SUDAKOV LOG'S
in
ELECTROWEAK PROCESSES

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1) Motivation

large one–loop corrections at high energies

2) NLO for SU(2) Toy Model

A) form factor
B) scattering

3) Standard Model: W, Z, γ

1) MOTIVATION (one-loop!)

example: $e^+ e^- \to t\bar{t}$ FIG

boxes + vertex corrections dominate

running coupling: $\sim \ln(s)$

boxes, vertices: $\sim \ln^2(s/M^2)$

example: massive U(1)

$$\Rightarrow \quad \text{Born} * \left(1 - 2\frac{\alpha}{\pi}\left(\frac{1}{4}\ln^2\frac{s}{M^2} + \ldots\right)\right)$$

magnitude ($\alpha_w/\pi = 10^{-2}$)

$\left(\frac{s}{M^2}\right)$	$-\frac{1}{4}\ln^2\frac{s}{M^2}$	$+\frac{3}{4}\ln\frac{s}{M^2}$	$-\frac{\pi^2}{12}+\frac{7}{8}$	Σ	$*2\frac{\alpha_w}{\pi}$
$\left(\frac{1000}{100}\right)^2$	- 5.30	+ 3.45	- 1.70	-3.5	-7%
$\left(\frac{2000}{100}\right)^2$	- 8.97	+ 4.49	- 1.70	-6.2	-12%

large negative correction,
compensated by real radiation in inclusive rate!

massive boson:
exclusive rate physically meaningfull!

higher orders, leading log:

$$\Big(1 - \frac{1}{2}\underbrace{\frac{g^2}{16\pi^2}\ln^2\frac{s}{M^2}}_{\mathsf{L}}\Big) \Rightarrow e^{-\frac{1}{2}\mathsf{L}}$$

(per leg) in amplitude

nonabelian theory: $e^{-\frac{C_F}{2}\mathsf{L}}$

numerically: $\mathsf{L}_{ew} = \begin{matrix} 0.07 \\ 0.11 \end{matrix}$ for $\begin{matrix} 1\ \text{TeV} \\ 2\ \text{TeV} \end{matrix}$

$C_F = \frac{3}{4}$ for SU(2)

expected correction for $f_L \bar{f}_L \to f'_l \bar{f}'_L$

$$\sim \underset{\text{rate}}{2} \cdot \overset{\text{legs}}{4} \cdot \frac{3}{4} \cdot \frac{1}{2}\mathsf{L}_{ew} \sim 20 - 30\%$$

EW theory; 1 Loop

$f_L \bar{f}_L \to f'_l \bar{f}'_L$

$$\mathcal{M}_{\text{Born}} = \frac{ig^2}{s}\left(T^3_f T^3_{f'} + \tan^2\Theta_w \frac{Y_f Y_{f'}}{4}\right)$$
$$\left(M_Z^2/s \to 0\right)$$

one loop: explicit calculation $\surd$

dominant correction: $* \left(1 - F^f_{\text{L,R}}\right)$ with

$$F^f_{\text{L}} = \Big(\underbrace{\frac{1}{2}}_{W} + \underbrace{\frac{1}{4c^2_{\text{w}}} + \tan^2_{\text{w}}(s^2_{\text{w}} Q^2_f - 2T^3_f Q_f)}_{Z}\Big) L$$

$$F^f_{\text{R}} = \Big(\underbrace{\tan^2_{\text{w}} s^2_{\text{w}} Q^2_f}_{Z}\Big) L$$

(per line)

2) NLO for SU(2) Toy Model

A) Form Factor

$$F_{\text{Born}} = \bar{\psi}(p_2)\gamma_\mu \psi(p_1)$$

$$\frac{\partial}{\partial \ln Q^2} F = \left[\int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2)) \right] F$$

Collins, Sen

$\alpha(Q^2) =$ running coupling; γ, ζ, ξ in LO and NLO

from one loop calculation:

$$F = F_B \left[1 + \frac{\alpha}{2\pi} \left(-\ln^2 \frac{Q^2}{M^2} + 3 \ln \frac{Q^2}{M^2} \right) \right]$$

$$\Rightarrow F = F_0(\alpha(M^2)) * \exp \left\{ \int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \left[\int_{M^2}^{x} \frac{\mathrm{d}x'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2)) \right] \right\}$$

$$\underset{\text{NLO}}{\simeq} F_B \exp \left\{ \frac{1}{2\pi} \left[- \int_{M^2}^{Q^2} \frac{\mathrm{d}x}{x} \int_{M^2}^{x} \frac{\mathrm{d}x'}{x'} \alpha(x') + 3\alpha \ln (Q^2/M^2) \right] \right\}$$

with $\gamma(\alpha) = -\frac{\alpha}{2\pi}$ and $\zeta(\alpha) + \xi(\alpha) = 3\frac{\alpha}{4\pi}$

B) four fermion scattering $\quad f + f' \to f + f'$

$$\frac{ig^2}{s}\left(\bar{\psi}_2 t^a \gamma^\mu \psi_1\right)\left(\bar{\psi}_4 t^a \gamma_\mu \psi_3\right)$$
$$\equiv \frac{ig^2}{s} A^\lambda$$

one loop:

$$\frac{ig^2(Q^2)}{s}\frac{\alpha}{2\pi}\left[\left\{-C_F\left(\ln^2\left(\frac{-s}{M^2}\right) - 3\ln\left(\frac{-s}{M^2}\right)\right) + \left(-C_A \ln\left(\frac{u}{s}\right) + 2\left(C_F - \frac{T_F}{N}\right)\ln\left(\frac{u}{t}\right)\right)\ln\left(\frac{-s}{M^2}\right)\right\} A^\lambda + \left\{2\frac{C_F T_F}{N}\ln\left(\frac{u}{t}\right)\ln\left(\frac{-s}{M^2}\right)\right\} A^d\right]$$

with $\quad A^d = \left(\bar{\psi}_2 \gamma^\mu \psi_1\right)\left(\bar{\psi}_4 \gamma_\mu \psi_3\right)$ diagonal!

$\Rightarrow$ evolution for $\tilde{A} = \begin{pmatrix} \tilde{A}^\lambda \\ \tilde{A}^d \end{pmatrix}$ $\qquad$ ($\tilde{A} = A *$collinear logs)

$$\frac{\partial}{\partial \ln Q^2}\tilde{A} = \chi\left(\alpha\left(Q^2\right)\right)\tilde{A}$$

$\chi_{\lambda\lambda}$, $\chi_{\lambda d}$ from above

$\chi_{d\lambda}$ analogous; $\quad \chi_{dd} = 0$

3) Standard Model: W, Z, γ

examine $f'\bar{f}' \to f\bar{f}$

$$A_B = \frac{ig^2}{s} \sum_{I,\ J=L,\ R} \left(T^3_{f'} T^3_f + t^2_W \frac{Y_{f'} Y_f}{4} \right) A^{f'f}_{IJ}$$

with $$A^{f'f}_{IJ} = \left(\bar{f}'_I \gamma_\mu f'_I \right) \left(\bar{f}_J \gamma_\mu f_J \right)$$

consider first a ficticious theory, photon mass $= M$
$\Rightarrow$ collinear factor:

$$\exp \left[- \left(T_f (T_f + 1) + t^2_W \frac{Y_f^2}{4} \right) (L(s) - 3l(s)) \right]$$

$$L(s) = \frac{g^2}{16\pi^2} \ln^2 \left(\frac{-s}{M^2} \right)$$

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right handed: $\chi = t^2_W Y_{f'} Y_f \ln \left(\frac{u}{t} \right)$

left handed:

$$\chi_{\lambda\lambda} = -4 \ln \left(\frac{u}{s} \right) + \left(t^2_W Y_{f'} Y_f + 2 \right) \ln \left(\frac{u}{t} \right)$$

$$\chi_{\lambda d} = \frac{3}{4} \ln \left(\frac{u}{t} \right) \qquad \chi_{d\lambda} = 4 \ln \left(\frac{u}{t} \right)$$

$$\chi_{dd} = t^2_W Y_{f'} Y_f \ln \left(\frac{u}{t} \right)$$

real world: $m_\gamma = 0$

combine virtual corrections
and real radiation (prescription of Fadin et al.)

$$\begin{aligned} \Rightarrow \chi &= \left(t_W^2 Y_{f'} Y_f - 4 s_W^2 Q_{f'} Q_f\right) \ln\left(\frac{u}{t}\right) \\ \chi_{\lambda\lambda} &= \ldots \\ &\vdots \quad \ddots \end{aligned}$$

result:

$$\begin{aligned} \sigma/\sigma_B(e^+e^- \to Q\bar{Q}) &= 1 + \ 5.30\, l(s) - 1.66\, L(s)) \\ &\quad -12.84\, l(s)L(s) + 1.92\, L^2(s) \\ \sigma/\sigma_B(e^+e^- \to q\bar{q}) &= 1 + 20.54\, l(s) - 2.17\, L(s)) \\ &\quad -53.72\, l(s)L(s) + 2.79\, L^2(s) \\ \sigma/\sigma_B(e^+e^- \to \mu^+\mu^-) &= 1 + 10.09\, l(s) - 1.39\, L(s)) \\ &\quad -21.66\, l(s)L(s) + 1.41\, L^2(s) \end{aligned}$$

$$\begin{aligned} L(s) &= 0.07 \quad (0.11) \\ l(s) &= 0.014 \quad (0.017) \end{aligned} \qquad \text{for } \sqrt{s} = 1 \text{ TeV } (2 \text{ TeV})$$

similar and even larger corrections for A_{LR}
smaller corrections for A_{FB}

SUMMARY

- large double logarithmic corrections
  e.g. $e^+e^- \to Q\bar{Q}$ (2 TeV), $M = M_W$ for IR cutoff

$$\sim \quad -18\% \quad \text{(one loop)}$$
$$\sim \quad +2.3\% \quad \text{(two loop)}$$

- large compensations from subleading terms

$$\sim \quad +9\% \quad \text{(one loop)}$$
$$\sim \quad -2.4\% \quad \text{(two loop)}$$

- large differences between different flavours

$$\sigma,\ A_{FB},\ A_{LR}$$