R_{had}

from Tagged Photon Events at Φ - or B - Factories

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based on
S. Binner; J.H. Kühn; K. Melnikov; PLB459(1999)279
H. Czyz; J.H. Kühn; HEP-PH/0008262

I MOTIVATION

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- Rates
- ISR vs FSR
- The Generator
- Results

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- The Form Factor
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- The Generator
- Results

IV SUMMARY

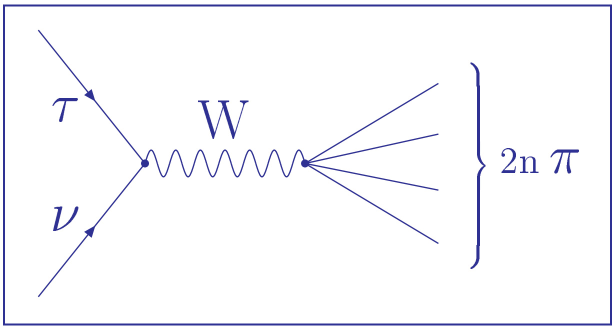
I MOTIVATION

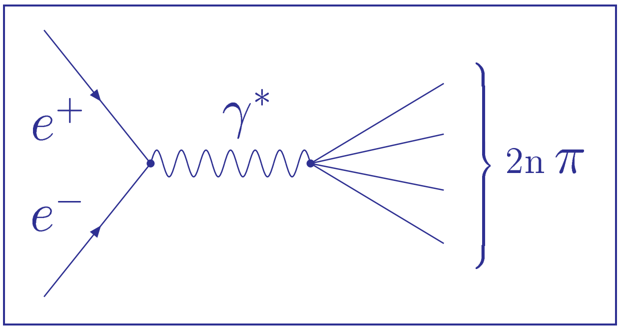
Important contributions to $\alpha(\, M_Z \,)$ and $(g-2)_\mu$ from R_{had} at low energies $(\, 2\, m_\pi \; - \; 2\, GeV \,)$

nonperturbative region

$\Rightarrow$ data

recently mild discrepancies between:





large isospin breaking ?

experimental problems ?

independent measurements ?

aim: - high precision for $\pi^+ \pi^-$
($<$ 1 %)
- good precision between 1 and 2 GeV for multi-meson-states ($<$ 3 %)

$\sqrt{s}$ fixed at "factories"

DAPHNE	1.02 GeV
B-Factories	10.5 GeV

consider

$e^+ e^- \rightarrow \gamma$ + hadrons
DAPHNE : $\gamma \quad \pi^+ \pi^-$

with photon in detector

$\Rightarrow$ kinematic constraints: $\vec{p}(\pi^+, \pi^-, \gamma) = 0$

II $e^+ e^- \to \gamma \pi^+ \pi^-$

Rates

$\pi^+ \pi^- \gamma$: $E_\gamma > 100 MeV$

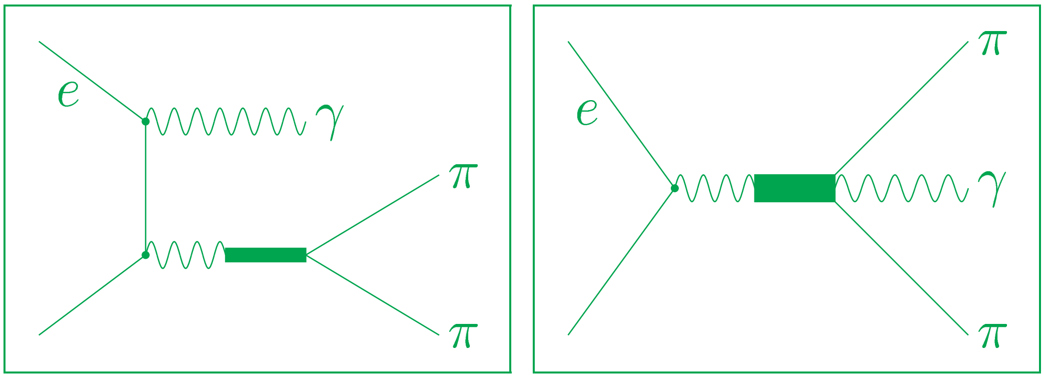
$\sqrt{s}$ $[GeV]$	$\int \mathcal{L}$ $[fb^{-1}]$	#events, $\theta_{min} = 7°$
1.02	1.35	$16 \cdot 10^6$
10.6	100	$3.5 \cdot 10^6$

multi-hadron-events (R $\equiv$ 2)

$\sqrt{s} = 10.6\, GeV$

Q^2-intervall $[GeV]$	#events, $\theta_{min} = 7°$
[1.5 , 2.0]	$9.9 \cdot 10^5$
[2.0 , 2.5]	$7.9 \cdot 10^5$
[2.5 , 3.0]	$6.6 \cdot 10^5$
[3.0 , 3.5]	$5.8 \cdot 10^5$

ISR vs FSR



Solutions:

1) select configuration with dominantly ISR

   $\Rightarrow$ small angle between γ and $e^{\pm}$

2) allow only configurations where FSR is well predicted

   $\Rightarrow$ γ soft, collinear with π

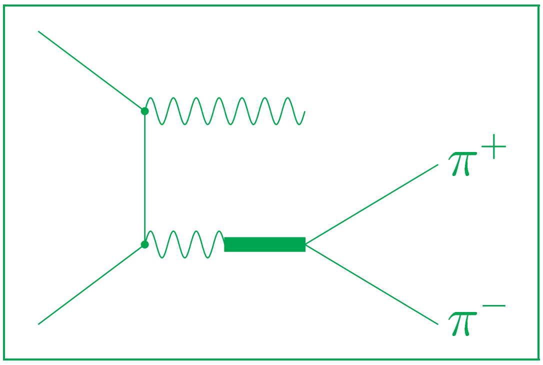
3) identify distributions which test model for FSR (angular distributions; charge asymmetry)

quantitative analysis $\Rightarrow$ MC program

The Generator

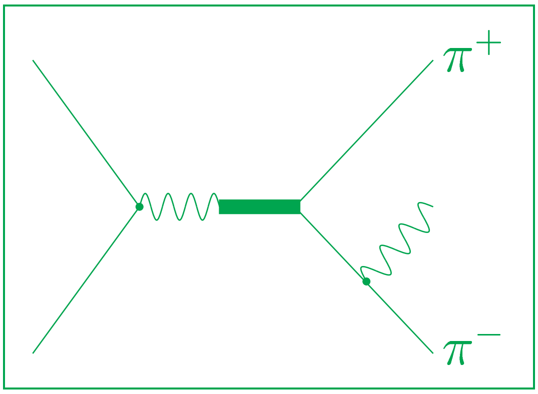
Ingredients:

- ISR



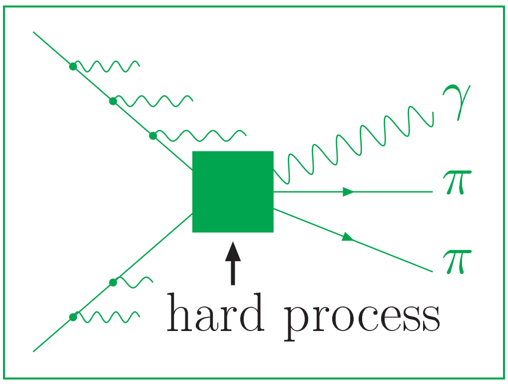
  pionformfactor $\rightarrow$ to be tested

- FSR



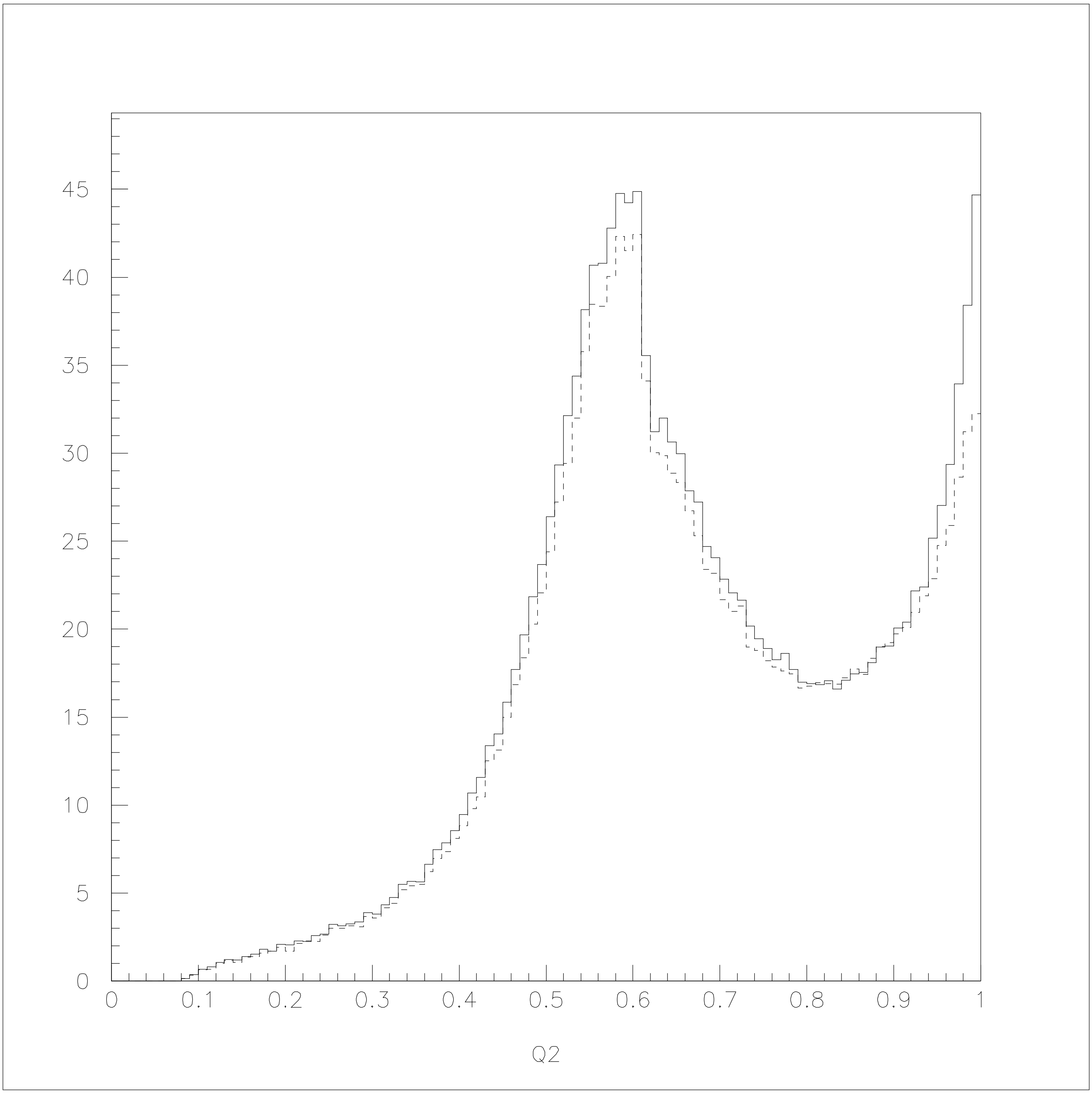
  radiation from pointlike pions
  ($\rightarrow$ probably overestimate)

- additional collinear radiation with structure function techniques



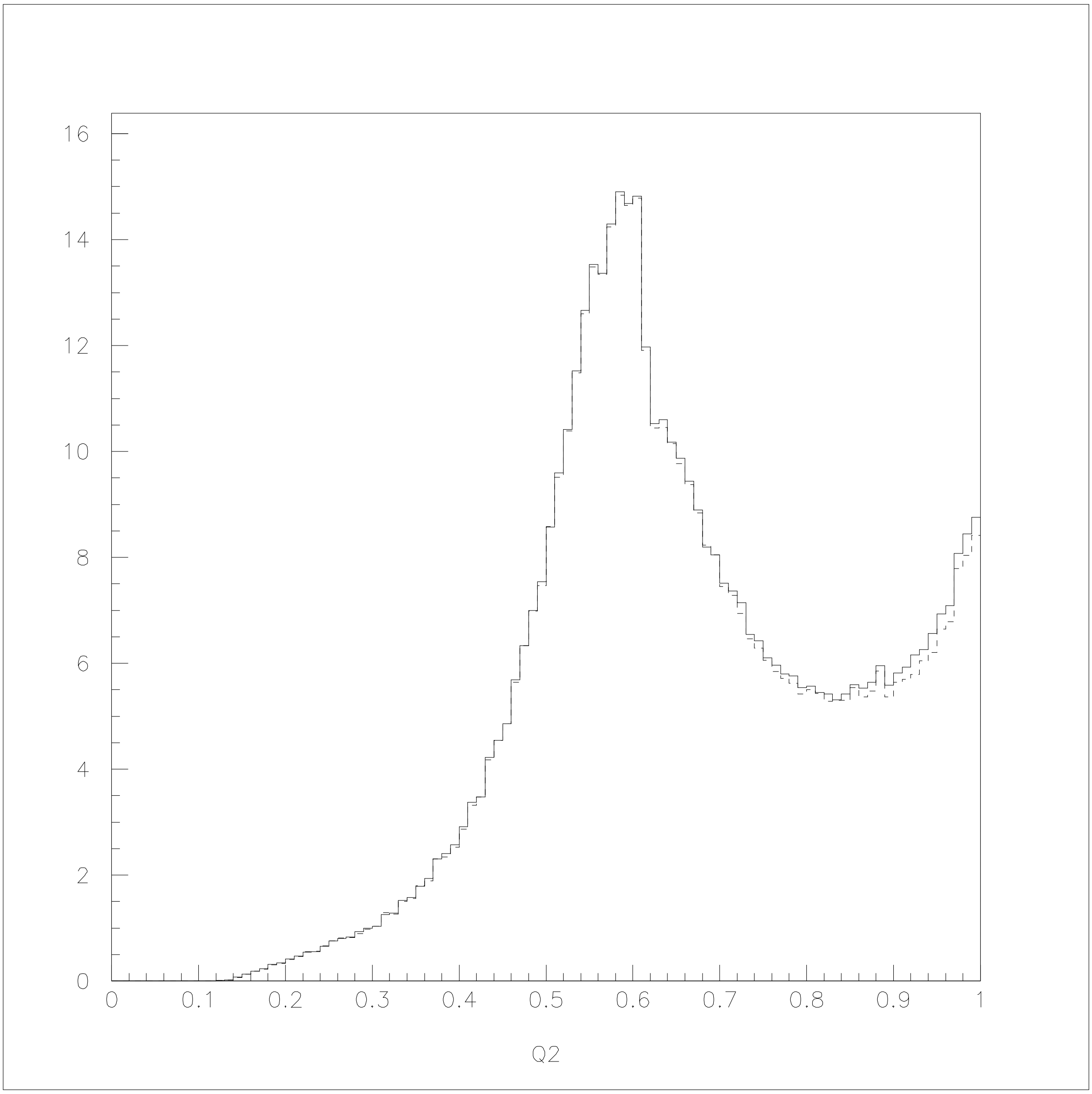
Results

- $\frac{d\sigma}{dQ^2}$ with / without coll. radiation



$d\sigma(\, e^+ e^- \;\rightarrow\; \pi^+ \pi^- \gamma\,)/dQ^2_{\pi^+\pi^-}$ in nb/GeV^2 vs. $Q^2_{\pi^+\pi^-}$ in GeV^2 for $\sqrt{s} = 1.02\, GeV$, without collinear ISR (solid line) - with collinear ISR (dashed line); for $7^\circ < \theta_\gamma < 173^\circ$ and $Q^2_{\pi^+\pi^-\gamma} > 0.9\ GeV^2$.

- select photons at small angles and pions in central region

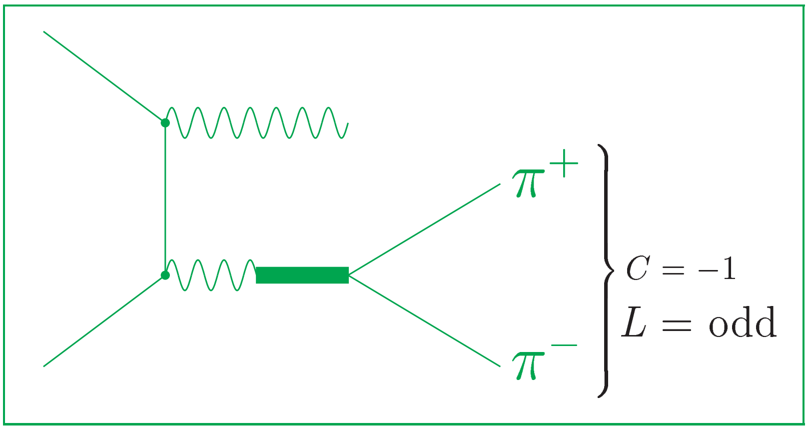


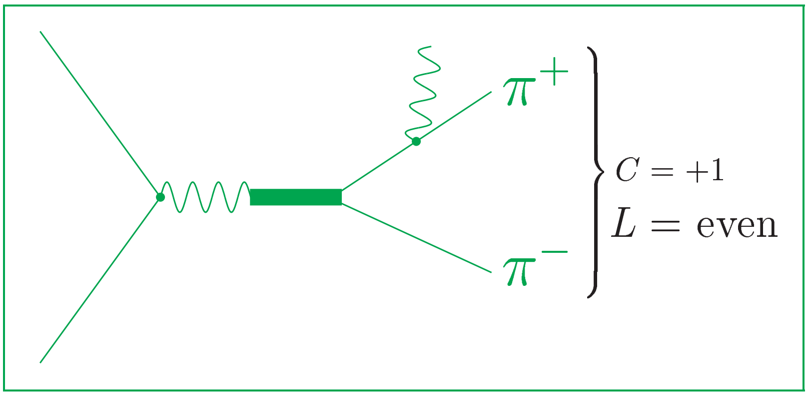
ISR contribution (dashed line) and complete result (solid line) of $d\sigma(e^+e^- \to \pi^+\pi^-\gamma)/dQ^2_{\pi^+\pi^-}$ in nb/GeV^2 vs. $Q^2_{\pi^+\pi^-}$ in GeV^2 for $\sqrt{s} = 1.02\ GeV$. Cuts: $7° < \theta_\gamma < 20°$ or $160° < \theta_\gamma < 173°$, $30° < \theta_\pi < 150°$, $\omega_\gamma > 0.02$ GeV and $Q^2_{\pi^+\pi^-\gamma} > 0.9\ GeV^2$.

$\Rightarrow$ sizeable rate left $\sim \frac{1}{3}$
relative importance of FSR small [dashed vs. solid]

- Test of FSR model?

interference:

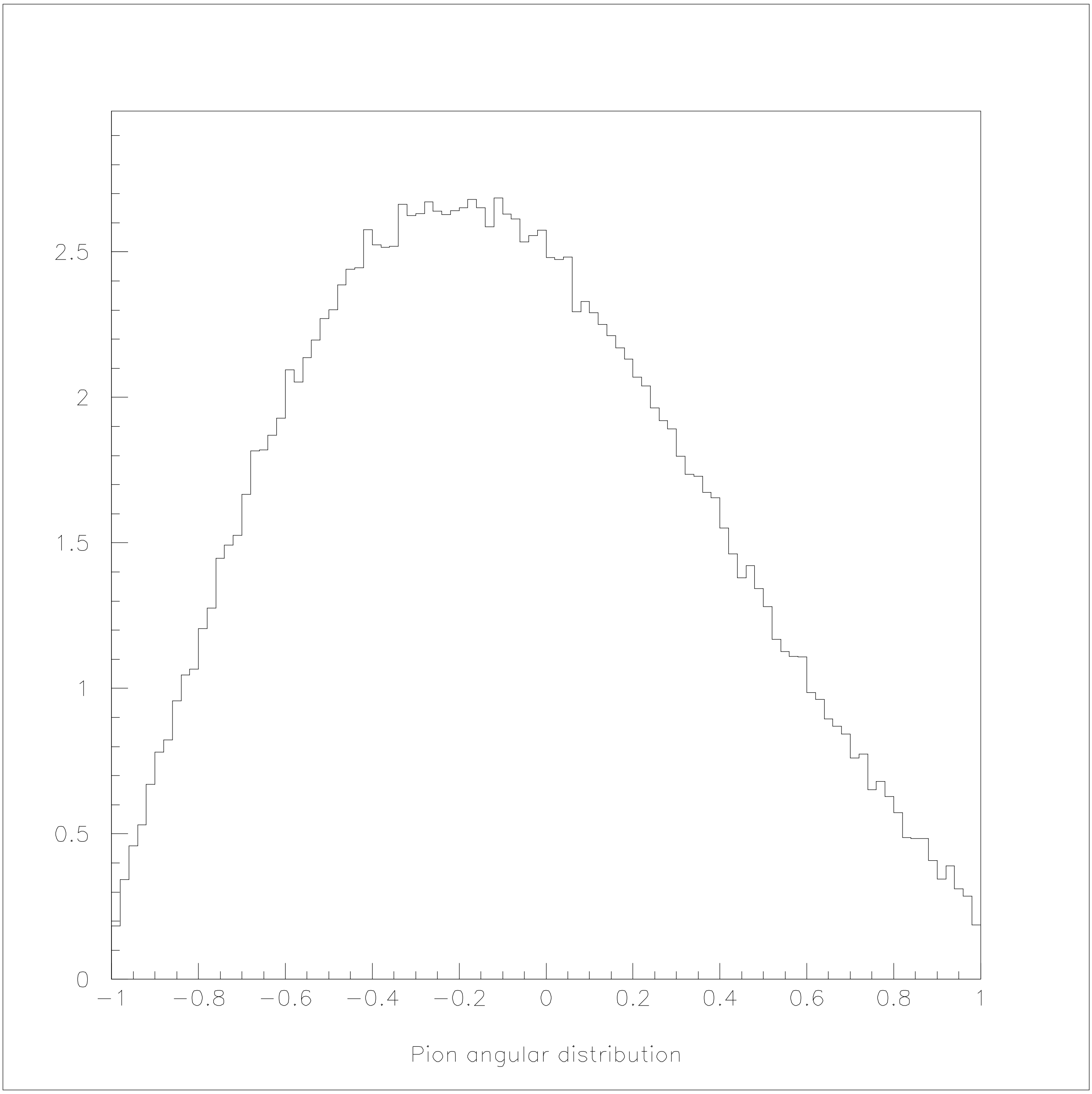




$\Rightarrow$ interference odd under $\pi^+ \leftrightarrow \pi^-$

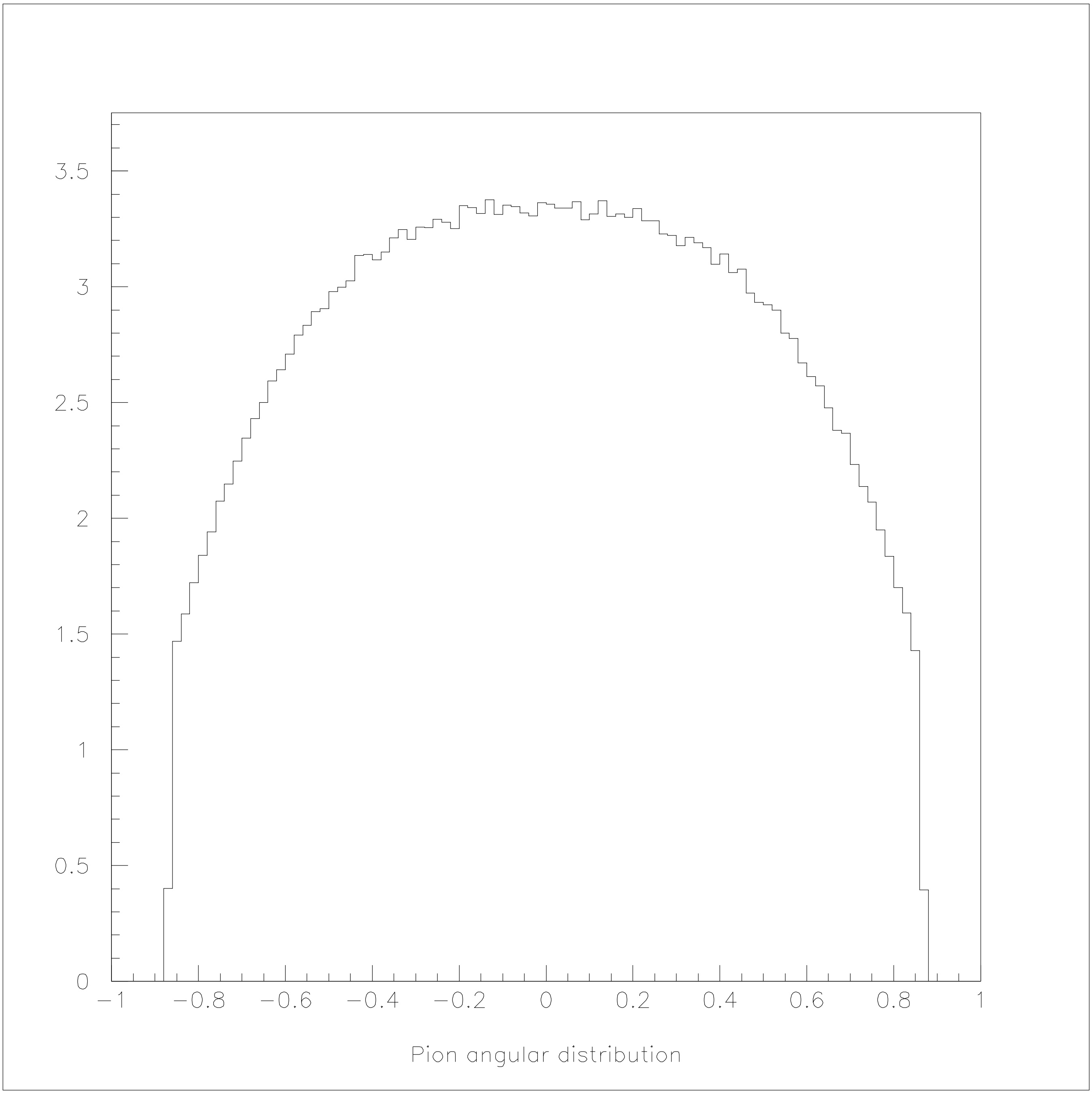
$\Rightarrow$ asymmetric differential distribution $\int$ interf. $= 0$

- Large FB asymmetry of $\pi^{\pm}$ for events with central γ ($\sim$ 20%!)



Pion (π^+) angular distribution in the laboratory frame. The cut on the photon angle is $60° < \theta_\gamma < 120°$.

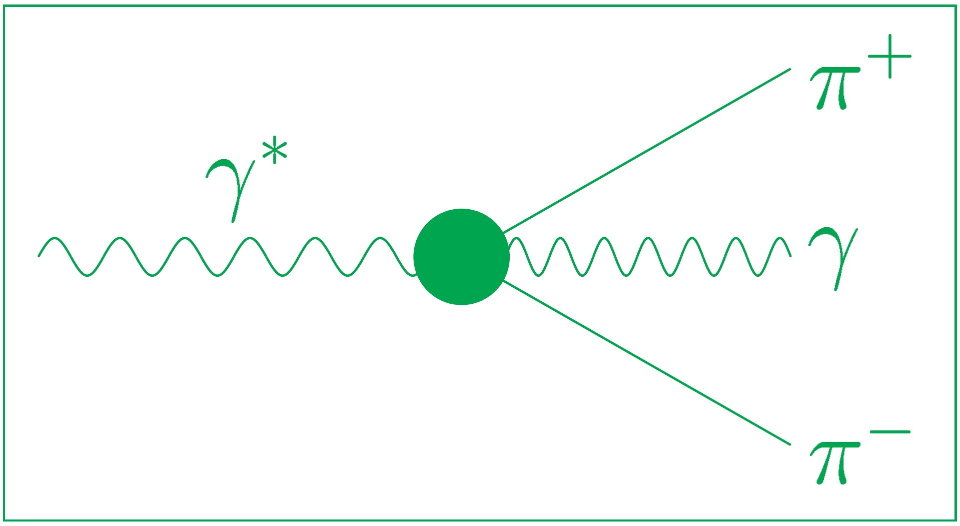
- small FB asymmetry for events with small angle photons and central pions ($\sim 1.5\%$)



Pion (π^+) angular distribution in the laboratory frame. The photon angle is restricted to $7° < \theta_\gamma < 20°$ or $160° < \theta_\gamma < 183°$. The cut on the pion angle is $30° < \theta_{\pi^+,\pi^-} < 150°$.

$\Rightarrow$ crucial test of FSR model !

$\overset{?}{\Rightarrow}$ interesting physics

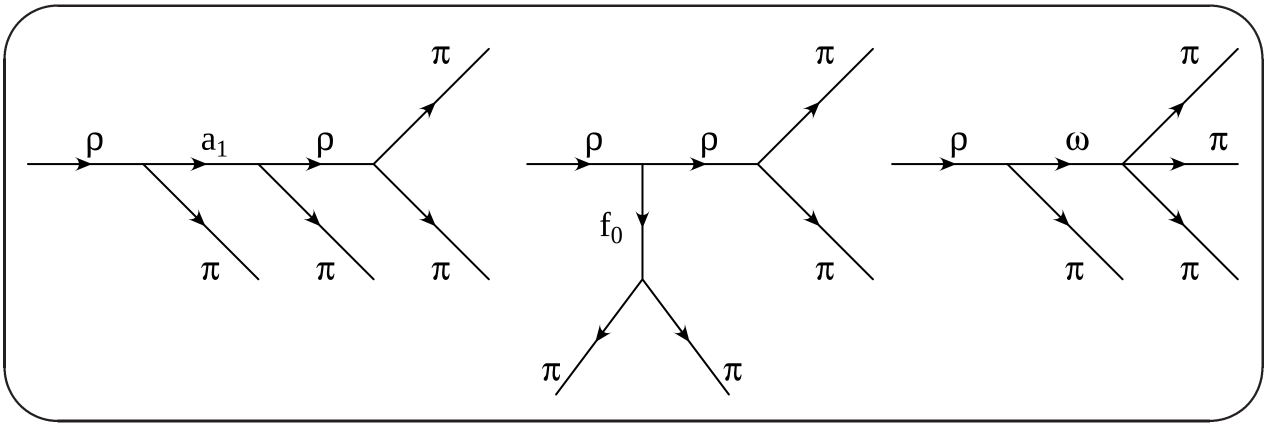


related work: Melnikov, Nguyen, Valeriani, Venanzoni
PLB 477 (2000) 114

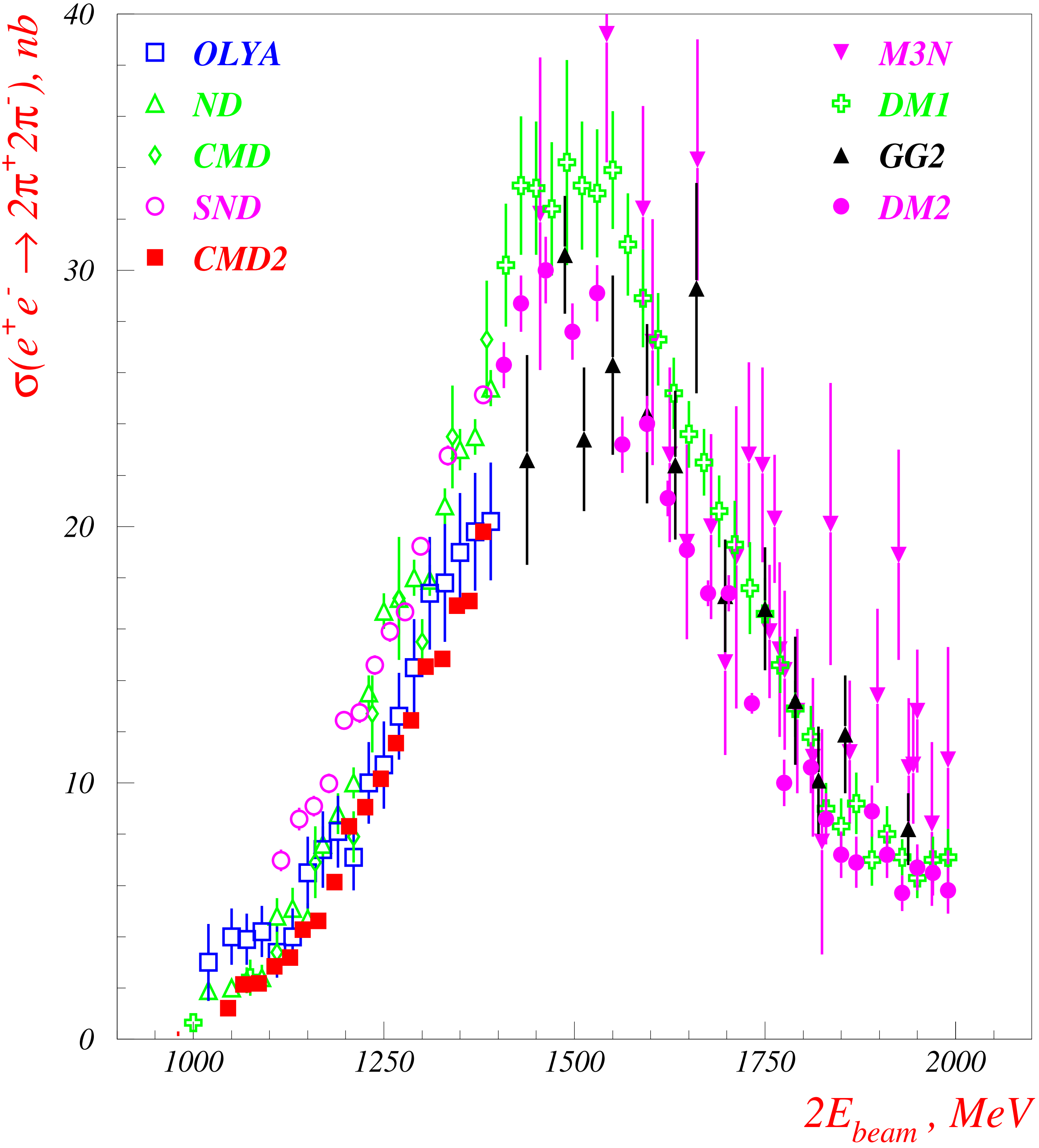
III $e^+ e^- \rightarrow \gamma\, 4\,\pi$

The Form Factor

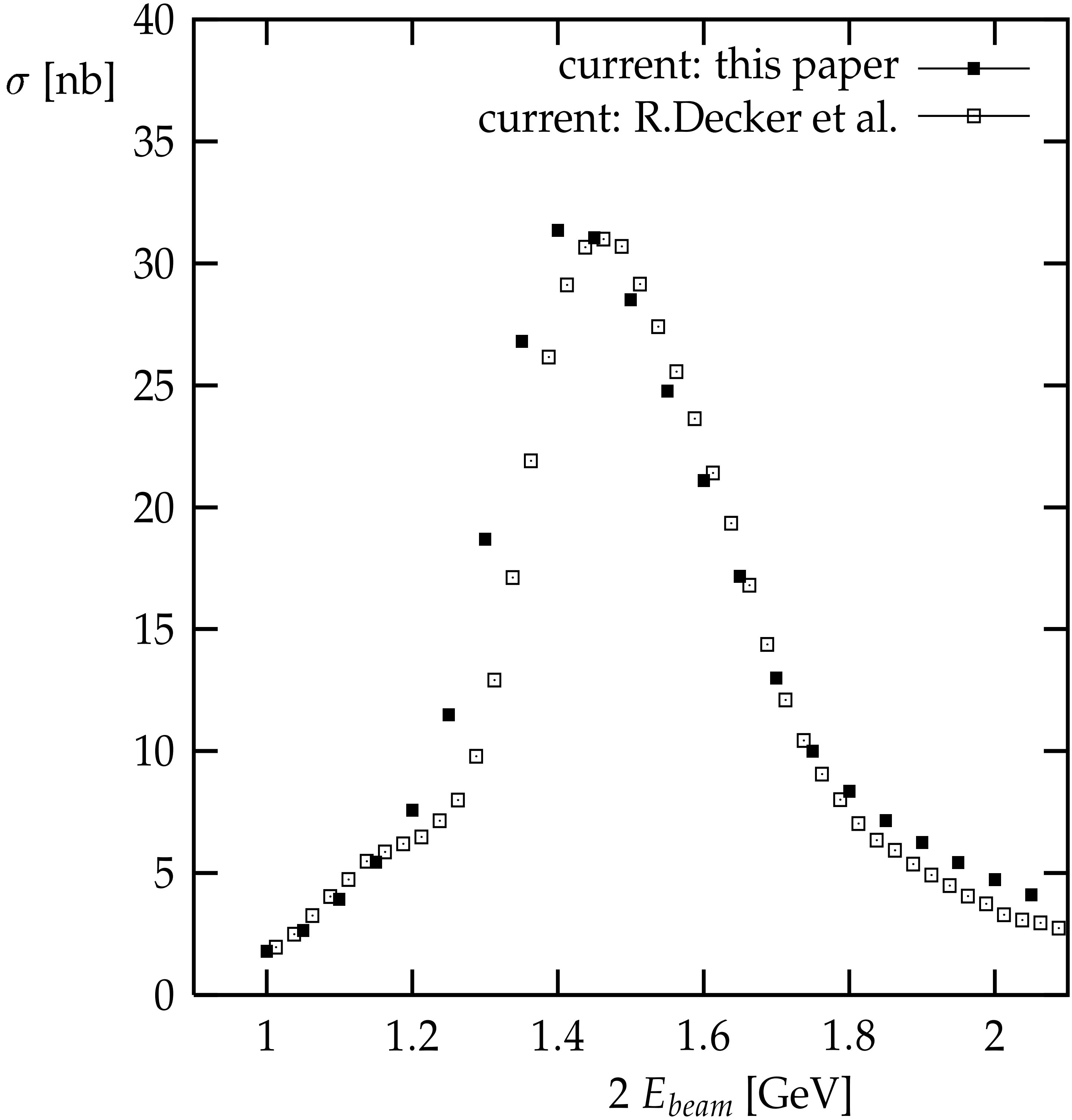
modified/corrected version of model by Decker et al. ZPC70(247)1996

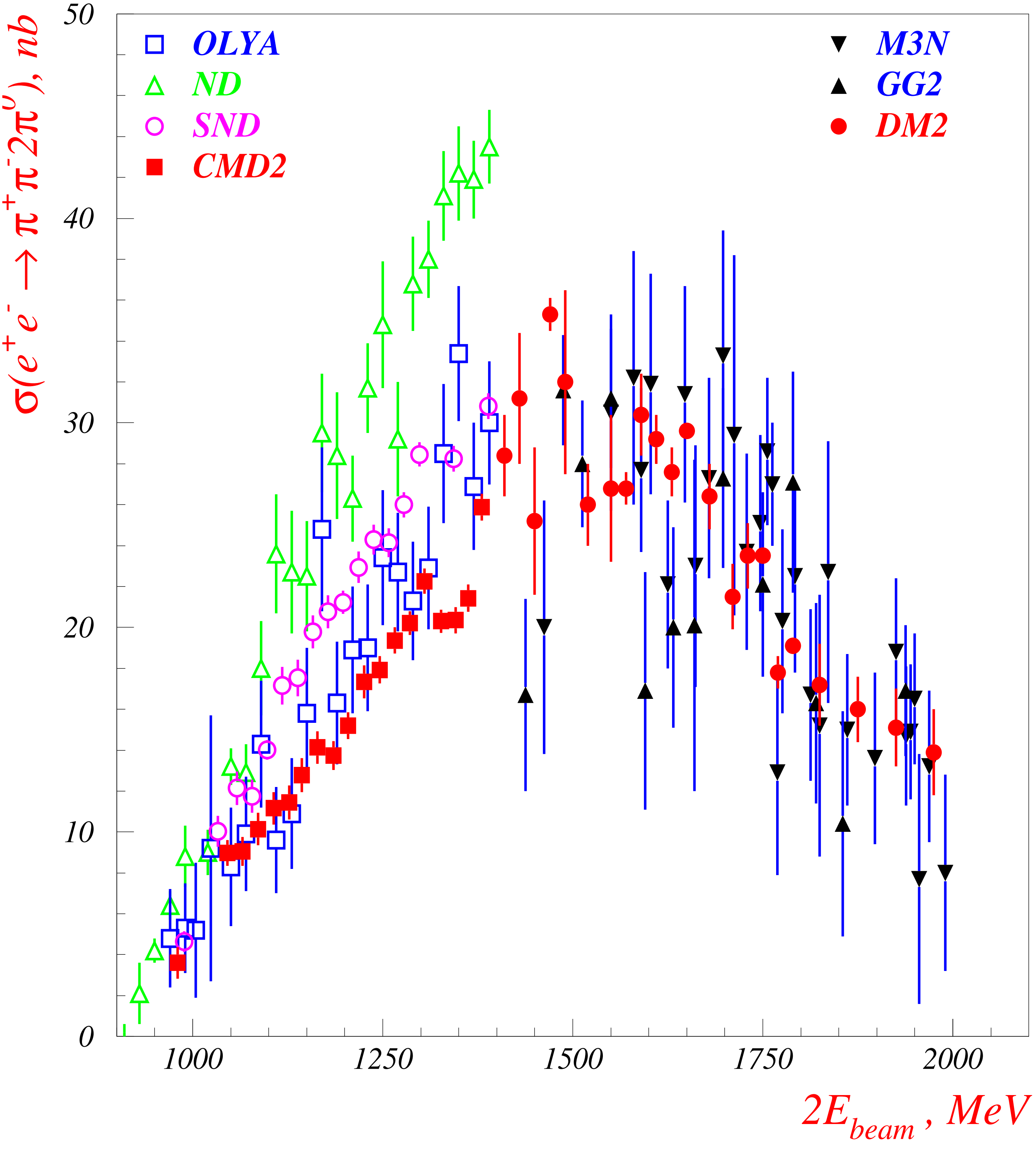


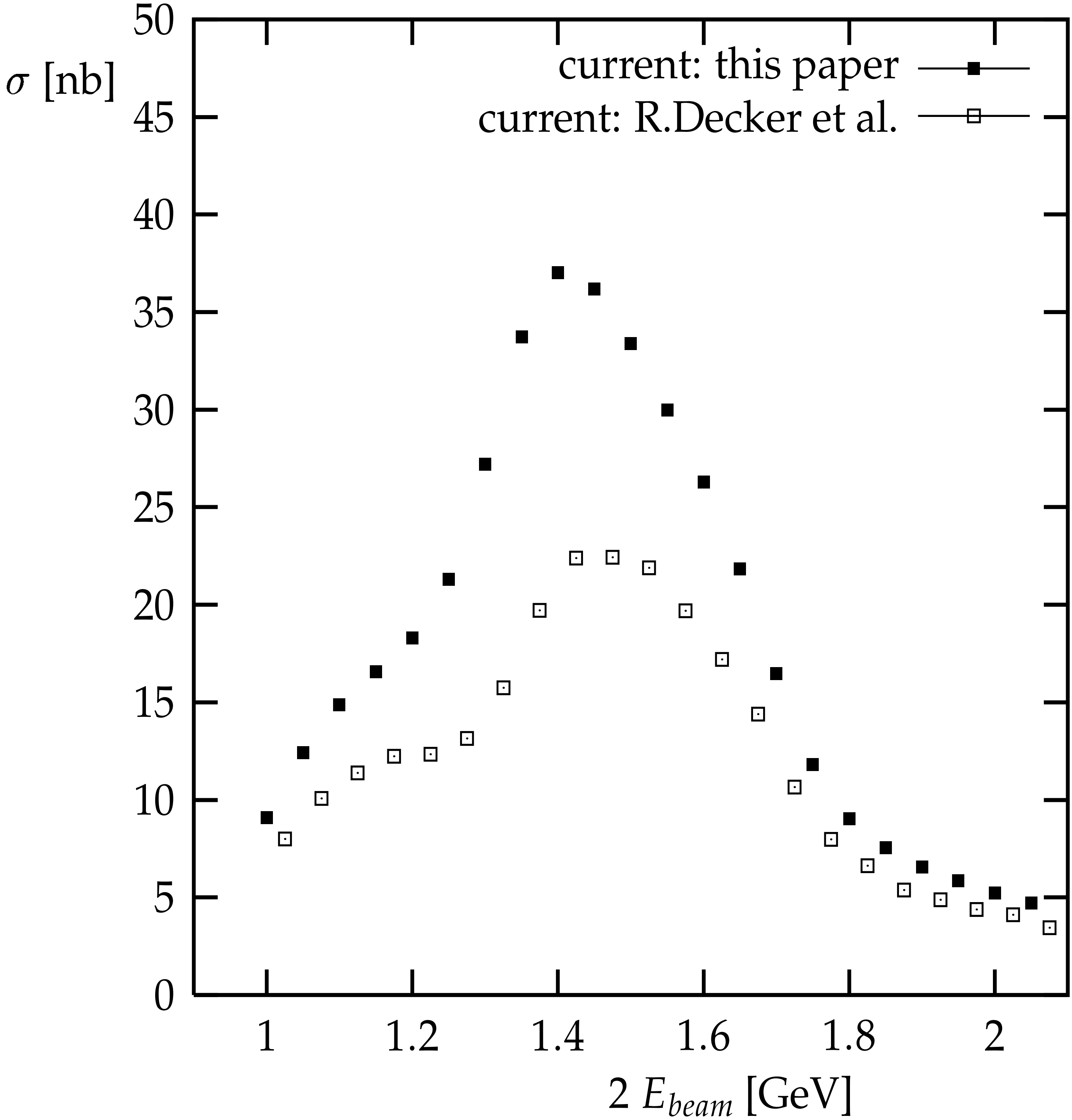
Diagrams contributing to the hadronic current.



$\sigma(e^+e^- \to 2\pi^+2\pi^-)$, nb
$2E_{beam}$, MeV
OLYA
ND
CMD
SND
CMD2
M3N
DM1
GG2
DM2
0
10
20
30
40
1000
1250
1500
1750
2000


σ [nb]
current: this paper
current: R.Decker et al.
0
5
10
15
20
25
30
35
40
1
1.2
1.4
1.6
1.8
2
2 E_beam [GeV]


OLYA
ND
SND
CMD2
M3N
GG2
DM2
$\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)$, nb
$2E_{beam}$, MeV
0
10
20
30
40
50
1000
1250
1500
1750
2000



50
45
40
35
30
25
20
15
10
5
0
σ [nb]
current: this paper
current: R.Decker et al.
1
1.2
1.4
1.6
1.8
2
2 Ebeam [GeV]

Isospin Relations

for amplitudes!

$e^+e^- \to 4\pi$

$$
\begin{array}{lcl}
+\,-\,0\,0 & : & J^\mu(p^+, p^-, p_1, p_2) \\
+\,+\,-\,- & : & J^\mu(p_1^+, p_1^-, p_2^+, p_2^-) \;+\; J^\mu(p_2^+, p_1^-, p_1^+, p_2^-) \\
 & & +\; J^\mu(p_1^+, p_2^-, p_2^+, p_1^-) \;+\; J^\mu(p_2^+, p_2^-, p_1^+, p_1^-)
\end{array}
$$

$\tau \to 4\pi\nu$

$$
\begin{array}{lcl}
-\,0\,0\,0 & : & J^\mu(p^-, p_1, p_2, p_3) \;+\; J^\mu(p^-, p_2, p_1, p_3) \\
 & & +\; J^\mu(p^-, p_3, p_1, p_2) \\
-\,-\,+\,0 & : & J^\mu(p_1, p^0, p^+, p_2) \;+\; J^\mu(p_2, p^0, p^+, p_1)
\end{array}
$$

one (!) amplitude determines all four matrix elements

relation for rates

$$R^\tau(-\;0\;0\;0) = \frac{1}{2}\,R(+\;+\;-\;-)$$

$$R^\tau(-\;-\;+\;0) = \frac{1}{2}\,R(+\;+\;-\;-) + R(+\;-\;0\;0)$$

is simple consequence

The Generator

Hard photon from ISR

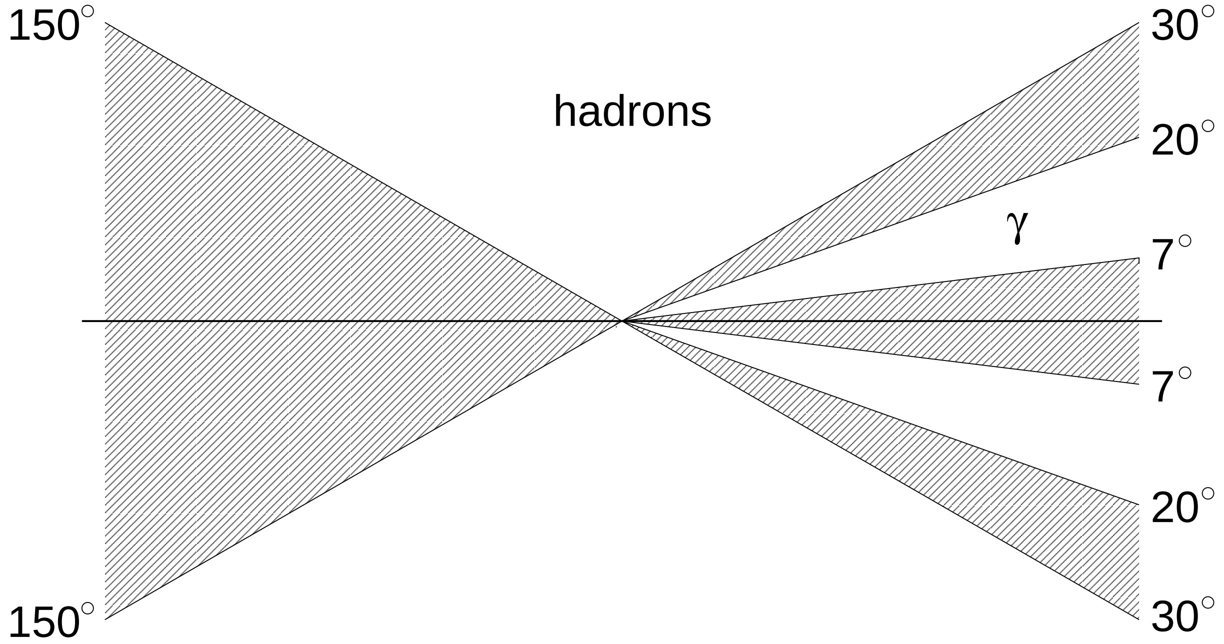
$+$ collinear radiation

modular structure

(separation of leptonic and hadronic tensor)

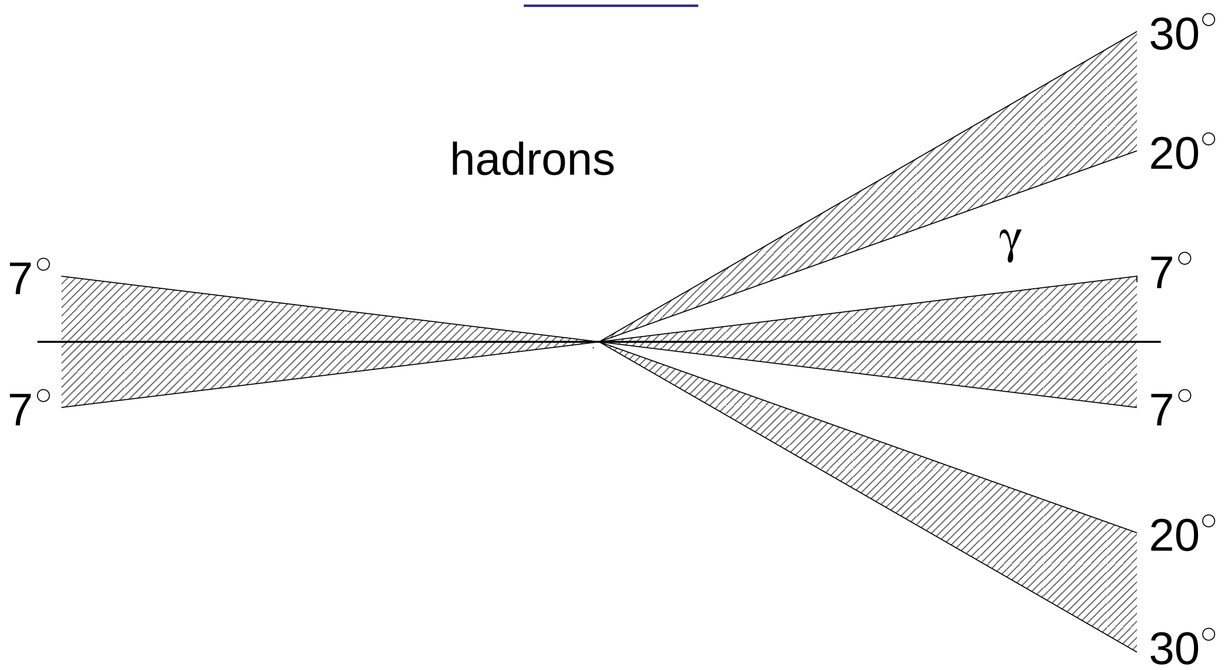
Results

cuts1

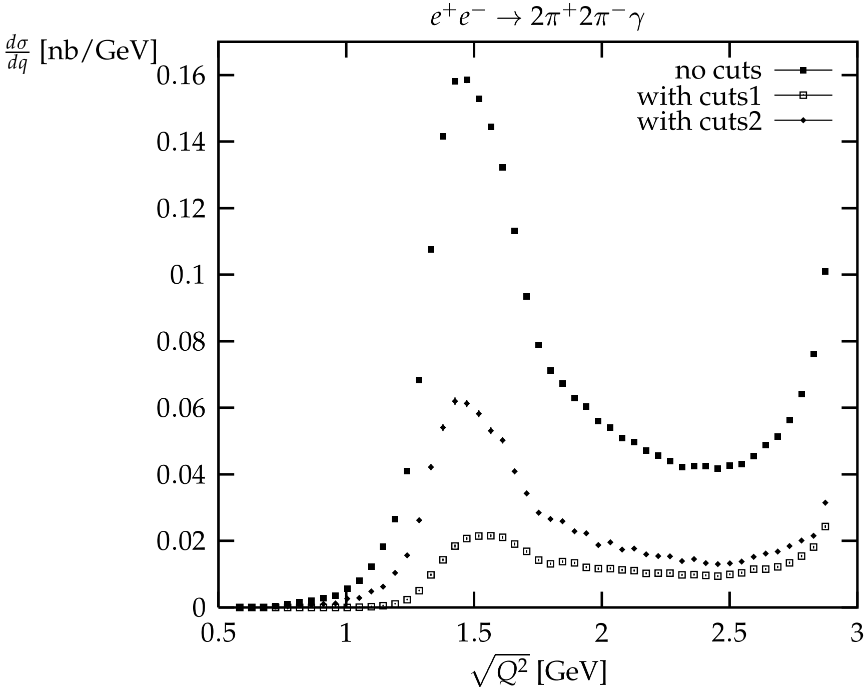


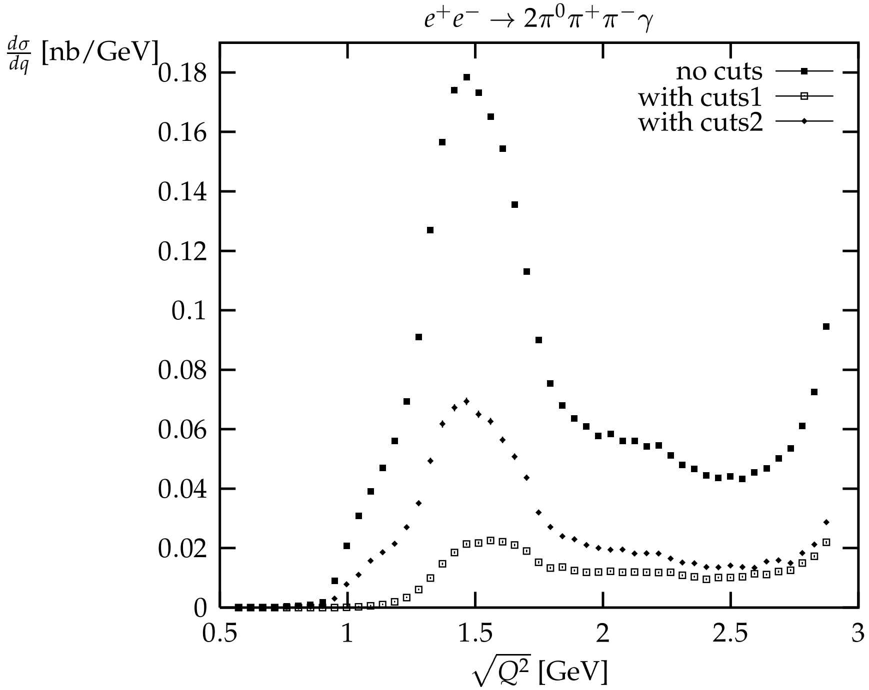
+ reflection

cuts2

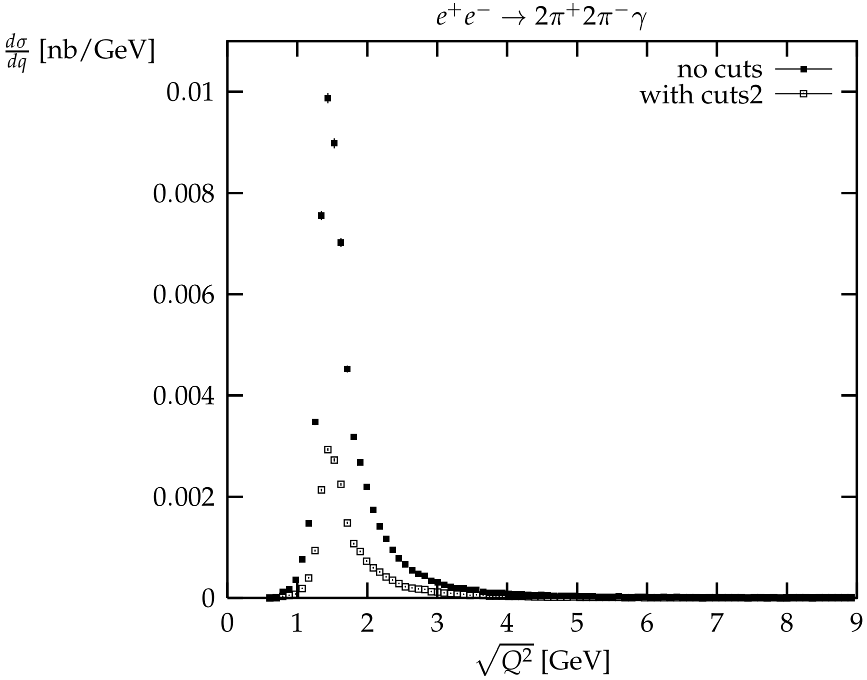


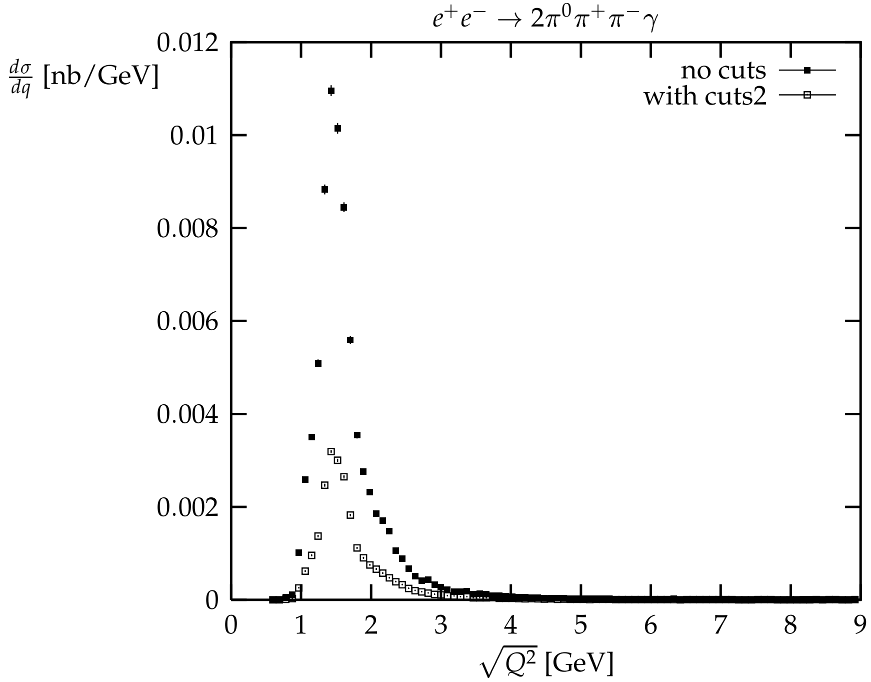
+ reflection





The differential $e^+e^- \to 4\pi\gamma$ cross sections at beam energy 1.5 GeV with minimal photon energy equal 0.1 GeV with no angular cuts (no cuts) and two sets of angular cuts (with cuts1) and (with cuts2), where $q \equiv \sqrt{Q^2}$ is an invariant mass of the 4π system.





The differential $e^+e^- \to 4\pi\gamma$ cross sections at beam energy 5 GeV with minimal photon energy equal 0.2 GeV with no angular cuts (no cuts) and angular cuts (with cuts2), where $q \equiv \sqrt{Q^2}$ is the invariant mass of the 4π system.

Results

$7^\circ < \theta_\gamma < 173^\circ$

$$\boxed{E_{Beam} = 0.5 GeV \;\;,\;\; E_\gamma > 50 MeV}$$

$$\begin{aligned} \sigma(e^+e^- \to 2\pi^+\, 2\pi^-\, \gamma) &= 0.0317\, nb \\ cuts1: &= 0.0140\, nb \\ \sigma(e^+e^- \to 2\pi^0\pi^+\pi^-\gamma) &= 0.0320\, nb \\ cuts1: &= 0.0100\, nb \end{aligned}$$

For $\int L = 10^7\, s \cdot 10^{33}\, cm^{-2}s^{-1} = 10^7\, nb^{-1}$

$N_{ev} \sim 10^5$

$$\boxed{E_{Beam} = 1.5 GeV \;\;,\;\; E_\gamma > 100 MeV}$$

$$\begin{aligned} \sigma(e^+e^- \to 2\pi^+\, 2\pi^-\, \gamma) &= 0.130\, nb \\ cuts1: &= 0.022\, nb \\ cuts2: &= 0.046\, nb \end{aligned}$$

$N_{ev} \sim (1.3 - 1.5)\, 10^6$

cuts: $\frac{1}{3} - \frac{1}{5}$

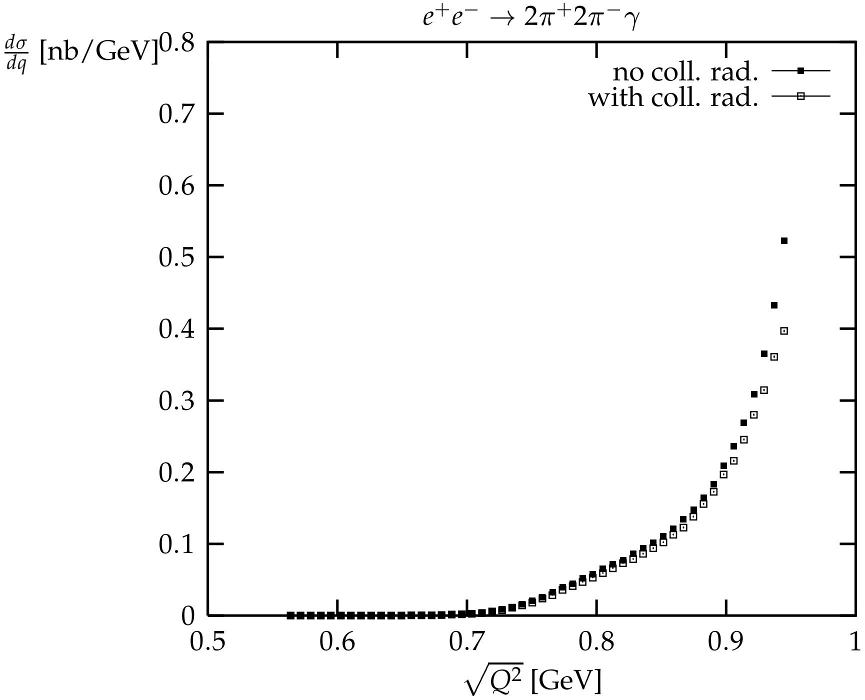
$$\boxed{E_{Beam} = 5GeV \ , \ E_\gamma > 200MeV}$$

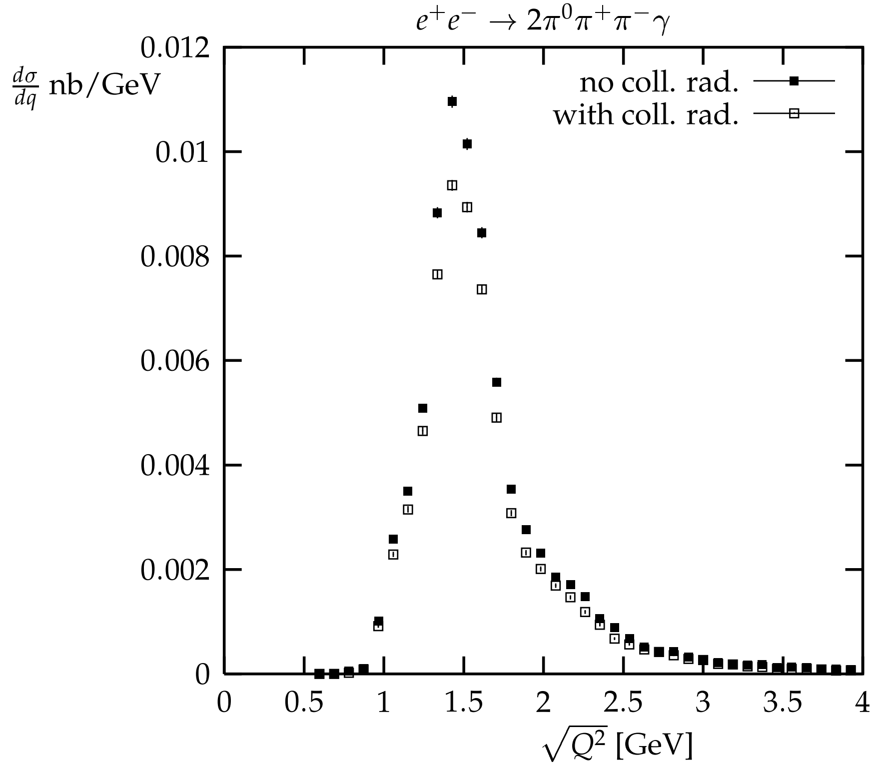
$$
\begin{aligned}
\sigma(e^+e^- \to 2\pi^+\, 2\pi^-\, \gamma) &= 0.00592\, nb \\
cuts2: &= 0.00186\, nb \\
\sigma(e^+e^- \to 2\pi^0\pi^+\pi^-\gamma) &= 0.00711\, nb \\
cuts2: &= 0.00217\, nb
\end{aligned}
$$

$$N_{ev} \sim (6-7)\, 10^4$$

cuts 2 : $*\frac{1}{3}$

collinear radiation $\Rightarrow$ reduction by 10 %





Effect of collinear radiation on the differential $e^+e^- \to 4\pi\gamma$ cross sections at beam energy 0.5 GeV (left) and 5 GeV (right) with minimal photon energy 0.05 and 0.2 GeV correspondingly. In the mode with collinear radiation minimal invariant masses of the $4\pi + \gamma$ systems were 0.95 GeV and 9.5 GeV. $\sqrt{Q^2}$ is the invariant mass of the 4π system.

IV SUMMARY

- rates for γ + had are sufficient for an accurate measurement or R
- systematics ?
  theory: higher orders
  expt. ?
- FSR can be controlled through FB-asymmetry and proper cuts
- Generators for $e^+e^- \to 2\pi\gamma$ and $e^+e^- \to 4\pi\gamma$ exist.
- The 4π reaction is accessible at DAPH-NE for a limited region of phase space. Interesting possibility for B-factories!
- Improvements of 4π-formfactor desirable!