcal{A}}$ from complete one-loop result diagonalize matrix χ

$$\tilde{\mathcal{A}} = \sum_i \tilde{\mathcal{A}}_{0i}(\alpha(M^2)) \exp\left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \chi_i(\alpha(x))\right]$$

starting values for $\tilde{\mathcal{A}}$ from one-loop result (including constant terms)

Angular dependence is important.

result:

$$\sigma = \sigma_{\mathrm{B}} + \frac{\alpha}{4\pi}\sigma^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \sigma^{(2)} + \ldots$$

$u\bar{u} \to u\bar{u}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{80}{3}\ln\left(\frac{s}{M^2}\right) - \left(\frac{22}{9} + 3\pi^2\right)\right]\sigma_{\mathrm{B}}$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{449}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{2414}{9} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_{\mathrm{B}}$$

$u\bar{u} \to d\bar{d}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{26}{3}\ln\left(\frac{s}{M^2}\right) + \left(\frac{221}{9} - 3\pi^2\right)\right]\sigma_{\mathrm{B}}$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) - \left(\frac{1625}{18} - \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_{\mathrm{B}}$$

U(1) theory

$$\sigma^{(1)} = \left[-4\ln^2\left(\frac{s}{M^2}\right) + 12\ln\left(\frac{s}{M^2}\right) - \left(\frac{376}{9} - \frac{4\pi^2}{3}\right)\right]\sigma_{\mathrm{B}}$$

$$\sigma^{(2)} = \left[8\ln^4\left(\frac{s}{M^2}\right) - \frac{532}{9}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{1130}{3} + \frac{16\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_{\mathrm{B}}$$

3) Subleading n_f Terms

Stability, subleading terms?

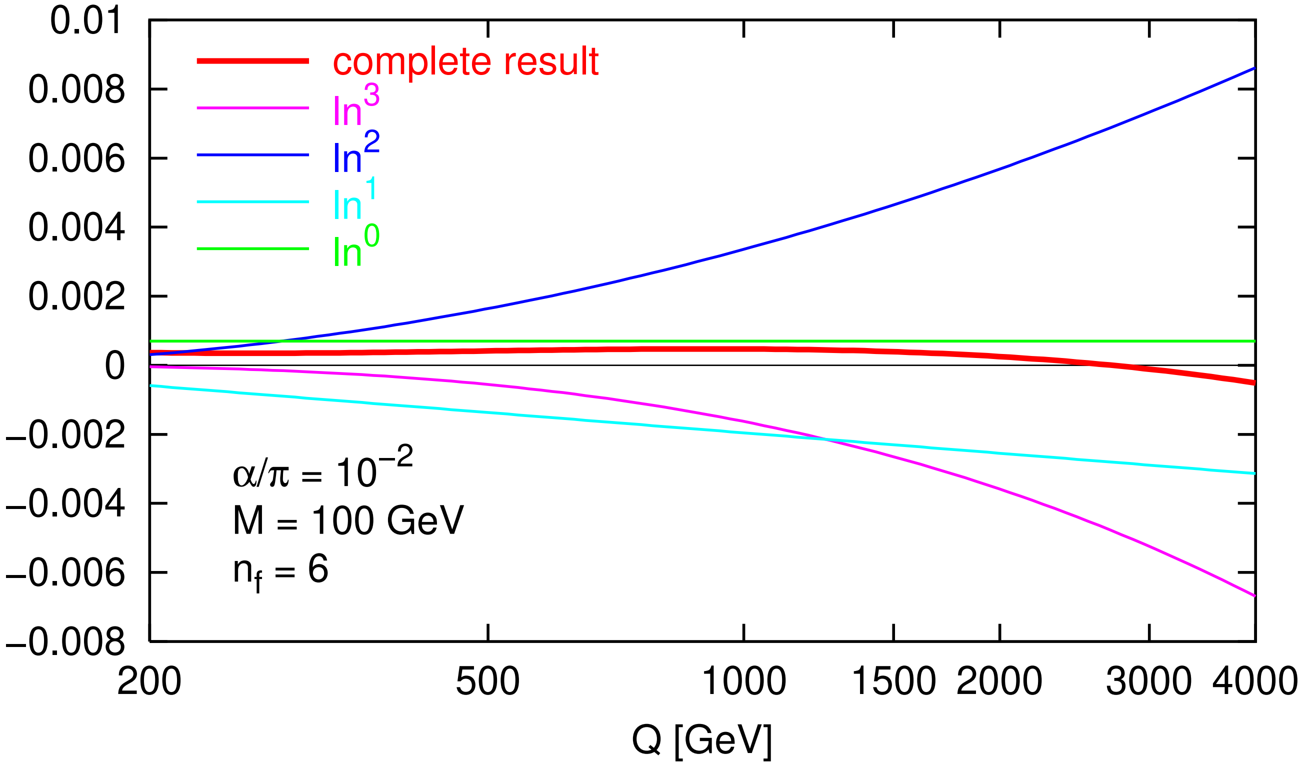
consider the full series of the n_f terms
for the form factor

$$\mathcal{F} = \left\{1 + \left(\frac{\alpha}{4\pi}\right)^2 n_f \left[-\frac{4}{9}\ln^3\left(\frac{Q^2}{M^2}\right) + \frac{38}{9}\ln^2\left(\frac{Q^2}{M^2}\right) - \frac{34}{3}\ln\left(\frac{Q^2}{M^2}\right) + \frac{16\pi^2}{27} + \frac{115}{9}\right]\right\}\mathcal{F}_{\mathrm{B}}$$

Feucht, Kühn, Moch

$\ln^3$ and $\ln^2$ consistent with NNLL,
$\ln^1$ and $\ln^0$ new;
alternating sign!
increasing coefficients!

energy dependence of correction:



4) Standard Model: W, Z, γ

examine $f'\bar{f}' \to f\bar{f}$

$$A_B = \frac{ig^2}{s} \sum_{I,J=L,R} \left(T^3_{f'} T^3_f + t^2_W \frac{Y_{f'} Y_f}{4} \right) A^{f'f}_{IJ}$$

with $A^{f'f}_{IJ} = \left(\bar{f}'_I \gamma^\mu f'_I \right) \left(\bar{f}_J \gamma_\mu f_J \right)$

combine virtual corrections and
real radiation
(prescription of Fadin et al.)

define

$$L(s) = \frac{g^2}{16\pi^2} \ln^2 \left(\frac{s}{M^2} \right) = 0.07 \quad (0.11)$$

$$l(s) = \frac{g^2}{16\pi^2} \ln \left(\frac{s}{M^2} \right) = 0.014 \ (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

for $\sqrt{s} = 1$ TeV (2 TeV)

result:

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to Q\bar{Q}) =$$
$$1 - 1.66\,L(s) + 5.31\,l(s) - 15.53\,a$$
$$+1.93\,L^2(s) - 9.43\,L(s)l(s) + 28.79\,l^2(s)$$

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to q\bar{q}) =$$
$$1 - 2.18\,L(s) + 20.58\,l(s) - 36.00\,a$$
$$+2.79\,L^2(s) - 50.06\,L(s)l(s) + 293.95\,l^2(s)$$

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to \mu^+\mu^-) =$$
$$1 - 1.39\,L(s) + 10.12\,l(s) - 30.99\,a$$
$$+1.42\,L^2(s) - 18.43\,L(s)l(s) + 99.04\,l^2(s)$$

similar and even larger corrections for A_{LR}
smaller corrections for A_{FB}

5) Questions about Experimental Setup

- How exclusive are "exclusive" four-fermion processes?

- Can collinear ISR of W, Z be detected at TESLA or at LHC?

- Can collinear FSR of W, Z be detected in quark jets?
  If we constrain the jet mass,
  will we encounter QCD Sudakov logs?

- Interplay?

SUMMARY

- large double logarithmic corrections
  e.g. $e^+e^- \to q\bar{q}$ (2 TeV), $M = M_W$ for IR cutoff

  $\sim$ -24% (one-loop) LL
  $\sim$ $+3.4\%$ (two-loop) LL

- large compensations from subleading terms

  $\sim$ $+35\%$ (one-loop) NLL
  $\sim$ -9.3% (two-loop) NLL

- again alternating sign in NNLL order

  $\sim$ -10% (one-loop) NNLL
  $\sim$ $+8.5\%$ (two-loop) NNLL

- corresponding pattern in subleading n_f terms

- large differences between different flavours

- results for σ, A_{FB}, A_{LR} available